Institute of Electrical and Microengineering
Biomedical and neuromorphic microelectronic systems

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v1.3	A. Schmid	2024, November 6	v1.4	A. Schmid	2025, October 24			

BIOELECTRONICS AND BIOMEDICAL MICROELECTRONICS

This documents gathers complements of information to the course that results from issues and questions of general interest, as well as corrections to the course and exercises. Its most recent version is available for download from the Moodle web site of the course.

Thank you very much to all contributors who participate to the improvement of the course by their comments and corrections.

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1. PRESENTATION TRANSPARENCIES

1.1 EQUILIBRIUM ELECTROCHEMISTRY

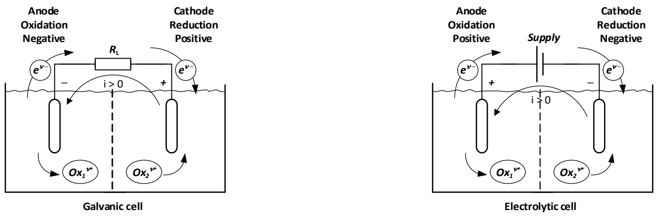
In an electrochemical cell, two electrodes are physically isolated such that oxidation and reduction effects occur at different compartments or electrodes [1]. Oxidation is defined as

$$Red_1 \rightarrow Ox_1 + ve^-$$

and reduction is defined as

$$Ox_2 + ve^- \rightarrow Red_2$$

By definition, the electrode where oxidation occurs is the anode and the electrode where reduction occurs is the cathode. The reactions spontaneously take place in a galvanic cell (Figure 1.1.a), whereas they are forced to take place in an electrolytic cell where a supply voltage is applied (Figure 1.1.b).



(a) (b)

Figure 1.1 Electrochemistry of a (a) galvanic cell and (b) electrolytic cell.

Oxidation occurs at the anode of a galvanic cell. Electrons are extracted from the electrode by the effect of oxidation thereby creating a negative voltage at the electrode. Electrons flow into the external circuit and reach the cathode. Reduction occurs at the cathode creating an excess of positive charge. The cathode has thus higher potential than the anode. Current flows in the external circuit reversely with respect of electron flow. In an electrolytic cell, oxidation occurs at the anode by definition. Nevertheless, the process must be forced and electrons must be withdrawn from the anode creating a positive charge. The reduction is driven by these electrons that reach the cathode, creating an excess of negative charges. Thus, in an electrolytic cell, the anode is positive and the cathode is negative.

1.2 LECTURE L02

1.2.1 COMMENT ON SLIDE #15

R is the gas constant (not Boltzmann's constant)

1.2.2 COMMENT ON SLIDE #17 AND SLIDE #18

The sign difference originate from the fact that internal and external are permuted:

$$\ln(a/b)=\ln(a)-\ln(b)=-\ln(b/a)$$

1.2.3 Comment on Slide #22

The A$^-$ ionic specie that reads at the bottom of the table is not really relevant as it does not appear extracellular. Only potatium, sodium and to some extent chloride are relevant ionic species to the cell equilibrium and action potential. The reference from which this Table is extracted [2] actually expresses A$^-$ as a large and impermeable ion.

1.3 Lecture L04

1.3.1 Comments of Slide #18

Slide #18 presents an approximation of the closed-loop gain of the inverting configuration in presence of finite open-loop gain (of the amplifier). The approximation is presented in Figure 1.2, and two detail derivations of the approximation are presented in the following.

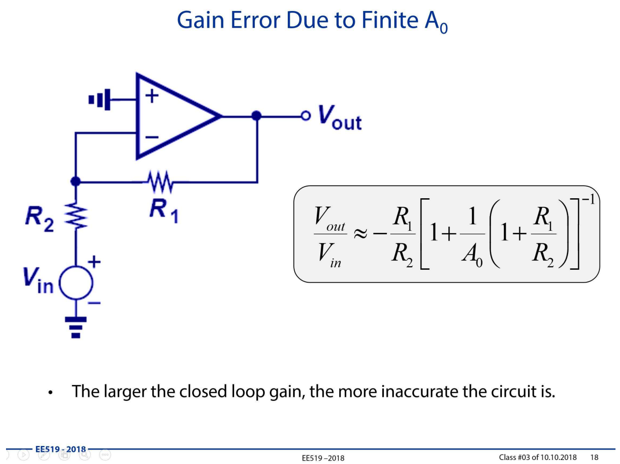


Figure 1.2 Lecture #4, slide #18.

