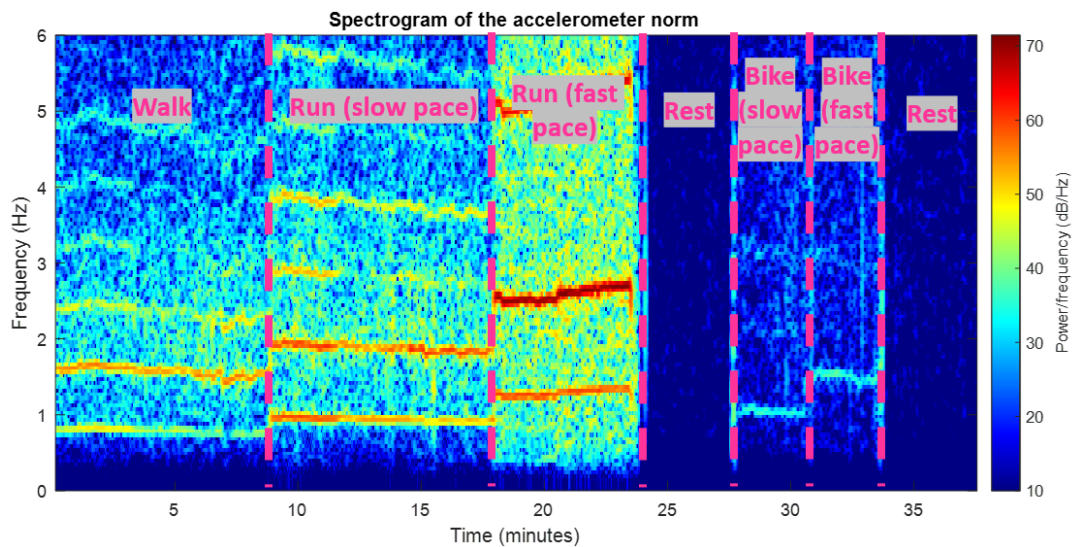


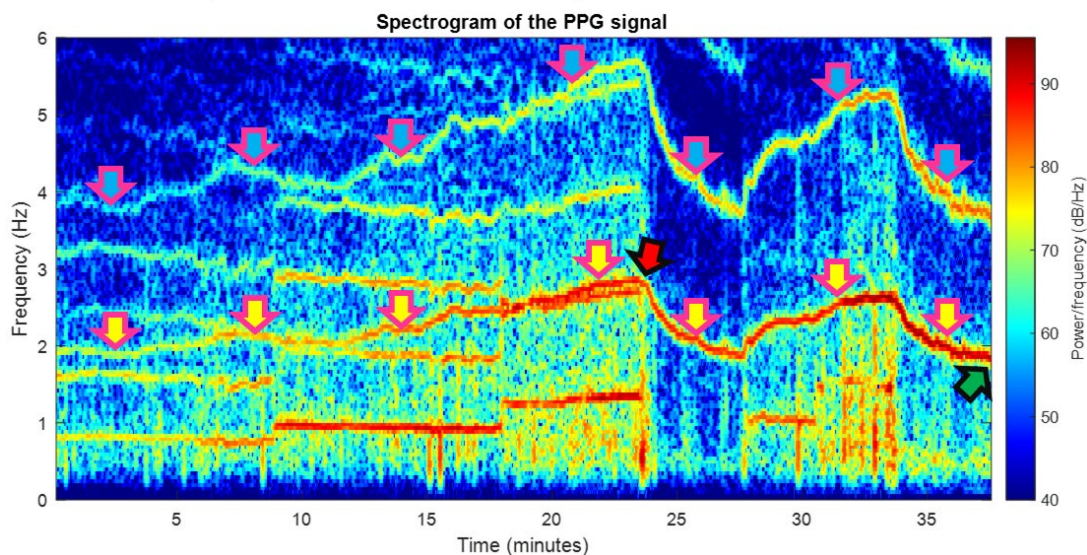
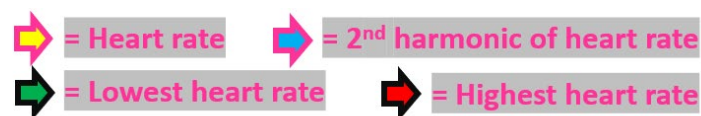
Practical session – Time & Frequency

Experiment 1: Heart rate estimation from a smartwatch-derived PPG signal during exercise

ANSWER TO QUESTION #1: The 7 phases of the protocol are highlighted in the spectrogram of the accelerometer norm signal below. The rest phases are the most obvious to identify first: there is little to no motion in those two phases, and therefore no power (blue color in the spectrogram). The transitions between the remaining phases can then be easily identified where there are clear and rapid changes in the frequency content (corresponding for instance to changes between walking, slow running and fast running paces).



