$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{KT}\right)}$$

$$g_c(E) = 4\pi \left(\frac{2m^*}{h^2}\right)^{3/2} \sqrt{E - E_c}$$

$$n = \int_{E_c}^{\infty} g_c(E) \cdot f(E)\, dE = N_c \cdot \frac{2}{\sqrt{\pi}} \cdot F_{1/2}\left(\frac{E - E_F}{KT}\right)$$

effective density of states in the conduction band

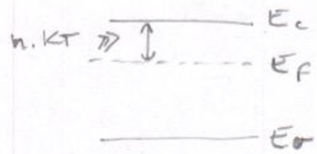
$$N_c = 2 \cdot \left(\frac{2\pi m_e^* \, KT}{h^2}\right)^{3/2}$$

Fermi-Dirac integral:

$$F_{1/2}(x) = \int_{E_c}^{\infty} \frac{x^{1/2}}{1 + e^{x - x_F}}\, dx$$

Non-degenerate semiconductors :

- Doping $\ll N_c$
- $E_c - E_F \gg$ a few KT




then we can approximate the fermi-integral by :

$$F_{1/2}\left(\frac{E - E_c}{KT}\right) = \frac{\sqrt{\pi}}{2} \cdot \exp\left(-\frac{E_c - E_F}{KT}\right)$$

thus :

$$n = N_c \cdot \exp\left(-\frac{E_c - E_F}{KT}\right)$$

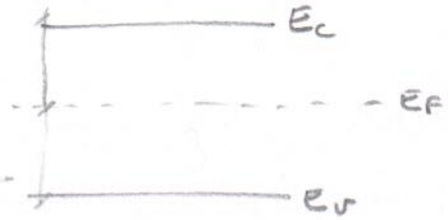
* For degenerate semiconductors $\Rightarrow$ the correct function $F_{1/2}$ needs to be used.

For conduction band

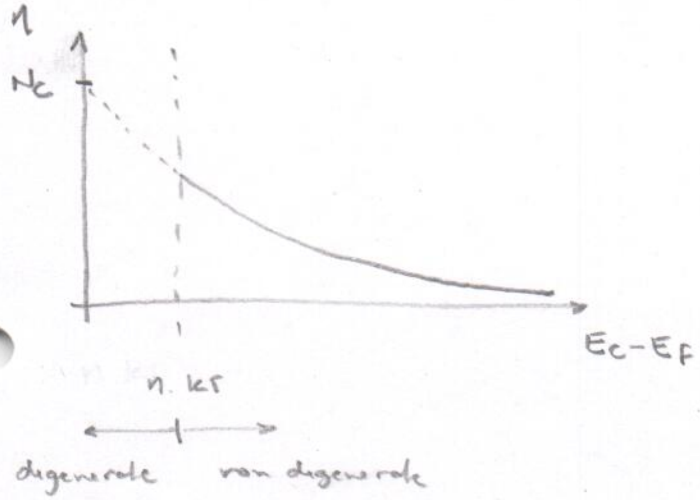
$$n = N_c \exp\left(-\frac{E_c - E_F}{kT}\right)$$

For valence band

$$p = N_v \exp\left(-\frac{E_F - E_v}{kT}\right)$$




Simplified equations can be obtained by maxwell-Boltzmann statistics.




for Silicon at 300 K:

$$N_c = 2\left(\frac{2\pi\, m^*_{de} \cdot kT}{h^2}\right)^{3/2}$$

$$= 2 \cdot \left(\frac{2\pi \cdot 1.09 \times 5.69 \times 10^{-16}\ eV\, s^2/cm^2 \times 0.0259\ eV}{(4.14 \times 10^{-15}\ eV \cdot s)^2}\right)^{3/2} = 2.9 \times 10^{19}\ cm^{-3}$$

$$N_v = 3.1 \times 10^{19}\ cm^{-3}$$

* please use these numbers from now on!

what is the position of $E_F$ in intrinsic semiconductors ?

Substracting Eqs. [1] and [2] :

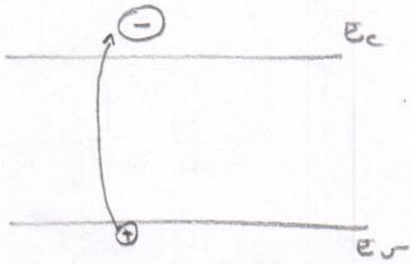
[1] $E_C - E_F = kT \ln \left( N_c / n \right)$

[2] $E_F - E_v = kT \ln \left( N_v / p \right)$

$$E_F - E_V - E_C + E_F = kT \ln \left( N_c / n \right) - kT \ln \left( N_v / p \right)$$

$$E_F = \frac{E_v + E_c}{2} + \frac{kT}{2} \ln \left( \frac{N_v}{N_c} \cdot \frac{n}{p} \right)$$

what can we say about $n$ and $p$ ?




$n$ must be equal to $p$ in intrinsic semiconductors

$n = p$

thus

$$E_F = \frac{E_v + E_c}{2} + \frac{kT}{2} \ln \left( \frac{N_v}{N_c} \right) \equiv E_i$$

or

$$E_F = \frac{E_v + E_c}{2} + \frac{3}{4} kT \ln \left( \frac{m^*_{dh}}{m^*_{de}} \right)$$

$E_F$ lies <u>very close</u> , but not exactly , at the middle of the band gap.

