4.2a Electrodynamic transducers

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Goals and Prerequisites

Goals

The objectives of this lecture are:

- to present in a straightforward manner the **physical phenomena** responsible of the electrodynamic transduction;
- to describe these phenomena through **coupling equations** linking the electrical quantities (U,i) and mechanical quantities (F,v);
- to translate these equations into **circuit representations**.

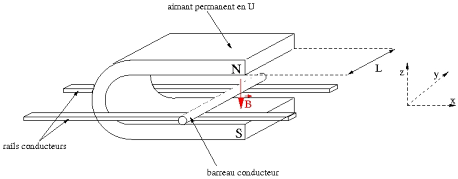
Prerequisites

The following should be already known:

- basic notions of electromagnetics,
- electrotechnics,
- electrical-mechanical representations.

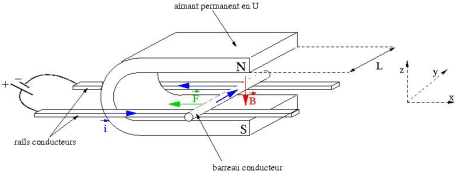
Simple Experiment

⋆ Let's consider the simple system below, constituted of a conducting bar susceptible to roll over 2 conducting rails, along $\vec{x}$ axis. A U-shaped magnet is placed between the rails, so that the bar is within a constant magnetic field $\vec{B}$ directed towards $-\vec{z}$ axis.



⋆ This forms an electromagnetic transducer, capable of behaving as an actuator (the bar being set in movement when an electrical current is circulating through it), or as a sensor (the movement of the bar induces an electrical voltage at its extremities).

Laplace Force (1)



⋆ Let's first consider an electrical current $\vec{i}$ circulating through the conducting bar. Under the combined effect of the magnetic field $\vec{B}$ and current $\vec{i}$, an electromagnetic force (called Laplace force) $\vec{F}$ is applied on the bar.

⋆ The magnitude of this force is proportional to the current i, the magnetic field B, and the length ℓ of the conducting bar, and $\vec{F}$ is oriented towards a direction so that the 3 vectors $\vec{i}$, $\vec{B}$ and $\vec{F}$ constitute a direct orthogonal basis:

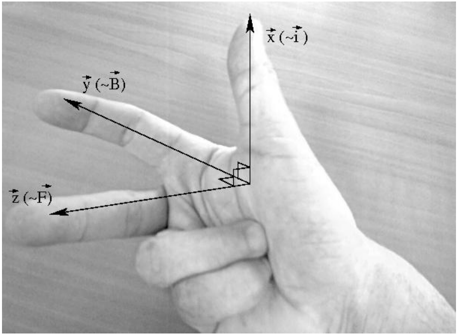
$$\vec{F} = \ell \vec{i} \times \vec{B}.$$

⋆ In the following, the Laplace force will result in the translational movement of the bar, this movement being eventually compensated by other forces applied to the bar (damping, stiffness, inertia).

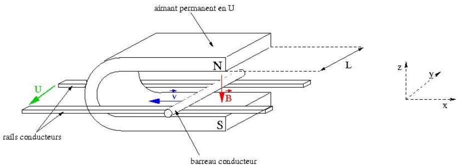
Note: The considered length ℓ corresponds to the portion of bar within the magnetic field.

Laplace Force (2)

Several methods allow to easily find the direction of the Laplace force $\vec{F} = \ell\vec{i} \times \vec{B}$, including the so-called rule "right hand".



Lenz Law



⋆ Let's now consider a moving bar along the $\vec{x}$ axis under the effect of an external force. Under the combined effect of the magnetic field $\vec{B}$ and the velocity $\vec{v}$ of the bar, a voltage U called "electromotive force" is induced across the bar.

