# EE512 – Applied Biomedical Signal Processing

## Introduction to neural networks

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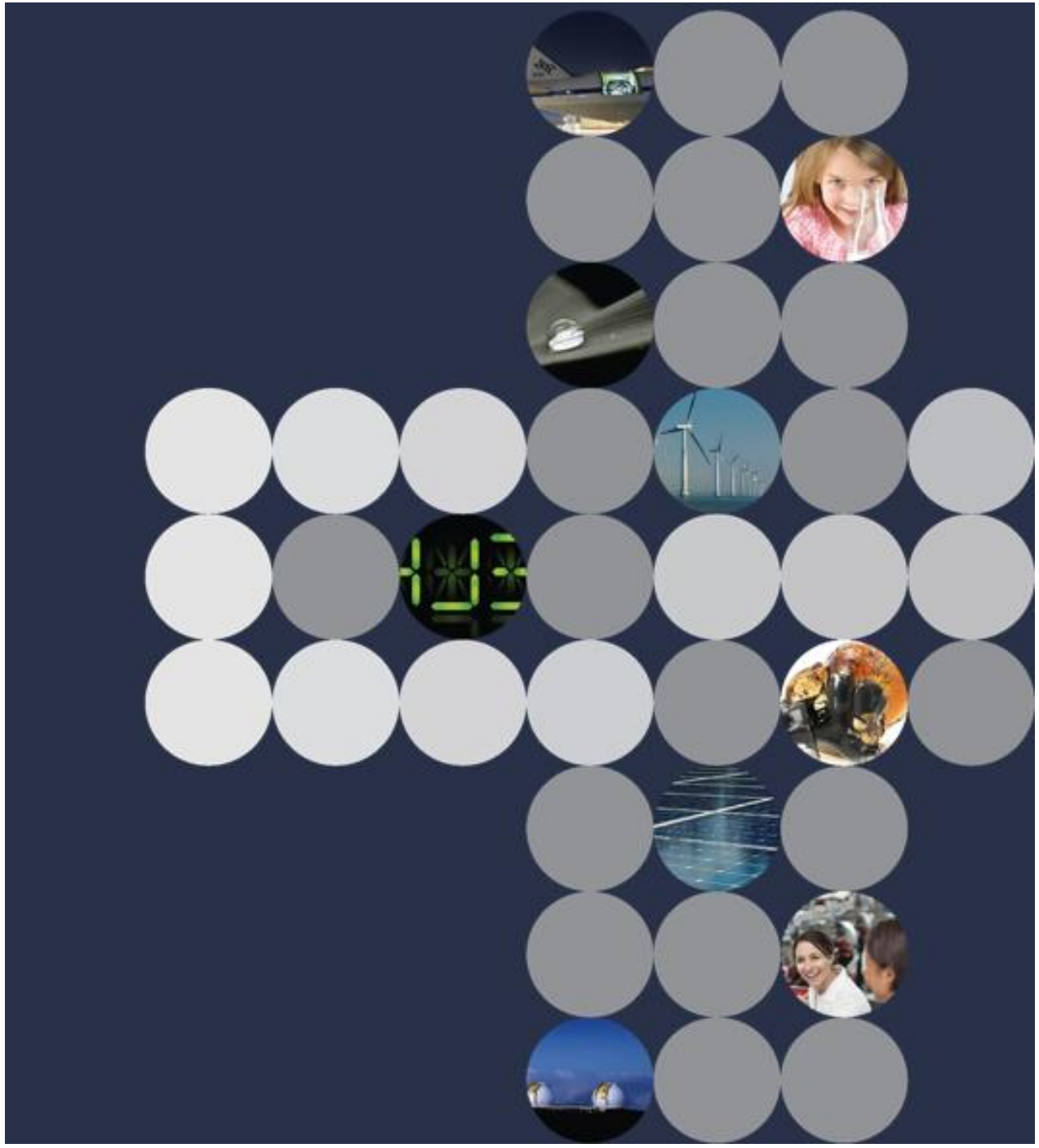
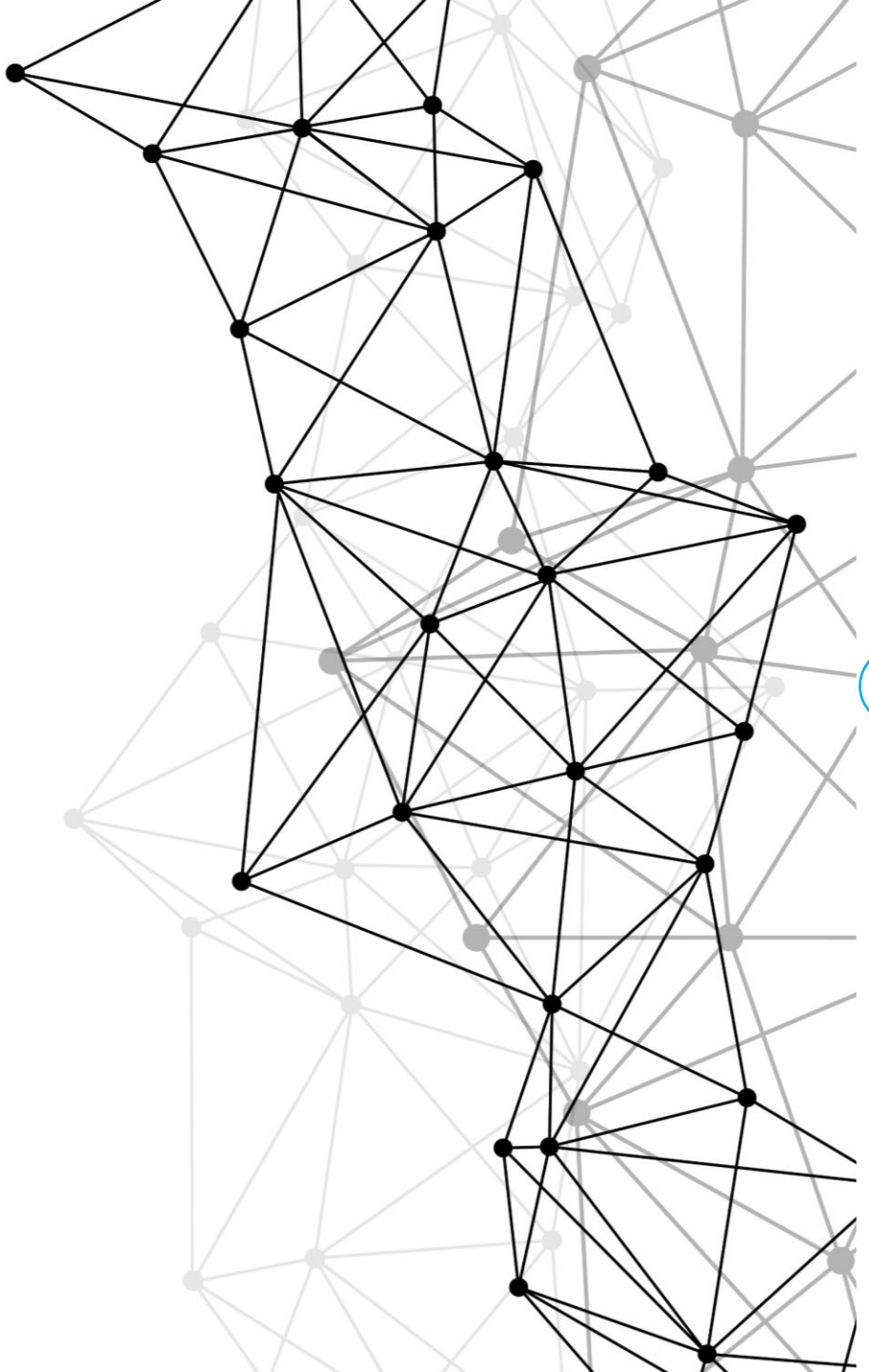
CSEM Signal Processing & AI Group



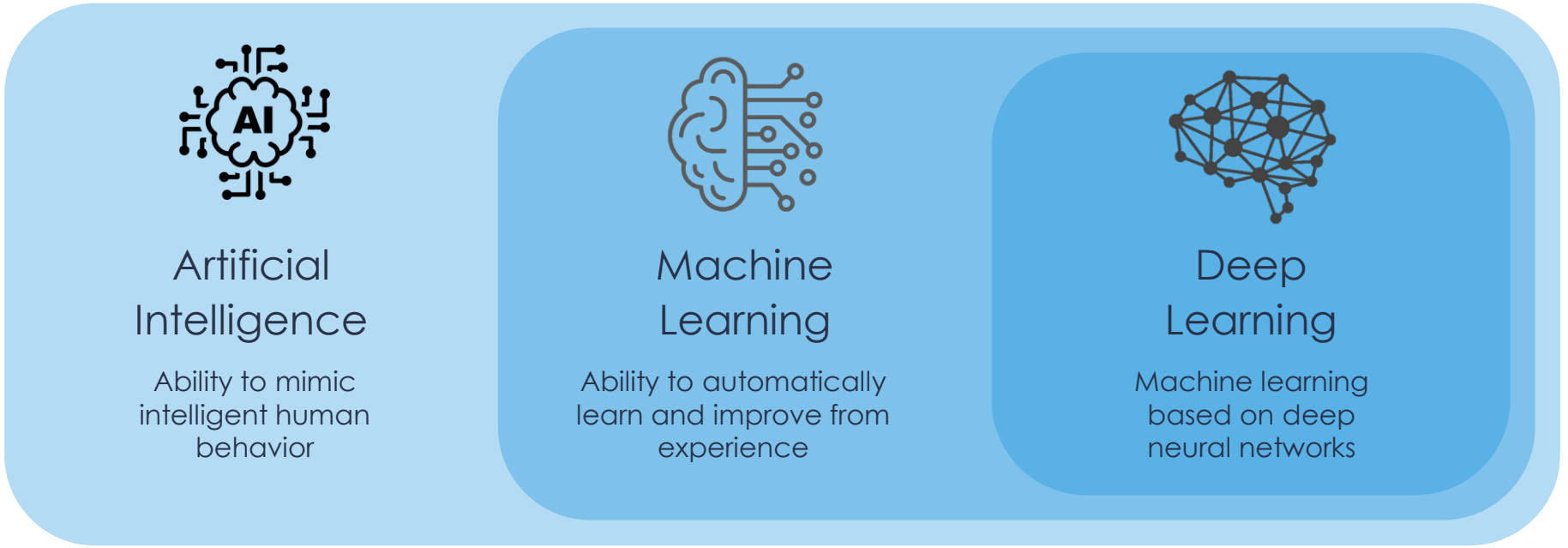
EPFL :: csem

# Content

- Introduction
- Perceptron
- Multilayer perceptron (MLP)
- Loss function
- Gradient descent
- Backpropagation
- Convolutional neural network
- Recurrent neural network
- Bias-variance trade-off
- Data



# Introduction – What is deep learning?




## Examples – Non-exhaustive list

Rule-based systems
Depth-first search algorithm
Breadth-first search algorithm
Propositional calculus
Predicate calculus logic

Linear regression
Logistic regression
Support vector machine
Decision trees
Gradient boosting
Principal component analysis
K-means clustering

Recurrent neural networks
Convolutional neural networks
Deep reinforcement learning
Generative adversarial networks

# Introduction – Brief history

**McCulloch and Pitts**
First mathematical model of artificial neuron

1943

**Widrow and Hoff**
ADALINE
Single layer artificial NN

1960

**Rumelhart, Hinton, Williams**
Multilayer perceptron (MLP)
Backpropagation

1986

**Krizhevsky, Sutskever, Hinton**
AlexNet

2012

**He et al.**
ResNet
Skip connection

2015

1958

**Rosenblatt**
Perceptron

1969

**Minsky and Papert**
XOR problem

1989

**LeCun et al.**
Convolutional NN

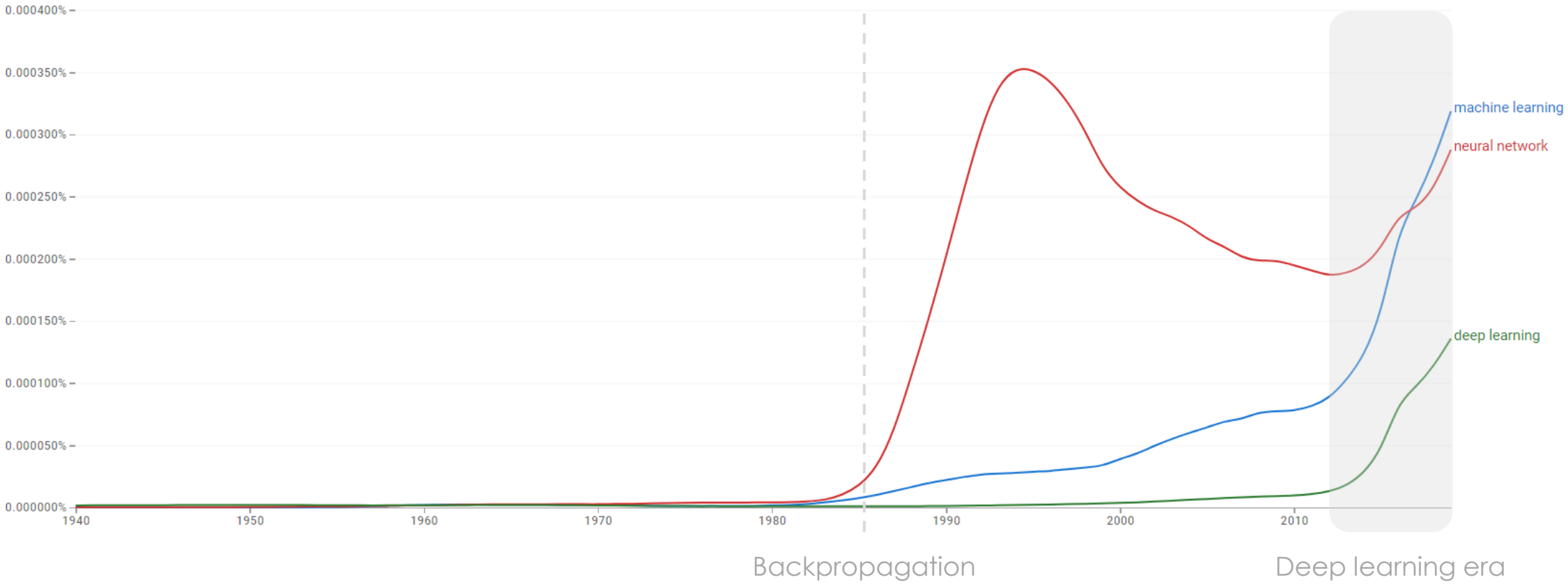
2014

**Goodfellow**
Generative adversarial networks

Deep learning era

# Introduction – Interest

## NGRAM – Frequency of words found in printed sources



# Introduction – Why Now?

Neural networks date back decades. What has changed?

## Big Data

- Larger datasets
- Easier collection and storage

## Hardware

- Graphics processing units (GPUs)
- Parallelization


## Software

- Improved techniques
- New models
- Toolboxes

PYTORCH
TensorFlow
Caffe
MODELS

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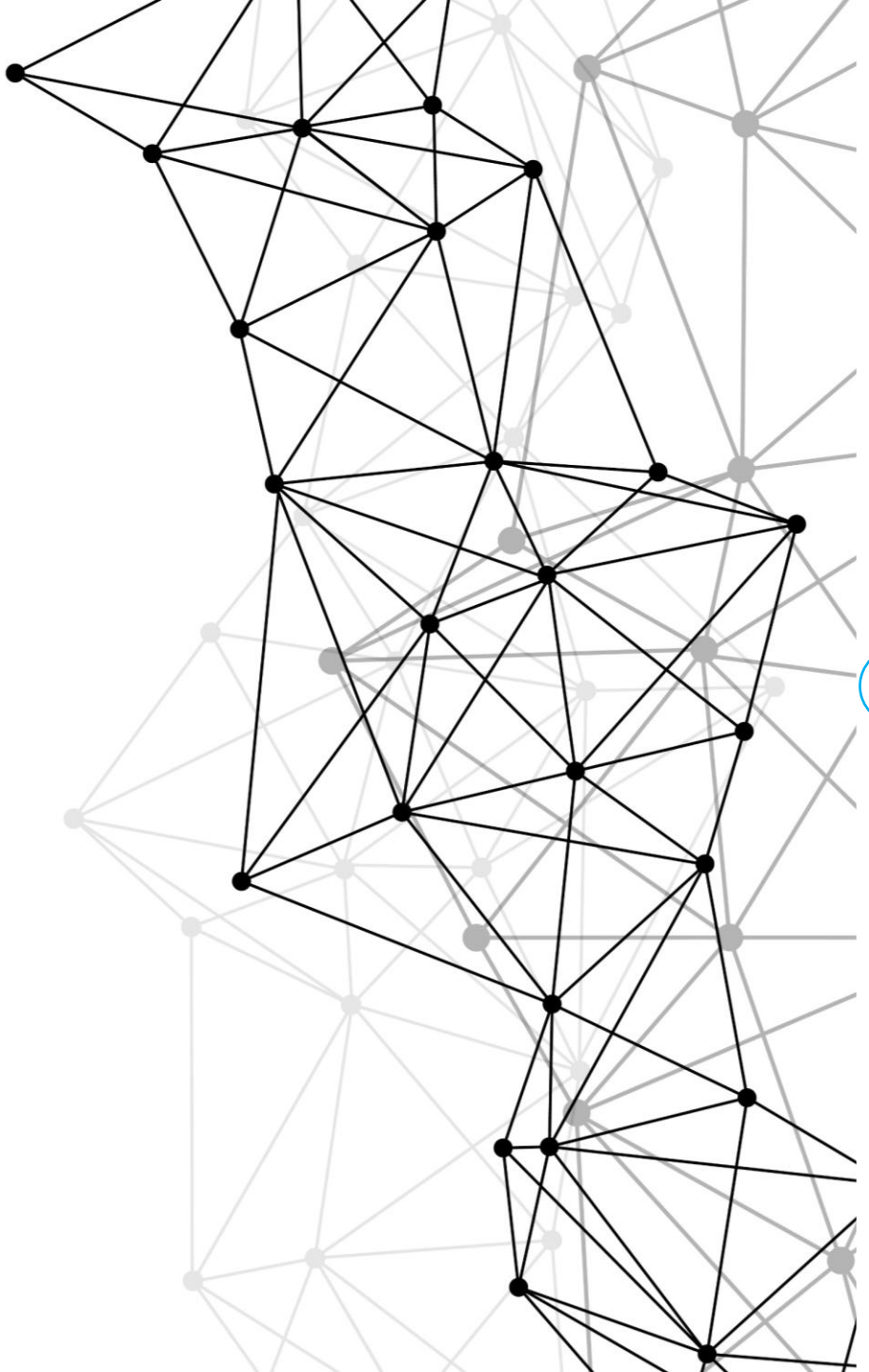
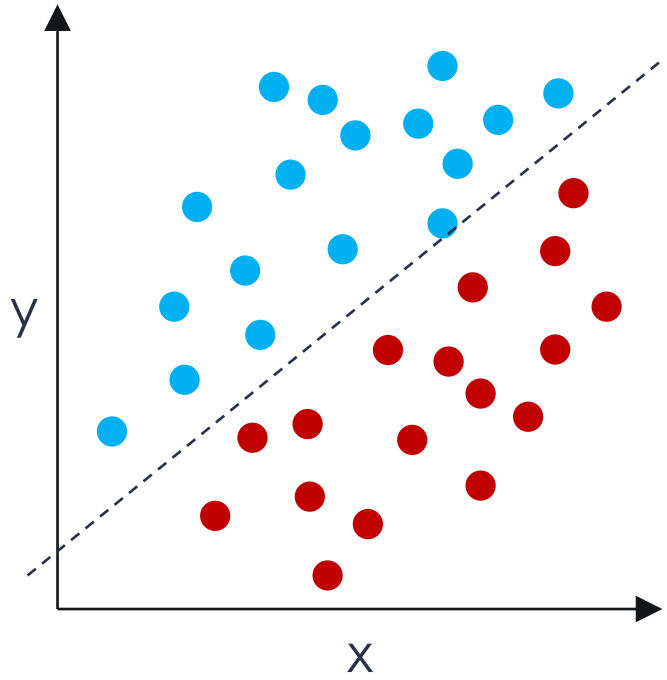


# Perceptron

- Developed in 1958 by Rosenblatt
- Built based on McCulloch-Pitts model of a neuron
- Simplest form of a neural network
- Inspired by brain structure and function
- Structural building block of deep learning
- Supervised learning
- Linear binary classifier



