EE512 – Applied Biomedical Signal Processing

Practical session – SVD

Correction

Experiment 1: 12-leads ECG

QUESTION #1.1: Limb leads and augmented limb leads

a) Explain the singular values of the six first columns, and of the last 6 columns.

The SVD of the first 6 columns (leads I, II, III, VR, VL, VF) yields to :

0.4396 0.3110 0.0000 0.0000 0.0000 0.0000

Only two singular values are non-zero. Normal, in the standard ECG three of the signals are just linear combinations of three other ones (electrodes I, II, III), which are themselves dependent with respect to the potential reference.

The same operation on the last 6 columns yields:

1.2328 0.7696 0.3568 0.2150 0.1709 0.1097

Which indicates a much weaker dependence as each signal comes from a different electrode.

b) Compute the effective rank with a threshold of 0.98.

You can compute the effective rank using the following implementation:

*csum= np.cumsum(S1*S1)/np.sum(S1*S1)*

which gives for the 6 first colums:

[0.7026 1.0000 1.0000 1.0000 1.0000 1.0000]
And thus an effective rank around 2.

which gives for the 6 last colums:

[0.6532 0.9077 0.9624 0.9823 0.9948 1.0000]

And thus an effective rank around 4.

Experiment 2: Singular values and process complexity

```
Singular values before: 1.000000, 0.000005, 0.000003, 0.000000
Singular values during: 1.000000, 0.102895, 0.073738, 0.067564
Singular values after: 1.000000, 0.208506, 0.187214, 0.088356
```

QUESTION #2.1: Data pre-processing

a) *Explain from their singular values the relationship between EEG leads before stimulation.*

Before stimulation, leads decomposition shows the 3 smallest singular values suggesting the signals between the leads are very correlated and so very similar.

b) *Implement pre-processing of the EEG signals before stimulation to normalize the data. Then re-compute the singular value decomposition of the pre-processed signals.*

Pre-processing consists in removing the mean and normalizing the variances of the EEG signals before stimulation:

pre_proc_signal = scipy.stats.zscore(signal)

The singular value after normalization gives:

```
Singular values before: 1.000000, 0.827413, 0.270988, 0.232586
Singular values during: 1.000000, 0.101275, 0.075675, 0.066337
Singular values after: 1.000000, 0.214136, 0.174770, 0.098511
```

c) *Explain the relationship between EEG leads based on these new singular values, and the difference observed with the ones computed in a).*

After zero-meaning and normalization, the singular values of the ECG signals before stimulation appears much higher suggesting that signals between the leads are very different and so independent from each other.

Subtracting the mean values allows one to cancel a common factor from the columns (which could have an influence on the singular values). Normalization (all signals scaled to unit variance) suppresses effects due to amplitude differences in the signals.

QUESTION #2.2: Correlation interpretation

a) *Based on the resulting singular values are the signals more correlated before, during or after stimulation.*

During stimulation, the singular values are very low compared to the maximum singular value. When evaluating the effective rank of the matrix during the stimulation we see that the rank is 1, suggesting the 4 leads are strongly correlated.

b) *Explain in terms of correlation the difference between each case ('before', 'during', 'after').*

First we can notice that all singular values are significantly non zero prior to stimulation. This indicates the signals are not linearly dependent, and thus not strongly correlated (activities on 4 electrodes).
Then, the 3 smallest singular values are indeed much smaller during stimulation. The signals become a lot more similar during stimulation, because the latter influences all electrodes.
Finally, we see that the situation after stimulation is in between the two former situations. It indicates that the effect of stimulation is still partly present.

Experiment 3: Drift cancellation and frequency component extraction

QUESTION #1: Drift cancellation

Find a window length L, so that the first component obtained using the function SSA_decomposition corresponds to the long-term drift.

The SSA decomposition shows strong first component with a very low frequency corresponding to a long-term drift with a window length of 10 seconds (Figure 1). One can observe the first component signal follows the baseline of the raw signal (Figure 2).