⋆ This potential difference U is proportional to the length ℓ of the moving conductor and the magnitudes of velocity $\vec{v}$ and magnetic field $\vec{B}$.

$$U = -B\ell v$$

Note: The sign '$-$' here comes from the projection of the vector v on the $\vec{x}$ axis, which leads to an emf opposed to the establishment of the velocity (in this case U will have a "positive" sign in the chosen convention, where the induced current flows through the conducting bar in the opposite direction to the $\vec{y}$ axis, and leads to a positive Laplace force in the $\vec{x}$ axis).

Electromagnetic Force vs. Electromotive Force?

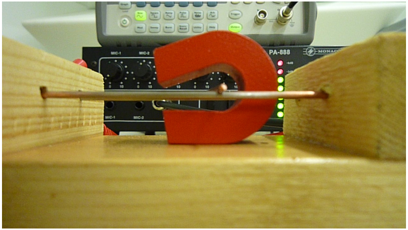
⋆ The phenomena discussed previously are inherently **inseparables**.
Let us consider a portion of the electrical conductor immersed in a magnetic field $\vec{B}$.

- When a current $\vec{i}$ flows through the conductor, it is subjected to an electromagnetic force ($\vec{F} = B\ell\vec{i}$), which puts in motion (velocity $\vec{v}$),
- This movement generates a voltage (**electromotive force**, $U = -B\ell v$) across the bar, **which has the effect of reducing the current that is the cause of movement**.

⋆ The two phenomena are clearly **linked** and are responsible for the **reversible** nature of electrodynamic transducers.

Oscillating System

⋆ Experiment: the conducting bar is subjected to a sinusoidal current.

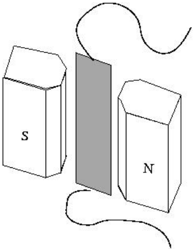


⋆ Coupling equations:

$$\begin{aligned} F(t) &= B\ell i(t) \\ U(t) &= -B\ell v(t) \end{aligned}$$

The ribbon transducer

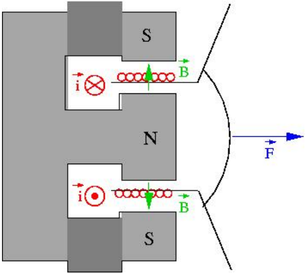
⋆ Replacing the bar with a conductive membrane is then obtained a **ribbon transducer** (the large area of the mechanical element provides a better mechanical-acoustical coupling).



Its use is still not widespread, as too bulky. The development of electrodynamic transducers led to axisymmetric geometries allowing more efficient transduction (equivalent compactness), more robust and less expansive. Nowadays, the vast majority of electrodynamic transducers are based on an axisymmetric geometry: the voice coil transducer.

How to make a more compact Electrodynamic Transducer?

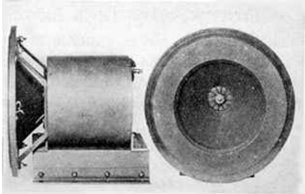
⋆ One way to improve the transduction efficiency is to increase the length ℓ of conductor which is immersed in the magnetic field $\vec{B}$, while maintaining a large radiation surface. To maintain a certain compactness of the object (in thickness), one solution is to wrap the driver in the form of a coil, which requires to think the spatial distribution of the magnetic field $\vec{B}$ again, which must have a radial structure centered in the axis of the coil.



Note: The radiation surface is however constrained by the "low-frequency" hypothesis ($ka << 1$ where a is the diaphragm radius).

Structure of a Voice Coil Transducer

⋆ In its conventional architecture dating from the Rice & Kellogg's patent filed in 1925, a voice coil transducer comprises a coil joined together to the diaphragm, and immersed in a radial magnetic field.



Picture of an electrodynamic loudspeaker developed and patented by Chester W. Rice and Edward W. Kellogg (Patent No. 1,812,389).
http://www.aes.org/aeshc/docs/recording.technology.history/rice-kellogg.html

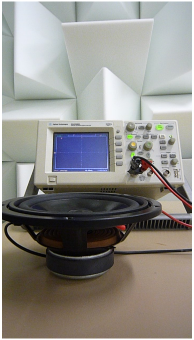
The Transducer as actuator

⋆ An electrodynamic loudspeaker is connected to a power amplifier, which is connected to a low-frequency generator, whose the input signal is sinusoidal. On the one hand we can remark the movement magnitude of the diaphragm depends on the excitation frequency. On the other hand the sound is not noticeable at low frequencies (5 Hz and 20 Hz).