1.3.2 Comments of Slide #34

Slide #34 among other presents C_{ox} used within equations of the MOSFET. In order to understand C_{ox}, we consider its definition which originates from the theory of semiconductor devices. A MOSFET can be modeled as a parallel-plate capacitor (MOS capacitor) in which the gate forms one plate and the substrate forms the second plate of the capacitor. The gate oxide forms the dielectric.

Consequently, the following expression applies.

Where $\varepsilon_{ox} = 3.9\ \varepsilon_0 = 3.9\ (8.85 \times 10^{-12}$ F/m); ε_0 is the relative permittivity of free space and the relative permittivity of silicon dioxide ($\varepsilon_{r(ox)}$) is equal to 3.9. A is the area under the gate $W \times L$, and t_{ox} is the gate-oxide thickness.

$$C_{gate} = \frac{\varepsilon_{ox} A}{t_{ox}} \quad \text{in} \quad \text{F} \tag{1.1}$$

Because ε_{ox}/t_{ox} are purely technological parameters over which the circuit designer cannot take influence, it is customary to simplify equations by expressing the gate oxide capacitance per unit area as follows. The latter is often shortly named “oxide capacitance,” somewhat abusively in the sense that the formulation eludes the area contribution.

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} \quad \text{in} \quad F/m^2 \quad \text{or} \quad F/\mu m^2 \tag{1.2}$$

1.3.3 Comments of Slide #60

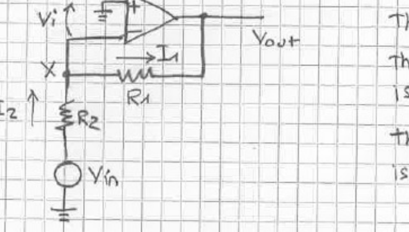
Slide 60 presents an expression of CMRR that reads as follows.

$$CMRR = \left|\frac{A_{DM}}{A_{CM-DM}}\right| \tag{1.3}$$

A_{CM-DM} is the same as A_{CM} that appears in the correction of exercise 3.1, and denotes the common-mode to differential-mode gain. Hence, CMRR is a ratio of the desired gain over the undesired gain. Expressed differently, the undesired differential output component resulting from variations of the input common mode (A_{CM-DM}) normalize the desired differential output resulting from amplification (A_{DM}).

1.3.4 EFFECT OF FINITE GAIN ON AN AMPLIFIER CONFIGURATION

Method #1 (Gray, 2001)



This is an inverting circuit configuration. The open-loop gain (of the opamp) is equal to A_0.

The closed-loop gain (of the circuit) is obtained from V_{out}/V_{in}

Applying KCL at node X yields

$$\frac{V_{in} - V_i}{R_2} + \frac{V_{out} - V_i}{R_1} = 0 \qquad (1)$$

In addition, we have

$$V_i = -\frac{V_{out}}{A_0} \qquad (2)$$

Inserting (2) into (1), and applying algebraic manipulations enable deriving an approximation as follows.

$$R_1 V_{in} - R_1 V_i + R_2 V_{out} - R_2 V_i = 0$$

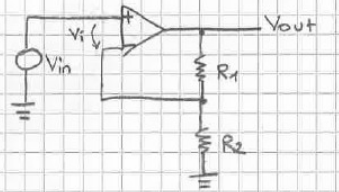
$$R_1 V_{in} + \frac{V_{out}}{A_0}(R_1 + R_2) + R_2 V_{out} = 0$$

Thus: $R_1 V_{in} = -\frac{V_{out}}{A_0}(R_1 + R_2) - R_2 V_{out} = -\frac{V_{out}}{A_0}(R_1 + R_2 + A_0 R_2)$

$$\frac{V_{out}}{V_{in}} = -\frac{A_0 R_1}{R_1 + R_2 + A_0 R_2} = -\frac{R_1}{R_2} \frac{1}{\frac{R_1}{R_2}\frac{1}{A_0} + \frac{1}{A_0} + 1}$$

$$= -\frac{R_1}{R_2} \frac{1}{1 + \frac{1}{A_0}\left(1 + \frac{R_1}{R_2}\right)}$$

Method #2 (Razavi, 2006)



This is a non-inverting amplifier configuration circuit

The closed-loop gain (of the circuit) is $\frac{V_{out}}{V_{in}} \approx 1 + \frac{R_1}{R_2}$ (3)