# Perceptron

1. Inputs: $\boldsymbol{x} \in \mathbb{R}^m$ with $m$ features
2. Weights and bias

   Strength of connection between units
3. Weighted sum

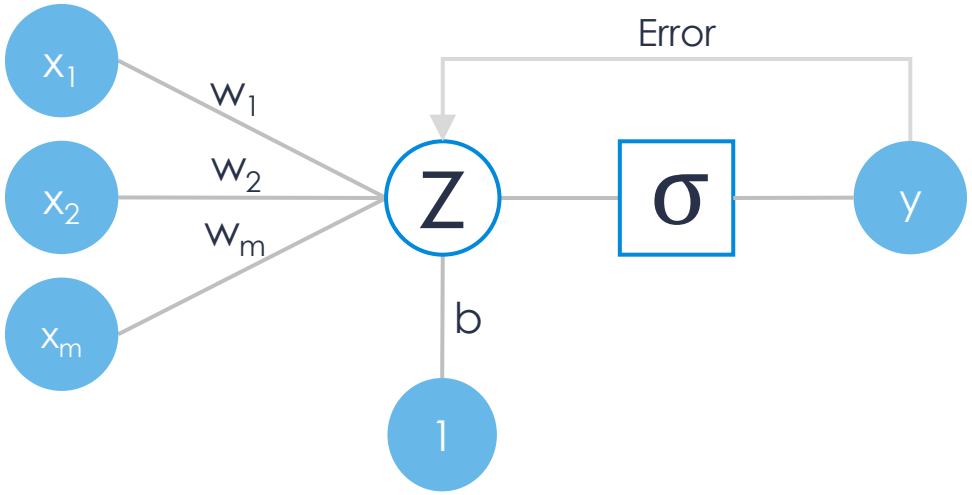
   $z = w_1 \cdot x_1 + \ ... + w_m \cdot x_m + b$

   $z = \sum_1^m w_i \cdot x_i + b$

   $z = \boldsymbol{w}^T \boldsymbol{x} + b$

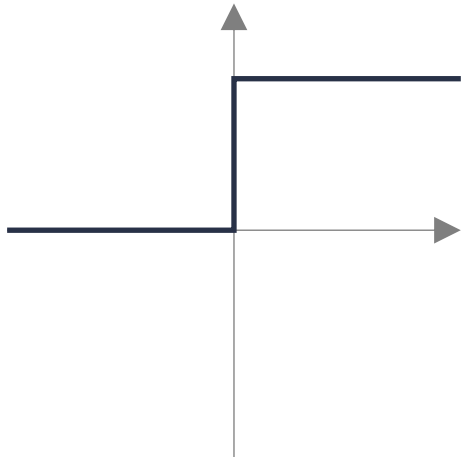
   $\boldsymbol{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_m \end{bmatrix} \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}$
4. Activation function

   $\hat{y} = \sigma(z)$



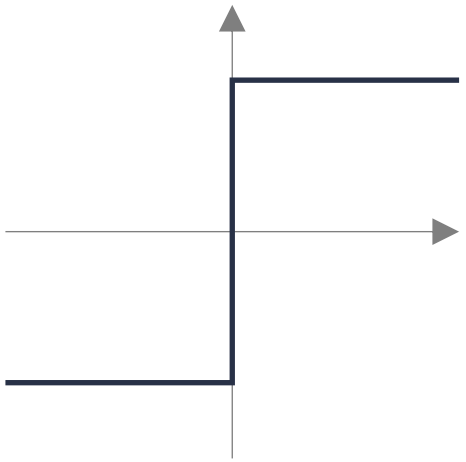
# Perceptron – Activation function

- Determine whether neuron fires or not
- Map weighted sum into desired range



Step function

$$\sigma(z) = \begin{cases} 0 & if\ z \leq 0 \\ 1 & if\ z > 0 \end{cases}$$

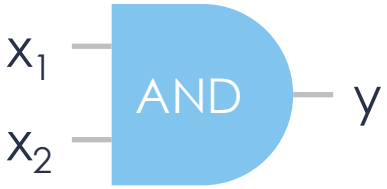


Sign function

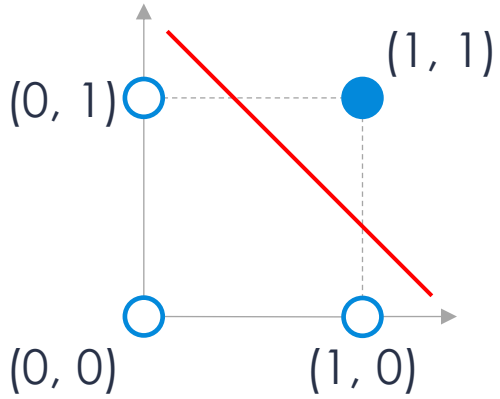
$$\sigma(z) = \begin{cases} -1 & if\ z \leq 0 \\ 1 & if\ z > 0 \end{cases}$$

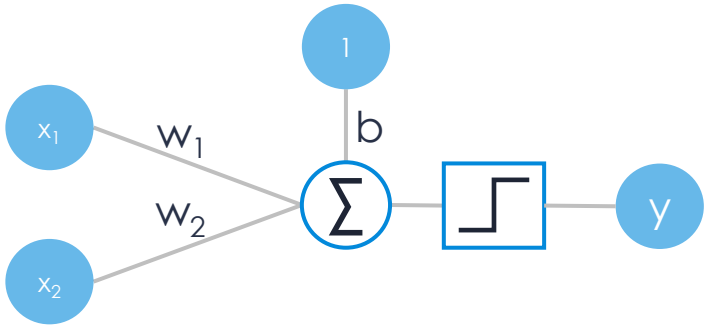
# Perceptron example – Logic gates

- Building blocks of digital system - **AND**




| $x_1$ | $x_2$ | y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |




$$z = w_1 \cdot x_1 + w_2 \cdot x_2 + b$$

Example with $b = -0.8$, $w_1 = 0.5$, $w_2 = 0.5$

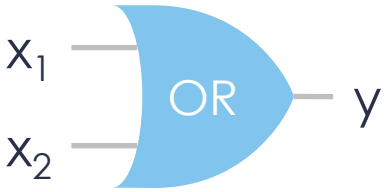
$x_1 = 0, x_2 = 0$
$z = 0.5 \cdot 0 + 0.5 \cdot 0 + (-0.8) = -0.8 < 0$
$\hat{y} = \sigma(z) = \sigma(-0.8) = 0$

$x_1 = 0, x_2 = 1$
$z = 0.5 \cdot 0 + 0.5 \cdot 1 + (-0.8) = -0.3 < 0$
$\hat{y} = \sigma(z) = \sigma(-0.3) = 0$

$x_1 = 1, x_2 = 1$
$z = 0.5 \cdot 1 + 0.5 \cdot 1 + (-0.8) = 0.2 > 0$
$\hat{y} = \sigma(z) = \sigma(0.2) = 1$

# Perceptron example – Logic gates

- Building blocks of digital system - **OR**




| $x_1$ | $x_2$ | y |
|---|---|---|
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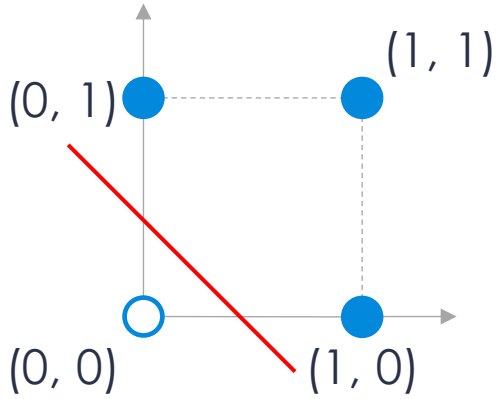
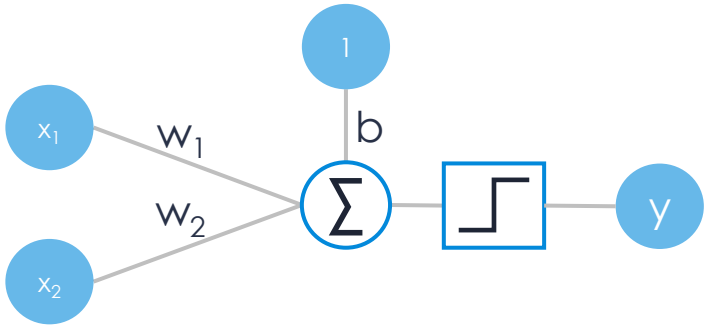
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |




$$z = w_1 \cdot x_1 + w_2 \cdot x_2 + b$$

Example with $b = -0.3$, $w_1 = 0.5$, $w_2 = 0.5$

$x_1 = 0, x_2 = 0$
$z = 0.5 \cdot 0 + 0.5 \cdot 0 + (-0.3) = -0.3 < 0$
$\hat{y} = \sigma(z) = \sigma(-0.3) = 0$

$x_1 = 0, x_2 = 1$
$z = 0.5 \cdot 0 + 0.5 \cdot 1 + (-0.3) = 0.2 > 0$
$\hat{y} = \sigma(z) = \sigma(0.2) = 1$

$x_1 = 1, x_2 = 1$
$z = 0.5 \cdot 1 + 0.5 \cdot 1 + (-0.3) = 0.7 > 0$
$\hat{y} = \sigma(z) = \sigma(0.7) = 1$

# Perceptron algorithm

- Learn weights to draw decision boundary
- Converge if there exists a separating hyperplane between the two classes

**Perceptron Learning Algorithm**

**Input**: $D = \{\boldsymbol{x}_j, y_j\}_{j=1}^{n}$ training set of $n$ samples with $\boldsymbol{x}_j \in \mathbb{R}^m$ ($m$ features)

\# Weights and bias initialization

$\boldsymbol{w}^{(0)} = \mathbf{0}$

$b^{(0)} = 0$

$t \leftarrow 0$

**While** *no convergence* **do**

  **for** every $(\boldsymbol{x}_j, y_j) \in D$ **do**

    \# Compute prediction

    $\hat{y}_j = \sigma\left(\boldsymbol{x}_j{}^T \boldsymbol{w}^{(t)} + b\right)$

    \# Compute error

    $error = y_j - \hat{y}_j$

    \# Update weights and bias

    **if** $error\ != 0$ **then**

      $\boldsymbol{w}^{(t+1)} \leftarrow \boldsymbol{w}^{(t)} + \eta \cdot error \cdot \boldsymbol{x}_j$

      $b^{(t+1)} \leftarrow b^{(t)} + \eta \cdot error$

    **else**

      $\boldsymbol{w}^{(t+1)} \leftarrow \boldsymbol{w}^{(t)}$

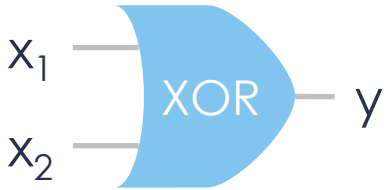
      $b^{(t+1)} \leftarrow b^{(t)}$

  $t \leftarrow t + 1$

**Output:** $\boldsymbol{w}$, $b$

# Perceptron limitations

- Only solve linearly separable problems
- Cannot solve **XOR problem**

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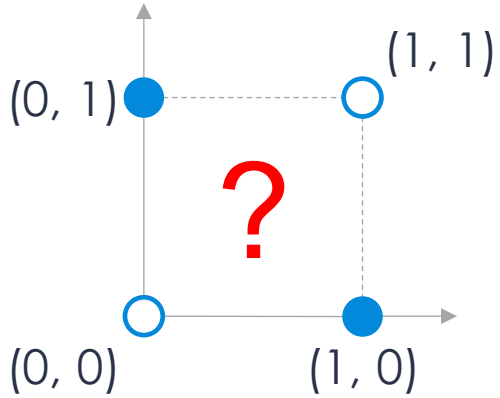

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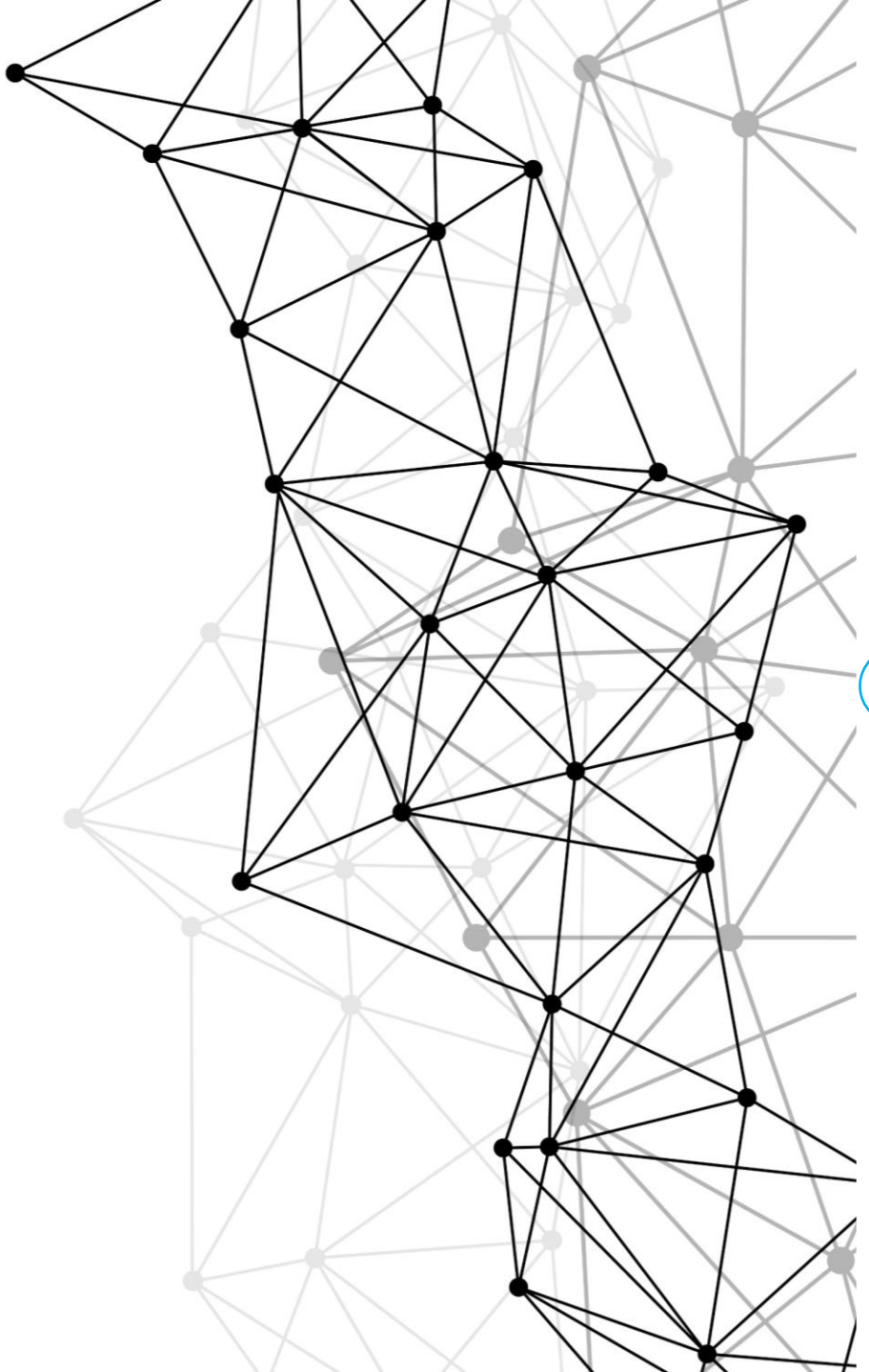
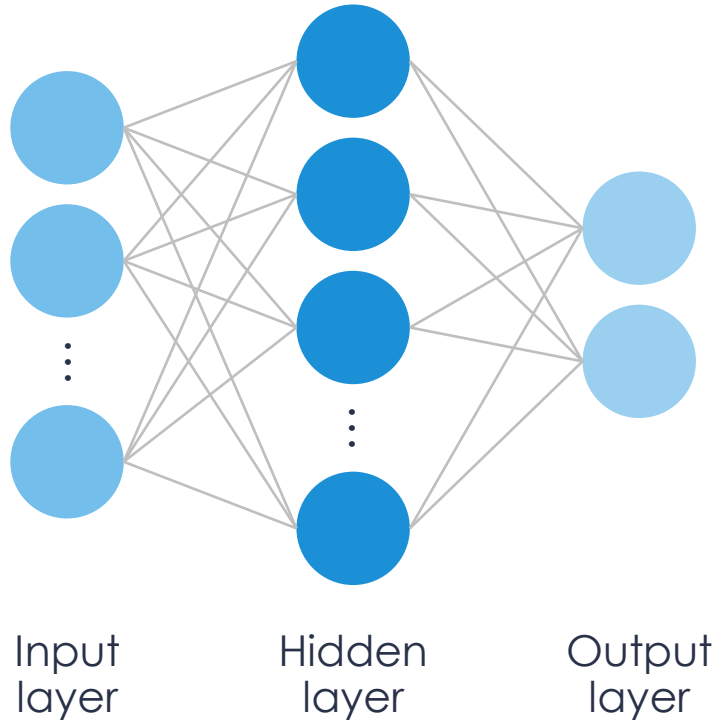


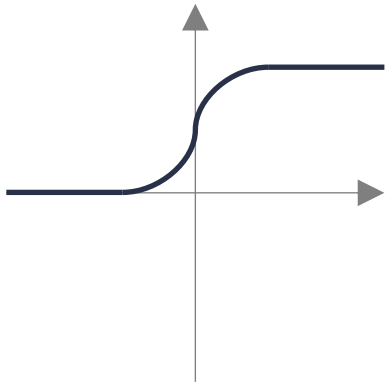
# Multilayer perceptron (MLP)

- Solve non-linear problems
- Feedforward artificial neural network
- 3 types of layers:
  - Input layer: Feed in input features
  - Hidden layers: Weighted sum and activation function
  - Output layer
- Each node in a layer is connected to all nodes in next layer → Fully connected
- Each connection has a weight
- Classification and regression



# MLP – Activation function

- Introduce non-linearity

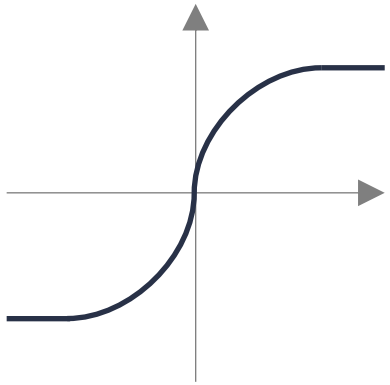


Sigmoid

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\frac{\partial\sigma}{\partial z}(z) = \sigma(z)\big(1-\sigma(z)\big)$$

PyTorch `torch.nn.Sigmoid()`

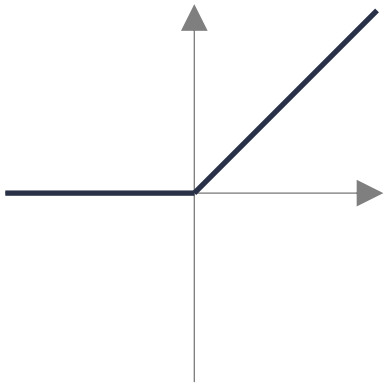


Hyperbolic tangent (tanh)

$$\sigma(z) = \frac{1-e^{-2z}}{1+e^{-2z}}$$

$$\frac{\partial\sigma}{\partial z}(z) = \big(1-\sigma(z)\big)^2$$

`torch.nn.Tanh()`

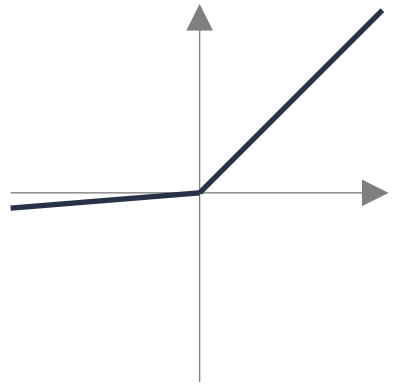


Rectified linear (ReLU)

$$\sigma(z) = \max(0, z)$$

$$\frac{\partial\sigma}{\partial z}(z) = \begin{cases} 0 & if\ z < 0 \\ 1 & if\ z \geq 0 \end{cases}$$

`torch.nn.ReLU()`



Leaky ReLU

$$\sigma(z) = \begin{cases} \alpha z & if\ z < 0 \\ z & if\ z \geq 0 \end{cases}$$

$$\frac{\partial\sigma}{\partial z}(z) = \begin{cases} \alpha & if\ z < 0 \\ 1 & if\ z \geq 0 \end{cases}$$

