Introduction to Acoustics

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MA1 - Audio Engineering

EPFL

Introduction

What is "acoustics" ?

- Science of sounds : branch of physics that addresses emission, propagation, and perception of sounds
- Quality of a space / room

Introduction

What is a *sound* ?

- variation of pressure
  (rapid movement of particles back and forth)
- that can produce an auditive sensation

Introduction

What is a *noise* ?

(General language)

- acoustic phenomenon (sound) that is not wanted
- it is SUBJECTIVE

But it also denotes a a random signal (sound)

Sound is vibration

A **vibration** is:

- local variation of pressure / stress
- vibratory movement of the medium around an equilibrium position

Longitudinal vibration (fluids & solids) :

Transverse vibration (solids only) :

We are mainly concerned with **longitudinal vibrations** in the **air**.

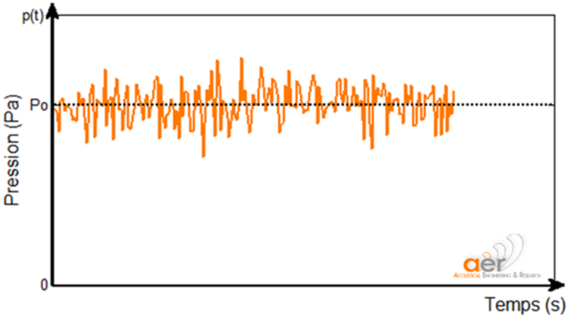
Acoustic pressure

Acoustic vibrations = small fluctuations of pressure around the mean value.

Total pressure P_{tot}
Atmospheric pressure P_0 (constant)
Acoustic pressure $p(t)$ (time dependent)

$$\boxed{P_{tot} = P_0 + p(t)}$$

Unit : *Pascal* (*Pa*)
Linear acoustic hypothesis :
$p(t) << P_0$

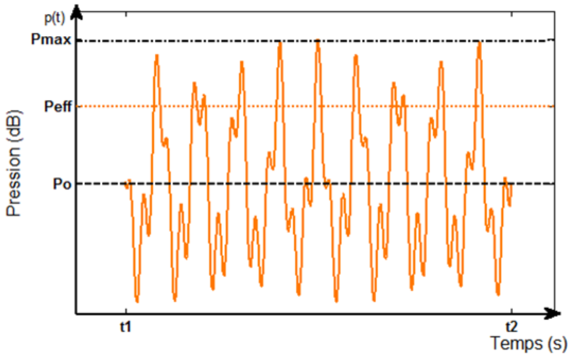


Rms pressure

Quantity related to the acoustic pressure and to the *level* of sound

Given acoustic pressure $p(t)$ between t_1 and t_2 :

$$p_{rms}^2 = \frac{1}{t_2 - t_1}\left(\int_{t_1}^{t_2} p(t)^2 dt\right)$$
$$= < p^2(t) >$$



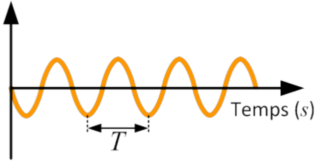
Period, frequency

Even though pure sinusoidal signals are rarely found in nature, sounds as signals are linear combinations of sinusoidal signals. Therefore sound signal description rely of Fourier analysis.

Period T : duration of one oscillation

Frequency f : number of oscillations per second

$$\boxed{f = 1/T}$$



Relation with *pitch*
Units : period in seconds (s) and frequency in Hertz ($Hz = 1/s$)
Angular frequency : ω (in rad/s) $\boxed{\omega = 2\pi f}$

Human ear sensitive to frequencies between $20Hz$ and $20kHz$:

$<$ 20Hz	20Hz to 200Hz[1]	200Hz[1] to 2kHz[1]	2 kHz[1] to 20kHz	$>$ 20kHz
infrasounds	bass	medium	treble	ultrasounds

[1] Note: those frequency bounds should be considered indicative...