The open-loop gain (of the amplifier) is expressed as

$$V_{out} = A_0 V_i \quad \rightsquigarrow \quad A_0 = \frac{V_{out}}{V_i} = \frac{V_{out}}{(V_{in,+} - V_{in,-})} \tag{4}$$

$$V_{in,-} = \frac{R_2}{R_1+R_2} V_{out} \tag{5}$$

substituting (4) into (5) yields

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1+\frac{R_2}{R_1+R_2}A_0} = \left(1+\frac{R_1}{R_2}\right)\frac{1}{1+\left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0}}$$

the right-most term is obtained from dividing the upper and lower parts of the fraction by the inverse of $\left(\frac{R_2}{R_1+R_2}A_0\right)$

the approximation $(1-\varepsilon)^{-1} \approx 1-\varepsilon$ if $\varepsilon \ll 1$ is used

$$\frac{V_{out}}{V_{in}} \approx \left(1+\frac{R_1}{R_2}\right)\left(1-\left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0}\right) \tag{6}$$

Comparing (3) (no gain error) with (6) enables interpreting the term $\left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0}$ as a gain error that should be minimized

1.3.5 OUTPUT IMPEDANCE OF A CASCODE STAGE

Slides #44 is reprinted in Figure 2.3. The detail derivation of R_{out} is provided in the following.

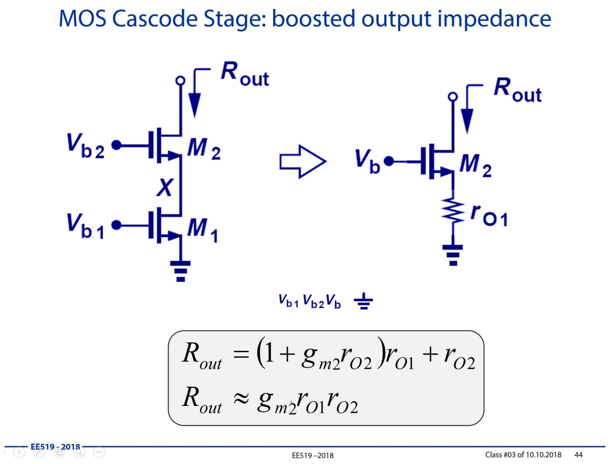
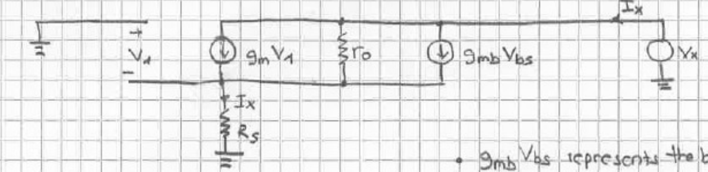


Figure 2.3 Lecture #4, slide #44.

CS stage with source degeneration (details, adapted from Razavi, 2016)

Calculation of the output resistance



- $g_{mb} V_{bs}$ represents the body effect (drain current as a function of bulk voltage)
- load resistor to V_{DD} temporarily excluded

V_{bs} can be assumed equal to V_1 if the bulk is grounded

$V_1 = -I_x R_S$ because the current flowing through R_S is I_x

the current flowing through r_o is expressed as follows

$$I_x - (g_m + g_{mb}) V_1 = I_x + (g_m + g_{mb}) I_x R_S$$

V_x consists of the sum of the voltage drops across r_o and R_S

$$V_{out} = V_x = (I_x + (g_m + g_{mb}) I_x R_S)\, r_o + I_x R_S$$

$$R'_{out} = \frac{V_x}{I_x} = (1 + (g_m + g_{mb}) R_S)\, r_o + R_S = (1 + (g_m + g_{mb}) r_o) R_S + r_o$$

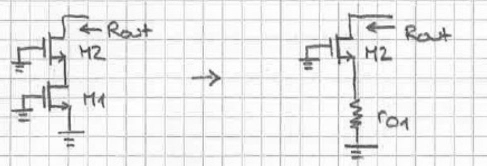
Assuming that $(g_m + g_{mb})\, r_o \gg 1$ yields

$$R'_{out} \approx (g_m + g_{mb})\, r_o R_S + r_o = (1 + (g_m + g_{mb}) R_S)\, r_o \quad (1)$$

If the system has a load R_D towards V_{DD}, then R_D must be considered in parallel to R'_{out} in order to obtain the overall circuit R_{out}

if $R'_{out} \gg R_D$ is acceptable, then $R_{out} \approx R_D$

Calculation of the output impedance of a cascode stage



(a) (b)

The cascode structure in (a) is equivalent to a cascode stage with a source degeneration resistor equal to r_{o1}.