`torch.nn.LeakyReLU()`

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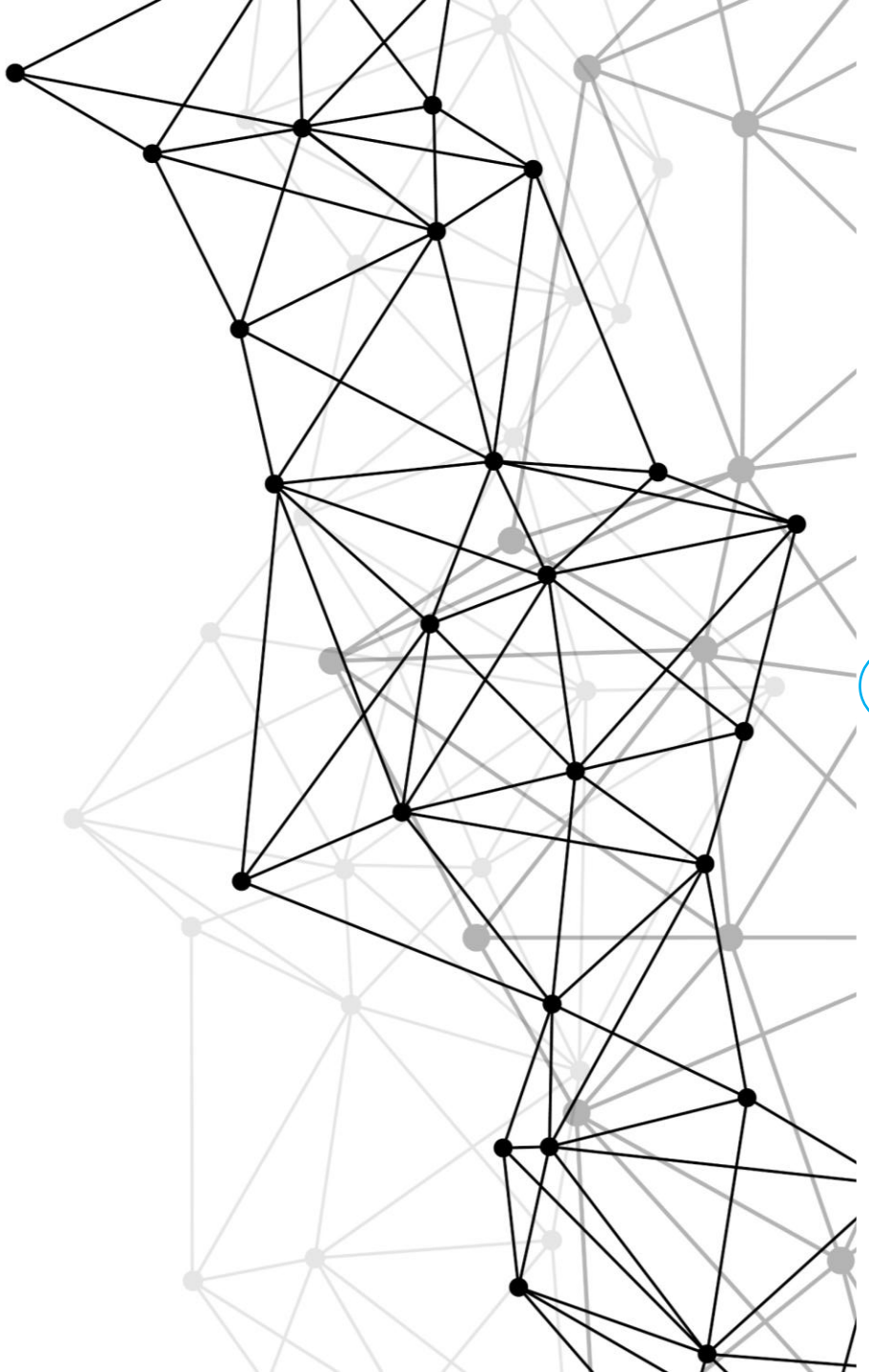


# Loss

- Measure the cost of incorrect predictions
- How close a predicted value is to the actual value → $l(y_i, \hat{y}_i)$
- Guide the model training process towards correct predictions
- Different loss functions for different problems



- **Empirical loss** measures the total loss over the entire dataset
- Also known as objective function, cost function, or empirical risk

$$L(W, b) = \frac{1}{n} \sum_{i=0}^{n} l(y_i, \hat{y}_i)$$

# Classification loss function

- Discrete values
  - **Cross entropy loss**
    - Classification with $k$ classes (> 2 classes) and output probabilities
    - Models with *Softmax* output

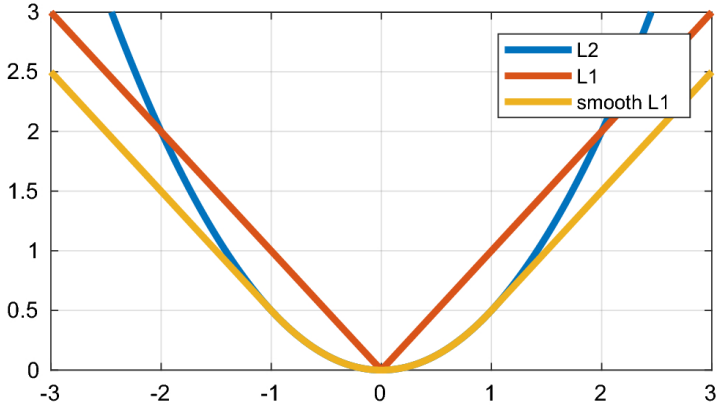
$$l(y_i, \hat{y}_i) = -\sum_{j=1}^{k} y_{ij} \cdot log\hat{y}_{ij}$$

    - PyTorch : `torch.nn.functional.cross_entropy(input, target)`
  - **Binary cross entropy loss**
    - Binary classification $y \in \{0, 1\}$ with output probabilities
    - Models with *Sigmoid* output

$$l(y_i, \hat{y}_i) = -[\, y_i \cdot log\hat{y}_i + (1 - y_i) \cdot log(1 - \hat{y}_i)]$$

    - PyTorch : `torch.nn.functional.binary_cross_entropy(input, target)`

# Regression loss functions




- Continuous values
  - **Mean absolute error** (MAE) or L1 loss
    - Robust to outliers
    - Gradient not differentiable at 0 → Unstable optimization

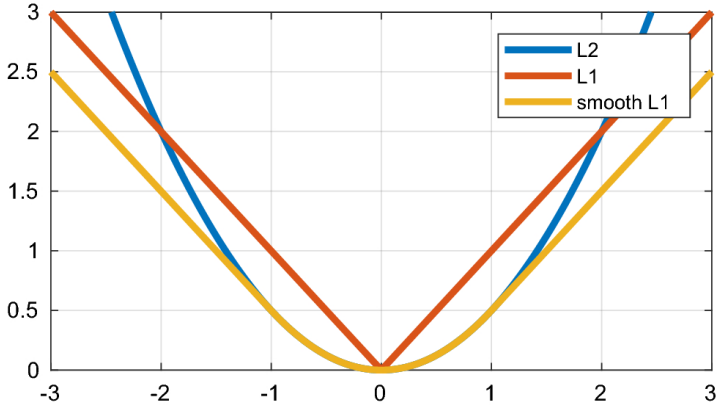
$$l(y_i, \hat{y}_i) = |y_i - \hat{y}_i|$$

    - PyTorch : `torch.nn.functional.l1_loss(input, target)`
  - **Mean square error** (MSE) or L2 loss
    - Sensible to outliers
    - Smooth and continuous gradient → Easy optimization

$$l(y_i, \hat{y}_i) = (y_i - \hat{y}_i)^2$$

    - PyTorch : `torch.nn.functional.mse_loss(input, target)`

# Regression loss functions




- Continuous values
  - **Smooth L1 loss**

$$l(y_i, \hat{y}_i) = \begin{cases} 0.5 \cdot (y_i - \hat{y}_i)^2 / \beta & if\, |y_i - \hat{y}_i| < \beta \\ |y_i - \hat{y}_i| - 0.5 \cdot \beta & otherwise \end{cases}$$

    - Combines benefits of MSE loss and MAE loss
    - More stable than MAE and less sensitive to outliers than MSE
    - Beta ($\beta$): threshold at which to switch between L1 and L2 loss
    - PyTorch : `torch.nn.functional.smooth_l1_loss(input, target, beta=1.0)`

# Loss optimization

- Find the network weights and bias that achieve the lowest loss

$$W^*, b^* = argmin_{W,b} \frac{1}{n} \sum_{i=0}^{n} l\big(y_i, f(x_i; W, b)\big) = argmin_{W,b} L(W, b)$$

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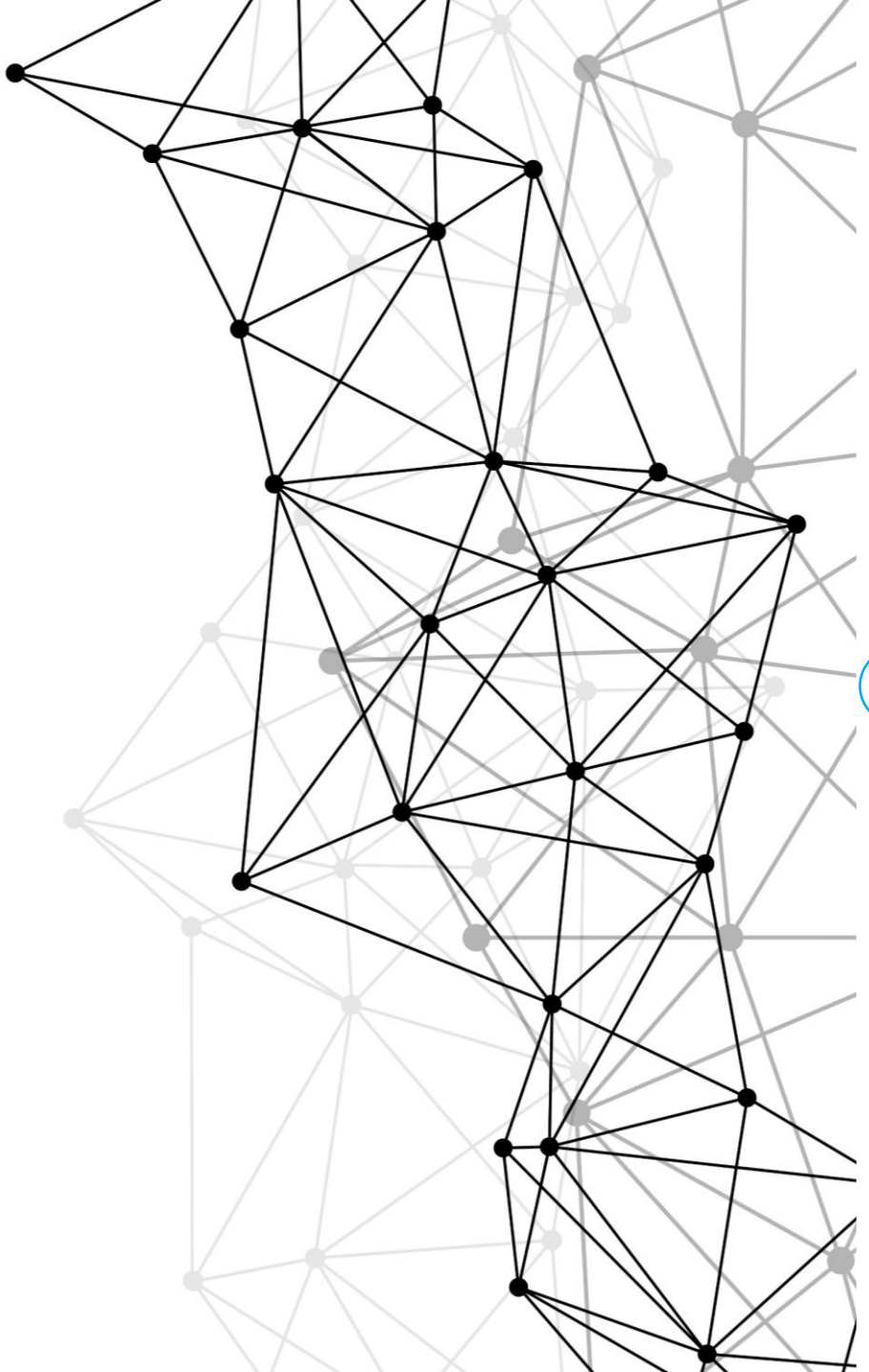
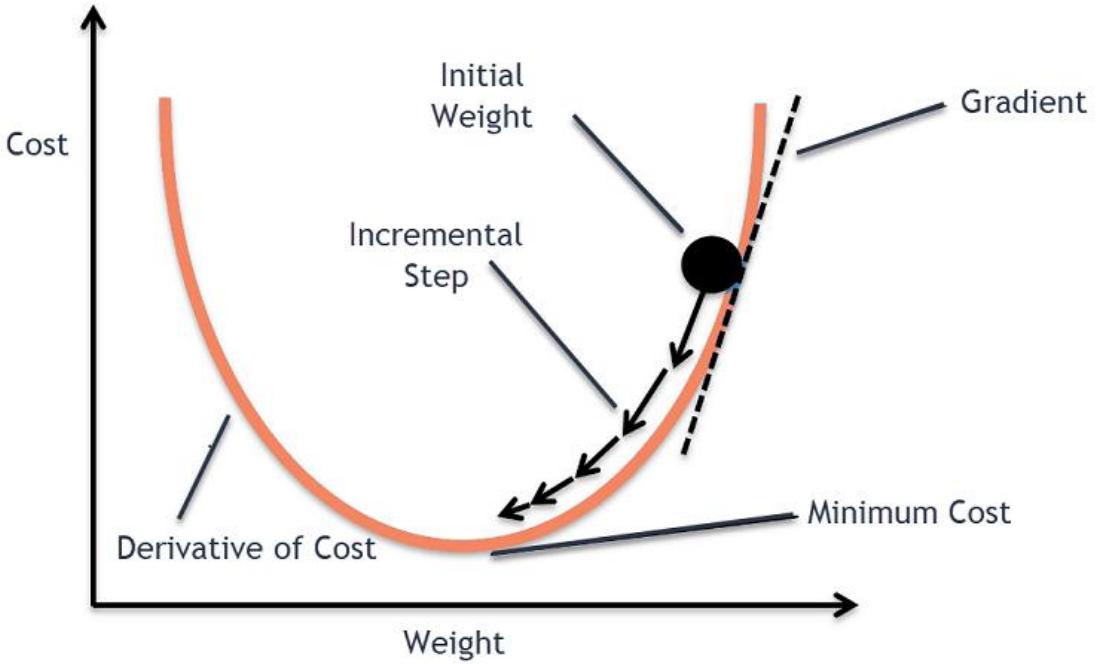


# Gradient descent

- Proposed by Cauchy in 1847
- Iterative first-order optimization algorithm
- Iteratively adjusts parameters to minimize the cost function
- Function requirements
  - Differentiable – has a derivative for each point in its domain
  - Convex – line connecting two points does not cross the curve



# Gradient descent

1. Initialize parameters (weights and biases) with random values
2. Calculate the empirical loss

$$L(W,b) = \frac{1}{n}\sum_{i=0}^{n} l(y_i, \hat{y}_i)$$

3. Calculate the derivative of the empirical loss (gradient)

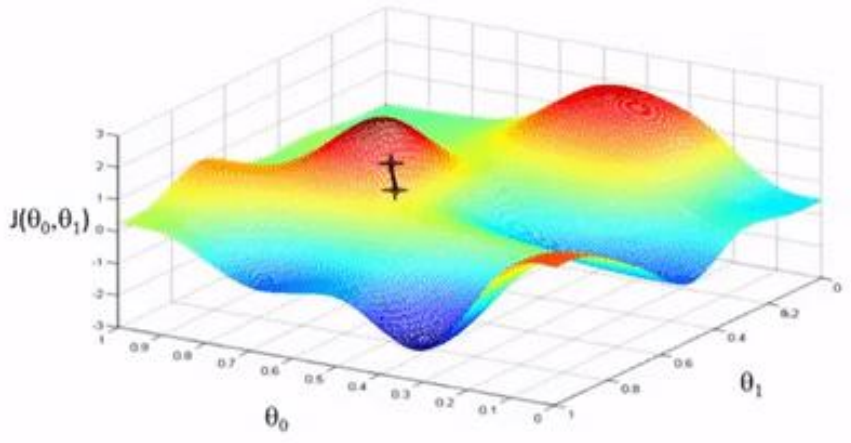
$$\frac{\partial L(W,b)}{\partial W} \quad \text{and} \quad \frac{\partial L(W,b)}{\partial b}$$

4. Make a step in the opposite direction of the gradient. Update parameters

$$W \leftarrow W - \eta \frac{\partial L(W,b)}{\partial W}$$

$$b \leftarrow b - \eta \frac{\partial L(W,b)}{\partial b}$$

5. Repeat steps 2, 3 and 4 until convergence

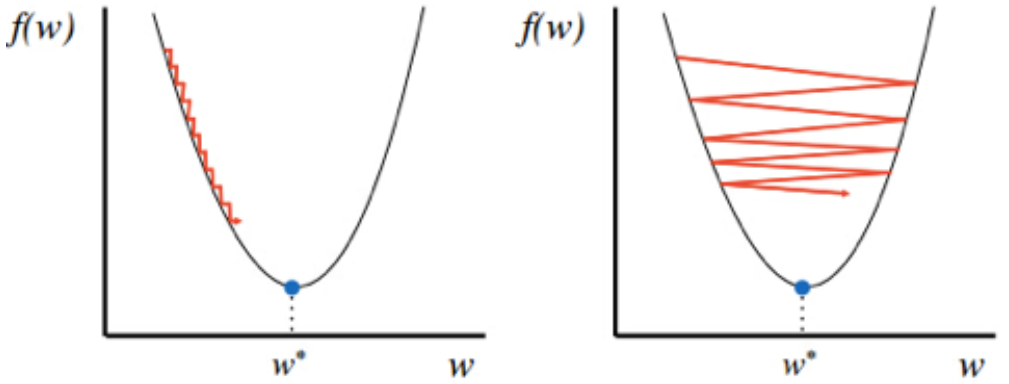


# Gradient descent – Learning rate ($\eta$)

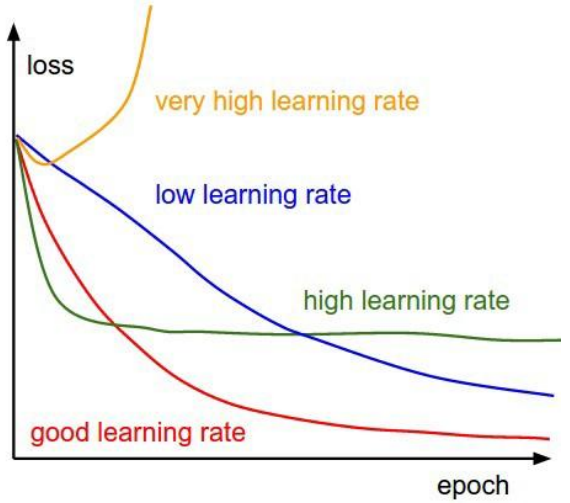
- Size of steps taken to reach the minimum
- Must be chosen carefully!

