# EE465 - W9
# PHASE LOCKED LOOP
# PLL - PART 1

**Prof. D. Dujic**

École Polytechnique Fédérale de Lausanne

Power Electronics Laboratory

Switzerland

To be able to correctly exchange energy with the grid, knowledge about grid conditions is required:

- grid voltages, frequency, phase
- grid conditions are not constant and various disturbances are to be expected
- grid monitoring and synchronization must be fast and robust against disturbances
- phase-angle of the grid voltage is required to transform grid variable from $abc$-frame to $dq$-frame

**Phase Locked Loop** (PLL) is effective way for grid monitoring and synchronization

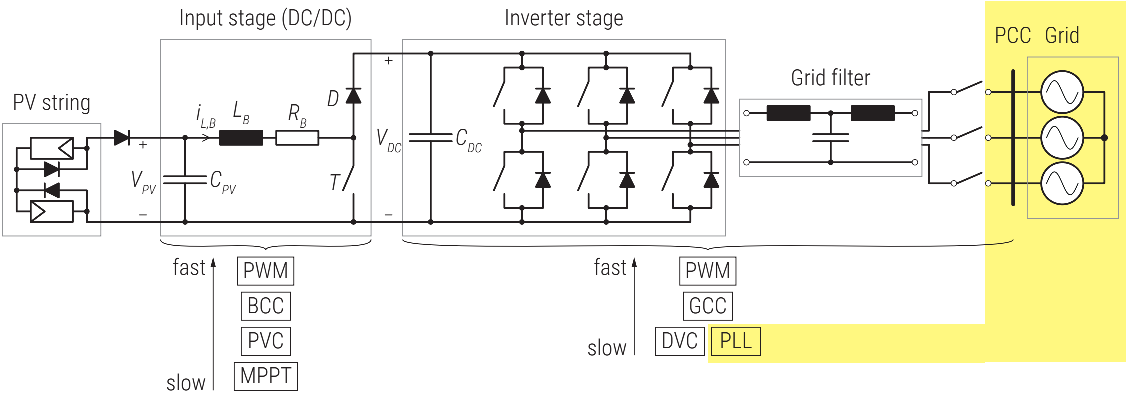


**Figure 1** PV double-stage grid connected converter.

# GRID SYNCHRONIZATION

*Various techniques are available...*

# GRID SYNCHRONIZATION TECHNIQUES

Normally, we are mostly interested into grid voltage magnitude and phase, assuming that frequency is relatively stable

Yet, often grid may be polluted with other harmonic components and they need to be correctly detected and tracked

Grid synchronization techniques can be classified into:

**Frequency-domain detection methods**: mostly based on some kind of form of Fourier Transform implementation

- the Fourier series
- the Discrete Fourier Transform (DFT)
- the Recursive Discrete Fourier Transform (RDFT)
- frequency domain analysis assume that the fundamental frequency of the processed signal is well-known

**Time-domain detection methods**: based on some kind of adaptive loop that is able to track components of interest of input signal

- the Phase-Locked Loop (PLL)
- the Frequency-Locked Loop (FLL)

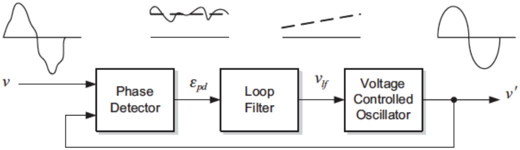


**Figure 2** Basic structure of a PLL.

Consider basic trigonometric relations:

$$A\cos(n\omega t)\cos(n\omega t) = A\cos^2(n\omega t) = \frac{A}{2} + \frac{A\cos(2n\omega t)}{2}$$

$$B\sin(n\omega t)\sin(n\omega t) = B\sin^2(n\omega t) = \frac{B}{2} - \frac{B\cos(2n\omega t)}{2}$$

Multiplication of unknown sine/cosine signal by an unitary since/cosine signal gives:

- DC part - if the frequencies are the same (otherwise is zero)
- AC part at double the frequency, if the frequencies are the same

We could probe unknown signal with unitary sine/cosine signals to detect its amplitude, frequency and phase

A generic periodic signal $v(t)$ can be expressed as a sum of the following terms (*Fourier*):

$$v(t) = a_0 + \sum_{n=1}^{\infty}(a_n cos(n\omega t) + b_n sin(n\omega t))$$

where:

$$a_0 = \frac{1}{T}\int_0^T v(t)dt$$

$$a_n = \frac{2}{T}\int_0^T v(t)\cos(n\omega t)dt = \frac{1}{\pi}\int_{-\pi}^{\pi} v(\theta)\cos(n\theta)d\theta$$

$$b_n = \frac{2}{T}\int_0^T v(t)\sin(n\omega t)dt = \frac{1}{\pi}\int_{-\pi}^{\pi} v(\theta)\sin(n\theta)d\theta$$

