Battery (equivalent circuit) models

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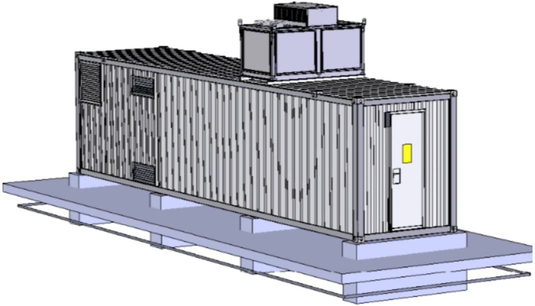
Recap of (second-to-) last class

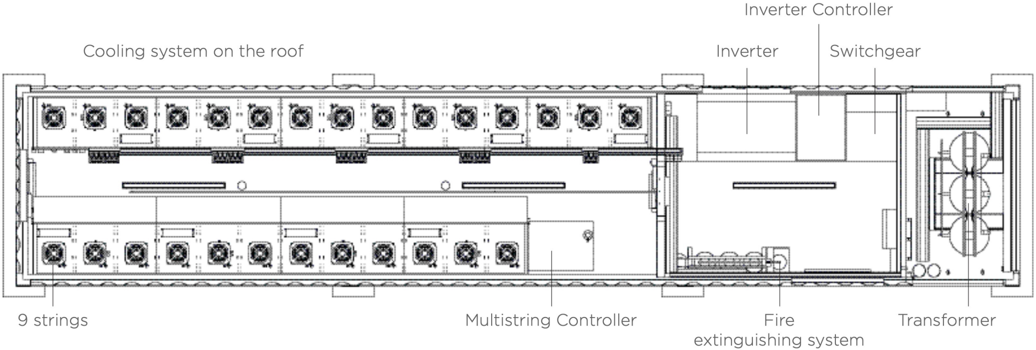
- We started the discussion on Battery Management Systems (BMS), which we will continue today in the second part of the class.
- We presented the internal structure of a battery system and illustrated its main components and their functionalities: battery pack, DC bus, and power converter.
- We then focused more specifically on battery cells, discussing in particular:
  - The cell voltage and its dependence on the state of charge (SoC).
  - A simple battery cell model consisting of a voltage source in series with an internal resistance (we will explore this model in more detail today).
  - The introduction of a new definition of state of charge, the Coulombic state of charge, which differs from the previously discussed Energetic state of charge model.

Bonus content: Working example, the BESS of EPFL



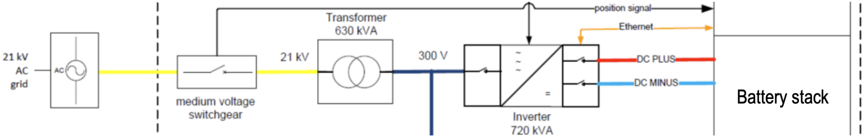
Power rating and energy capacity: 0.72 MVA, 0.56 MWh.
Technology: Lithium-titanate-oxide (LTO).
Container size: 12 x 3 x 2 m, $\approx$ 20 tonnes.

Bonus content: Inside the container



Three main systems: battery and battery management system, grid-interfacing components, and auxiliaries (fire-extinguishing system, air cooling, lightning)

Bonus content: BESS's electrical stack



From right to left: battery stack , DC (direct current) bus, Power converter, Step-up transformer, Circuit breaker, grid connection.

Cell stack

Composition: cell ⟶ module ⟶ rack ⟶ stack.

Cell information:

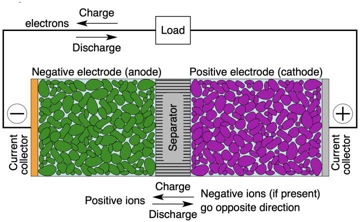
- Nominal voltage: 2.3 V
- Min/max voltage: 1.7 V, 2.7 V
- Current capacity: 30 Ah
- Similar cell datasheet

- Module: 20s3p (20 cells in series, 3 in parallel)
- Modules in a rack: 15s
- Stack: 9 modules in parallel
- Total cells: $20 \times 3 \times 15 \times 9 = 8'100$
- System voltage: $600 \div 800$ V (DC bus)

Series composition of cells increases the voltage; parallel composition increases the current capacity (Ah).

