Midterm 2024 - Solutions

Problem 1:

General charge neutrality:

$$n + N_A^- = p + N_D^+$$

In this case $N_D^+ = N_D = n_i$, the charge neutrality could be simplified as:

$$n = p + N_D^+$$

Combined with $np = n_i^2$:

$$n(n - N_D^+) = n_i^2$$

Carrier concentration n and p could be solved as:

$$n = \frac{N_D^+ \pm \sqrt{{N_D^+}^2 + 4n_i^2}}{2} = \frac{n_i \ \pm \sqrt{n_i^2 + 4n_i^2}}{2} = \frac{n_i \ (1 \pm \sqrt{5})}{2}$$

$$n = 2.2361 \times 10^{10} cm^{-3}$$

$$p = \frac{n_i^2}{n} = 4.4721 \times 10^9 \ cm^{-3}$$

Thus:

$$E_F - E_C = kT \ln\left(\frac{n}{N_C}\right) = -0.5456 \ eV$$

Thus $E_F - E_v = 1.12 - 0.5456 = 0.5744$

Notes

1. The values of n and p are not that different in this case.
2. In this case, E_F is 14 meV higher than the mid-gap. For Intrinsic Si, this value is 1 meV.

Problem 2. Solution

(a) Donor concentration at the n-side: $N_D = 2 \text{ x } 10^{18} cm^{-3}$
Acceptor concentration at the p-side: $N_A = 3 \text{ x } 10^{16} cm^{-3}$

Reason:
From the electric field profile, the depletion width $|x_1| << |x_2|$, this is one-side heavily-doped pn junction.

Note that for the majority carrier density at each side, whose mobilities are 400 $cm^2/V \cdot s$ and 200 $cm^2/V \cdot s$, there're two possible cases:

(i) The hole mobility at p-side μ_h^p = 400 $cm^2/V \cdot s$, the electron mobility at n-side μ_e^n = 200 $cm^2/V \cdot s$.

(ii) The hole mobility at p-side μ_h^p = 200 $cm^2/V \cdot s$, the electron mobility at n-side μ_e^n = 400 $cm^2/V \cdot s$.

From plot 1 (Mobility vs. Doping):
If the doping follows case (i), the corresponding doping densities at p-side and n-side are:
$N_A = 3 \text{ x } 10^{16} cm^{-3}$ and $N_D = 2 \text{ x } 10^{18} cm^{-3}$.
This matches with the one-side heavily-doped condition.

If the doping follows case (ii), the corresponding doping densities at p-side and n-side are:
$N_A = 5 \text{ x } 10^{17} cm^{-3}$ and $N_D = 4 \text{ x } 10^{17} cm^{-3}$.
This doesn't match with the one-side heavily-doped condition.

Therefore, N_D should be 2 x $10^{18} cm^{-3}$ and N_A should be 3 x $10^{16} cm^{-3}$. This is n+p junction.

(b) $\phi_{bi} = \frac{kT}{q} ln \frac{N_A N_D}{n_i^2} = 0.026 \cdot ln \frac{3\text{x}10^{16} \cdot 2\text{x}10^{18}}{10^{20}} = 0.88\, V$

The fact that:

$$N_A >> N_D$$

Thus $x_p = \sqrt{\frac{2\varepsilon\phi_{bi}}{qN_A}} = \sqrt{\frac{2 \cdot 11.7 \cdot 8.85 \cdot 10^{-14} \cdot 0.88}{1.6 \cdot 10^{-19} \cdot 3\text{x}10^{16}}} = 1.95 \text{ x } 10^{-5}$ cm or 195 nm

$x_n = \sqrt{\frac{2\varepsilon N_A \phi_{bi}}{qN_D(N_A+N_D)}} = \sqrt{\frac{2 \cdot 11.7 \cdot 8.85 \cdot 10^{-14} \cdot 0.88 \cdot 3\text{x}10^{16}}{1.6 \cdot 10^{-19} \cdot 2\text{x}10^{18} \cdot (3\text{x}10^{16}+2\text{x}10^{18})}} = 2.90 \text{ x } 10^{-7}$ cm or 2.9 nm

Or more easily: $x_p \cdot N_A = x_n \cdot N_D$ $\quad x_n = x_p \frac{N_A}{N_D}$ = 195 nm x $\frac{3\text{x}10^{16}}{2\text{x}10^{18}}$ = 2.9 nm

Therefore, x_1= 2.9 nm and x_2= 195 nm

$|E_{max}(V = 0)| = \frac{qN_A x_p}{\varepsilon} = \frac{1.6 \cdot 10^{-19} \cdot 3\text{x}10^{16} \cdot 1.95 \text{ x } 10^{-5}}{11.7 \cdot 8.85 \cdot 10^{-14}}$ = 9.0 x 10^4 V/cm or 90 kV/cm

Therefore, y_1= 90 kV/cm.