- **Too high value** → Large steps, risk of overshooting minimum
- **Too small value** → Small steps, take more time and computations to reach minimum

- Typically use adaptive learning rates

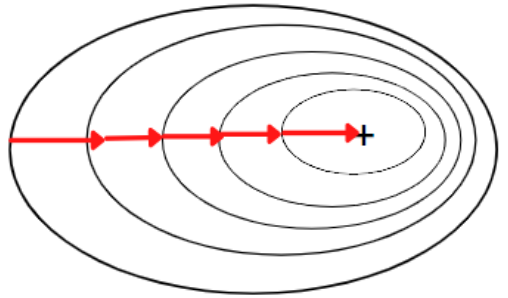



Too small Too large

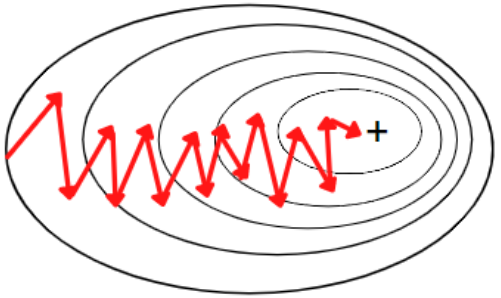


# Gradient descent - Variants

1. Batch gradient descent
   - Update parameters after all training samples
   - Stable gradient and convergence
   - Can be stuck in local minimum instead of global one
   - Long processing time for large training dataset

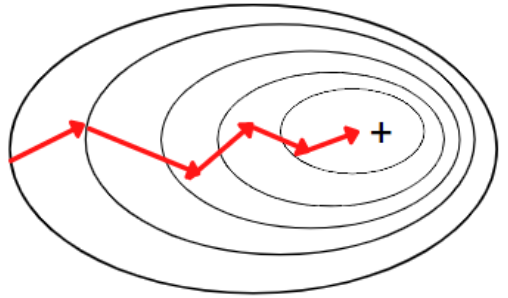


2. Stochastic gradient descent (SGD)
   - Updates parameters after each training sample
   - Faster
   - Can escape local minimum
   - Loss in computational efficiency
   - Noisy gradient due to frequent updates



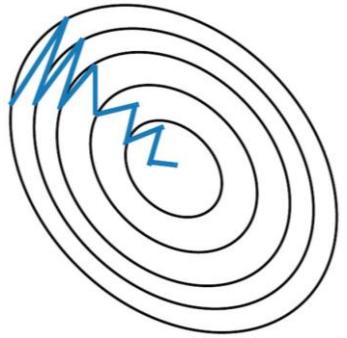
# Gradient descent - Variants

3. Mini-batch gradient descent
   - Combination of batch gradient descent and stochastic gradient descent
   - Split training dataset into small batches (e.g. 32, 64, 128) and updates parameters on each of these batches
   - Balance between computational efficiency and speed
   - Less noisy than SGD
   - Faster batch GD



# Gradient descent - Variants

4. Momentum
   - Reduce high variance in SGD
   - Faster convergence in relevant direction
   - Reduce fluctuation in irrelevant direction
   - Escape local minima and saddle points
   - Exponential moving average over the past gradients
   - Assign greater weights on most recent values



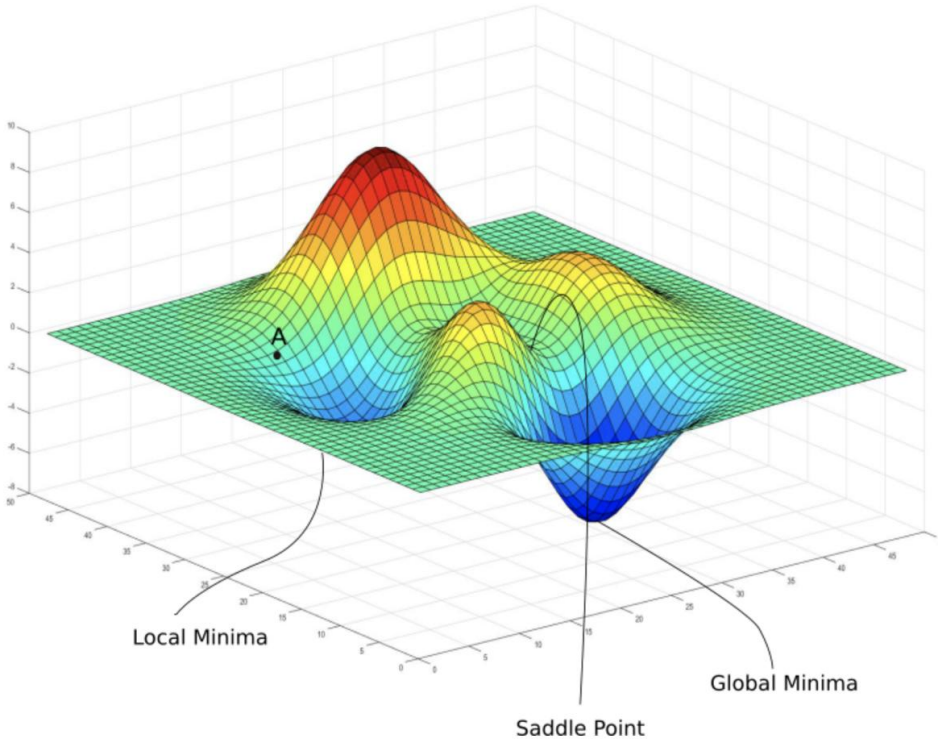
5. Adam (adaptive moment estimation)
   - Works with momentum of first and second order
   - Adaptive learning rate for each parameter
   - Reduce speed to avoid jumping over minimum

# Gradient descent - Variants

| Variant | Gradient computation | Convergence stability | Computational cost | Noise in updates | Memory usage | Best suited for |
|---|---|---|---|---|---|---|
| Batch GD | Full dataset | Stable | High | Low | High | Small datasets |
| Stochastic GD | 1 sample | Noisy | Low per iteration | High | Low | Large datasets, online learning |
| Mini-Batch GD | Small batch | Moderate | Moderate | Moderate | Moderate | Most practical applications |
| Momentum | History + gradient | Stable | Moderate | Reduced | Moderate | Problems with oscillations, Deep network |
| ADAM | Adaptive per parameter | Stable | Moderate to high | Low | Moderate | Deep learning, large dataset |

# Gradient descent – Challenges

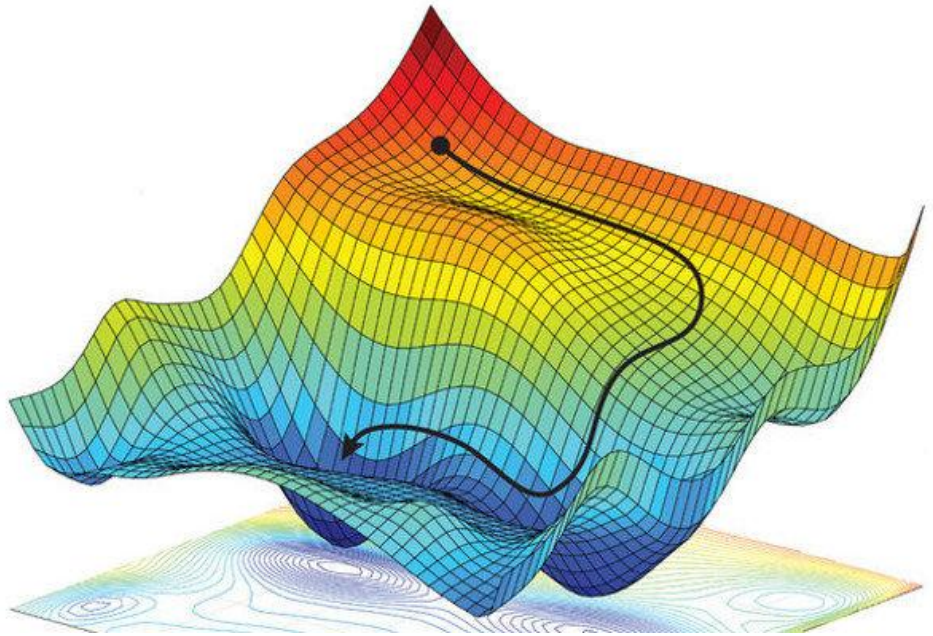
1. Local minima and saddle points
   - Learning stops when slope of cost function is at or close to 0
   - Gradient descent can struggle to find the global minimum
2. Vanishing gradient descent
   - Gradients get smaller and smaller, and approach 0
   - Parameters updates become insignificant
   - Stop learning and never converges
3. Exploding gradients
   - Gradients get larger and larger
   - Big parameter updates
   - Gradient descent diverges



# PyTorch – Gradient-based optimization algorithm

- Stochastic gradient descent (SGD)
  - *torch.optim.SGD(params, lr)*
- ADAM
  - *torch.optim.Adam(params, lr)*
- Taking an optimization step

```
for input, target in dataset:
    optimizer.zero_grad()
    output = model(input)
    loss = loss_fn(output, target)
    loss.backward()
    optimizer.step()
```



# Content

- Introduction
- Perceptron
- Multilayer perceptron (MLP)
- Loss function
- Gradient descent
- Backpropagation
- Convolutional neural network
- Recurrent neural network
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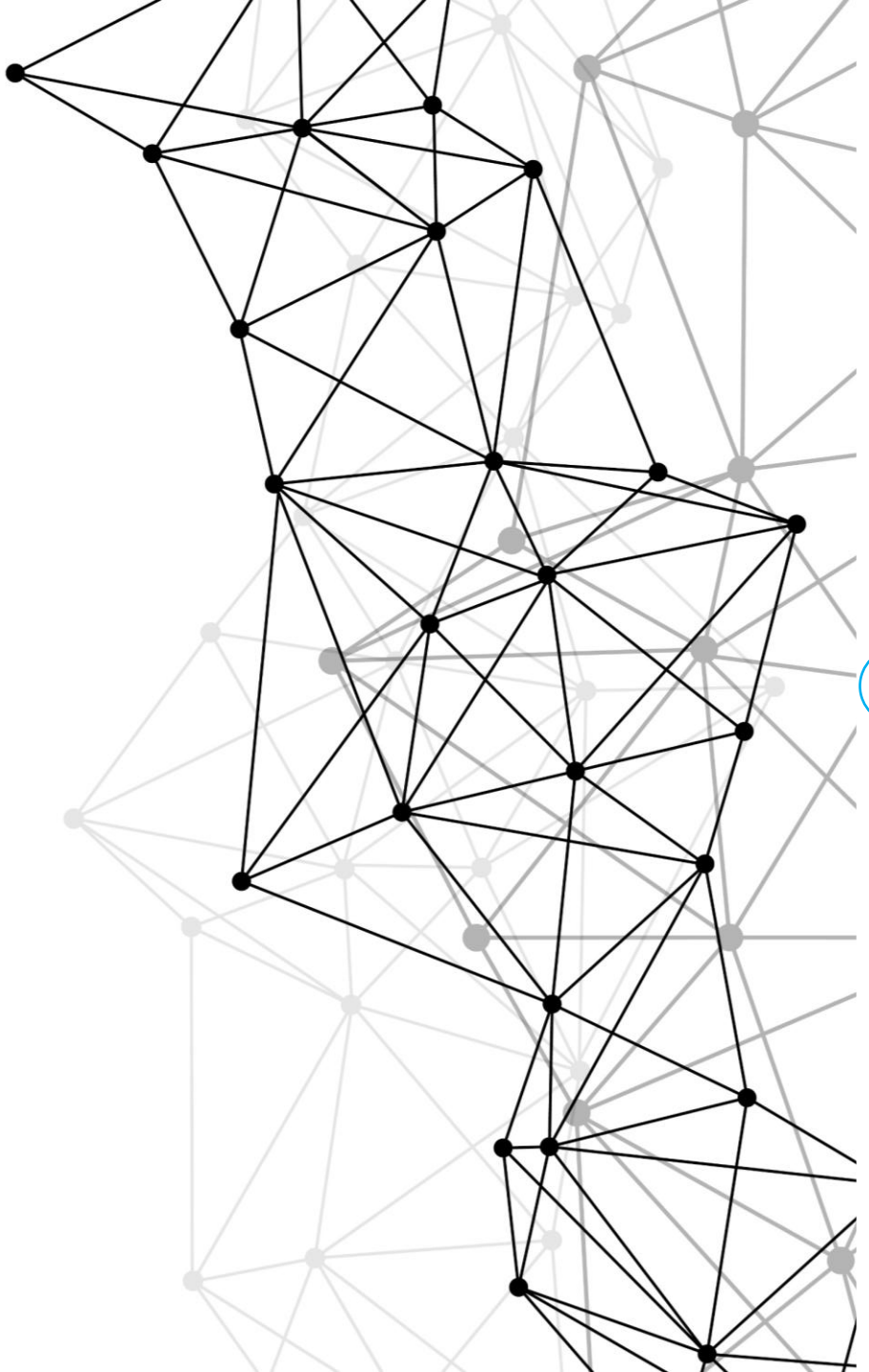
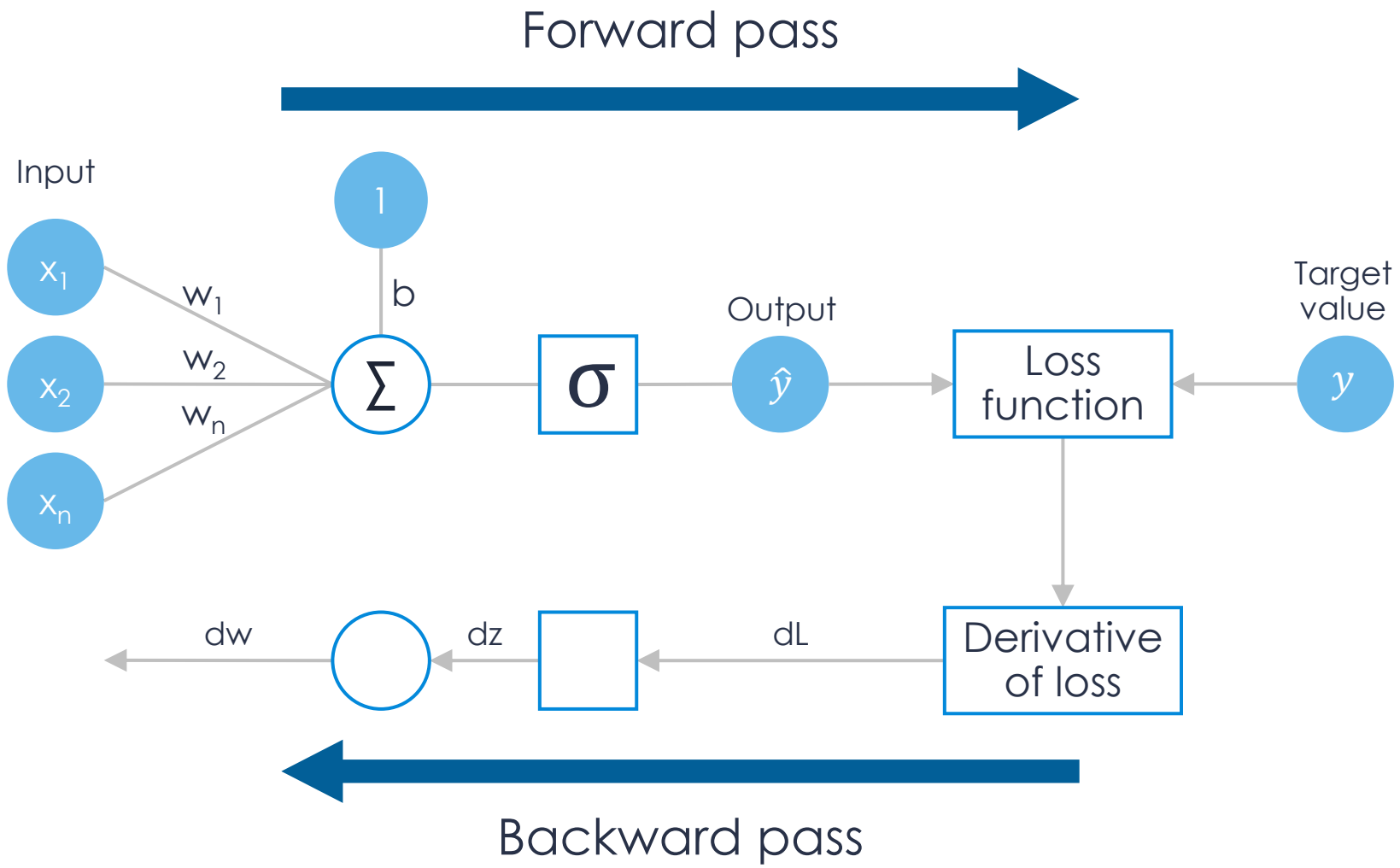


# Backpropagation

**How to compute the gradient?**

- Two main passes
  - Forward pass
    - Compute the predicted values from input data
  - Backward pass
    - Essential part in the training
    - Compare predicted and target values
    - Compute the gradient of the loss function with respect to each parameter
    - Adjust model parameters to decrease error and get closer to target values

# Backpropagation

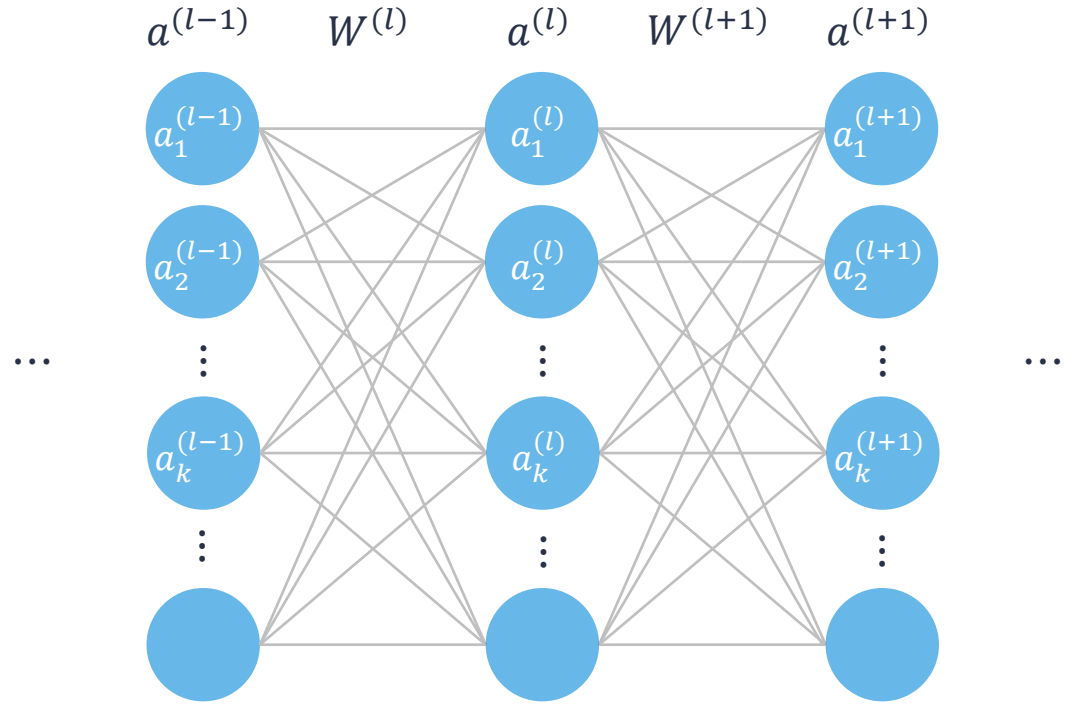


# Backpropagation

- Forward pass

$$x \xrightarrow{w,b} z^{(1)} \xrightarrow{\sigma} a^{(1)} \xrightarrow{w,b} z^{(2)} \xrightarrow{\sigma} a^{(2)} \longrightarrow \cdots \xrightarrow{w,b} z^{(L)} \xrightarrow{\sigma} a^{(L)} = f(x; w, b) = \hat{y}$$

| | |
|---|---|
| Input | $a^{(0)} = x$ |
| Prediction | $f(x; w, b) = a^{(L)} = \hat{y}$ |
| Layer | $\forall\, l = 1, \ldots, L$ |
| Weighted sum | $z^{(l)} = w^{(l)} a^{(l-1)} + b^{(l)}$ |
| Activation | $a^{(l)} = \sigma\left(z^{(l)}\right)$ |



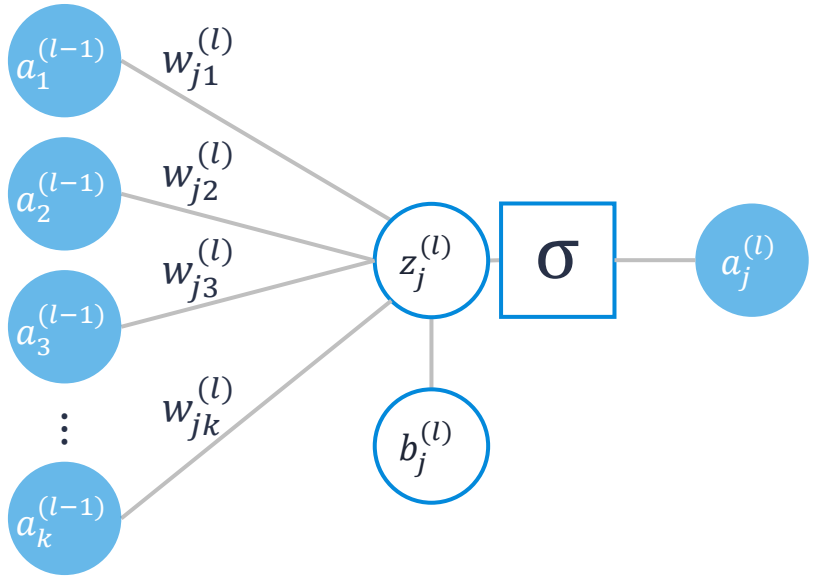
# Backpropagation

- Forward pass

$$x \xrightarrow{w,b} z^{(1)} \xrightarrow{\sigma} a^{(1)} \xrightarrow{w,b} z^{(2)} \xrightarrow{\sigma} a^{(2)} \longrightarrow \cdots \xrightarrow{w,b} z^{(L)} \xrightarrow{\sigma} a^{(L)} = f(x; w, b) = \hat{y}$$

$$z_j^{(l)} = \sum_k w_{jk}^{(l)} a_k^{(l-1)} + b_j^{(l)}$$

$$a_j^{(l)} = \sigma\left(z_j^{(l)}\right)$$



# Backpropagation – Weight update

- Update one weight $w_{jk}^{(l)}$
- Compute derivative of cost function with respect to this parameter $\frac{\partial L(W,b)}{\partial w_{jk}^{(l)}}$
- Apply **chain rule** to break a large problem into smaller ones

$$\frac{\partial L}{\partial w_{jk}^{(l)}} = \frac{\partial L}{\partial z_j^{(l)}} \frac{\partial z_j^{(l)}}{\partial w_{jk}^{(l)}}$$

$$\frac{\partial L}{\partial w_{jk}^{(l)}} = \frac{\partial L}{\partial z_j^{(l)}} a_k^{(l-1)}$$