Hence, R_{out} can be calculated using expression (1).

$$R_{out} = (1 + (g_{m2} + g_{mb2})\, r_{o2})\, r_{o1} + r_{o2}$$

Neglecting the body-bias effect yields

$$R_{out} = (1 + g_{m2} r_{o2})\, r_{o1} + r_{o2}$$

$$\approx g_{m2} r_{o2} r_{o1}$$

2. EXERCISES

2.1 SYSTEMATIC METHOD FOR DERIVING THE INPUT-REFERRED NOISE POWER DENSITY

(1) drawing the small signal equivalent circuit

(2) condition 1: The source of the circuit is a voltage source:

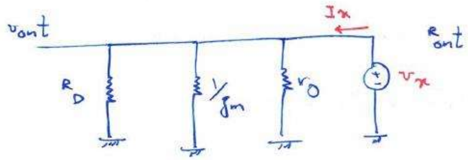
VoltageGain calculation: $A_v = \left.\frac{v_{out}}{v_{in}}\right|_{I_{out}=0}$

condition 2: The source of the circuit is a current source:

→ Gm calculation: $G_m = \left.\frac{I_{out}}{v_{in}}\right|_{v_{out}=0}$

(3) calculating R_{out}:

Ex:



$R_{out} = \left.\frac{v_x}{I_x}\right|_{v_{in}=0}$

(4) calculating the contribution of each element → transistors, → resistors

transistors: $I_n^2 = 4KT\,\frac{2}{3}\,g_m$

condition 1: The transistor plays a role in output Gain →

R_{out} is not equal to R_{out} of step 3.

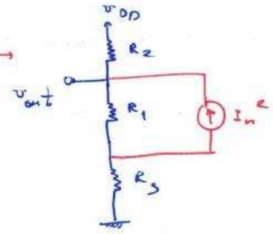
$R_{out(new)} = \frac{A_v}{G_m}$ ($G_m = g_m$)

→ considering only the contribution of that transistor

condition 2: The transistor doesn't play role in output Gain →

R_{out} is equal to the R_{out} of step 3

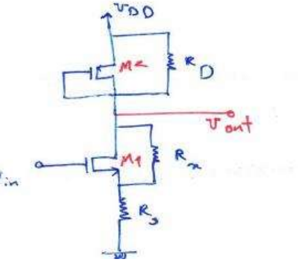
Resistors: $I_n^2 = 4KT \times \frac{1}{R}$ —Ex:→



(5) Summing up the noise contributions of all elements

$v_{n,out}^2 = v_{n,O_1}^2 + v_{n,O_2}^2 + v_{n,O_3}^2 + \dots$

(6) $v_{n,in}^2 = \frac{v_{n,out}^2}{|A_v|^2}$ Example:



2.2 EXERCISES RELATED TO NOISE ANALYSIS, QUESTIONS RELATED TO 5.1(A) ON PP. 3/11 OF THE SOLUTIONS

Preliminary remark: this page presents the calculation of the output-referred noise of the diverse component of the small-circuit noise model. Each component's contribution is considered individually.

Upon calculating the individual contribution to the output noise power of each individual component, how should the output resistance of the circuit be considered ?

R_{out} is expressed for each individual contributor to the output noise power in Exercise 5.1(a), specifically, on pp. 3/11 and the top of pp. 4/11 (solution sheet). In exercise 5.1(b) however, the output resistance is taken identical in the calculation of both contributor to the output-referred noise power.

During the calculation of the noise contribution of one element, this latter is replaced by its component's small-signal noise model (*e.g.*, a current source placed in parallel to a noiseless resistance model a noisy resistor). All other components are replaced by their noiseless model. Transistors can be replaced by their equivalent access resistance (common-source: $R_{out} = 1/g_{ds}$, common-gate: $R_{out} = 1/g_{ds} + g_m R_s/g_{ds}$, source follower: $R_{out} = 1/g_m$).

