MICHAELIS - MENTEN

$$\boxed{E + S \underset{K_d}{\overset{K_a}{\rightleftarrows}} ES \xrightarrow{K_p} E + P}$$

↳ Rate of Product formation: $\frac{d([P])}{dt} = K_p[ES]$

↳ Rate of Complex formation: $\frac{d([ES])}{dt} = K_a[E][S]$

↳ Rate of Complex breakdown (complex separation): $\frac{d([E][S])}{dt} + \frac{d[P]}{dt} = (K_d + K_p)[ES$

⟹ At steady state the complex breakdown ≡ enzymatic reaction

⇓

$$\underbrace{\boxed{\frac{d([E][S])}{dt} + \frac{d[P]}{dt}} }\ \boxed{=\ \underbrace{\frac{d[ES]}{dt}}}$$

$$(K_d + K_p)[ES] = K_a[E][S]$$

$$\rightarrow \quad [ES] = \frac{K_a}{K_d + K_p} \cdot [E][S]$$

[ES]: concentration of the complex

$$\frac{K_a}{K_d + K_p} \rightarrow \boxed{K_M} = \text{Michaelis - Menten constant}$$

$$\boxed{K_M = \left(\frac{K_a}{(K_d + K_p)}\right)^{-1}}$$

→ it shows how efficient is the enzyme.

→ Let's now consider $[E_0]$ as initial concentration of total enzyme

↳ enzyme "still" $\underbrace{\text{avaible}}_{[E]}$ will be: $([E_0] - [ES]) = [E]$

↳ We can re-write the concentration of the complex:

before → $[ES] = \frac{[E][S]}{K_M}$

now → $[ES] = \frac{([E_0]-[ES])[S]}{K_M}$ → $[ES] = [E_0]\cdot\frac{[S]}{[S]+K_M}$

we can rewrite as ↗

How ??

Demostration:

$$[ES] = \frac{([E_0]-[ES])[S]}{K_M} = \frac{[E_0][S]}{K_M} - \frac{[ES][S]}{K_M}$$

$$\rightarrow \underbrace{[ES] + \frac{[ES][S]}{K_M}} = \frac{[E_0][S]}{K_M}$$

$$[ES]\left(1+\frac{[S]}{K_M}\right) = \frac{[E_0][S]}{K_M} \rightarrow [ES]\left(\frac{K_M+[S]}{\cancel{K_M}}\right) = \frac{[E_0][S]}{\cancel{K_M}}$$

$$\rightarrow [ES](K_M+[S]) = [E_0][S] \Rightarrow \boxed{[ES] = [E_0]\frac{[S]}{[S]+K_M}}$$ ✓

- Now, let's think in terms of VELOCITY V

→ Velocity of [P] production → $V = \frac{d[P]}{dt}$ → $V_{max} = K_p[E_0]$

Recall: $\frac{d[P]}{dt} = K_p[ES]$ and $[ES] = [E_0]\cdot\frac{[S]}{[S]+K_M}$

$$\hookrightarrow \underbrace{\frac{d[P]}{dt}}_{V} = \underbrace{K_p[E_0]}_{V_{max}}\frac{[S]}{[S]+K_M} \Longrightarrow \boxed{V = V_{max}\frac{[S]}{[S]+K_M}}$$

Michaelis-Menten Equation