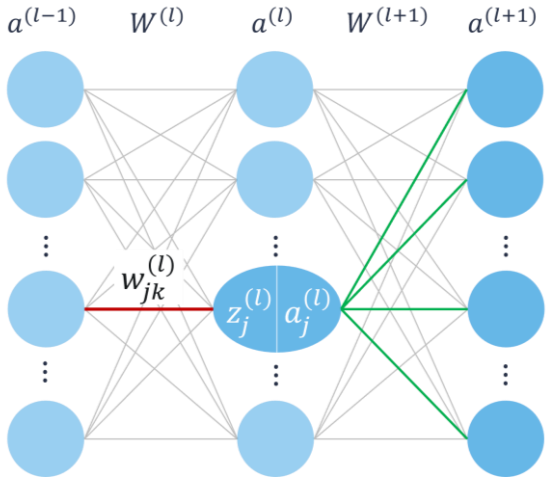
$$z_j^{(l)} = \sum_i w_{ji}^{(l)} a_i^{(l-1)} + b_j^{(l)}$$

$$\frac{\partial z_j^{(l)}}{\partial w_{jk}^{(l)}} = \frac{\partial}{\partial w_{jk}^{(l)}} \left( \sum_i w_{ji}^{(l)} a_i^{(l-1)} + b_j^{(l)} \right)$$

$$\frac{\partial z_j^{(l)}}{\partial w_{jk}^{(l)}} = \frac{\partial}{\partial w_{jk}^{(l)}} \left( \sum_i w_{ji}^{(l)} a_i^{(l-1)} \right)$$

$$\frac{\partial z_j^{(l)}}{\partial w_{jk}^{(l)}} = \frac{\partial}{\partial w_{jk}^{(l)}} \left( w_{jk}^{(l)} a_i^{(l-1)} \right)$$

$$\frac{\partial z_j^{(l)}}{\partial w_{jk}^{(l)}} = a_k^{(l-1)}$$



# Backpropagation – Weight update

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$$\frac{\partial L}{\partial w_{jk}^{(l)}} = \frac{\partial L}{\partial z_j^{(l)}} \frac{\partial z_j^{(l)}}{\partial w_{jk}^{(l)}}$$

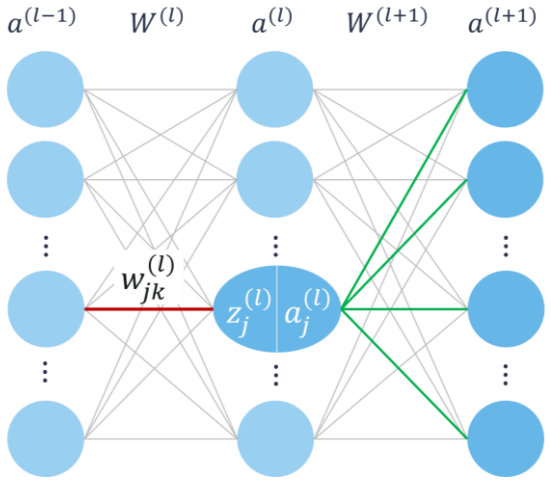
$$\frac{\partial L}{\partial w_{jk}^{(l)}} = \boxed{\frac{\partial L}{\partial z_j^{(l)}}} a_k^{(l-1)}$$

$$\frac{\partial L}{\partial w_{jk}^{(l)}} = \frac{\partial L}{\partial a_j^{(l)}} \boxed{\frac{\partial a_j^{(l)}}{\partial z_j^{(l)}}} a_k^{(l-1)}$$

$$\frac{\partial L}{\partial w_{jk}^{(l)}} = \frac{\partial L}{\partial a_j^{(l)}} \sigma'\left(z_j^{(l)}\right) a_k^{(l-1)}$$

$$a_j^{(l)} = \sigma\left(z_j^{(l)}\right)$$

$$\frac{\partial a_j^{(l)}}{\partial z_j^{(l)}} = \sigma'\left(z_j^{(l)}\right)$$



# Backpropagation – Weight update

- Update one weight $w_{jk}^{(l)}$
- Compute derivative of cost function with respect to this parameter $\frac{\partial L(W,b)}{\partial w_{jk}^{(l)}}$
- Apply **chain rule** to break a large problem into smaller ones

$$\frac{\partial L}{\partial w_{jk}^{(l)}} = \frac{\partial L}{\partial a_j^{(l)}} \sigma'\left(z_j^{(l)}\right) a_k^{(l-1)}$$

$$\frac{\partial L}{\partial w_{jk}^{(l)}} = \left(\sum_m \frac{\partial L}{\partial z_m^{(l+1)}} \frac{\partial z_m^{(l+1)}}{\partial a_j^{(l)}}\right) \sigma'\left(z_j^{(l)}\right) a_k^{(l-1)}$$

$$\frac{\partial L}{\partial w_{jk}^{(l)}} = \left(\sum_m \frac{\partial L}{\partial z_m^{(l+1)}} w_{mj}^{(l+1)}\right) \sigma'\left(z_j^{(l)}\right) a_k^{(l-1)}$$

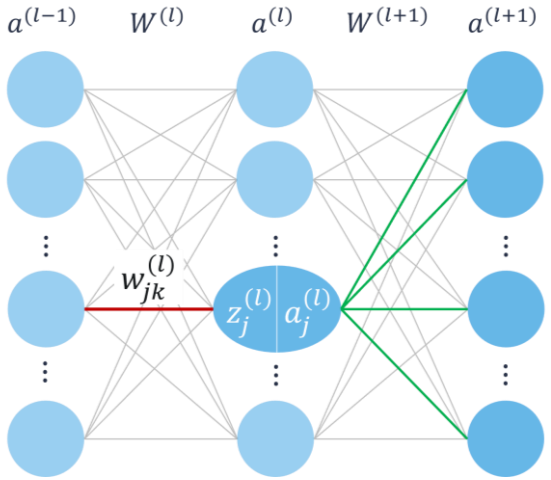
$$z_m^{(l+1)} = \sum_i w_{mi}^{(l+1)} a_i^{(l)} + b_m^{(l+1)}$$

$$\frac{\partial z_m^{(l+1)}}{\partial a_j^{(l)}} = \frac{\partial}{\partial a_j^{(l)}}\left(\sum_i w_{mi}^{(l+1)} a_i^{(l)} + b_m^{(l+1)}\right)$$

$$\frac{\partial z_m^{(l+1)}}{\partial a_j^{(l)}} = \frac{\partial}{\partial a_j^{(l)}}\left(\sum_i w_{mi}^{(l+1)} a_i^{(l)}\right)$$

$$\frac{\partial z_m^{(l+1)}}{\partial a_j^{(l)}} = \frac{\partial}{\partial a_j^{(l)}}\left(w_{mj}^{(l+1)} a_i^{(l)}\right)$$

$$\frac{\partial z_m^{(l+1)}}{\partial a_j^{(l)}} = w_{mj}^{(l+1)}$$



# Backpropagation – Weight update

$$\frac{\partial L}{\partial w_{jk}^{(l)}} = \frac{\partial L}{\partial z_j^{(l)}} a_k^{(l-1)} = \left( \sum_m \frac{\partial L}{\partial z_m^{(l+1)}} w_{mj}^{(l+1)} \right) \sigma'\left(z_j^{(l)}\right) a_k^{(l-1)}$$

- Error signal of neuron $j$ in layer $l$

$$\delta_j^{(l)} = \frac{\partial L}{\partial z_j^{(l)}} = \left( \sum_m \frac{\partial L}{\partial z_m^{(l+1)}} w_{mj}^{(l+1)} \right) \sigma'\left(z_j^{(l)}\right)$$

$$\frac{\partial L}{\partial z_m^{(l+1)}} = \delta_m^{(l+1)}$$

$$\delta_j^{(l)} = \left( \sum_m \delta_m^{(l+1)} w_{mj}^{(l+1)} \right) \sigma'\left(z_j^{(l)}\right)$$

$$\frac{\partial L}{\partial w_{jk}^{(l)}} = \delta_j^{(l)} a_k^{(l-1)}$$

- Update weight

$$w_{jk}^{(l)} \leftarrow w_{jk}^{(l)} - \eta \cdot \delta_j^{(l)} \, a_k^{(l-1)}$$

# Backpropagation – Bias update

- Update one bias $b_j^{(l)}$
- Compute derivative of cost function with respect to this parameter $\frac{\partial L(W,b)}{\partial b_j^{(l)}}$
- Apply **chain rule** to break a large problem into smaller ones

$$\frac{\partial L}{\partial b_j^{(l)}} = \frac{\partial L}{\partial z_j^{(l)}} \frac{\partial z_j^{(l)}}{\partial b_j^{(l)}}$$

$$\frac{\partial L}{\partial b_j^{(l)}} = \frac{\partial L}{\partial z_j^{(l)}} \cdot 1$$

$$\delta_j^{(l)} = \frac{\partial L}{\partial z_j^{(l)}}$$

$$\frac{\partial L}{\partial b_j^{(l)}} = \delta_j^{(l)}$$

$$z_j^{(l)} = \sum_i w_{ji}^{(l)} a_i^{(l-1)} + b_j^{(l)}$$

$$\frac{\partial z_j^{(l)}}{\partial b_j^{(l)}} = \frac{\partial}{\partial b_j^{(l)}} \left( \sum_i w_{ji}^{(l)} a_i^{(l-1)} + b_j^{(l)} \right)$$

$$\frac{\partial z_j^{(l)}}{\partial b_j^{(l)}} = \frac{\partial}{\partial b_j^{(l)}} \left( b_j^{(l)} \right)$$

$$\frac{\partial z_j^{(l)}}{\partial b_j^{(l)}} = 1$$

- Update bias $b_j^{(l)} \leftarrow b_j^{(l)} - \eta \cdot \delta_j^{(l)}$

# MLP limitations

- Fully connected → Many parameters
- No spatial or structural awareness
- Hard to train when deep architecture
  - Vanishing/exploding gradients
  - Weight initialization
- Require large labeled dataset
  - Overfitting

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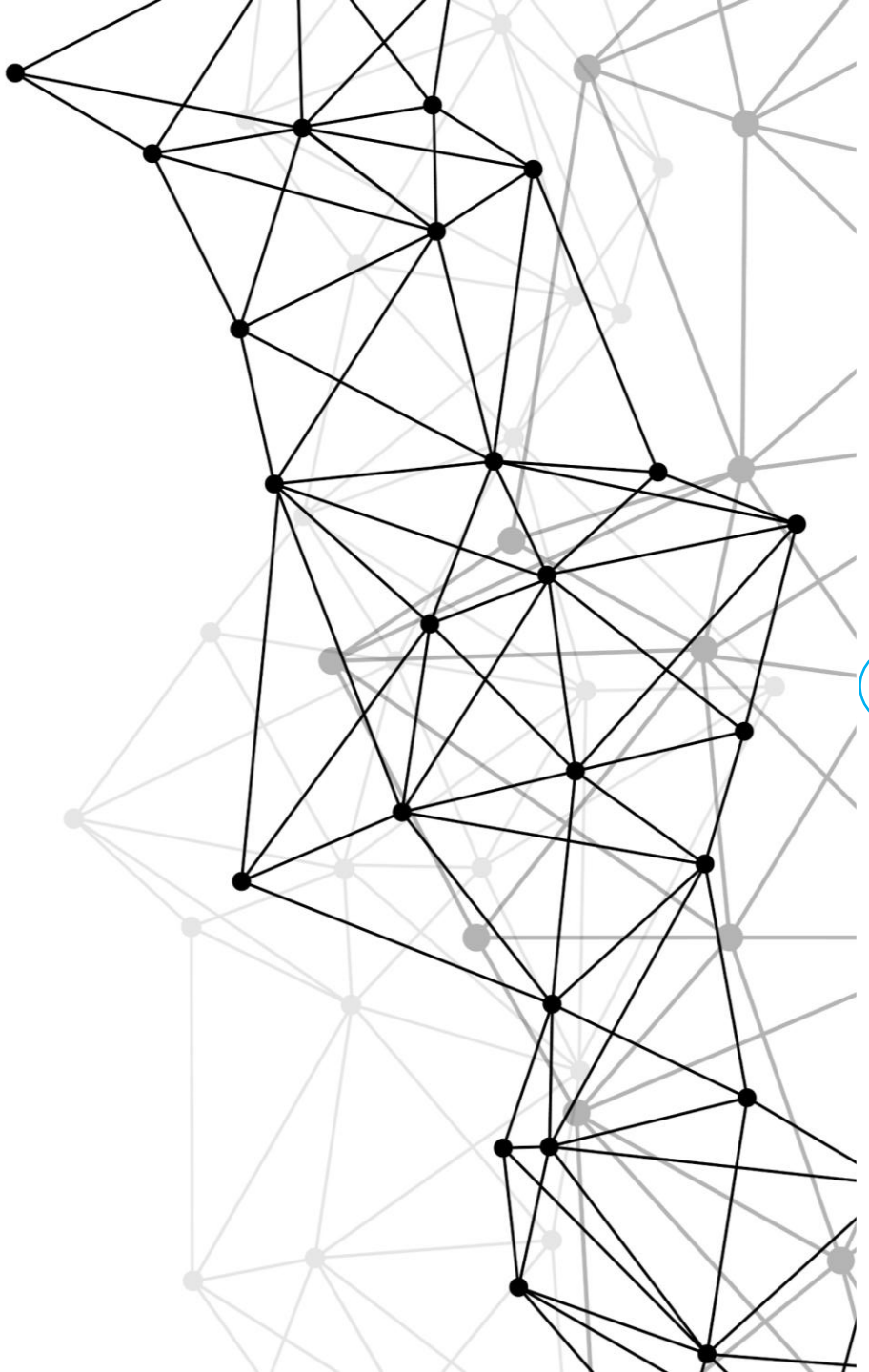
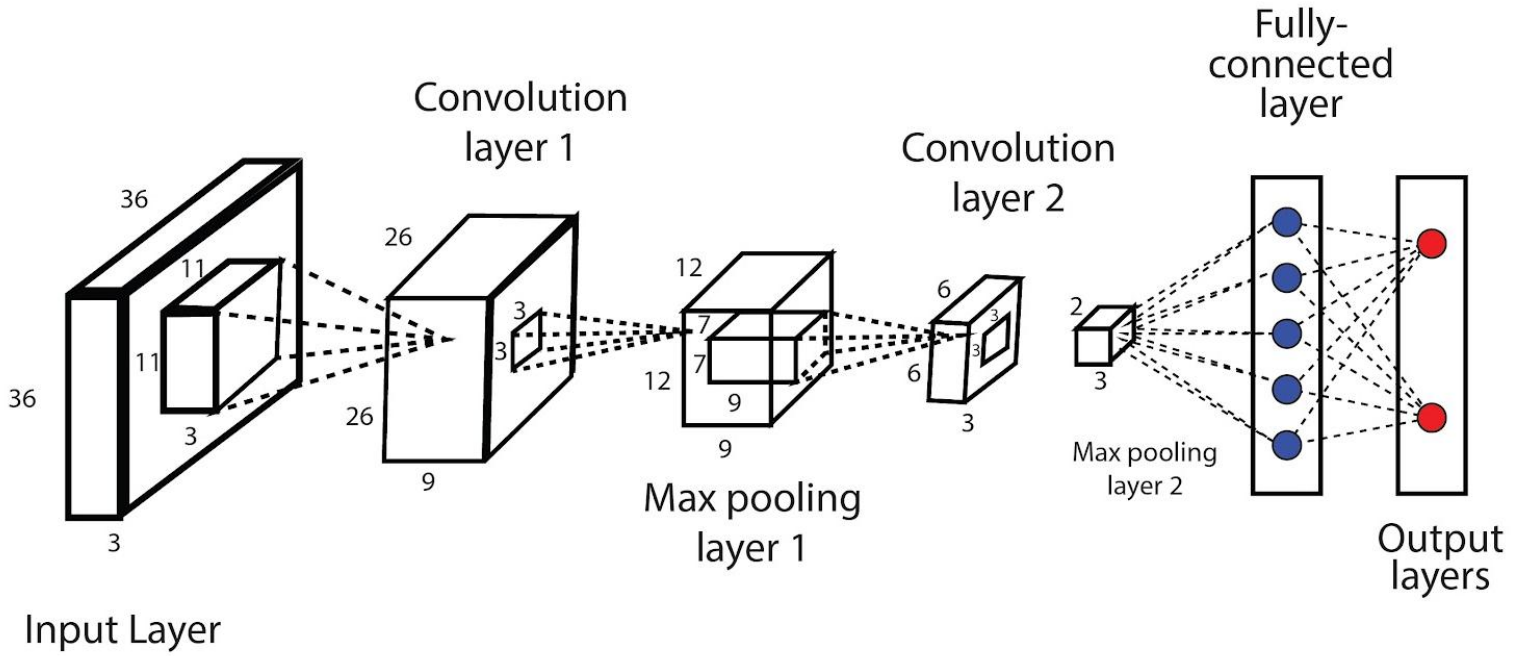


# Convolutional neural network (CNN)

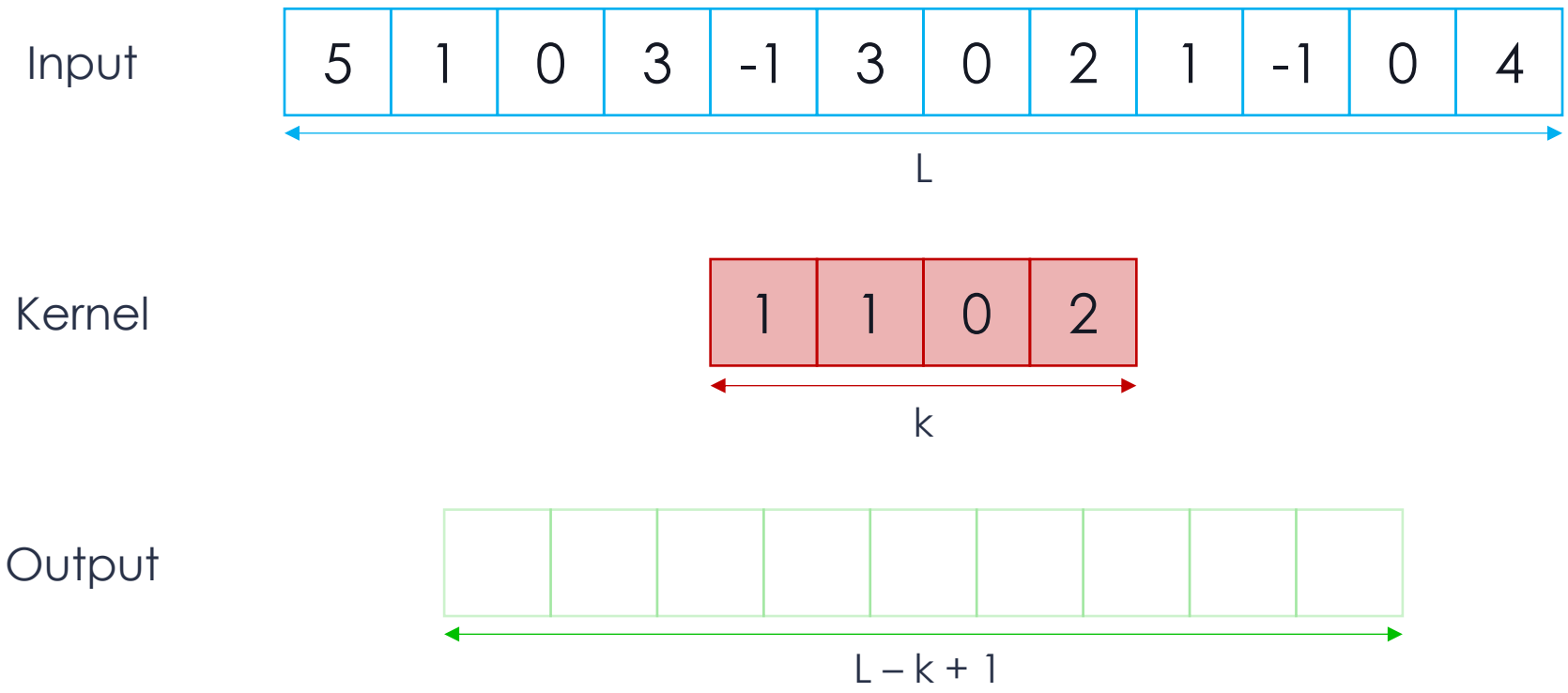
- 3 main types of layers
  - Convolutional layers
  - Pooling layers
  - Fully connected layers



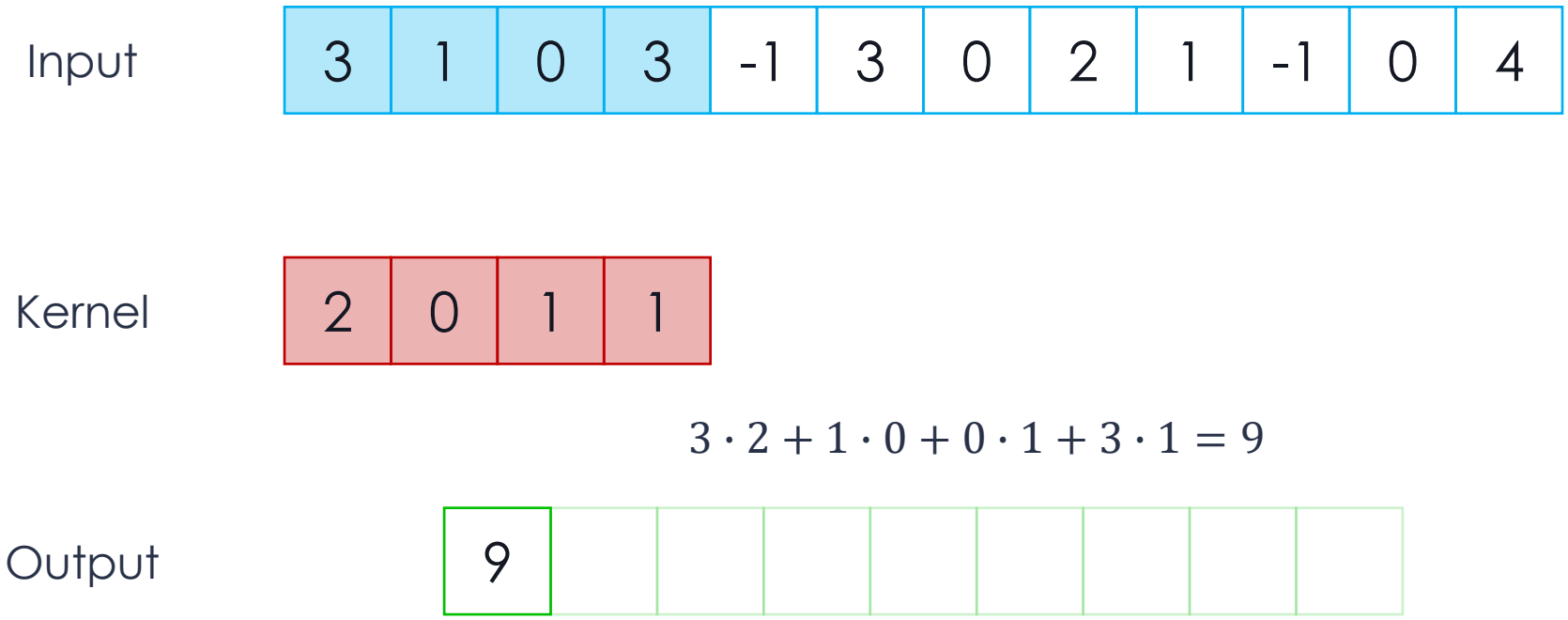
# CNN – Convolutional layer

- Core building block of a CNN
- Use filters that perform convolution operations as it is scanning the input
- Kernel → Field of view of the convolution
- Weight sharing
- Activation map or output feature map
- Receptive field → Area of the input that the kernel can see
- Generalize to multidimensional input → Convolution 1D, 2D, or 3D
- Preserve signal structure
  - 1D signal remains a 1D signal after convolutional layer
  - 2D signal remains a 2D signal after convolutional layer
  - Etc.