Frequency : properties

An *octave* is a 2-fold increase of frequency :

$$f \rightarrow 2f$$

A *decade* is a 10-fold increase in frequency :

$$f \rightarrow 10f$$

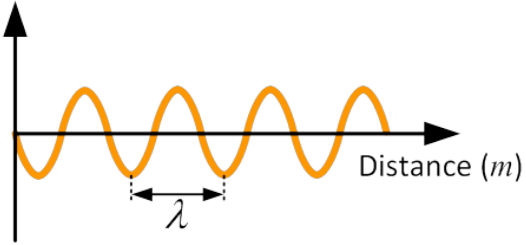
E.g. :

- the standard A is at $440Hz$, the A an octave higher is $880Hz$
- the audible frequency range covers 3 decades : $20Hz$ to $200Hz$, $200Hz$ to $2kHz$, $2kHz$ to $20kHz$

Wave length : spatial period

The wave length λ is the characteristic length of periodicity in *space* of a sound :

$$\boxed{\lambda = c/f}$$



Length of propagation of a wave during one period

Units : meter (m)

Wave number : spatial frequency

Wave number k is the frequency in *space* of a vibration : $\propto$ number of oscillations per unit length.

spatial angular frequency of the vibration $\boxed{k = \omega / c = 2\pi / \lambda}$

Units : radian per meter (rad/m)

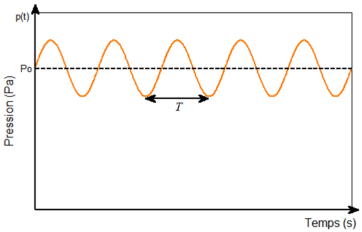
Pure sound

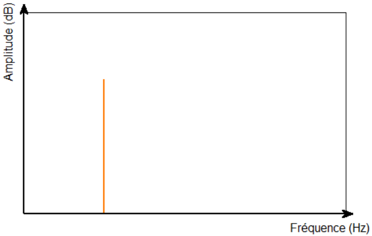
Sound with only one frequency

Pressure $p(t)$ is a pure sine : Play

$$p(t) = P_{max} sin(\omega t + \phi)$$

T : period (s)
$f = 1/T$: frequency (Hz)
$\omega = 2\pi f$: angular freq. (rad/s)

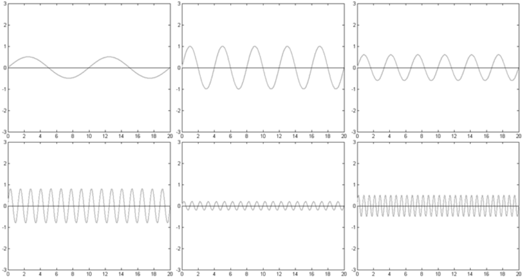




Complex (harmonic) sound

Sum of several pure sounds, the frequencies of which are in *harmonic* relation (integer multiple or rational)

$$p(t) = \sum_n P_n sin(\omega_n t + \phi_n)$$



Play

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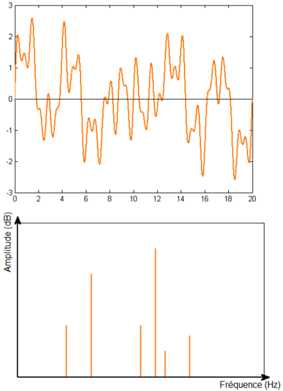


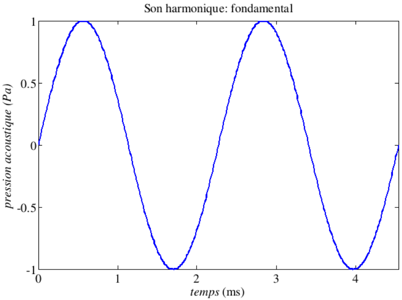
Play

Complex (harmonic) sound

Sum of several pure sounds, the frequencies of which are in *harmonic* relation (**integer** multiple or rational)

$$p(t) = \sum_{n}^{N} P_n sin(\omega_n t + \phi_n)$$

$\omega_n = n\omega_0 = n.2\pi.f$
f_0 = fundamental frequency (Hz)



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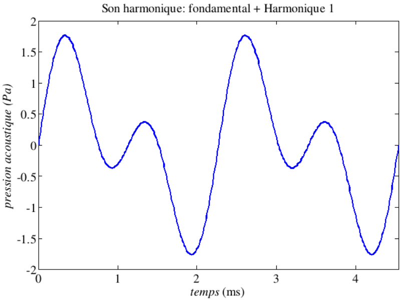