(c) The physical meaning of the shaded area is the built-in potential ϕ_{bi} of the pn junction.
By applying a reverse bias, the electric profile will change from the solid line to the dashed line.

The maximum area is reached when the reverse voltage reaches to breakdown voltage and it corresponds to $\phi_{bi} + qV_{br}$. The breakdown happens when the electric field reaches to critical electric field $E_{critical} = 300\ kV/cm$

$$|E_{max}(V)| = |E_{max}(V=0)| \cdot \sqrt{1 - \frac{V_{br}}{\phi_{bi}}}$$

$$300\text{ kV/cm} = 90\text{ kV/cm} \cdot \sqrt{1 - \frac{V_{br}}{0.88}}$$

V_{br} = -9 V

Therefore, the maximum area that can be reached is (9 + 0.88) eV.

(d) At p-side: $E_f - E_V = -kT\ ln\left(\frac{p}{N_V}\right) = -kT\ ln\left(\frac{N_A}{N_V}\right)$ = - 0.026 x $ln\left(\frac{3\text{x } 10^{16}}{3.1\text{ x } 10^{19}}\right)$ = 0.18 eV

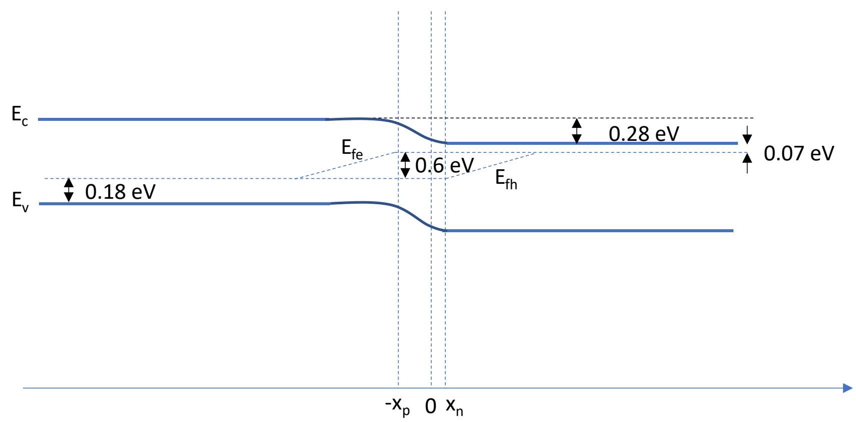
At n-side: $E_C - E_f = -kT\ ln\left(\frac{n}{N_C}\right) = -kT\ ln\left(\frac{N_D}{N_C}\right)$ = - 0.026 x $ln\left(\frac{2\text{ x } 10^{18}}{2.9\text{ x } 10^{19}}\right)$ = 0.07 eV

x_p = 195 nm

x_n = 2.9 nm

$\phi_{bi} = 0.88\ V$

Band diagram under 0.6 V:



Problem 3. Solution

a. In this situation, electrons are excited under low-level injection only in the central region. The larger concentration of electrons produce diffusion to the sides, as indicated by the inclined Efe in both ends.

- thermal equilibrium / **out of equilibrium**
- **uniformly doped** / non-uniformly doped
- n-type / **p-type**
- Is there an electric field: Yes / **No**
- Is there electron current: **Yes** / No

b.

$$N_a \approx p_0 = N_V e^{\left(-\frac{E_{fh}-E_V}{kT}\right)} = 3.1e19 e^{\left(-\frac{0.1}{0.026}\right)} = 6.6\text{e}17\text{ cm}^{-3}$$

$n_0 = \frac{n_i^2}{p0} = 151$ cm^{-3}

In the middle of the bar: From $E_c - E_{fn}$ in the central region is 0.3 eV, we can calculate the amount of e-h pair injected using $n = N_c e^{(-\frac{E_c - E_{fn}}{kT})}$ =2.83e14 cm^{-3}, which is more than 3 order of magnitude smaller than p_0 and much larger than n_0.

The excess electron concentration at the laser point is n' = n-n_0 ~ n = 2.83e14 cm^{-3}. n_o << n' << p_o which confirms the low level injection (LLI).

c.