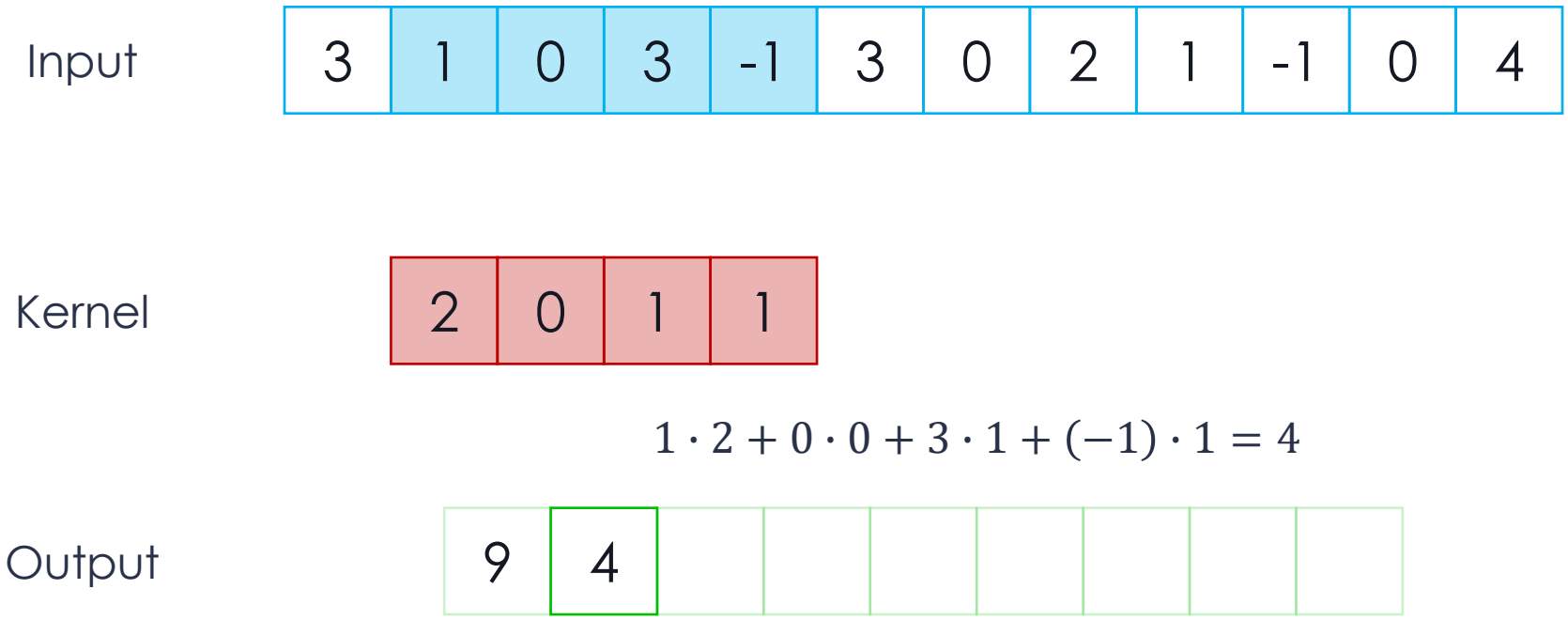
# CNN – Convolutional layer



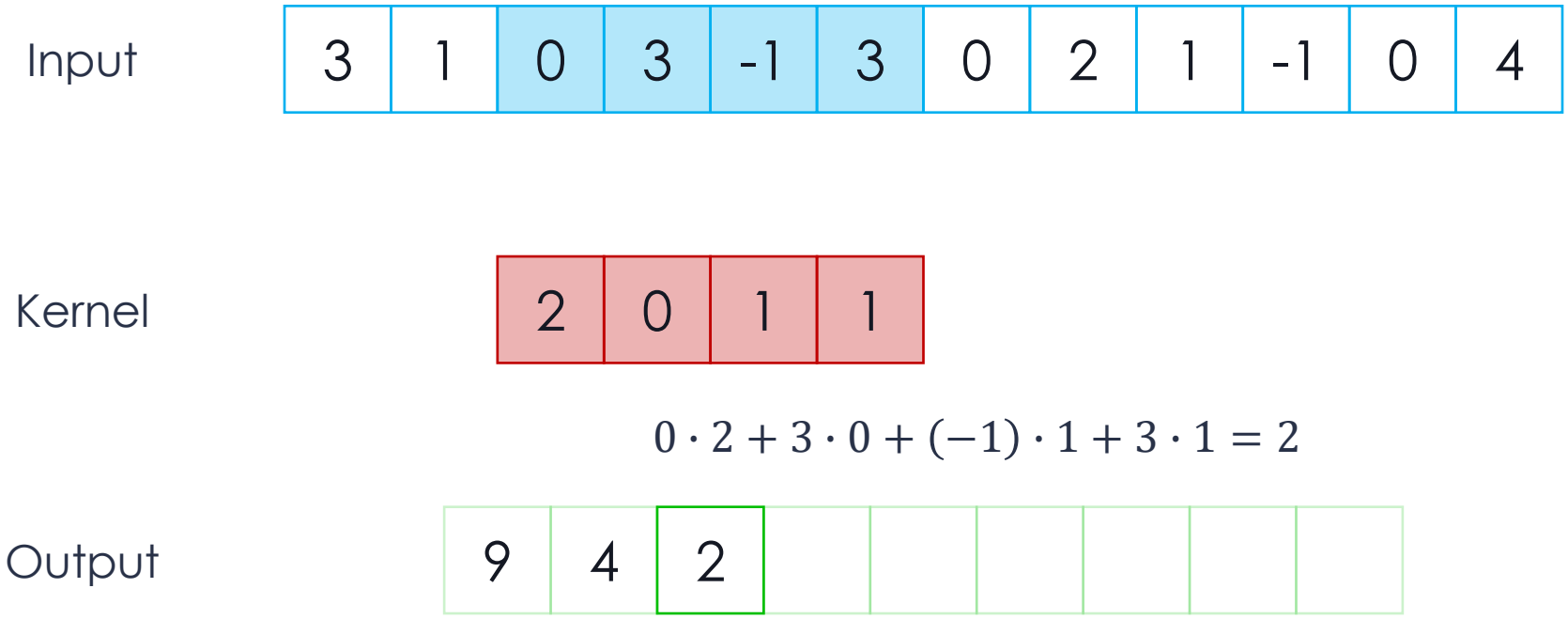
# CNN – Convolutional layer



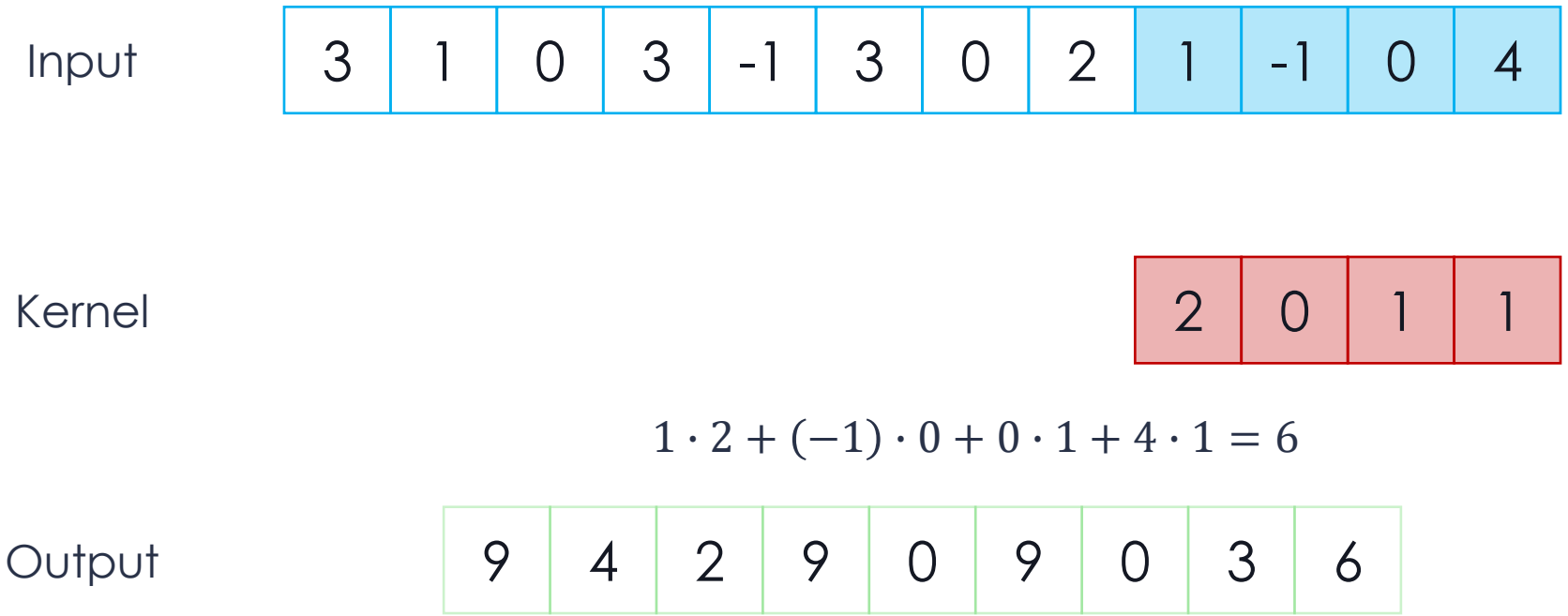
# CNN – Convolutional layer



# CNN – Convolutional layer

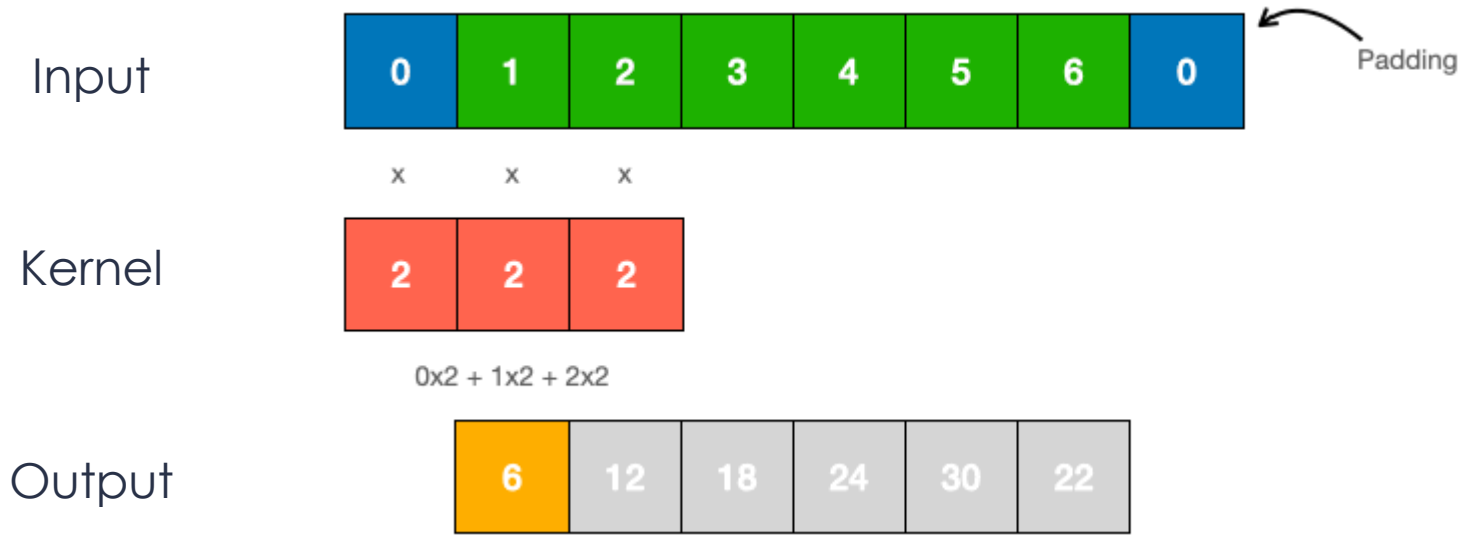


# CNN – Convolutional layer



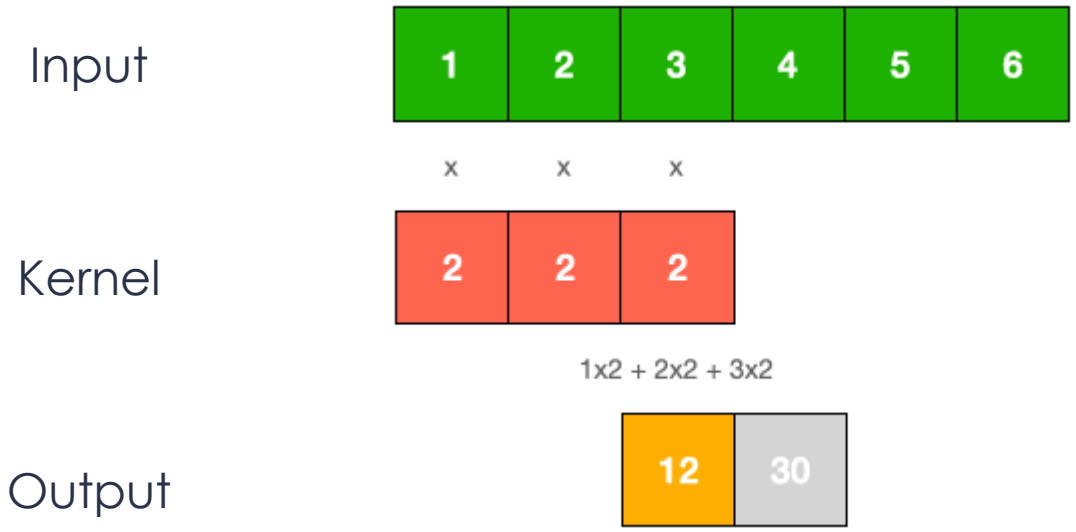
# CNN – Padding

- How the border of a sample is handled.
- Padded convolution keeps the output of same length as the input
- Unpadded convolution with kernel >1 crops some border of the sample



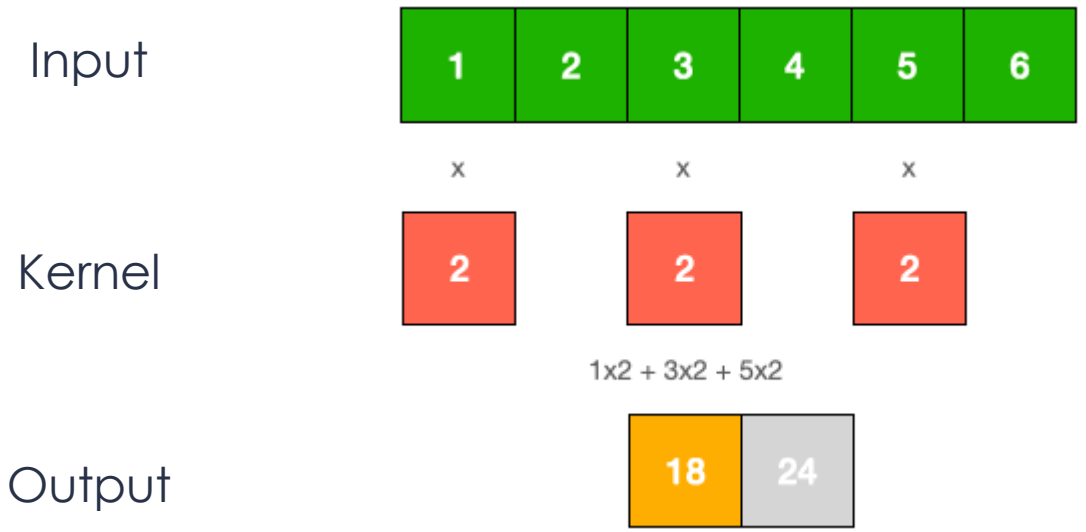
# CNN – Stride

- Step size when moving kernel across signal
- By default, slide the kernel by 1 step a time
- Increase stride to downsample the input vector



# CNN – Dilatation

- Insert spaces between kernel elements
- Expansion of the receptive field while preserving resolution
- Same computational and memory costs



# CNN – Convolutional layer PyTorch

- Convolutional layer 1D
  - `torch.nn.Conv1d(in_channels, out_channels, kernel_size, stride, padding, dilatation)`
  - Input: $(N, C_{in}, L_{in})$
  - Output: $(N, C_{out}, L_{out})$

$$L_{out} = \frac{L_{in} + 2 \cdot padding - dilation \cdot (kernel\_size - 1) - 1}{stride} + 1$$

- Convolutional layer 2D
  - `torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride, padding, dilatation)`
  - Input: $(N, C_{in}, H_{in}, W_{in})$
  - Output: $(N, C_{out}, H_{out}, W_{out})$

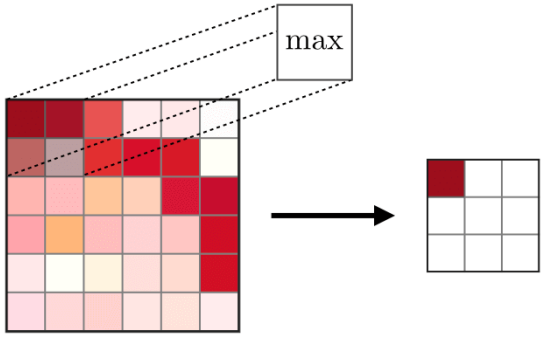
$$H_{out} = \frac{H_{in} + 2 \cdot padding[0] - dilation[0] \cdot (kerne\_size[0] - 1) - 1}{stride[0]} + 1$$

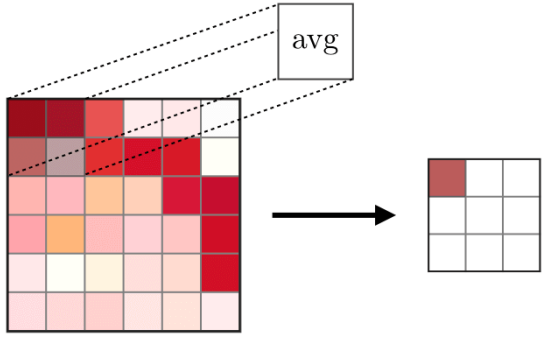
$$W_{out} = \frac{W_{in} + 2 \cdot padding[1] - dilation[1] \cdot (kerne\_size[1] - 1) - 1}{stride[1]}$$