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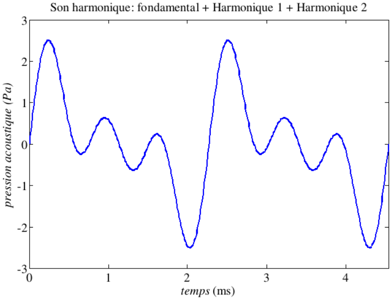
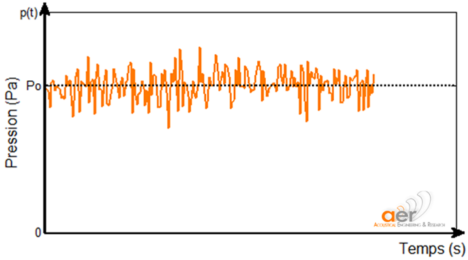


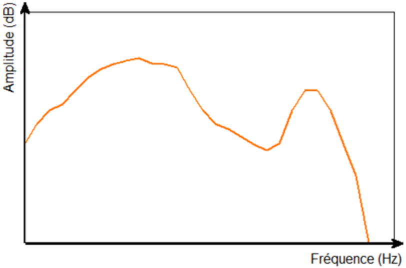
Random sound (random noise)

Random variations of acoustic pressure

- rich spectral content
- random vibrations

Complex sound with no time structure :





Play

Complex notation of sound signals

In harmonic regime (which applies for most sounds), the complex notation ("phasor") is used:

$$\underline{p}(t) = P_{max} e^{j(\omega t + \Phi)}$$

To simplify the notation, we only use the complex amplitude $\underline{p} = P_{max} e^{j\Phi}$.
With this notation, the following mathematical operations become:

	Time domain	Phasor notation
Derivation	$\frac{\partial \underline{p}}{\partial t}$	$j\omega \times \underline{p}$
Integration	$\int \underline{p} dt$	$\frac{\underline{p}}{j\omega}$

Speed of sound c

Speed of sound waves

Depends on physical properties of the medium :

- density
- temperature
- ambient pressure ...

Does *not* depend on :

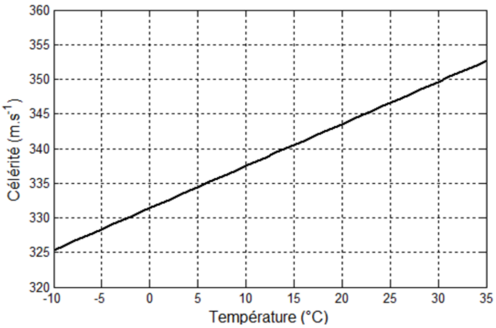
- amplitude
- frequency

Usually $c \approx 340 m/s$
(in dry ambient air)

Speed of sound c

Units : meter per second (m/s)

Approximate dependence on temperature :



$$\boxed{c_a(T) = 331.6 + 0.607\,T} \quad (\text{T in } {}^\circ C)$$

WARNING :
speed of sound
$\neq$
velocity of fluid particles !!

Acoustic velocity

Acoustic velocity v : at any given position $\vec{r}$ and time t, it represents the local perturbation of fluid particles velocity.

$$\vec{v}(\vec{r}, t) = \vec{V_t}(\vec{r}, t) - \vec{v_m}(\vec{r}, t)$$

In a medium at rest :

$$v_m = < V_t(\vec{r}, t) > = 0$$

Acoustic *volume* velocity

Acoustic volume velocity (or flow velocity) : integration of the acoustic velocity over a surface

$$\boxed{\vec{q} = \int_S \vec{v} d\vec{S}}$$

In pipe (1D, directing vector $\vec{e_x}$), S is the cross sectional area.

$$\boxed{\vec{q} = q\vec{e_x} = Sv\vec{e_x}}$$

In free field or room (3D), for a volume source, S is the contour surface of the source.

Acoustic power W

Quantity of energy through a given surface per unit time

$$W = \int_S < p.v > dS =< p.q >$$

Does not depend on the distance from the source

Units : Watt (W)

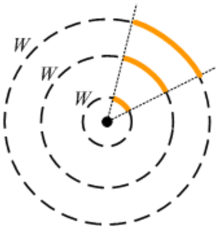
The power is linked to:

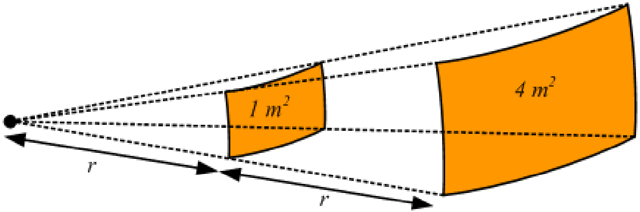
- acoustic velocity of the source (quadratic)
- radiation conditions (pipe, free field...)