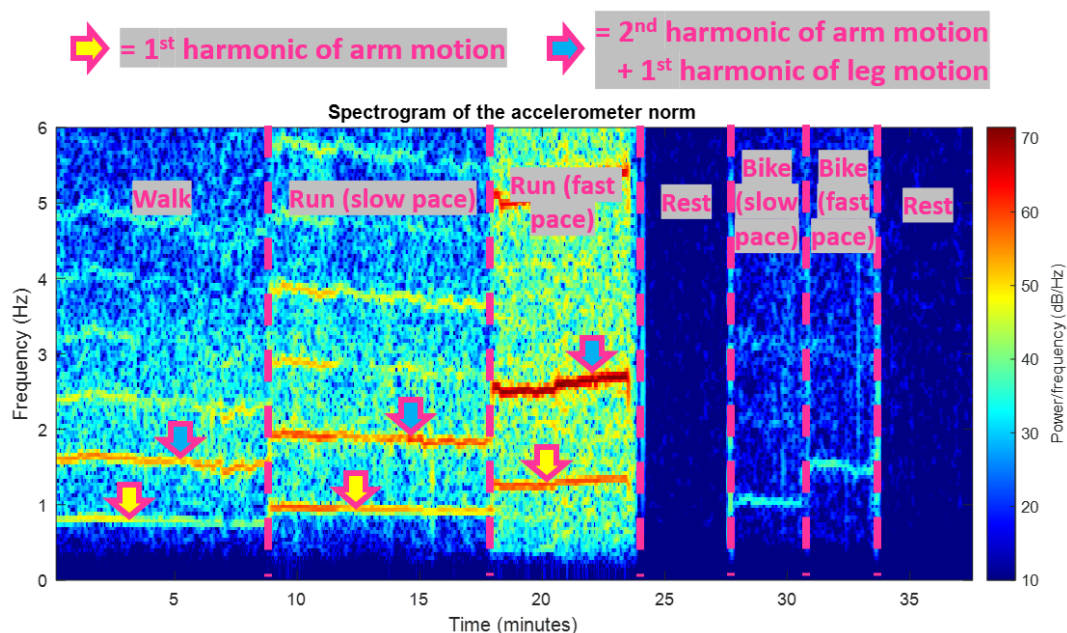
ANSWER TO QUESTION #2: Comparing the spectrogram of the PPG signal with that of the accelerometer norm and knowing that the motion frequencies also appear in the PPG signal, the heart rate frequency (and its second harmonic) can be identified, as shown below. The heart rate goes down to approximately 113 bpm (1.88 Hz) at the very end of the recording and goes up to approximately 173 bpm (2.88 Hz) at the end of the running (fast pace) session.



ANSWER TO QUESTION #3: During biking, the hands generally remain on the handlebar. The amount of motion is therefore much more limited than when the subject is running, where the hands generally move much more in a swinging motion. The acceleration created at the wrist by pedaling is also much weaker than that created by the impact of the feet on the ground when walking/running, where the whole body (and therefore the wrist as well) undergoes a downwards acceleration. From the accelerometer norm spectrogram, it appears clearly that the power (i.e. color 'redness') at the motion frequencies is much lower during biking.

ANSWER TO QUESTION #4: At the beginning of the fast running session, it appears clearly in the spectrogram of the PPG signal that the heart rate is more or less at the same frequency as the second harmonic of arm motion (and dominant frequency of leg motion). By filtering out the motion frequencies and their harmonics, one would also filter out the heart rate-related signal from the PPG signal, leaving us with just noise.

ANSWER TO QUESTION #5: As explained in the comments of the Matlab script, when walking/running, the arms swing at half the walking pace (try it if you are not convinced). Therefore, the lowest motion frequency visible in the accelerometer norm spectrogram at the beginning of the recording (~0.8 Hz) is the arm motion frequency. The walking frequency is therefore twice that value, i.e. ~1.6 Hz, or 96 steps/minute. This is also why the 2nd motion frequency component in the walking/running phases has more power (is 'more red') than the 1st (i.e., lowest) frequency component: it contains both the dominant leg motion frequency (the pace) and the second harmonic of swinging arm motion. Moreover, the impact of the feet hitting the ground generally causes a higher acceleration than the swinging motion of the arm, particularly when running.