# CNN – Pooling layer

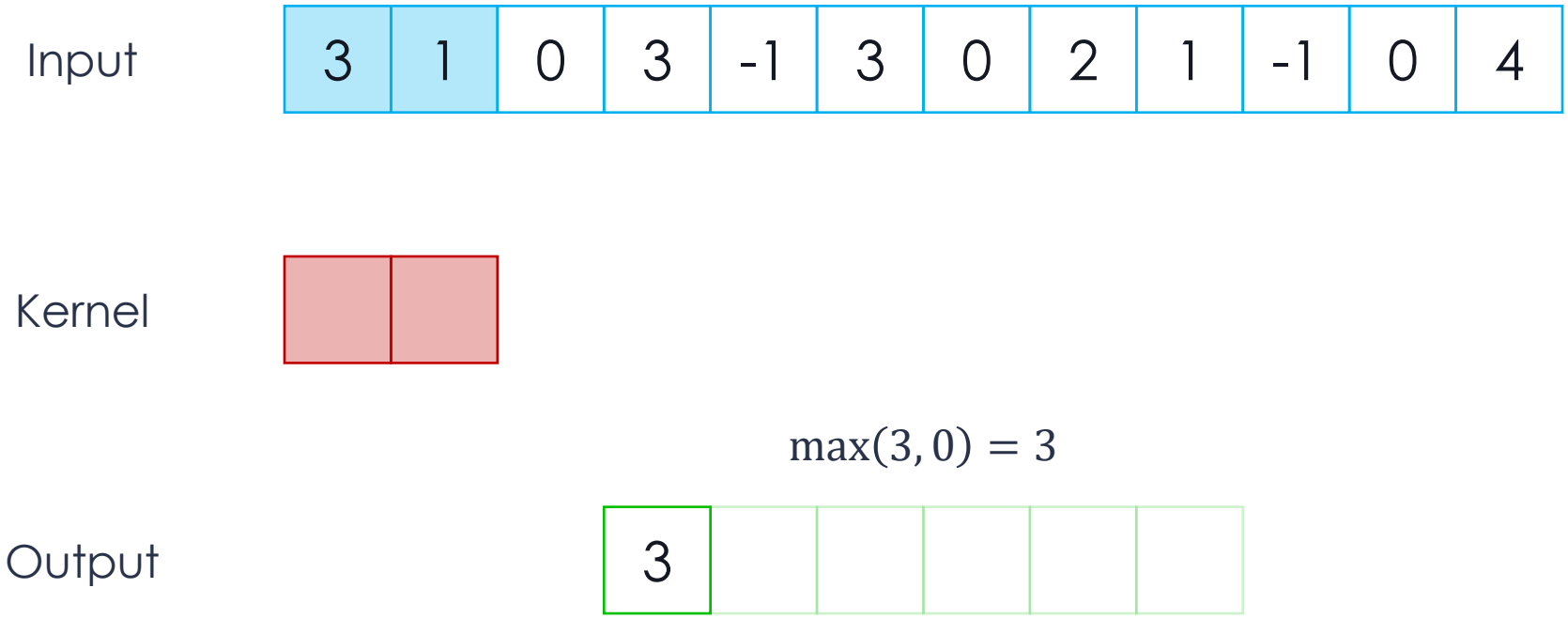
- Downsampling operation
- Grouping several activations into a more meaningful one
- Sweep filter across the entire input but no weights
- Typically applied after convolutional layer
- Provide invariance to deformations
- Most common:
  - Max pooling: each operation selects the maximum value of the current view
  - Average pooling: each operation averages the values of the current view



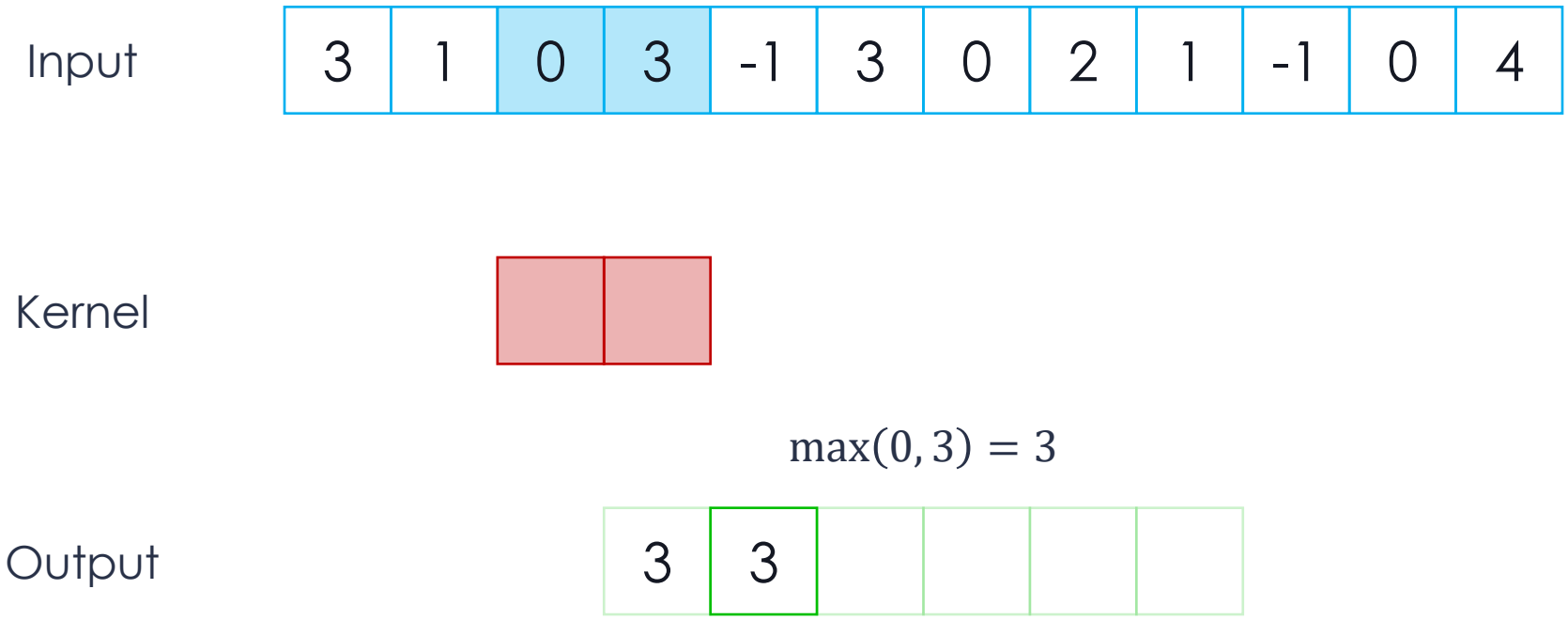


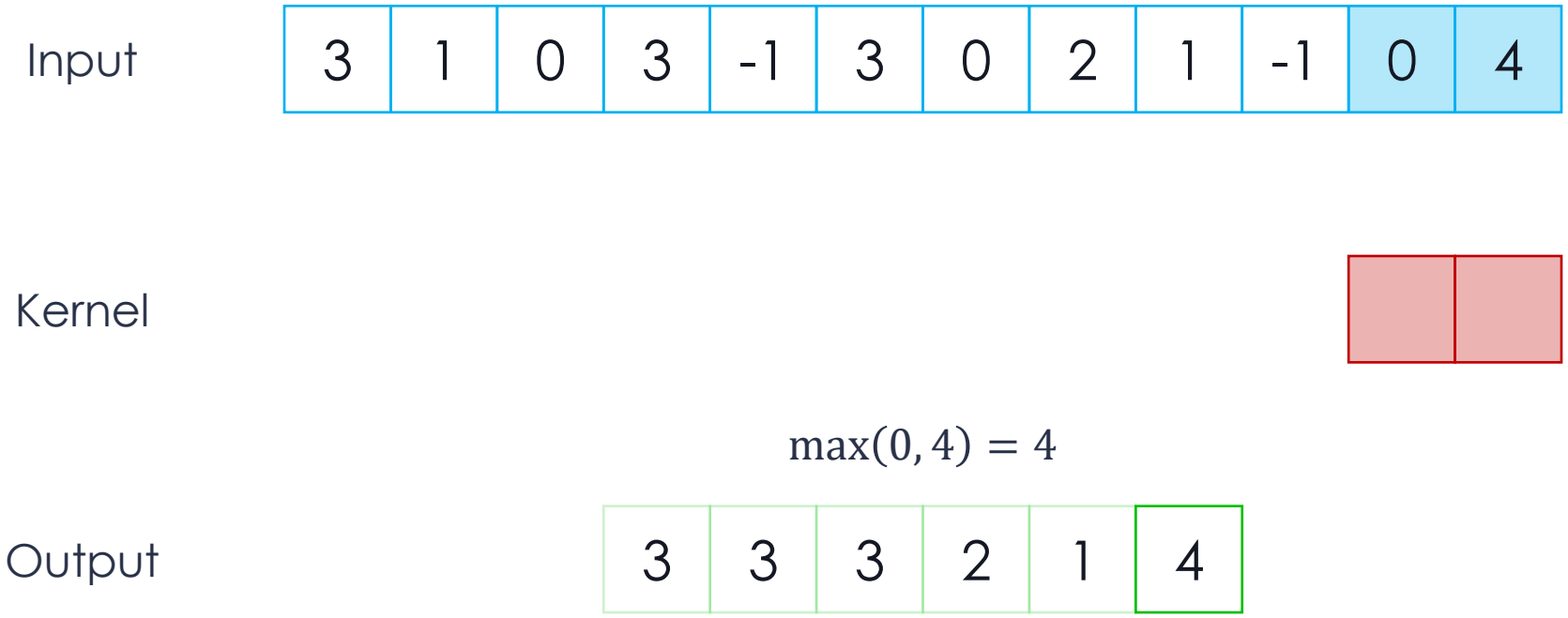
# CNN – Maxpooling layer



# CNN – Maxpooling layer



# CNN – Maxpooling layer



# CNN – Pooling layer PyTorch

- Max pooling layer 1D
  - `torch.nn.MaxPool1d(kernel_size, stride, padding)`
  - Input: $(N, C_{in}, L_{in})$
  - Output: $(N, C_{out}, L_{out})$

$$L_{out} = \frac{L_{in} + 2 \cdot padding - kernel\_size}{stride} + 1$$

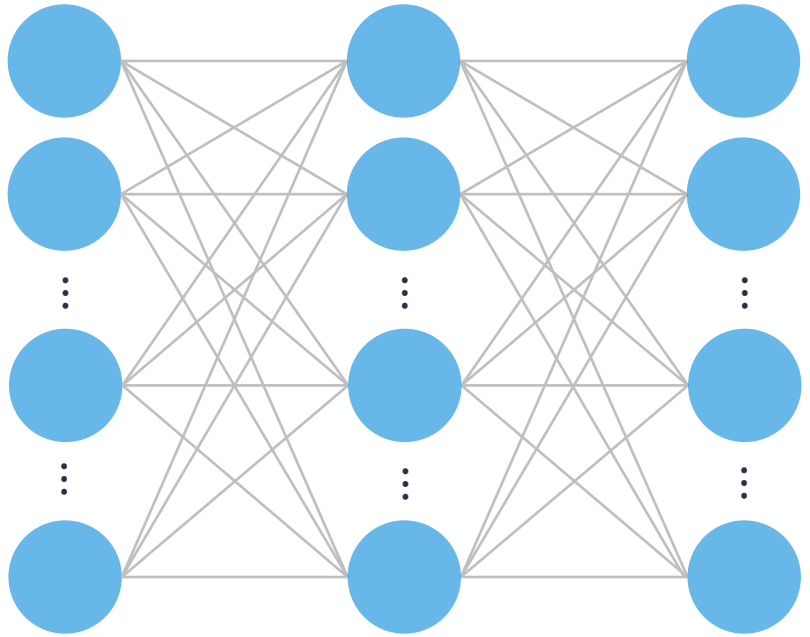
- Max pooling layer 2D
  - `torch.nn.MaxPool2d(kernel_size, stride, padding)`
  - Input: $(N, C_{in}, H_{in}, W_{in})$
  - Output: $(N, C_{out}, H_{out}, W_{out})$

$$H_{out} = \frac{H_{in} + 2 \cdot padding[0] - kernel\_size[0]}{stride[0]} + 1$$

$$W_{out} = \frac{W_{in} + 2 \cdot padding[1] - kernel\_size[1]}{stride[1]} + 1$$

# CNN – Fully-connected layer

- Operates on flattened input
- Every input influences every output
- At the end of CNN architecture
- PyTorch
  - `torch.nn.Linear(in_features, out_features)`
  - Input: $(N, H_{in})$
  - Output: $(N, H_{out})$



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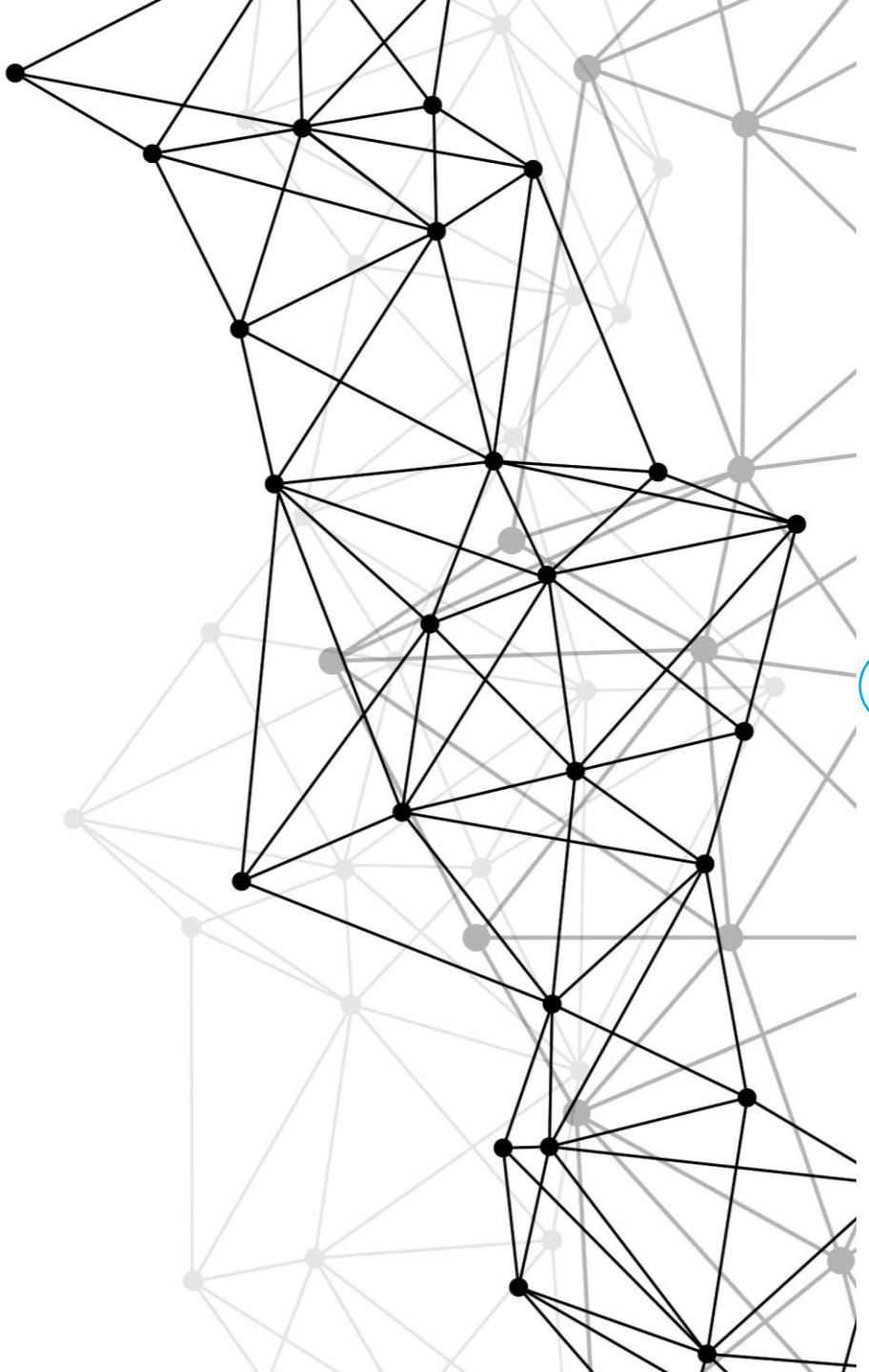
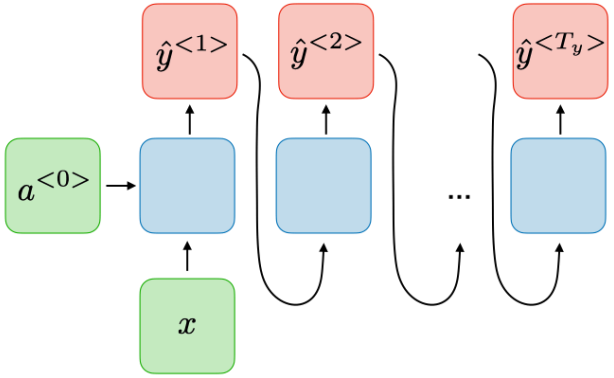


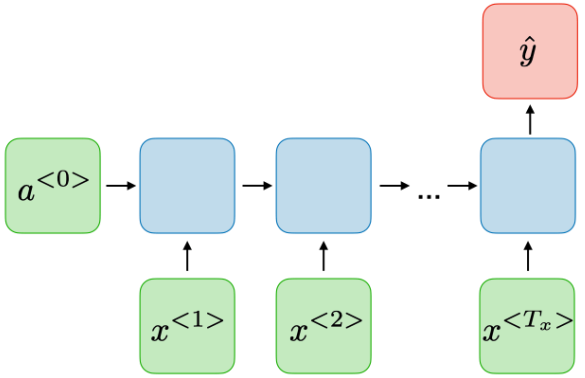
# Recurrent neural network (RNN)

- Problems dealing with sequential or time series data
- Capture temporal representation
- Typical applications: language translation, natural language processing, speech recognition, image captioning
- Take into account historical information
- Possibility of processing input of any length
- Model size not increasing with input size
- Weights are shared across time

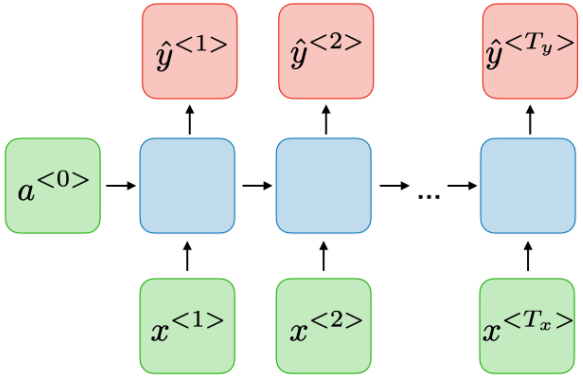
# RNN different applications



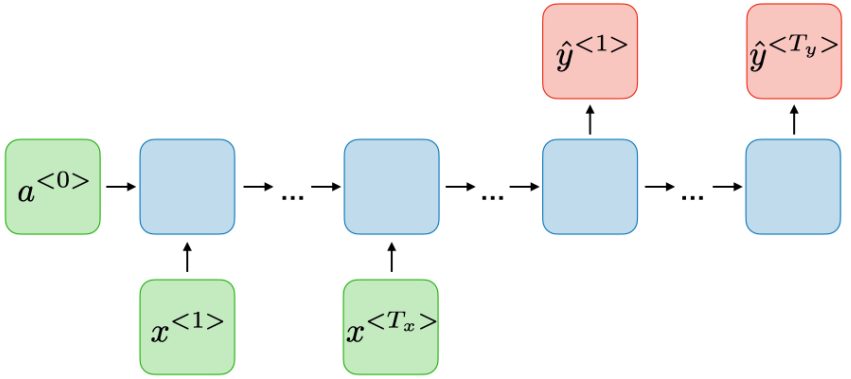

One-to-many
$T_x = 1, T_y > 1$




Many-to-one
$T_x > 1, T_y = 1$




Many-to-many
$T_x = T_y$




Many-to-many
$T_x \neq T_y$

# RNN

- Empirical loss
  - Loss of all time steps defined based on loss at every time step

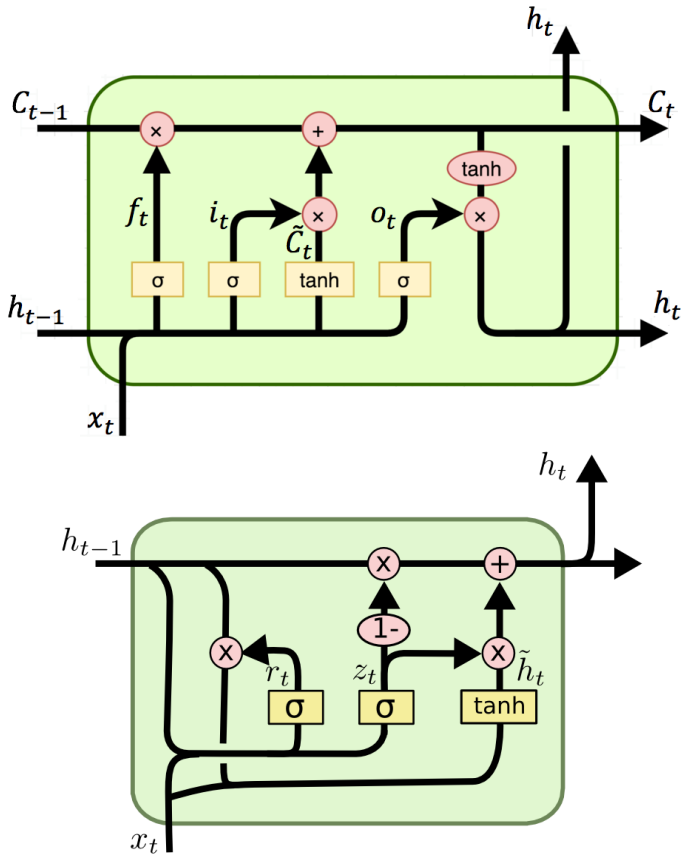
$$L(y, \hat{y}) = \sum_{t=1}^{T_y} L(y^{<t>}, \hat{y}^{<t>})$$