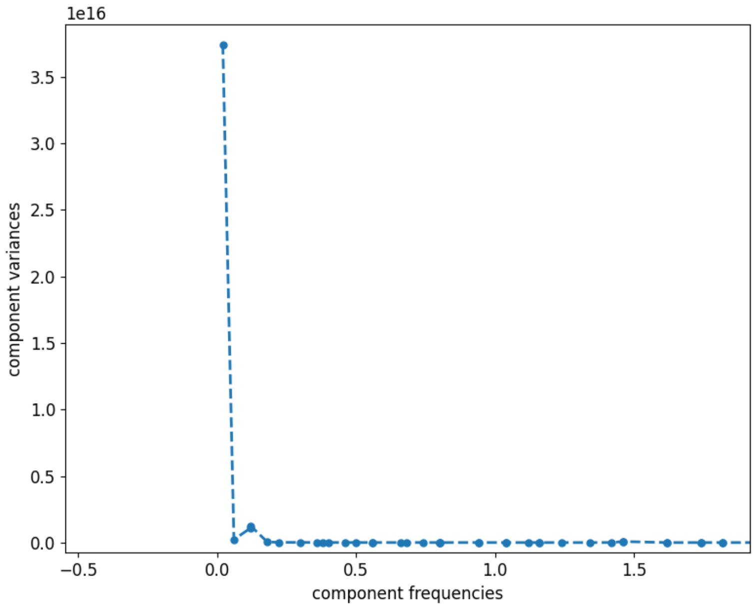

Figure 1. SSA components singular values and their corresponding main frequency.

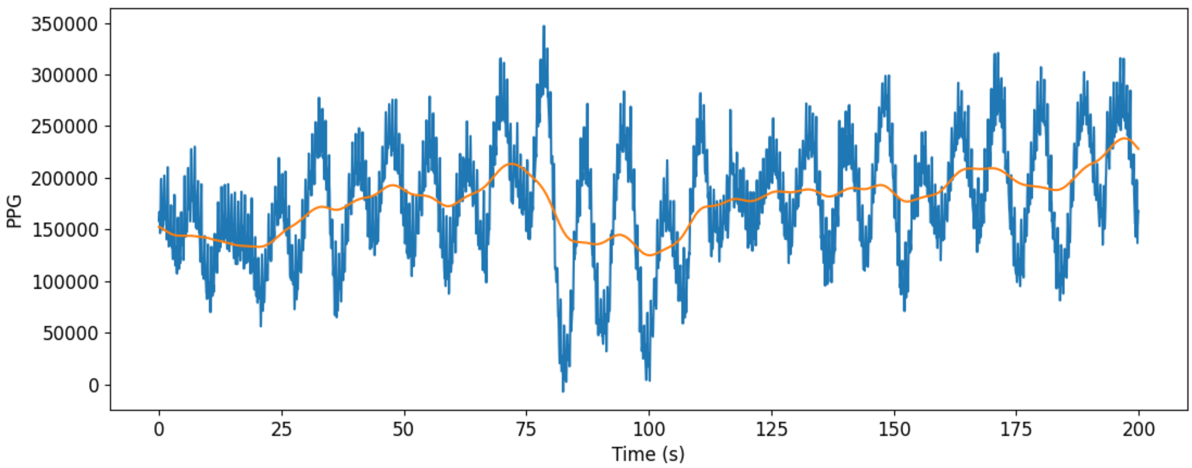

Figure 2. First component signal (orange) and ppg signal (blue).

QUESTION #2: Components extraction

The baseline is subtracted from the signals and decomposed using *SSA_decomposition*.

a) Find the component(s) related to the respiration in the PPG signal. Plot their sums.

The two first components at 0.12 Hz correspond to the respiration (Figure 3), as we can observe when plotting their sum on the raw signal (Figure 4).