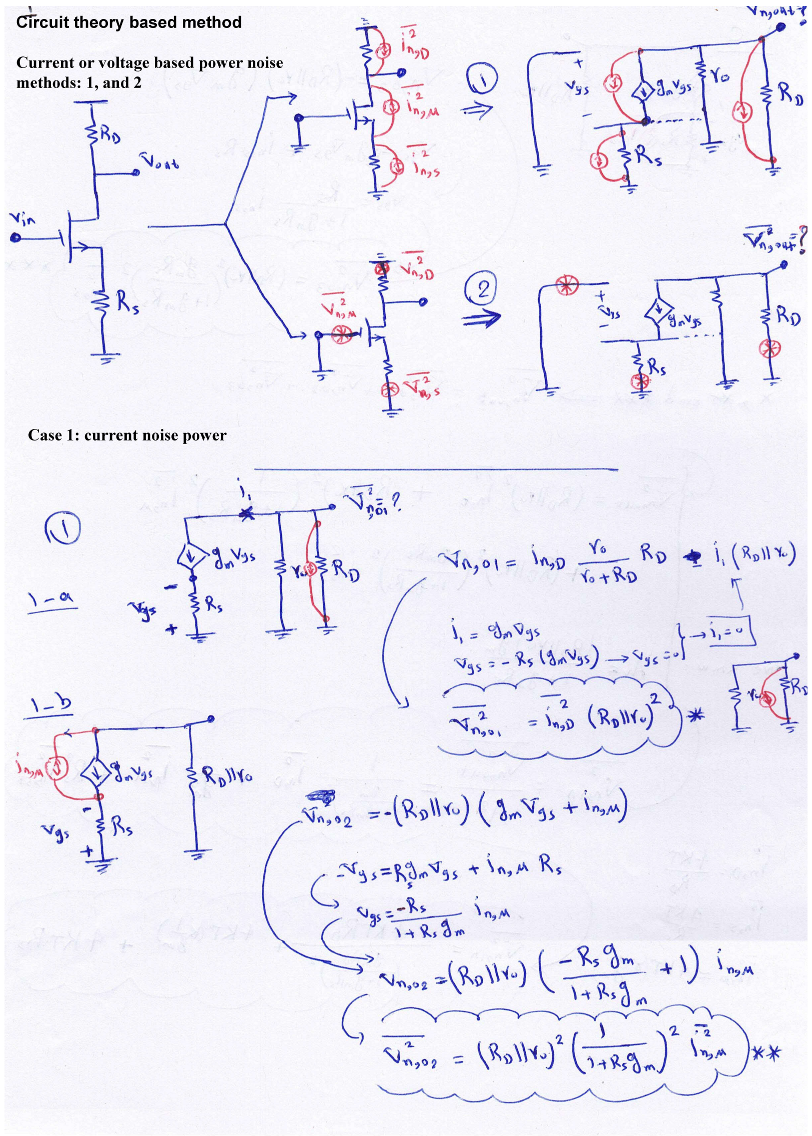
Comments regarding some simplifications that have been made in the solution:

- The modelling of the transistor as $1/g_m$ is used in Exercise 5, and could also have been applied to the calculations of Exercise 4 in order to yield a more accurate result.
- Along the same line, a model that is similar to the model used on the bottom of page 3/11 (degenerescence-source transistor noise) could also have been applied in the calculations of the two noise power models that read higher on the same page (drain resistance noise, transistor noise), yielding a more accurate result.

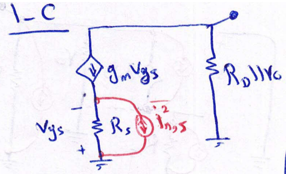
2.3 KCL-BASED METHOD FOR CIRCUITS IN EXERCISE 5 SPECIFICALLY 5(A)

The method presented in the correction sheet is based on Ohm's law, $V_{out} = R_{out} I^2_{n,out}$. A more detailed solution is proposed in the following that is based on Kirchhof's Current Laws.

As power noise sources, a current or voltage power noise model can be used, creating two cases. The derivations are presented in the following.



1-C



$v_{n,o3} = -(R_D \| r_o)(g_m v_{gs})$

$-v_{gs} = R_s g_m v_{gs} + i_{n,s} R_s$

$v_{gs} = \frac{R_s}{1+g_m R_s} i_{n,s}$

$$\overline{v^2_{n,o3}} = (R_D \| r_o)^2 \left(\frac{g_m R_s}{1+g_m R_s}\right)^2 \overline{i^2_{n,s}} \quad ***$$