- Backpropagation through time
  - Done at each point in time

$$\frac{\partial L^{(T)}}{\partial W} = \sum_{t=1}^{T} \frac{\partial L}{\partial W}\bigg|_{(t)}$$

# RNN variants

- Long Short-Term Memory units (LSTM)
  - Address the problem of long-term dependency
  - Internal memory cell
  - 3 gates (input gate, output gate and forget gate) to control the flow of information
- Gated Recurrent Unit (GRU)
  - Fewer parameters and faster training
  - 2 gates (reset gate and update gate) to control how much and which information to retain



# RNN variants

| Aspect | LSTM | GRU |
|---|---|---|
| Architecture | Memory cell and three gates (forget, input, output) | Two gates (update, reset) |
| State handling | Separate cell state and hidden state | Single combined hidden state |
| Complexity | More parameters (computationally intensive) | Fewer parameter (efficient) |
| Performance | Handles long-term dependencies better | Performs well on shorter sequences or datasets |
| Training time | Slower due to complexity | Faster due to simplicity |
| Uses cases | Tasks with complex dependencies like time-series forecasting, speech recognition, etc. | Tasks with simpler dependencies like text classification, sentiment analysis, etc. |

# RNN – PyTorch

- LSTM
  - *torch.nn.LSTM(input_size, hidden_size, num_layers, bidirectional)*
  - Input: $(L, N, H_{in})$
  - h_0: $(D * num\_layers, H_{out})$
  - c_0: $(D * num\_layers, H_{cell})$
  - Output: $(L, N, D * H_{out})$
- GRU
  - *torch.nn.GRU(input_size, hidden_size, num_layers, bidirectional)*
  - Input: $(L, N, H_{in})$
  - h_0: $(D * num\_layers, H_{out})$
  - Output: $(L, N, D * H_{out})$

$N$: batch size
$L$: sequence length
$D$: 2 if bidirectional and1 otherwise
$H_{in}$: input_size
$H_{cell}$: hidden_size
$H_{out}$: hidden_size

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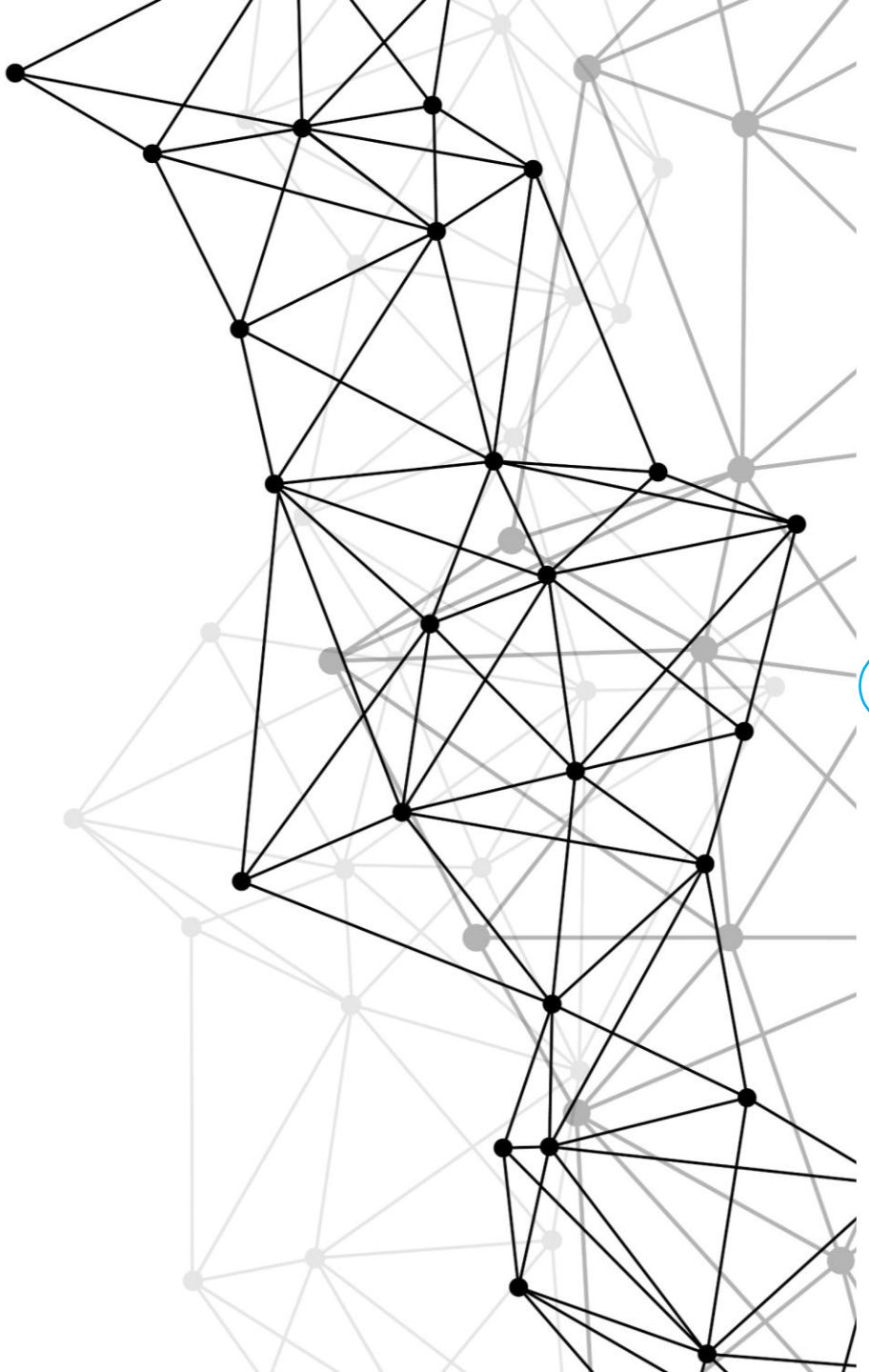
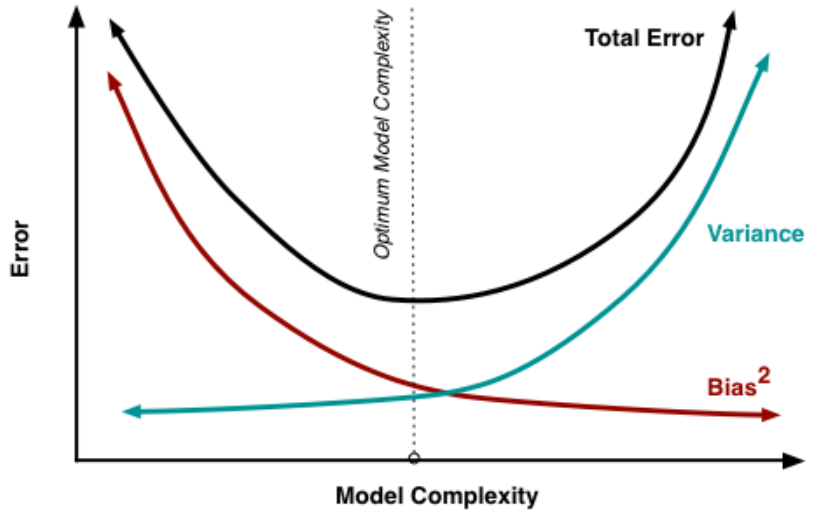


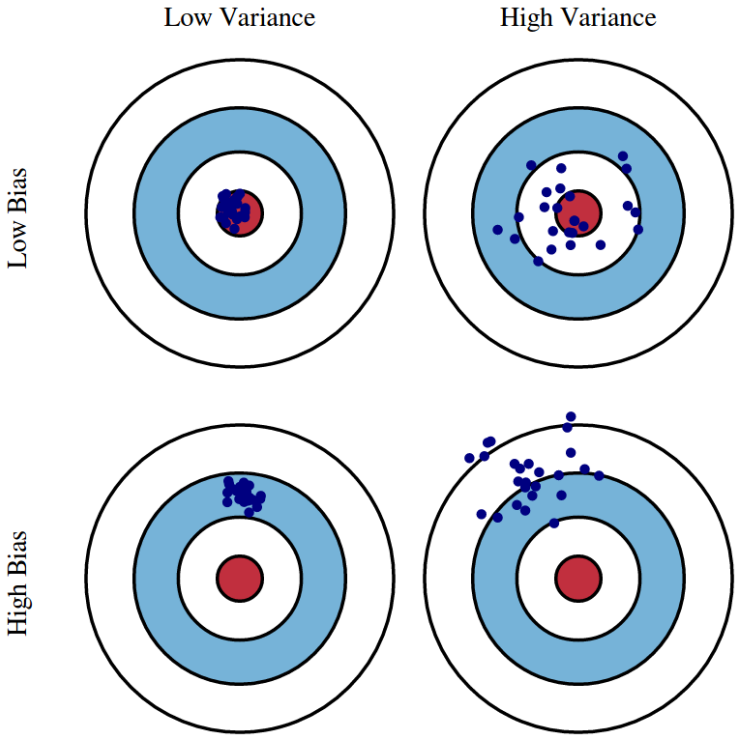
# Bias-variance trade-off

- Bias → Difference between average prediction of model and correct value
- Variance → How much an estimate varies around its average
- Simple model → High bias and low variance
- Complex model → Low bias and high variance

$$Err(x) = \left(E\left[\hat{f}(x)\right] - f(x)\right)^2 + E\left[\left(\hat{f}(x) - E\left[\hat{f}(x)\right]\right)^2\right] + \sigma_e^2$$

$$Err(x) = bias^2 + variance + noise$$



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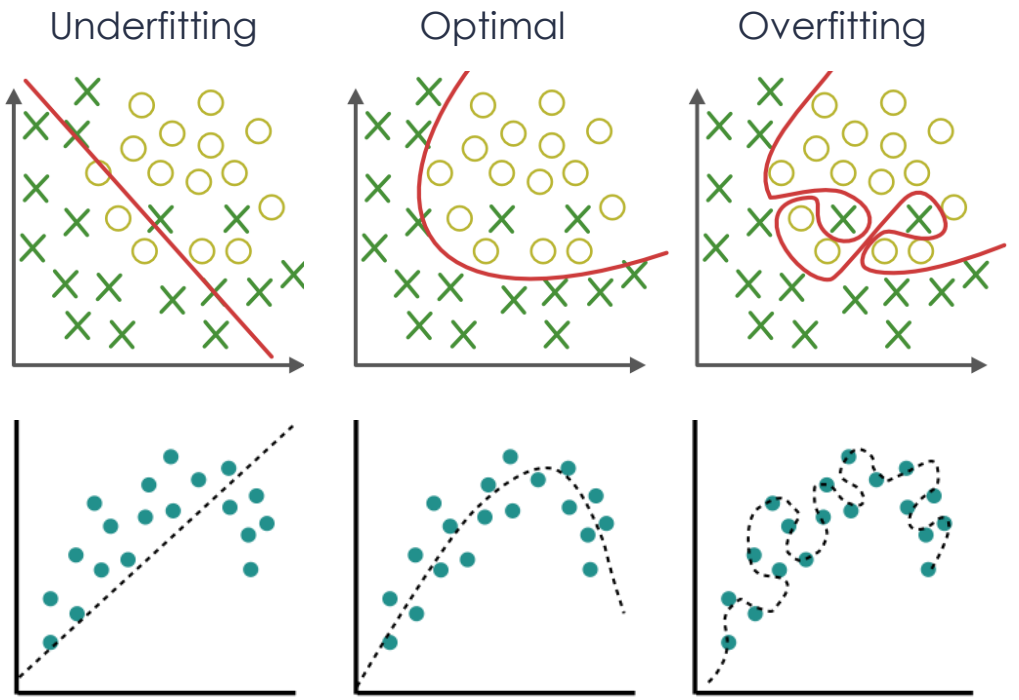
# Bias-variance trade-off

- Underfitting
  - Model is unable to capture the underlying pattern of the data
  - High bias and low variance
  - Small amount of data, or linear model for non-linear data
  - High error in training and test sets
- Overfitting
  - Model captures the noise with the underlying pattern in data
  - Low bias and high variance
  - Complex model
  - Low error in training set and high error in test set



# Prevent overfitting

1. Get more data
2. Use a model that has the right capacity
   - Enough to fit data
   - Not too much to fit noise
   - Parameter tuning
3. Average many different models
   - Models with different forms
   - Trained on different subsets
4. Use specific regularization techniques

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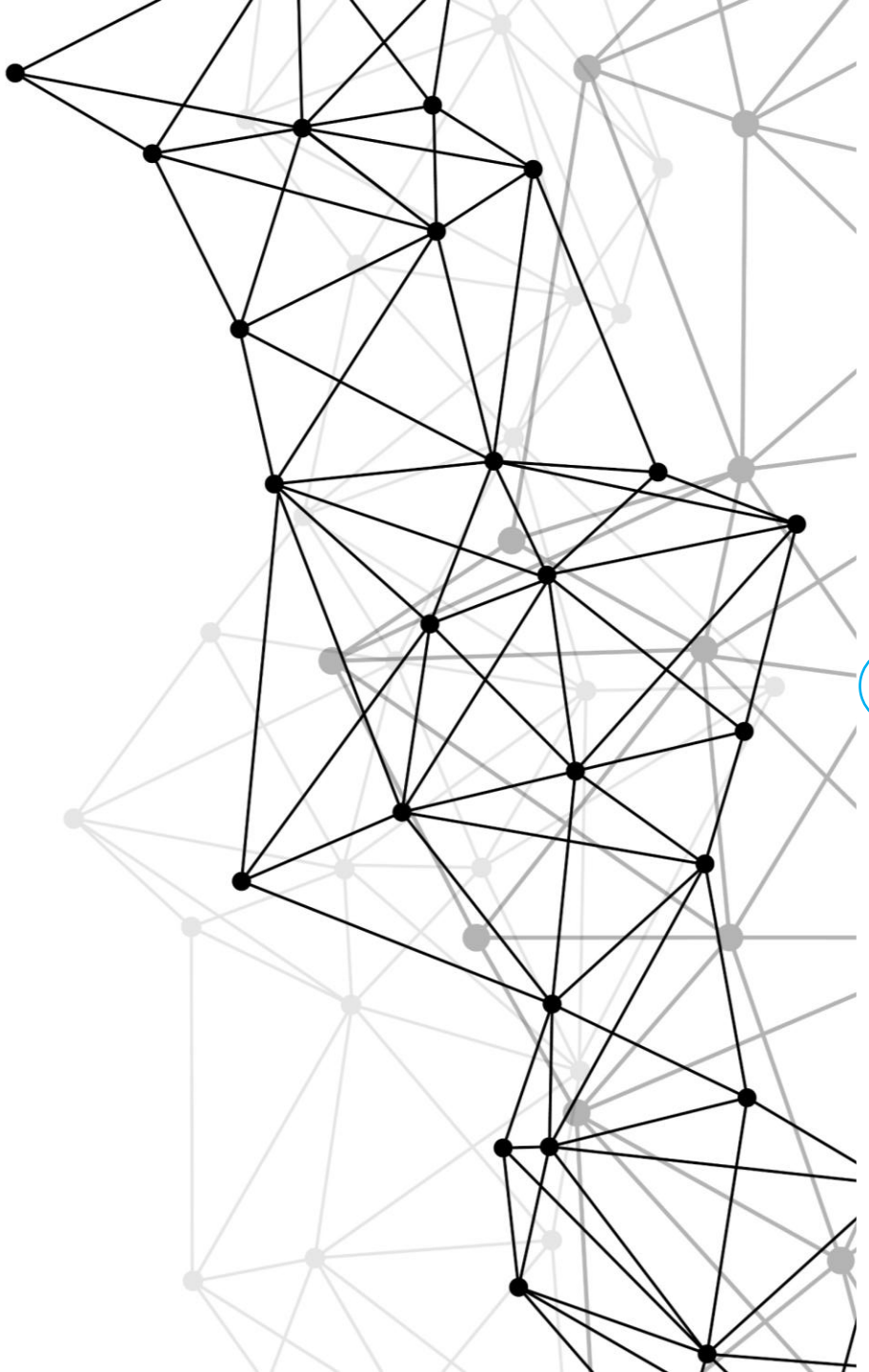
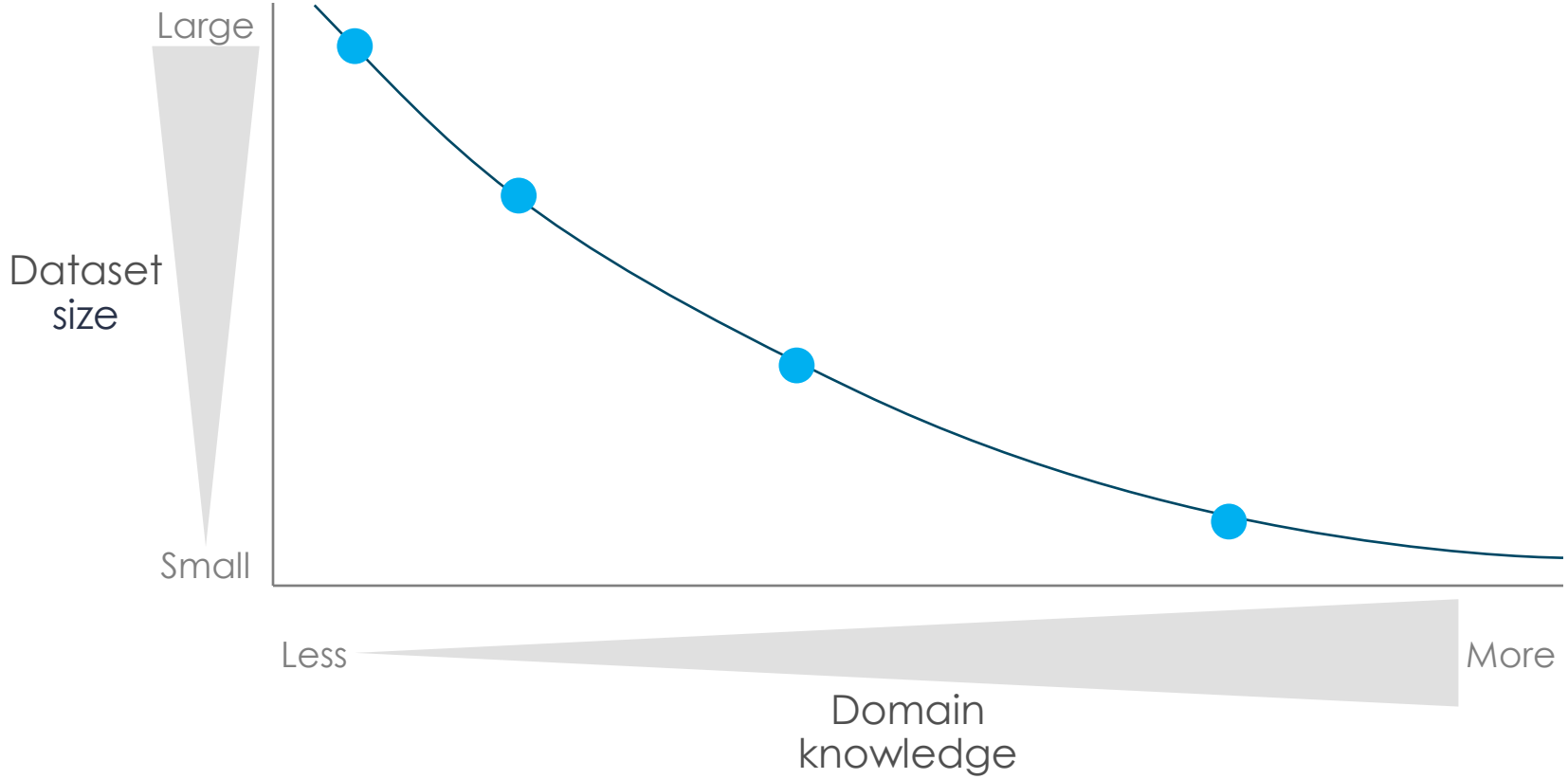


# Data

- Very data hungry
- Rule-based and neural network both need data
- Why is so much training data necessary for neural network?



# Data

- Quality vs quantity
- Model is as good as the data provided
- Data covering the entire solution space