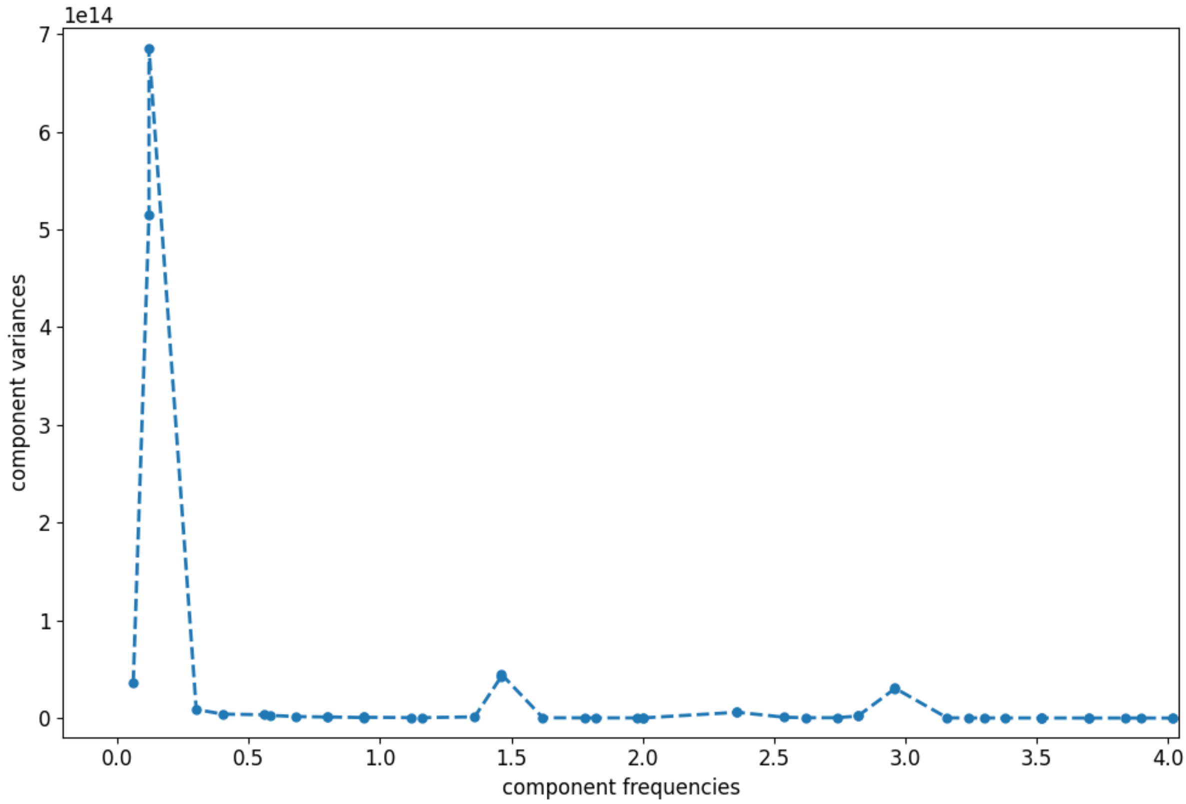

Figure 3. SSA components singular values and main frequency of signal without long term drift.The two first components correspond to the respiration.

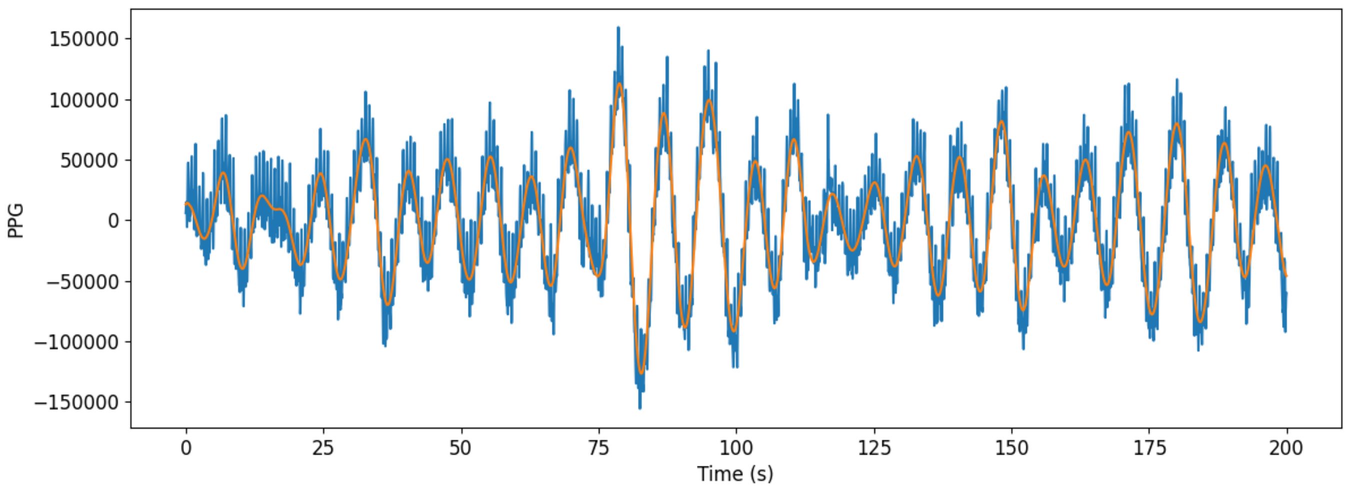

Figure 4. Respiratory signal without long term drift (blue), and the respiratory bursts signal (orange).

b) Find the component(s) related to the cadence in the accelerometer signal and give the corresponding frequency in steps per minute (Hint: in general cadence is about 180 steps per minute).