Left to right: videos of a loudspeaker stimulated with different frequencies: 5 Hz (left), 20 Hz (center) et 100 Hz (right)

The Transducer as sensor

⋆ An electrodynamic transducer is reversible: it can therefore be used as sensor (although its design is not optimized for this function). In the following experiment the electrodynamic loudspeaker is simply connected to the input of an oscilloscope, and the diaphragm is stimulated by an external sound source. It can be observed the voltage induced with the help of the oscilloscope. An interesting point is the presence of a damped sine curve after the initial pulse, which reflects the mechanical behaviour of the moving diaphragm (free oscillations after the initial excitation).



Video of an electrodynamic loudspeaker used as sensor (i.e. microphone)

Coupling equations (1)

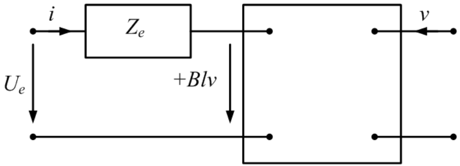
⋆ The coupling equations of an electrodynamic transducer are directly derived from the laws of behaviour outlined above. We define the velocity v in order to present a pattern of symmetrical coupling. The voltage is then expressed as: $U = B\ell v$

Electrical side:

The electrodynamic transducer comprises a voice coil which is a passive dipole (resistance R_e and inductance L_e in serie), to be taken into account in the coupling. If we consider a driving voltage U_e at the electrical terminals of the transducer, we get:

$$U_e - B\ell v = Z_e i$$

where $Z_e = R_e + jL_e\omega$ is the electrical impedance of the voice coil (resitance R_e and inductance L_e).



Coupling Equations (2)

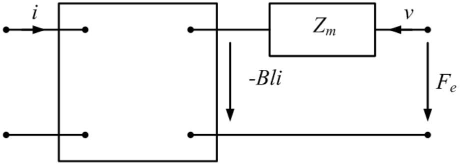
⋆ The electromagnetic force is expressed as: $F = B\ell i$

Mechanical:

The diaphragm of the electrodynamic transducer is a mechanical resonator (mechanical resistance R_m, mass M and mechanical compliance C_m) to be taken into account in the expression of the Newton law. If we consider the diaphragm is subjected to external forces F_e (apart the Laplace force), we get:

$$F_e + B\ell i = Z_m v$$

where $Z_m = R_m + j\omega M + \frac{1}{j\omega C_m}$ is the mechanical impedance of the diaphragm.



Coupling Equations (3)

⋆ The coupling equations of the electrodynamic transducer are:

$$\begin{aligned} U_e - Z_e i &= B\ell v, \\ F_e - Z_m v &= -B\ell i, \end{aligned}$$

$B\ell$ is the force factor of the transducer (ℓ is the length of the coil and B is the magnetic field in the gap).

Representation by a Transformer (1)

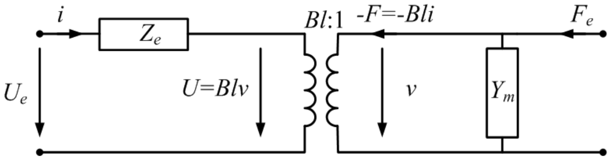
If we reintroduce $U = U_e - Z_e i = B\ell v$, and $-F = F_e - Z_m v = -B\ell i$, the coupling matrix is expressed as:

$$\begin{pmatrix} U \\ i \end{pmatrix} = \begin{pmatrix} B\ell & 0 \\ 0 & \frac{1}{B\ell} \end{pmatrix} \begin{pmatrix} v \\ -F \end{pmatrix},$$

The 2x2 diagonal matrix represents the electrical transformer.