Acoustic intensity I

Quantity of acoustic power *per unit surface*

Depends on the distance !!





Units : Watt per square meter (W/m^2)

$$I = < p.v > = < p^2 > /(\rho c) = \rho c < v^2 >$$

Relation between W and I

Intensity = acoustic power per unit surface
Power = integration of intensity over the surface :

$$\boxed{W = SI}$$

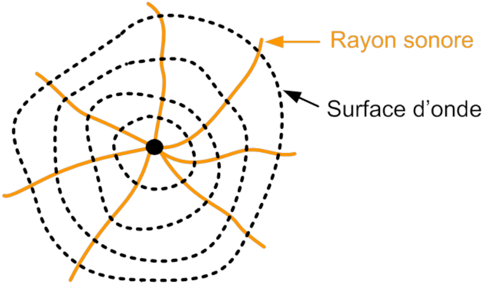
Spherical waves :

$$\boxed{W = 4\pi r^2 I(r)}$$

Intensity decreases with distance
(in $1/r^2$ for spherical waves)

Wave front and acoustic ray

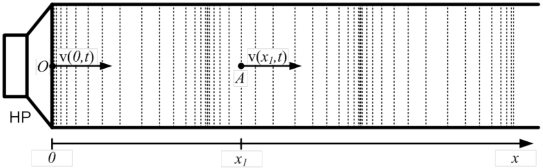
Wave front definition : surface of equal phase
Property : perpendicular to the direction of propagation
Acoustic ray : line starting from the source, following the propagation direction.
Always perpendicular to wave fronts.



Plane waves : definition

Wave fronts = parallel, plane surfaces
Acoustic rays = parallel, straight lines

Wave fronts are perpendicular to the direction of propagation



The acoustic wave equation (on p):

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

Plane waves : definition

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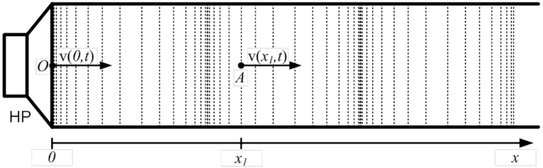

In 1D:

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

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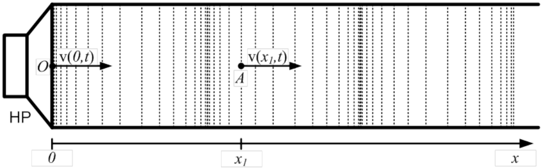


Solutions (in 1D):

$$p(x,t) = \left(p_+ e^{-jkx} + p_- e^{+jkx}\right) e^{j\omega t}$$

+ boundary conditions (if the medium is finite)

Plane waves : properties

Considering infinite tube (no reflected wave, only the $p_+ e^{-jkx}$ term), pressure p and velocity v are proportional :

$$\boxed{p(x,t) = Z_c v(x,t)}$$

where $Z_c = \rho c$ is defined as the characteristic impedance of the medium.
Amplitude of p (resp. v) independent of distance

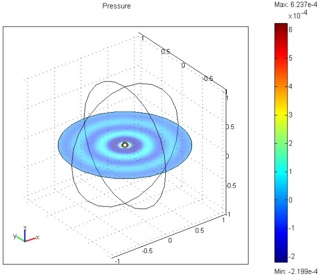
Intensity proportional to square of rms pressure :

$$\boxed{I = < pv > = \frac{p_{rms}^2}{\rho c}}$$

Spherical waves : definition

Wave front are concentric spheres

Amplitude and phase only dependent on distance from the source

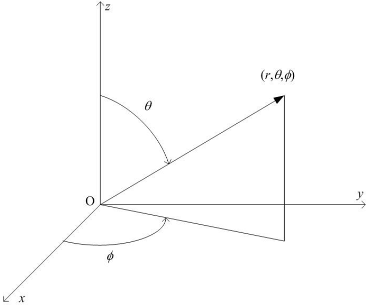


Ideal spherical wave source : solid, pulsating sphere (point source)