*, ** and *** $\Longrightarrow$ $\overline{v^2_{n,out}} = \overline{v^2_{n,o1}} + \overline{v^2_{n,o2}} + \overline{v^2_{n,o3}}$

$$\overline{v^2_{n,out}} = (R_D \| r_o)^2 \overline{i^2_{n,D}} + (R_D \| r_o)^2 \left(\frac{1}{1+g_m R_s}\right)^2 \overline{i^2_{n,M}} + (R_D \| r_o)^2 \left(\frac{g_m R_s}{1+g_m R_s}\right)^2 \overline{i^2_{n,s}}$$

we know: $A_v = \frac{(R_D \| r_o) g_m}{1+g_m R_s}$

$$\overline{v^2_{n,in}} = \frac{\overline{v^2_{n,out}}}{A^2} = \frac{1}{\left(\frac{g_m}{1+g_m R_s}\right)^2} \overline{i^2_{n,D}} + \frac{1}{g_m^2} \overline{i^2_{n,M}} + R_s^2 \overline{i^2_{n,s}}$$

$\overline{i^2_{n,D}} = \frac{4KT}{R_D}$

$\overline{i^2_{n,s}} = \frac{4KT}{R_s}$

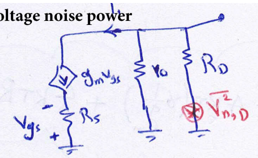
$\overline{i^2_{n,M}} \simeq 4KT(\gamma g_m)$

$$\overline{v^2_{n,in}} = \frac{4KT R_D}{\left(\frac{g_m R_D}{1+g_m R_s}\right)^2} + 4KT\left(\frac{\gamma}{g_m}\right) + 4KT R_s$$

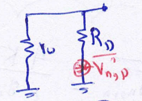
Case 2: voltage noise power

(2)

2-a

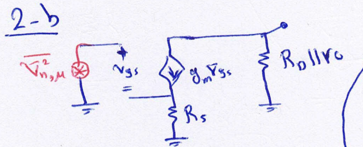


based on 1-a → $i_1 = 0$



$$\overline{v^2_{n,o1}} = \overline{v^2_{n,D}} \left(\frac{r_o}{r_o+R_D}\right)^2 \quad *$$

2-b



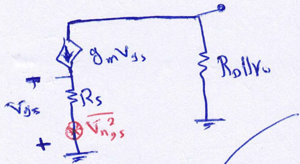
$v_{n,o2} = -(R_D \| r_o)(g_m v_{gs})$

KVL: $-v_{n,M} + v_{gs} + R_s(g_m v_{gs}) = 0$

$v_{gs} = \frac{v_{n,M}}{1+R_s g_m}$

$$\overline{v^2_{n,o2}} = (R_D \| r_o)^2 \left(\frac{g_m}{1+R_s g_m}\right)^2 \overline{v^2_{n,M}} \quad **$$

2-C



$v_{n,o3} = -(R_D \| r_o)(g_m v_{gs})$

$-v_{gs} = R_s(g_m v_{gs}) + v_{n,s}$

$v_{gs} = \frac{v_{n,s}}{1+R_s g_m}$

$$\overline{v^2_{n,o3}} = (R_D \| r_o)^2 \left(\frac{g_m}{1+R_s g_m}\right)^2 \overline{v^2_{n,s}} \quad ***$$

*, ** and *** $\Longrightarrow$ $\overline{v^2_{n,in}} = \frac{\overline{v^2_{n,o1}} + \overline{v^2_{n,o2}} + \overline{v^2_{n,o3}}}{A_v^2}$

and $A_v = \frac{(R_D \| r_o) g_m}{1+g_m R_s}$

$$\overline{v^2_{n,in}} = \frac{\overline{v^2_{n,D}}}{\left(\frac{R_D g_m}{1+g_m R_s}\right)^2} + \overline{v^2_{n,M}} + \overline{v^2_{n,s}}$$