Representation by a Transformer (2)

⋆ A possible equivalent electrical representation of the electrodynamic transduction involves a transformer of which the ratio is $B\ell$.



where $Y_m = 1/Z_m$ is the mechanical admittance of the diaphragm.

⋆ This representation, however, is rarely used because it does not retain the same type of analogy on both sides of the transformer. The variable pair of F and v is represented by analogy admittance (or inverse analogy), the force F is represented by a current, and speed v by a voltage. We generally prefer to use a gyrator to represent the electrodynamic transduction.

Representation by a Gyrator (1)

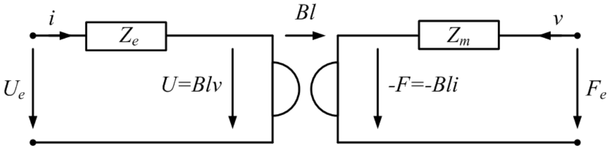
If we reintroduce $U = U_e - Z_e i = B\ell v$, and $-F = F_e - Z_m v = -B\ell i$, the coupling matrix is expressed as:

$$\begin{pmatrix} U \\ i \end{pmatrix} = \begin{pmatrix} 0 & B\ell \\ -\frac{1}{B\ell} & 0 \end{pmatrix} \begin{pmatrix} -F \\ v \end{pmatrix},$$

The 2x2 matrix represents the electrical gyrator.

Representation by a Gyrator (2)

⋆ The coupling representation of a gyrator is shown in the figure below, where $B\ell$: 1 represents the rate of gyration of the two-port network.



⋆ The gyrator, of which the physical reality is less obvious than that of a transformer, has the advantage of respecting the physics of the problem in the sense that, like the electrical quantities, mechanical parameters are shown in direct analogy.

Representation without Two-Port Network (1) "Equivalent" Electrical Schematics

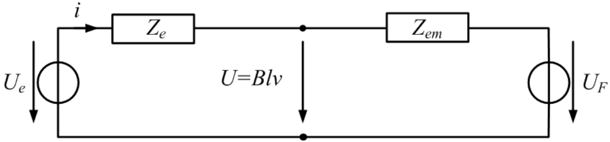
⋆ It is also possible to eliminate the two-port network, so as to retain only one representation of the transducer (electrical only or mechanical only).

Example 1: Electrical circuit representation of the transducer

$$\begin{aligned} U_e - Z_e i &= B\ell v, \\ v &= \frac{F_e}{Z_m} + \frac{B\ell i}{Z_m}, \end{aligned}$$

By introducing $U_F = \dfrac{B\ell . F_e}{Z_m}$ and $Z_{em} = \dfrac{(B\ell)^2}{Z_m}$, we obtain:

$$U_e = (Z_e + Z_{em})\, i + U_F$$



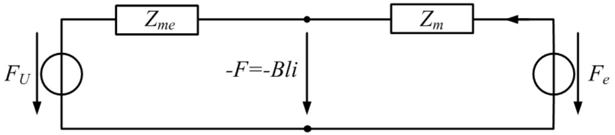
Representation without Two-Port Network (2) "Equivalent" Electrical Schematics

Example 2: Mechanical circuit representation (direct analogy) of the transducer

$$\begin{aligned} i &= \frac{U_e}{Z_e} - \frac{B\ell v}{Z_e}, \\ F_e - Z_m v &= -B\ell i, \end{aligned}$$

By introducing $F_U = -\frac{B\ell U_e}{Z_e}$ et $Z_{me} = \frac{(B\ell)^2}{Z_e}$, we obtain:

$$F_e = (Z_m + Z_{me})\, v + F_U$$



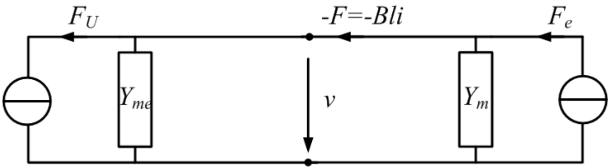
Representation without Two-Port Network (3) "Equivalent" Electrical Schematics