Based on the previous slide, selective band-pass filter can be implemented

- input signal is multiplied by sine/cosine basic functions
- constant grid frequency is assumed
- known grid voltage magnitude is assumed
- $n$ defines the order of harmonics to be extracted at the filter output

The amplitude and the phase-angle at frequency of interest are:

$$V_n^{'} = V_n \angle \theta_n \begin{cases} V_n = \sqrt{a_n^2 + b_n^2} \\ \theta_n = \arctan \dfrac{b_n}{a_n} \end{cases}$$

The mean value of the signal is obtained using low-pass filter (LPF)

- the lowest frequency of interest (e.g. grid frequency, 50Hz)
- at LPF input, the lowest frequency is double of that (e.g. 100Hz)
- LPF cut-off frequency should be order of magnitude lower (e.g. 10Hz)
- this implies very slow dynamic response of the system
- this may not be sufficient for real-time grid monitoring

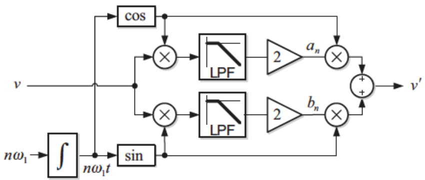


**Figure 3** Adaptive filter based on Fourier series decomposition.

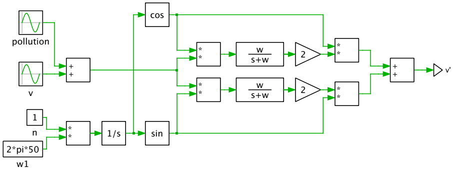


**Figure 4** PLECS implementation (example).

Using Euler's identities, the Fourier's coefficients can be calculated as

$$a_n = \frac{2}{T}\int_0^T v(t)\cos(n\omega t)dt = \frac{2}{T}\int_0^T v(t)\frac{e^{jn\omega t}+e^{-jn\omega t}}{2}dt$$

$$b_n = \frac{2}{T}\int_0^T v(t)\sin(n\omega t)dt = \frac{2}{T}\int_0^T v(t)\frac{e^{jn\omega t}-e^{-jn\omega t}}{2j}dt$$

Defining complex coefficient $c_n$:

$$c_n = \frac{1}{2}(a_n - b_n) = \frac{1}{T}\int_0^T v(t)e^{-jn\omega t}dt$$

we have ($c_n^*$ is the complex conjugate of $c_n$):

$$v(t) = a_0 + \sum_{n=1}^{\infty} c_n e^{jn\omega t} + \sum_{n=1}^{\infty} c_n^* e^{-jn\omega t}$$

Finally, considering $c_0 = a_0$, and $c_n^*|_{n<0} = c_n|_{n>0}$ we have:

$$v(t) = a_0 + \sum_{n=1}^{\infty} c_n e^{jn\omega t} + \sum_{n=1}^{\infty} c_n^* e^{-jn\omega t} = \sum_{n=0}^{\infty} c_n e^{jn\omega t} + \sum_{n=-1}^{-\infty} c_n e^{jn\omega t} = \sum_{-\infty}^{\infty} c_n e^{jn\omega t}$$

This is the compact complex form of the Fourier series (positive and negative frequencies are considered)

If we consider pulse function defined as:

$$v(t) = \begin{cases} 0, & -T \leq t < -\frac{\tau}{2} \\ \frac{1}{\tau}, & -\frac{\tau}{2} \leq t \leq \frac{\tau}{2} \\ 0, & \frac{\tau}{2} < t \leq T \end{cases}$$

Complex coefficients $c_n$ can be calculated as defined before:

$$c_n = \frac{1}{T}\int_0^T v(t)e^{-jn\omega t}dt$$

If we assume that:

- period $T$ is gradually made longer and pulse duration $\tau$ is kept constant
- value of the integral remains constant
- while the complex coefficients $c_n$ gradually become smaller $(1/T)$
- as $T \to \infty$, $v(t)$ becomes aperiodic and all $c_n$ are equal to 0

With $T = \infty$ we can redefine

$$V(\omega) = \mathcal{F}(v(t)) = \int_{-\infty}^{\infty} v(t)e^{-j\omega t}dt$$

This is known as *Fourier Transform* and it allows for continuous periodic and aperiodic functions in time domain to be represented as continuous functions in the frequency domain.