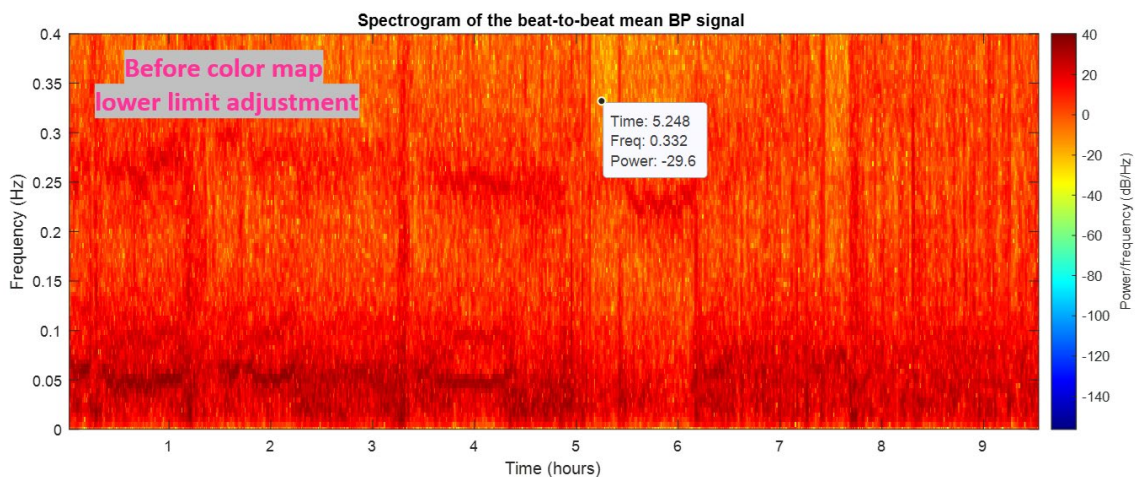


Experiment 2: Sympathovagal balance estimation from blood pressure variability during sleep

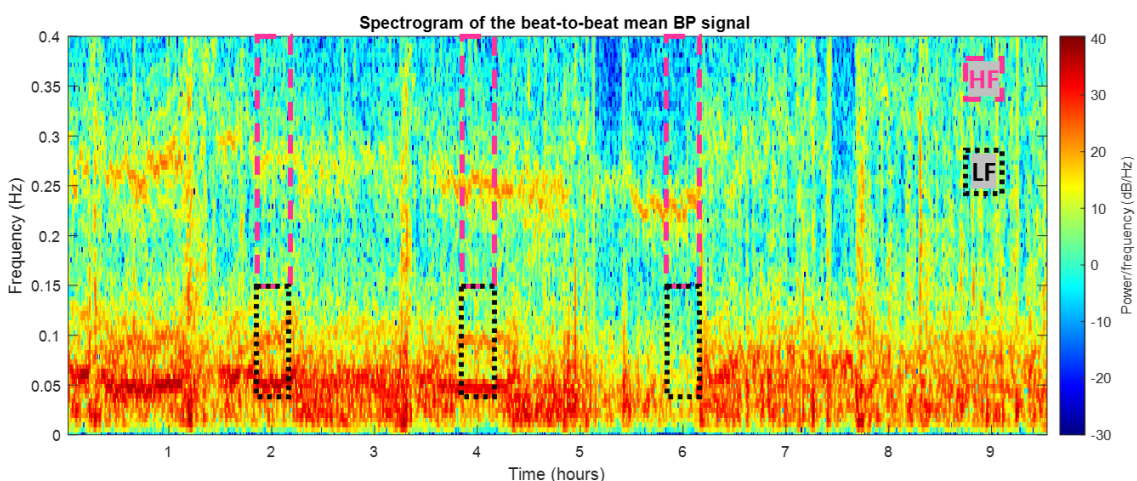
ANSWER TO QUESTION #1: We want to be able to estimate frequencies that can be as low as 0.04 Hz (lower bound of the LF frequency band) with a reasonable frequency resolution. A period of a sinusoid at 0.04 Hz has a duration of $1/0.04 = 25$ s. In order to encompass a few cycles of such a sinusoid without going to the other extreme of using a too long window where the non-stationarity assumption of the STFT is no longer valid most of the time, a possible choice can

be 150s (6 cycles at 0.04 Hz). We can safely assume that during sleep, most periods of 150s (2.5 minutes) are stationary. Many other choices – correctly justified – are possible, as long as they are short enough to comply with the assumption of stationarity, and long enough to measure the slowest frequency components.