The SSA decomposition of the accelerometer signal (Figure 5) shows a strong component around 3Hz corresponding to the cadence C = 3 * 60 =180 steps per minute. We distinguish the two strong first components at this frequency.

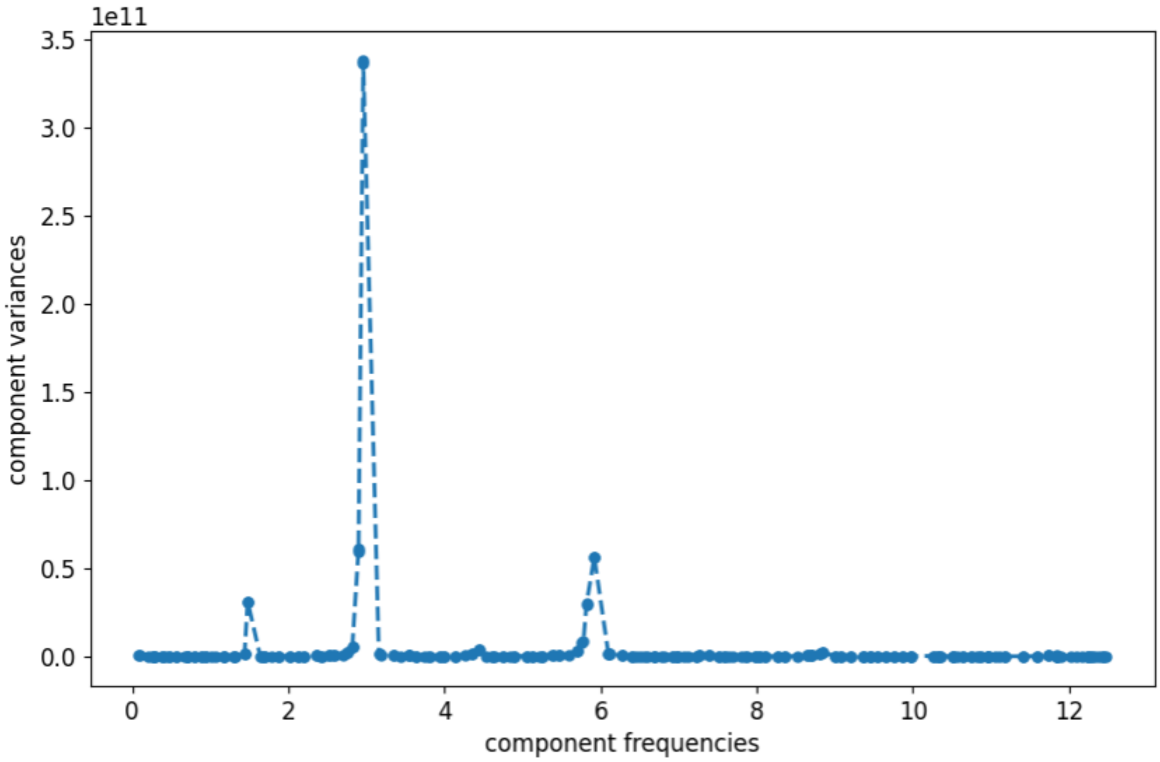

Figure 5. SSA components singular values and main frequency of the accelerometer signal..

c) Explain where the components of the SSA decomposition of the accelerometer signal at 1.5Hz and 6Hz come from. (Hint: c.f. Module 04 Time-Frequency Lab)

The component at 1.5Hz observed in the accelerometer, which shows less variability than the cadence component, corresponds to the movement of the arm which is acquired by the accelerometer at the wrist but with less amplitude than the cadence itself. The 6Hz component is the second harmonic of the cadence, at the double of the cadence frequency.