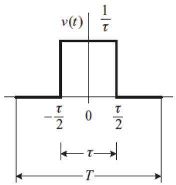


**Figure 5** Rectangular pulse - Time domain.

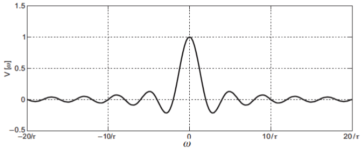


**Figure 6** Fourier Transform of a rectangular pulse - Frequency domain.

# THE DISCRETE FOURIER TRANSFORM

In a digital signal world, the integral becomes the sum of finite number of samples, equally spaced in time

Discrete input signal is defined as ($k = 0, 1, \ldots N - 1$):

$$v(kT_s) = v(t)\delta(t - kT_s)$$

where $\delta(x)$ is the Dirac's delta function, $T_s$ is the sampling interval, and $N$ is the number of samples to be processed.

$NT_s$ defines the duration of repetition pattern of input signal and usually matches the fundamental period of interest

Replacing integral with summation, the *Discrete Fourier Transform* (DFT) is defined as ($n = 0, 1, \ldots, N - 1$):

$$V(n) = \sum_{k=0}^{N-1} v(k)e^{-j2\pi \frac{k}{N} n}$$

The Inverse DFT is

$$V(k) = \frac{1}{N} \sum_{n=0}^{N-1} V(k)e^{j2\pi \frac{n}{N} k}$$

DFT is a transformation, in which $N$ equally spaced samples in time-domain are transformed into $N$ complex values in frequency-domain

The DFT could be used to extract fundamental frequency component of the grid voltage, however it is computationally demanding

*Fast Fourier Transform* (FFT) resolves computational burden, but is more suitable for grid monitoring than grid synchronization

# THE RECURSIVE DISCRETE FOURIER TRANSFORM

As DFT is computationally heavy, some simplifications are needed.

Consider DFT calculations to extract $n^{th}$ freq. component at instants $(k_s - 1)$ and $k_s$

$$V(n)_{k_s-1} = \sum_{k=k_s-N}^{k_s-1} v(k)e^{-j2\pi\frac{k}{N}n}, \qquad V(n)_{k_s} = \sum_{k=k_s-N+1}^{k_s} v(k)e^{-j2\pi\frac{k}{N}n}$$

Subtracting and reorganizing yields:

$$V(n)_{k_s} = V(n)_{k_s-1} + v(k_s)e^{-j2\pi\frac{k_s}{N}n} - v(k_s - N)e^{-j2\pi\frac{k_s-N}{N}n}$$

Since:

$$e^{-j2\pi\frac{k_s-N}{N}n} = e^{-j2\pi\frac{k_s}{N}n}$$

The RDTF can be reformulated as:

$$V(n)_{k_s} = V(n)_{k_s-1} + (v(k_s) - v(k_s - N))e^{-j2\pi\frac{k_s}{N}n}$$

and the $n^{th}$ harmonic component can be reconstructed as:

$$v(k) = \frac{2}{N}V(n)e^{j2\pi\frac{n}{N}k}$$

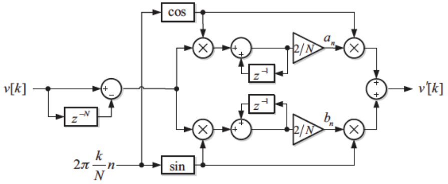


**Figure 7** Discrete adaptive filter based on the RDFT.

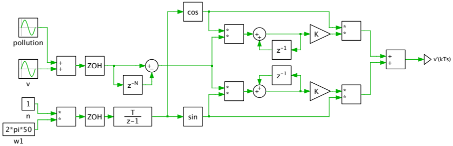


**Figure 8** PLECS implementation (example).

# PHASE LOCKED LOOP

*Basic operating principles*

A **PLL** is a closed-loop system in which an internal oscillator is controlled to keep the time of some external periodic signal through the feedback loop

The basic structure of a PLL consist of:

- **Phase Detector (PD)**: the output signal is proportional to the phase difference between input signal $v$ and the signal generated by an internal oscillator $v'$. Depending on the type of PD, in addition to DC phase-angle difference signal, high-frequency AC components appear as well.
- **Loop Filter (LF)**: This block has a low-pass characteristic and filters (attenuate) the high-frequency AC components from the PD output. Typically, this block is constituted by a first-order LPF or a PI controller.
- **Voltage Controlled Oscillator (VCO)**. This block generates at its output an AC signal whose frequency is shifted with respect to a given central frequency $\omega_c$ as a function of the input voltage provided by the LF.