What is p.n ?

$$p \cdot n = N_c \cdot N_v \cdot \exp\left(\frac{-E_c + E_F - E_F + E_v}{kT}\right)$$

$$= N_c \cdot N_v \cdot \exp\underbrace{\left(\frac{E_v - E_c}{kT}\right)}_{E_g/kT} = n_i^2$$ law of mass-action

thus : $\boxed{p \cdot n = n_i^2}$ it is a constant!

if degenerate $p \cdot n \neq n_i^2$

We can rearrange the terms:

$$n = n_i \left(\frac{E_F - E_i}{kT}\right)$$

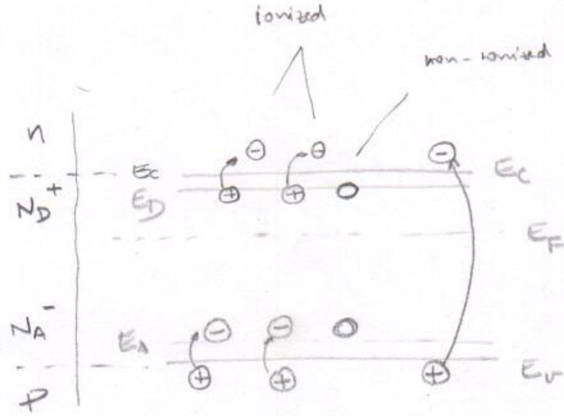
$$p = n_i \cdot \left(\frac{E_i - E_F}{kT}\right)$$

back to slides

⇒ **Doping**:

Donor: neutral / positive

Acceptor: neutral / negative




Not all dopants are necessarily ionized

$$N_D^+ = \frac{N_D}{1 + g_D \cdot \exp\left(-\frac{E_D - E_F}{kT}\right)}$$

$g_D$: degeneracy of donor level = 2 (spin of electrons)

$$N_A^- = \frac{N_A}{1 + g_A \exp\left(\frac{E_A - E_F}{kT}\right)}$$

$g_A$ = degeneracy of acceptor level = 4 = spin (2) × degeneracy of band at $k = 0$ (2)


in most semiconductors there are 2 bands degenerate at $k = 0$

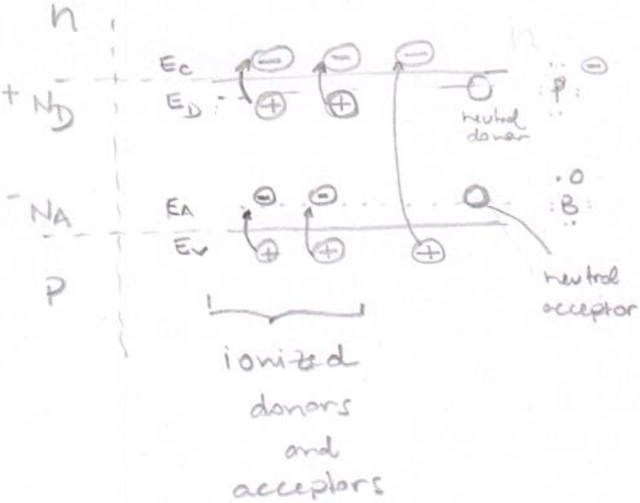
Due to charge neutrality :

$$n + N_A^- = p + N_D^+$$

the law of mass-action

$$n \cdot p = n_i^2$$

still applies ! (until degeneracy)

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How to calculate the Fermi level ?

for n-type Si :

$$n = p + N_D^+$$

p : intrinsic $\sim 10^{+10}\ cm^{-3}$

$N_D$ : doping $\sim 10^{18}\ cm^{-3}$

$$\Rightarrow n \simeq N_D^+$$

$$N_C \cdot \exp\left(-\frac{E_C - E_F}{kT}\right) \simeq N_D \cdot \frac{1}{1 + 2\exp\left(-\frac{E_D - E_F}{kT}\right)}$$

for a given $N_C, N_D, T, E_D$ $\Rightarrow$ we can calculate $\underline{\underline{E_F}}$ and then $\underline{\underline{n}}$.

$\Rightarrow$ use graphical method.

or

- n-type
  $N_D \gg N_A$ : $$n \approx \sqrt{\frac{N_D N_C}{2}} \cdot \exp\left[-\frac{(E_C - E_D)}{2kT}\right]$$

- if $N_A$ is non-negligible:
  $N_D > N_A$ : $$n \approx \left(\frac{N_D - N_A}{2N_A}\right) \cdot N_C \cdot \exp\left[-\frac{(E_C - E_D)}{kT}\right]$$

At relatively high temperatures, all donors and acceptors are ionized:

$$\boxed{n + N_A = p + N_D}$$

and since $n \cdot p = n_i^2$ $\Rightarrow$ $n = \frac{1}{2}\left[ (N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4n_i^2} \right]$

$$n^2 - (N_D - N_A)n - n_i^2 = 0$$

in n-type semiconductor: $N_D > N_A$

$$n_{no} = \frac{1}{2}\left[ (N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4n_i^2} \right] \approx N_D$$

if $|N_D - N_A| \gg n_i$

or $N_D \gg N_A$

$$p_{no} = \frac{n_i^2}{n_{no}} \approx \frac{n_i^2}{N_D}$$

Fermi level:

$$n_{no} = N_D = N_C \cdot \exp\left(-\frac{E_C - E_F}{KT}\right) = n_i \exp\left(\frac{E_F - E_i}{KT}\right)$$

we can solve numerically or graphically

for p-type semiconductor

- finish + exercises
- complete 1st list of exercises.

$$p = \frac{1}{2}\left[ (N_A - N_D) + \sqrt{(N_A - N_D)^2 + 4n_i^2} \right]$$

$$\simeq N_A$$

if $|N_A - N_D| \gg n_i$

or $N_A \gg N_D$

$$n = \frac{n_i^2}{N_A}$$

fermi-level:

$$p = N_A = N_V \cdot \exp\left(-\frac{E_F - E_V}{KT}\right) = n_i \cdot \exp\left(\frac{E_i - E_F}{KT}\right)$$