Example 2: Mechanical circuit representation (inverse analogy) of the transducer

$$\begin{aligned} i &= \frac{U_e}{Z_e} - \frac{B\ell v}{Z_e}, \\ F_e - Z_m v &= -B\ell i, \end{aligned}$$

By introducing $F_U = -\frac{B\ell U_e}{Z_e}$, $Y_{me} = \frac{Z_e}{(B\ell)^2}$, and $Y_m = \frac{1}{Z_m}$ we obtain:

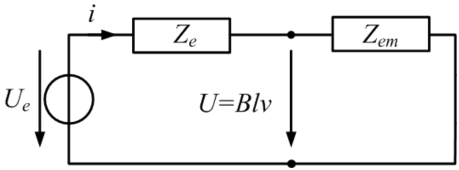
$$F_e = \left(\frac{1}{Y_m} + \frac{1}{Y_{me}}\right) v + F_U$$



Kinetic impedance (1)

The electrical impedance $Z_{em} = \frac{(B\ell)^2}{Z_m}$ corresponds to the mobility of the diaphragm.

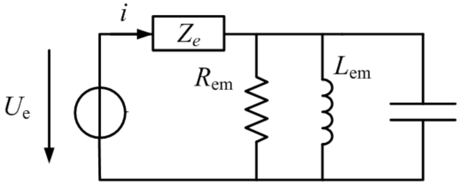
When the transducer is used as a speaker (generator behavior: $F_e = 0$), it can be observed on the measurement of the (electrical) input impedance of the transducer:



$$Z_{HP_{|F_e=0}} = \frac{U_e}{i_{|F_e=0}} = Z_e + \frac{(B\ell)^2}{Z_m}$$

$Z_{em} = \frac{(B\ell)^2}{Z_m}$ is the kinetic impedance (or motional impedance).

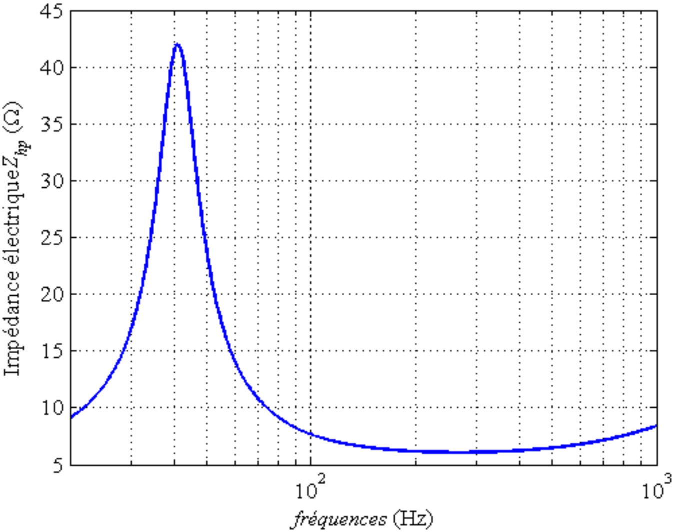
Kinetic impedance (2)



By writting $\frac{1}{Z_{em}} = \frac{R_m}{(B\ell)^2} + \frac{j\omega m}{B\ell)^2} + \frac{1}{j\omega(B\ell)^2 C_m}$, it is easily shown that it corresponds to a parallel electric resonator constituted of:

- a resistance $R_{em} = \frac{(B\ell)^2}{R_m}$,
- an inductance $L_{em} = (B\ell)^2 C_m$,
- a capacity $C_{em} = \frac{(B\ell)^2}{M}$.

Kinetic impedance (3)



References

M. Rossi, "Audio", chapitre 7.2, Presses Polytechniques et Universitaires Romandes, 2007.