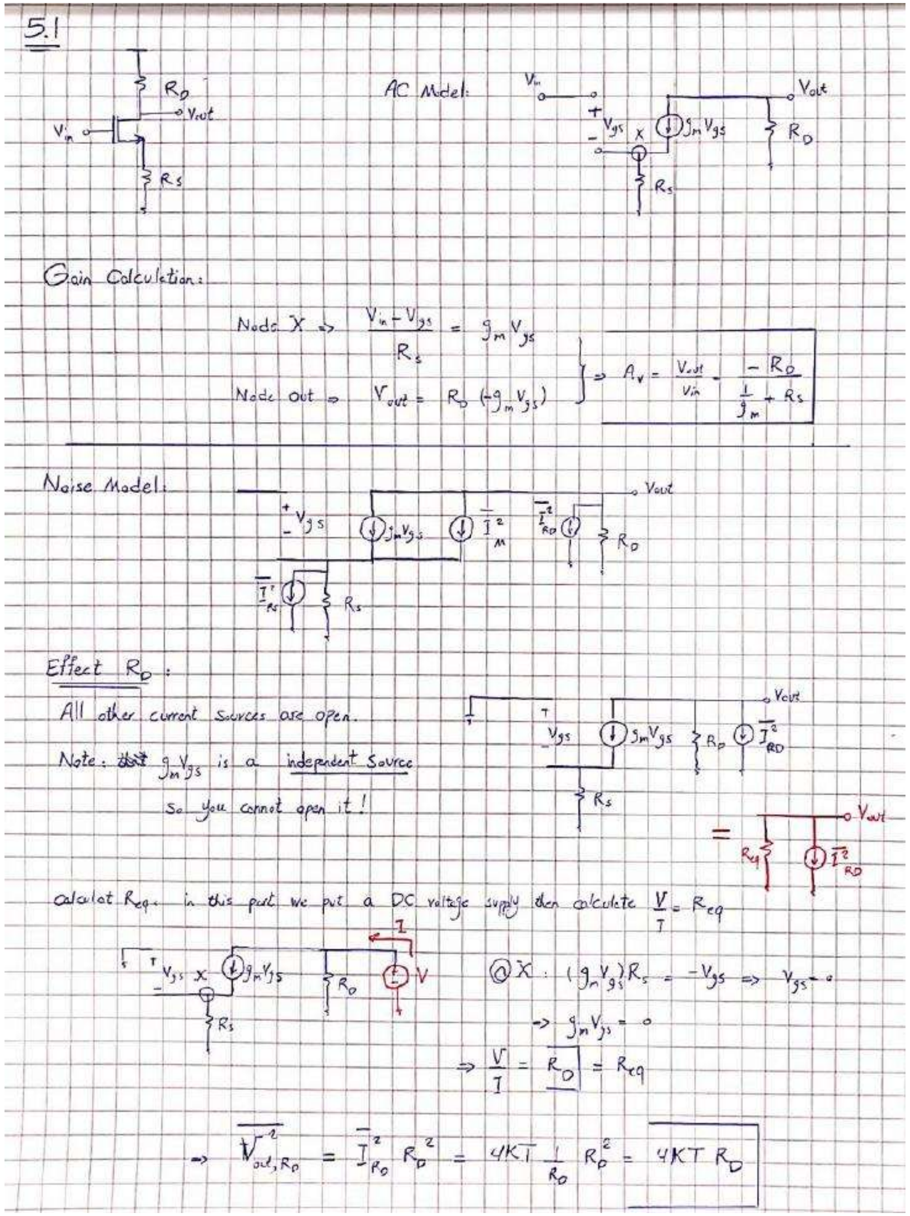
$\overline{V^2_{n,D}} = 4KTR_D$

$\overline{V^2_{n,s}} = 4KTR_s$

$\overline{V^2_{n,M}} = 4KT(\gamma \frac{1}{g_m})$

$$\overline{V^2_{n,in}} = \frac{4KTR_D}{\left(\frac{g_m R_D}{1+g_m R_s}\right)^2} + 4KT\left(\gamma\frac{1}{g_m}\right) + 4KTR_s$$

An alternate solution derivation is presented in the following based on the equivalent current power noise sources. The small-circuit noise models for all noise contributors individually to derive their analytical contribution to the output node. This method can always be applied and may also enable a better understanding.



5.1

AC Model:

Gain Calculation:

Node X => $\frac{V_{in} - V_{gs}}{R_s} = g_m V_{gs}$

Node out => $V_{out} = R_D(-g_m V_{gs})$

$\Rightarrow A_v = \frac{V_{out}}{V_{in}} = \frac{-R_D}{\frac{1}{g_m} + R_s}$

Noise Model:

Effect R_D:

All other current sources are open.