There are several possibilities to implement each block of a PLL, and some of them will be reviewed...

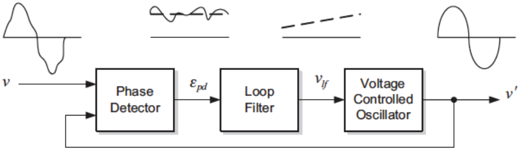


**Figure 9** Basic structure of a PLL.

Elementary PLL: the **PD** as a simple multiplier; the **LF** as a PI controller; the **VCO** as a sinusoidal function supplied by a linear integrator:

- input signal: $v(t) = V\sin(\theta) = V\sin(\omega t + \Phi)$
- VCO signal: $v'(t) = \cos(\theta') = \cos(\omega' t + \Phi')$
- PD output: $\epsilon_{pd} = Vk_{pd}\sin(\omega t + \Phi)\cos(\omega' t + \Phi') = \frac{Vk_{pd}}{2}\Big[\underbrace{\sin\big((\omega - \omega')t + (\Phi - \Phi')\big)}_{\text{low-frequency term}} + \underbrace{\sin\big((\omega + \omega')t + (\Phi + \Phi')\big)}_{\text{high-frequency term}}\Big]$

The LF will cancel high-frequency term, and assuming the VCO is well tuned ($\omega \approx \omega'$), the PD error DC signal relevant for analysis is:

$$\bar{\epsilon}_{pd} = \frac{Vk_{pd}}{2}\sin\big((\omega - \omega')t + (\Phi - \Phi')\big) \quad \Rightarrow \quad \bar{\epsilon}_{pd} = \frac{Vk_{pd}}{2}\sin(\Phi - \Phi')$$

While the multiplier PD produces nonlinear phase detection, for a very small phase error $\Phi \approx \Phi'$, we can linearize PD output around that operating point, since $\sin(\Phi - \Phi') \approx \sin(\theta - \theta') \approx (\theta - \theta')$. Thus, once PLL is locked relevant term of the phase error signal is:

$$\bar{\epsilon}_{pd} = \frac{Vk_{pd}}{2}(\theta - \theta')$$

Last expression can be used to implement small signal linearized model of the multiplier PD.

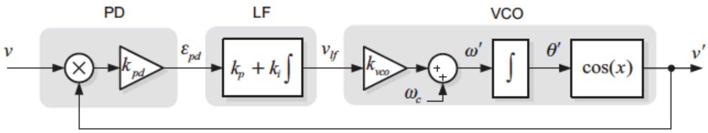


Figure 10 Block diagram of an elementary PLL.

The average frequency of the VCO is determined as ($\omega_c$ is the center frequency supplied to VCO; e.g. grid frequency):

$$\bar{\omega}' = (\omega_c + \Delta\bar{\omega}') = (\omega_c + k_{vco}\bar{v}_{lf})$$

Small signal variations in the VCO frequency are thus:

$$\tilde{\omega}' = k_{vco}\tilde{v}_{lf}$$

Variations in the phase-angle detected by the PLL can be written:

$$\tilde{\theta}'(t) = \int \tilde{\omega}' dt = \int k_{vco}\tilde{v}_{lf}\, dt$$

Derived set of equations can be easily translated to the complex frequency domain using the *Laplace Transform*.

Please note that we are free to provide the center frequency $\omega_c$ to PLL to speed up pull-in process.

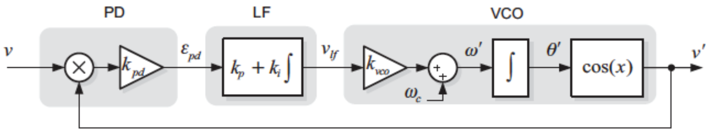


**Figure 11** Block diagram of an elementary PLL.

# PLL - LINEARIZED SMALL SIGNAL MODEL (I)

Using Laplace Transform, we have:

$$\overbrace{E_{pd}(s) = \frac{Vk_{pd}}{2}(\Theta(s) - \Theta'(s))}^{PD} \quad \overbrace{V_{lf}(s) = k_p(1 + \frac{1}{T_i s})E_{pd}(s)}^{LF} \quad \overbrace{\Theta'(s) = \frac{1}{s}K_{vco}V_{lf}(s)}^{VCO}$$