Because the energy capacity (kWh) is voltage times the current capacity, it scales with the number of cells regardless of composition (series | parallel).

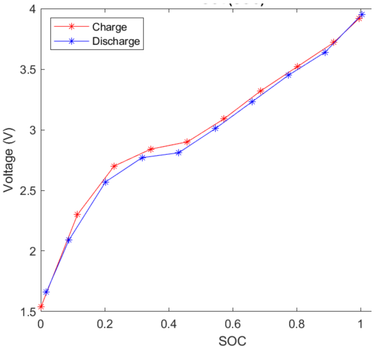
Bonus content: Internal structure of a battery cell



- A battery cell consist of 2 electrodes, a separator to keep them far apart, and an ionic conductor electrolyte.
- Batteries are realized by coupling an oxidizing element with a reducing element.
- Voltage levels are limited by the electrolyte, which decomposes when exposed to a large difference of potential (eg, 2 V for water-based electrolyte, as in lead-acid batteries)
- Higher voltage levels are achieved by combining cells in series, externally.

Recap: Cell voltage vs. state of charge (SOC)

The cell voltage varies with the cell state-of-charge, reflecting the different concentration of Li-ions in the electrodes following the charge and discharge of the battery.



The two (red and blue) curves (one referring to the charging process, the other to the discharging process) are to verify voltage hysteresis phenomena, which appears to be very limited.

In the laboratory session, your task will be to identify this relation in a battery cell by way of measurements.

Coulombic state of charge model for a lithium cell

For lithium-ion electrochemistries, the state of charge (SOC) can be approximated as:

$$SOC(t) = SOC(0) + \frac{1}{C_{nom}} \int_0^t i(\tau) d\tau$$

where C_{nom} is the cell energy capacity in Ah and $i(t)$ is the charging current over time. The charging efficiency is large (≥ 0.99) and can be omitted.

In discrete time:

$$SOC(t+1) = SOC(0) + \frac{\Delta_T}{C_{nom}} \sum_{\tau=0}^{t} i(\tau)$$

With abuse of notation, now t denotes the index of the discretized time interval with a step of Δ_T in hours.

Modelling battery voltage

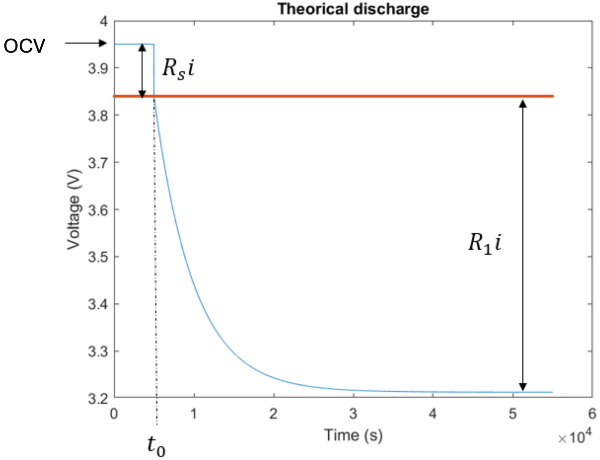
- Battery voltage reflects the concentration of Li-ions in the battery electrodes and electrolytes. As this concentration changes over time (eg, due to charge and discharge), voltage will also change.

- If one could model the interaction of each Li-ions particle within the battery, one could build a very detailed model of the voltage. Complex endeavor due to the many particles and interactions on three-dimensional (3D) surfaces (partial differential equations in time and space).

- Physical modeling of lithium-ion cells has been tackled with approximated models that simplify complex interactions in 3D space on 2D surfaces (pseudo 2D models), and single particle models, that assume that the electrodes (formed by a complex crystal structure) can be approximated as a single element.

- A (computationally tractable) alternative to physical models is Equivalent Circuit Models (ECMs). They (try to) describe the voltage of the battery as a function of the charging/discharging current, without however offering much interpretability of the parameters.