Note: $g_m V_{gs}$ is a independent source so you cannot open it!

calculat R_{eq}. in this part we put a DC voltage supply then calculate $\frac{V}{I} = R_{eq}$

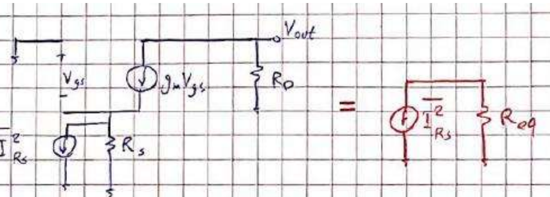
@X: $(g_m V_{gs}) R_s = -V_{gs} \Rightarrow V_{gs} = 0$

$\Rightarrow g_m V_{gs} = 0$

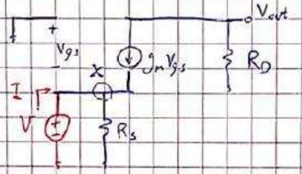
$\Rightarrow \frac{V}{I} = R_D = R_{eq}$

$\Rightarrow \overline{V^2_{out,R_D}} = \overline{I^2_{R_D}} R_D^2 = 4KT\frac{1}{R_D} R_D^2 = 4KTR_D$

Effect R_s:



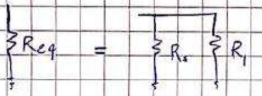
Calculate R_{eq}:



$$\frac{V}{I} = R_{eq}$$

@X: $V = -V_{gs}$

$$I + g_m V_{gs} = \frac{V}{R_s} \Rightarrow I - g_m V = \frac{V}{R_s} \Rightarrow \frac{V}{I} = R_{eq} = \frac{R_s}{1+g_m R_s}$$



$$R_{eq} = R_1 \| R_s, \quad \frac{R_s}{1+g_m R_s} = \frac{R_1 R_s}{R_1 + R_s} \Rightarrow R_1 = \frac{1}{g_m}$$

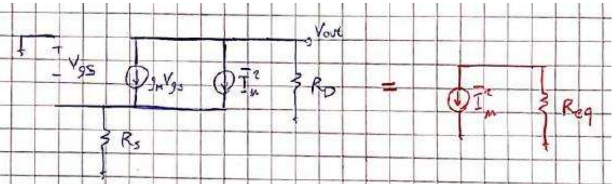
$$V_{out} = V_x \frac{R_D}{R_1} = V_x \frac{R_D}{\left(\frac{1}{g_m}\right)} = V_x (g_m R_D)$$

$$\Rightarrow \overline{V^2_{n,out}} = \overline{V^2_{n,X}} (g_m R_D)^2$$

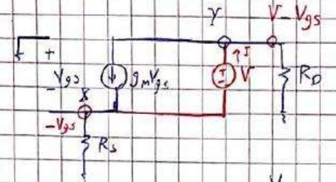
$$\overline{V^2_{n,X}} = \overline{I^2_{R_s}} (R_{eq})^2 = 4KT \frac{1}{R_s} \left(\frac{R_s}{1+g_m R_s}\right)^2$$

$$\Rightarrow \overline{V^2_{n,out}} = 4KT \frac{1}{R_s} \left(\frac{R_s}{1+g_m R_s}\right)^2 (g_m R_D)^2$$

Effect M_1:



Calculate R_{eq}:

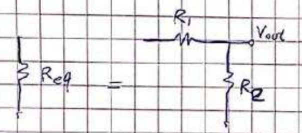


$$\frac{V_{gs}}{I} = \frac{R_s}{1+g_m R_s}$$

@Y: $I - g_m V_{gs} = \frac{V - V_{gs}}{R_D}$

@X: $g_m V_{gs} - I = \frac{-V_{gs}}{R_s}$

$$\Rightarrow \frac{V}{I} = \frac{R_s + R_D}{1+g_m R_s} = R_{eq}$$



$$R_1 = \frac{R_s}{1+g_m R_s}, \quad R_2 = \frac{R_D}{1+g_m R_s}$$

$$\overline{V^2_{out,n}} = \overline{I^2_{M_1}} R_2^2 = \left[4KT \frac{2}{3} g_m\right] \left(\frac{R_D}{1+g_m R_s}\right)^2$$

All Noises

$$\overline{V^2_{n,out}} = \overline{V^2_{n,out,R_D}} + \overline{V^2_{n,out,R_s}} + \overline{V^2_{n,out,M_1}}$$

$$\overline{V^2_{n,in}} = \frac{\overline{V^2_{n,out}}}{|A_v|^2}$$

2.4 EXERCISE 5

The $\gamma = 0$ that appears in Exercises 5.1 and 5.2 relates to the body effect coefficient. It should not be confused with the 2/3 coefficient that appears in the noise spectral power of transistors.

3. REFERENCES

[1] Atkins, P. W., Physical Chemistry, Oxford University Press, 1998

[2] Plonsey, R. and Barr, R.C., Bioelectricity A quantitative approach. 3rd Edition, Springer, New York, 2007