QUESTION #3.3: Heart rate estimation

The respiration components are subtracted from the PPG signal without baseline and decomposed using *SSA_decomposition*. The resulting signal is now mainly driven by the running cadence and the heart rate.

a) Based on the components observed from the accelerometer signal analysis, find the component related to the heart rate in the ppg signal, and give the corresponding heart rate in bpm (beats per minute)?

From the PPG SSA analysis (Figure 6), we see four main frequencies that represent our basic signal: low frequencies components (< 0.5Hz), 1.5Hz components, ~2.5Hz components, and 3Hz components.

The low frequency components correspond to remains of baseline and respiration. Though we remove the baseline and respiration components, part of this information is still in the resulting PPG signal but with much less impact on the signal variability.

Based on the analysis of the accelerometer signal, we know that the arm motion and the cadence respectively correspond to components at 1.5Hz and 3Hz.

Therefore, the heart rate component corresponds to the ~2.5Hz component, which gives a heart rate at 2.5*60 = 150 bpm.

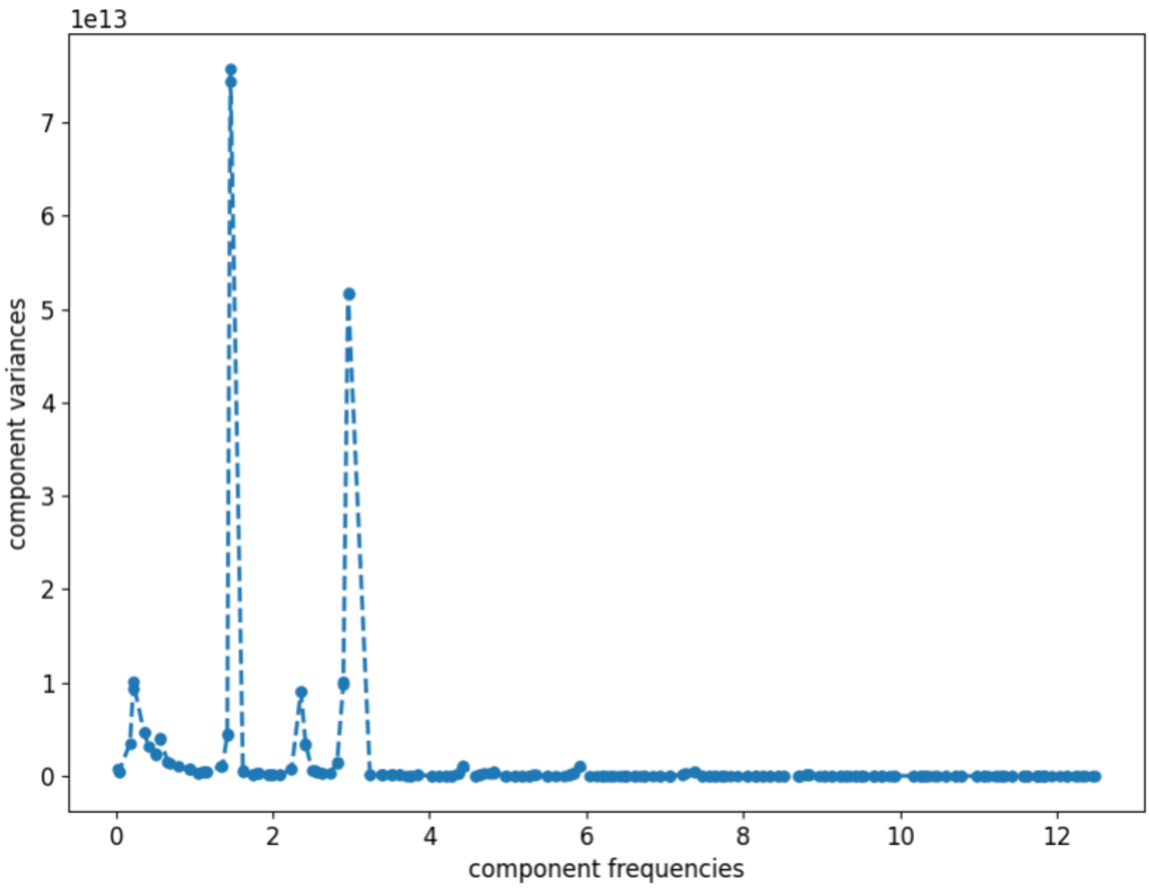

Figure 6. SSA analysis of the PPG signal after removal of baseline and respiration.