ANSWER TO QUESTION #2: In the [0.04, 0.4] Hz band, the lowest power values seem to fall more or less in the -30 dB/Hz color tone. While one could try to distinguish the frequencies of interest with the lowest power to set our threshold, the poor readability of the spectrogram is precisely what makes this task complicated. A simple choice is therefore to set the threshold at the aforementioned level of background noise (-30 dB/Hz), as we would otherwise risk cropping low-amplitude frequency components of interest out of the color range. (Other values correctly justified are of course possible.)

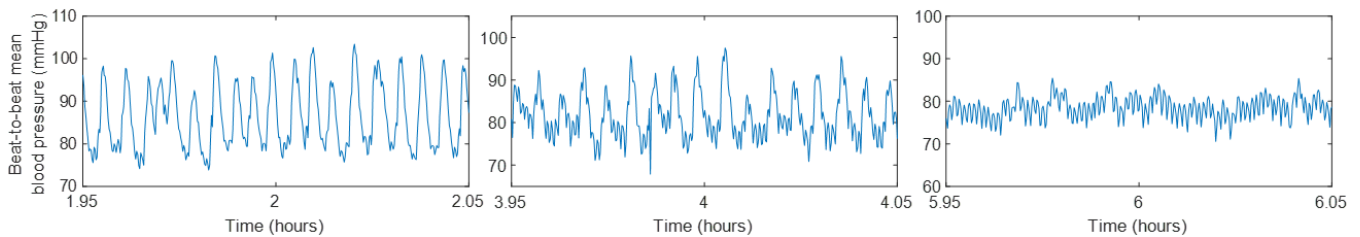


ANSWER TO QUESTION #3: As can be seen in the spectrogram, a strong LF component at 0.05 Hz (and its second harmonic at 0.1 Hz) is present around $t=2h$ and $t=4h$, indicative of a sympathetic activity. On the other hand, a distinct HF component at the frequency of respiration (~ 0.23 Hz) can be seen around $t=6h$ and slightly higher (~ 0.25 Hz) around $t=4h$, indicative of vagal activity. It is also partly present and visible around $t=2h$ at ~ 0.27 Hz. The LF-to-HF ratio (sympathovagal balance) is therefore higher around $t=2h$, then around $t=4h$, and it is lower around $t=6h$.



ANSWER TO QUESTION #4: The mean blood pressure signal around the three time points (± 3 minutes) is shown below. All plots have a vertical span of 40 mmHg for a meaningful comparison of the amplitudes. We clearly see a large and slow fluctuation in the plot on the left, also present in the middle plot, but not really in the rightmost plot. A faster fluctuation, of smaller amplitude, is clearly visible in the plot on the right, and also in the middle plot, but to a much lesser extent

on the left. Counting the number of fluctuations occurring in these 6-minute plots, we can roughly estimate their frequencies. For instance, we count ~18 oscillations in the leftmost plot, suggesting a dominant frequency at $18/(6 \cdot 60) = 0.05$ Hz, which falls in the LF band. In the rightmost plot, we count ~81 oscillations, suggesting a dominant frequency at $81/(6 \cdot 60) = 0.225$ Hz, which falls in the HF band. In this specific case (it is obviously not always the case with signals with a more complex frequency content), we could therefore indeed have reached, from those three time plots, the same conclusions we reached from the spectrogram on the sympathovagal balance. Using the spectrogram is simply much more practical and does not require manual and visual inspection of the time signal, as was done here.



Exercise: Respiration signal of a patient suffering from sleep apnea

ANSWER TO QUESTION #1: The signal consists of (stationary) segments of apneas, which can be as short as 20 seconds, and longer segments of normal respiratory activity, which are considered stationary.

Window length must be long enough to include at least one period of the lowest frequency of interest. The minimal respiratory frequency is 8 cycles/min = 0.133 Hz, corresponding to a period of respiration of $1/0.133 = 7.5$ s. Therefore, the window cannot be shorter than $L_{\min} = 7.5$ s.

On the other hand, window length must be short enough for the stationarity assumption to always be true. Therefore, the critical factor for the stationarity assumption is the duration of the shortest apneas, and therefore the window cannot be longer than $L_{\max} = 20$ s. With longer windows, apneas of 20 seconds will always include a portion of respiration.