Considering $k_{pd} = 2,\ k_{vco} = 1,\ V = 1$, we can write the open-loop phase transfer function:

$$H_{OL}(s) = PD(s) \cdot LF(s) \cdot VCO(s) = 1 \cdot k_p(1 + \frac{1}{T_i s}) \cdot \frac{1}{s} = \frac{k_p s + \frac{k_p}{T_i}}{s^2}$$

Closed-loop phase and error transfer functions are:

$$H_{CL}(s) = \frac{\Theta'(s)}{\Theta(s)} = \frac{H_{OL}(s)}{1 + H_{OL}(s)} = \frac{k_p s + \frac{k_p}{T_i}}{s^2 + k_p s + \frac{k_p}{T_i}} \qquad E_{CL}(s) = \frac{E_{pd}(s)}{\Theta(s)} = 1 - H_{CL}(s) = \frac{s^2}{s^2 + k_p s + \frac{k_p}{T_i}}$$

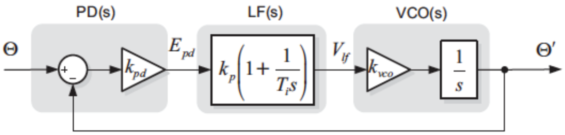


**Figure 12** Small signal model of an elementary PLL.

# PLL - LINEARIZED SMALL SIGNAL MODEL (II)

From these transfer functions we can conclude:

- $H_{OL}(s)$ reveal two poles at origin $(s^2)$; type 2 system, able to track even a constant slope ramp in the input phase-angle without any steady state error
- $H_{CL}(s)$ reveal that PLL has low-pass filtering characteristic (attenuation of noise and high-frequency signals)

These second-order transfer functions can be written as:

$$H_{CL}(s) = \frac{k_p s + \frac{k_p}{T_i}}{s^2 + k_p s + \frac{k_p}{T_i}} \Rightarrow H_{CL}(s) = \frac{2\xi\omega_n s + \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$E_{CL}(s) = \frac{s^2}{s^2 + k_p s + \frac{k_p}{T_i}} \Rightarrow E_{CL}(s) = \frac{s^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

with $\omega_n = \sqrt{\frac{k_p}{T_i}}$ and $\xi = \frac{\sqrt{k_p T_i}}{2}$

Tuning of PI can be done considering dynamic response of the second-order system:

- settling time: $t_s = 4.6\tau$ for $\pm 1\% FV$, where $\tau = 1/(\xi\omega_n)$
- PI regulator: $k_p = 2\xi\omega_n = 9.2/t_s$, and $T_i = 2\xi/\omega_n = t_s\xi^2/2.3$
- PI regulator parameter $k_p$ should be divided with magnitude of $V$, if $V \neq 1$

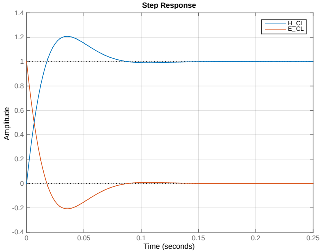


Figure 13 Step response of $H_{CL}$ and $E_{CL}$.

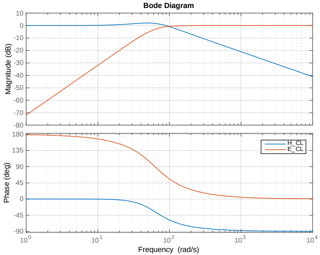


Figure 14 Frequency response of $H_{CL}$ and $E_{CL}$.

# PLL RESPONSE (I)

Simulations results show PLL response when tracking:

- single-phase grid voltage with $100V_{peak}$
- disturbed by angle jump ($+45^{\circ}$) and frequency jump (from 50 to 45 Hz) at $t = 100ms$
- settling time is $t_s = 100ms, \xi = 1/\sqrt{2}$
- $\omega_n = 2\pi 10\, rad/s$