Spherical waves : definition

Wave front are concentric spheres

Amplitude and phase only dependent on distance from the source



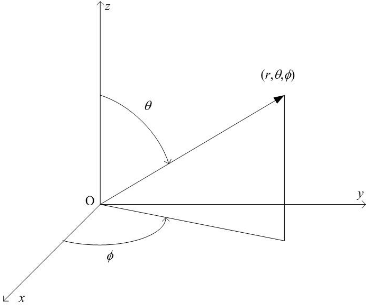
$$p(r, \phi, \theta, t) = p(r, t)$$

Ideal spherical wave source : solid, pulsating sphere (point source)

Spherical waves : definition

Wave front are concentric spheres

Amplitude and phase only dependent on distance from the source



In spherical coordinates:

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \cot\theta\frac{\partial}{\partial\theta} + \frac{\partial^2}{\partial\theta^2} + \frac{1}{\sin^2\theta}\frac{\partial}{\partial\phi}$$

Ideal spherical wave source : solid, pulsating sphere (point source)

Spherical waves : properties

Pressure p decreases like $1/r$:

$$p(r,t) = \frac{A}{r} e^{j(\omega t - kr)}$$

Intensity prop. to $1/r^2$:

$$I(r) = \frac{<p^2(r,t)>}{\rho c} = \frac{1}{\rho c}\frac{A^2}{r^2}$$

Power on concentric spheres is constant :

$$W = \frac{4\pi r^2 <p^2(r,t)>}{\rho c} = \frac{4\pi A^2}{\rho c}$$

Sound wave superposition (I)

Uncorrelated sources

N sources → resulting pressure field is the sum of each pressure wave (example : several cars in traffic, several instrument in a room...)

Property : the *interference* term is null

$$\begin{aligned} p_{rms}^2 &= < \left(\sum_n p_n(t)\right)^2 > \\ &= \sum_n < p_n^2(t) > + 2 \sum_i \sum_{j>i} < p_i(t) p_j(t) > \\ &= \sum_n < p_n^2(t) > \end{aligned}$$

The energy of each source adds up :

$$\boxed{I = \sum_{n=1}^{N} I_n} \text{ and } \boxed{p_{rms}^2 = \sum_{n=1}^{N} p_{rms,n}^2}$$

Sound wave superposition (II)

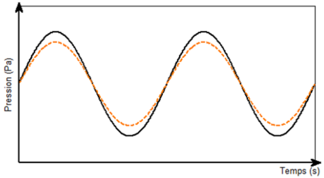
Correlated sources

Relations between sources (e.g. several speakers in a room, all playing the same signal)

Interferences are possible (application to active noise control)

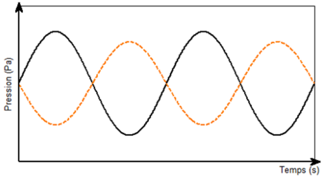
Case p_1 and p_2 in phase :

$$\begin{aligned} p(t) &= p_1(t) + p_2(t) = 2p_1(t) \\ p_{rms}^2 &= 4p_{rms,1}^2 \end{aligned}$$



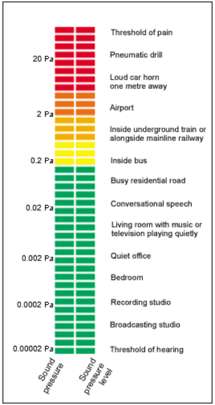
Case p_1 and p_2 out of phase :

$$\begin{aligned} p(t) &= p_1(t) + p_2(t) = 0 \\ p_{rms}^2 &= 0 \end{aligned}$$



Sound levels

From the weakest audible sound (hearing threshold p_0) to the pain threshold : factor 10^6 to 10^8 on p !!



$\Rightarrow$ we need a logarithmic scale !!