From plot 1:

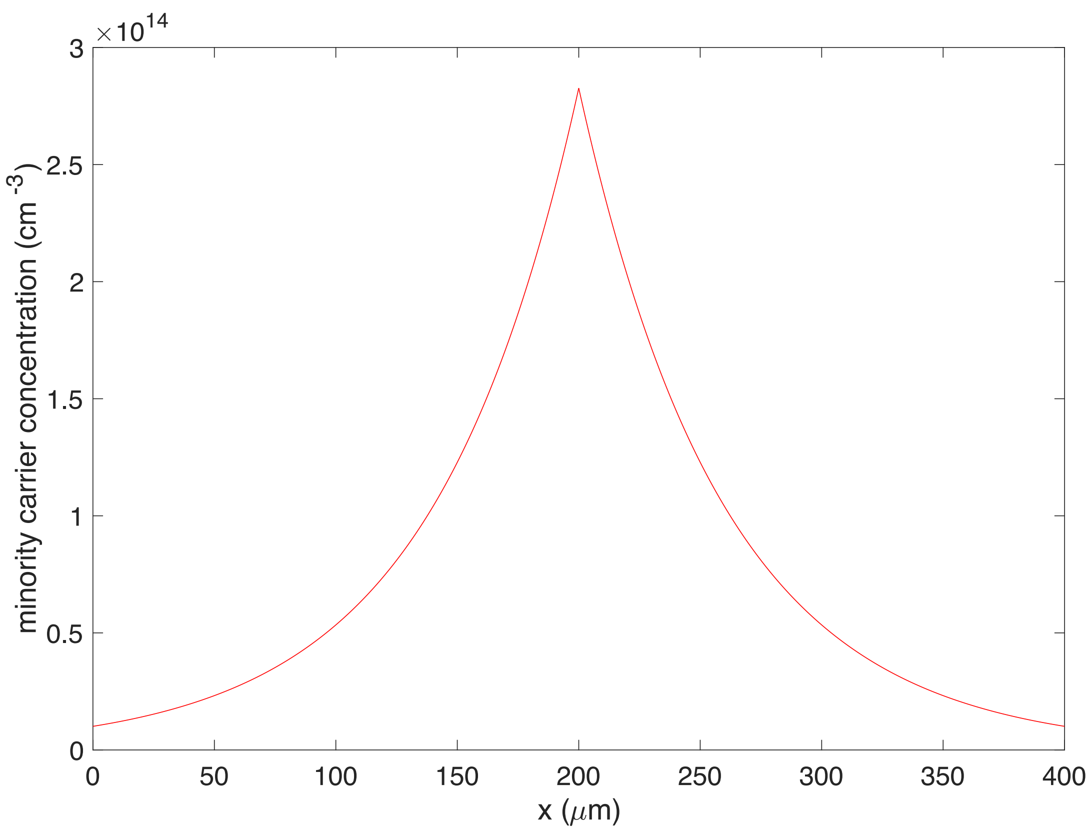
μ_n~400 cm^2/Vs for Na = 6.6e17 cm^{-3}

Thus the diffusion coefficient can then be determined as De = KT/q*μ_n = 10.4 cm^2/s

From plot 2, the minority carrier diffusion length is Le = 6e-3 cm = 60 um

Therefore the minority carrier lifetime is 3.5e-06s

d. The minority carrier concentration decreases exponentially with n(x) = n(200 um)*exp(-x/Le). The same can be determined from n=Nc*exp(-(Ec-Ef_e(x))/KT)



e. The electron current density at x = 200um:

From the quasi-Fermi level we have $J_n(200um) = qn(200um)\mu_n \frac{dE_{fn}}{dx}(200um)$, where:

- $n(200um) = 2.83e14$ cm^{-3}
- μ_n~400 cm^2/Vs from the Conwell diagram seen in class, given that Na = 6.6e17 cm^{-3}
- $\frac{dE_{fn}}{dx}(0-200) = \frac{E_{fn}(0)-E_{fn}(200\mu m)}{200\mu m} = \frac{0.8-0.7}{200E-4} = 5$ eV/cm (for 0-200um)

  And $\frac{dE_{fn}}{dx}(200-400) = -5$ eV/cm (for 200-400um)

Thus $J_n(200) = 2.83E14 * 400 * 5 * 1.6E-19 = 90.6$ mA/cm^2 (for 0-200um)

And
$J_n(200) = 2.83E14 * 400 * (-5) * 1.6E-19 = -90.6$ mA/cm^2 (for 200-400um)

f. After the laser is off the carriers will recombine, and the excess carriers will decrease as:

$n' = n'(0) \cdot (1 - e^{-t/\tau})$, where τ is the lifetime.

After a few $\tau's$, the wafer will reach thermal equilibrium with the fermi level at the