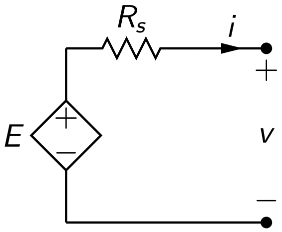
Voltage dynamics as a function of the charging/discharging current



Voltage response of a battery cell to a stepwise variation of the discharging current.

Equivalent Circuit Models (ECMs)

ECM of battery cell: internal resistance model



Symbols:

- i: cell current, supplied to an external load if positive, consumed (provided by a power supply) if negative. Measurable.
- E: cell internal voltage. It is a modelling abstraction and is not measurable (it can be estimated, though).
- v: cell terminal voltage. This is the voltage measurable at the cell terminals.
- R_s: cell resistance

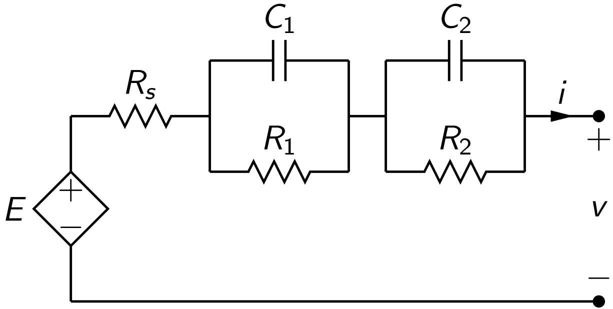
The cell internal voltage E depends on the battery SOC (as shown later), and that's why in the schematic is denoted as a variable voltage source.

A recall from electric circuit theory: the open-circuit voltage (OCV) denotes the value of the voltage v when $i = 0$.

ECM of battery cell: two-time-constant model

To capture voltage dynamics, one should add “dynamic” elements to the circuit above. In linear circuit theory, these are inductors and capacitors.

Two RC branches added in series to the series resistance are able to capture reasonably well voltage dynamics. This is known as“TTC model” (two-time-constant model).



Two-time-constant model: state-space model

The governing equations of the TTC model, in state-space form, are:

$$\begin{aligned} \dot{x} &= \mathcal{A}x + \mathcal{B}u \\ v &= \mathcal{C}x + \mathcal{D}u \end{aligned}$$

where the state and input vectors are

$$x = \begin{bmatrix} v_{C_1} & v_{C_2} \end{bmatrix}, u = \begin{bmatrix} i & 1 \end{bmatrix}^T$$

and the state-space matrices A, B, C, and D are:

$$\mathcal{A} = \begin{bmatrix} -1/(R_1 C_1) & 0 \\ 0 & -1/(R_2 C_2) \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 1/C_1 & 0 \\ 1/C_2 & 0 \end{bmatrix}, \mathcal{C} = \begin{bmatrix} 1 & 1 \end{bmatrix}, \mathcal{D} = \begin{bmatrix} R_s & E \end{bmatrix}.$$

Self-assessment of understanding and exercises

1. Explain the difference between the SOC models seen in weeks 1 and in this slide pack in terms of meaning, input, and parameters.
2. Losses (present in week 1's SOC model) are not present any longer in this slides' SOC model ($\eta \approx 1$). Explain where you expect them to appear again.
3. Explain the qualitative difference between a physical model of the cell voltage and an equivalent circuit model.
4. Explain the limitations of the internal-resistance model of the battery (slide 12) when applied to model the behaviour of the cell voltage of slide 10.
5. Demonstrate formally (ie, by way of equations) that a capacitor, in steady-state conditions, can be replaced by an open circuit. (not seen in this class; recall elements from linear circuit theory courses).
6. In the internal-resistance model, the cell internal voltage E can be estimated easily by measuring the battery terminal voltage v in open-circuit conditions (so called OCV): why? Can you say the same about the TTC model? With the TTC model, which conditions do you need to make sure that v reflects E?
7. (Extra:) Prove that the state-space formulation of the TTC model shown in slide 13 is as given in slide 14.