Sound levels

Acoustic *sound pressure level* (SPL) : $$L_p = 10log_{10}\left(\frac{p_{rms}^2}{p_0^2}\right) = 20log_{10}\left(\frac{p_{rms}}{p_0}\right)$$

where the reference pressure p_0 is $20\mu Pa$ (mean hearing threshold *in the air*)

Acoustic *intensity level* : $$L_I = 10log_{10}\left(\frac{I}{I_0}\right)$$

where $I_0 = \frac{p_0^2}{\rho c} = 10^{-12} W/m^2$

Note: generally, we can write $L_p = L_I$

Acoustic *power level* : $$L_W = 10log_{10}\left(\frac{W}{W_0}\right)$$

where $W_0 = I_0 \times 1m^2 = 10^{-12} W$

Power and pressure levels

Units : $dB \rightarrow$ need to detail what kind of level (L_p, L_I, or L_W)

Reference level : refer to a reference value used in the log ($20\mu Pa$, $1W$, ...)

Relation between L_p (L_I) and L_W for an omnidirectional source ($I(r) = \frac{W}{\pi r^2}$):

$$\boxed{L_p = L_I = L_W - 10log_{10}(4\pi r^2)}$$

two correlated sources of equal amplitude and in phase :

$$2 \times p \rightarrow L_p + 6dB$$

two uncorrelated sources of equal intensity :

$$2 \times p_{rms}^2 \text{ (or } 2 \times I) \rightarrow L_p + 3dB$$

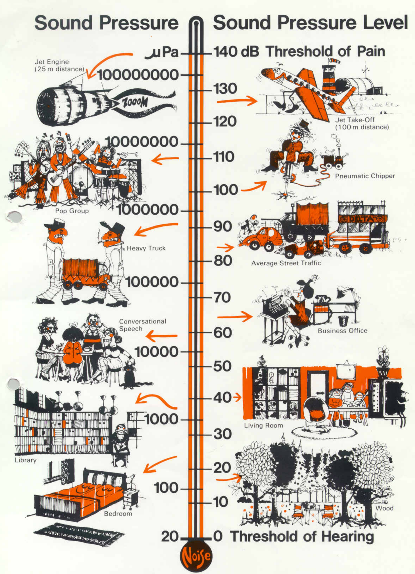
Logarithmic scale

Useful :

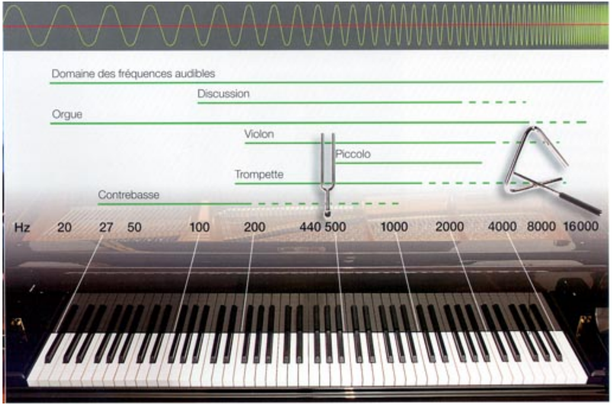
- hearing threshold $p_0 = 20\mu Pa \rightarrow 0dB$
- pain threshold $\sim 20Pa \rightarrow 120dB$

Relation to perception :

- human ear sensitive to the *log* of pressure
- doubling of sound pressure $6dB$
- just noticeable difference of level between 2 sounds $\approx 1dB$
- tripling of sound pressure $\approx 10dB$


Sound Pressure
Sound Pressure Level
µPa
140 dB Threshold of Pain
Jet Engine
(25 m distance)
100000000
130
Jet Take-Off
(100 m distance)
120
10000000
110
Pneumatic Chipper
100
Pop Group
1000000
90
Heavy Truck
80
Average Street Traffic
100000
70
Conversational
Speech
Business Office
60
10000
50
40
Living Room
1000
30
Library
20
100
10
Bedroom
Wood
20
0 Threshold of Hearing
Noise

Frequency scales



Frequency scales

Acoustical scales:

- *octave* ($f_0 \rightarrow 2f_0$)
- *semi-octave* ($f_0 \rightarrow \sqrt{2}f_0$) *full octave*
- *third-octave* ($f_0 \rightarrow \sqrt[3]{2}f_0$) *full octave*
- *decade* ($f_0 \rightarrow 10f_0$)
- etc.

Musical scales:

- octaves ($f_0 \rightarrow 2f_0$)
- *major third* ($f_0 \rightarrow 5/4f_0$)
- *minor third* ($f_0 \rightarrow 6/5f_0$)
- *perfect fifth* ($f_0 \rightarrow 3/2f_0$)
- etc.