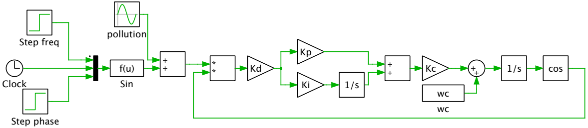


**Figure 15** Elementary PLL (PLECS example)

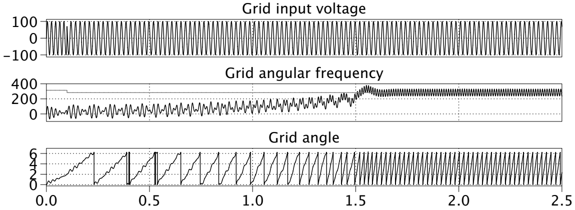


**Figure 16** Elementary PLL pull-in with $\omega_c = 0$

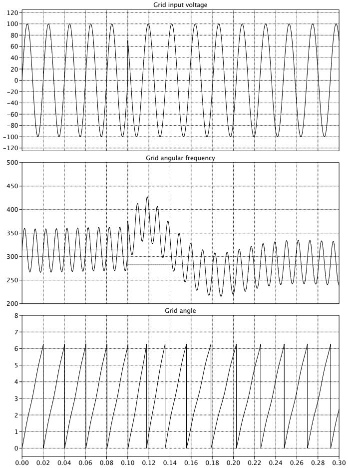


**Figure 17** Elementary PLL response ($\omega_c = 2\pi 50$)

# PLL RESPONSE (II)

Several parameters are often used to describe PLL performance:

**The hold range** $\Delta\omega_H$ is the frequency range at which PLL is able to keep itself phase locked:

$$\Delta\omega_H = k_{pd} \cdot k_{vco} \cdot LF(0)$$

where $LF(0)$ is the DC gain of the loop filter. For PI controller $LF(0) = \infty$

**The pull-in range** $\Delta\omega_P$ is the frequency range at which PLL will always became locked (process may become slow).
The time $T_P$ can be calculated as:

$$T_P \approx \frac{\pi^2}{16} \frac{\Delta\omega_{in}^2}{\xi\omega_n^3}$$

where $\Delta\omega_{in}$ is variation in input frequency

**The lock range** $\Delta\omega_L$ is the frequency range within which PLL locks within one single-beat note between the reference frequency and the output frequency.
The lock range and lock-in time for PI LF can be approximated as:

$$\Delta\omega_L \approx 2\xi\omega_n \approx 2\xi\sqrt{\frac{k_p}{T_i}}, \qquad T_L \approx \frac{2\pi}{\omega_n}$$

**The pull-out range** $\Delta\omega_{PO}$ is the dynamic limit for stable PLL.
If tracking is lost within this range, a PLL will become phase locked again after a time longer than $T_L$ but shorter than $T_P$.

$$\Delta\omega_{PO} \approx 1.8\omega_n(\xi + 1)$$

Bandwidth of a single-phase PLL has to be low to smooth oscillations in detected frequency and phase

Simulations show, that without center frequency supplied to VCO (e.g. $\omega_c = 0$), pull-in time is much longer (around 2s) than expected (around $300ms$)

Part of error comes from assumption that frequency signal to be phase-locked is much higher than the PLL bandwidth so that high frequency term can be neglected.

Grid frequency is very close to PLL cut-off frequency ($\approx 21.3Hz$), and double-frequency term is not properly attenuated ($100Hz$)

Thus, a simple multiplier PD is not sufficient to cancel out signal at twice the grid frequency

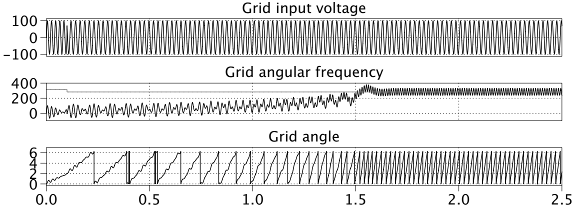


**Figure 18** Elementary PLL pull-in with $\omega_c = 0$

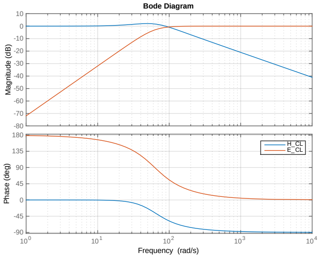


**Figure 19** Frequency response of $H_{CL}$ and $E_{CL}$.

# PLL BASED ON QUADRATURE SIGNAL GENERATOR (I)

Assume we have an input signal:

$$v(t) = V\sin(\theta) = V\sin(\omega t + \Phi)$$

If we are able to obtain in-quadrature signal, using Quadrature Signal Generator (QSG):

$$v_{iq}(t) = -V\cos(\theta) = -V\cos(\omega t + \Phi)$$

Then, PD based on QSG would cancel-out the oscillations at twice the input frequency:

$$\begin{aligned}\epsilon_{pd} &= V\sin(\omega t + \Phi)\cos(\omega' t + \Phi') - V\cos(\omega t + \Phi)\sin(\omega' t + \Phi') \\ &= V\sin((\omega - \omega')t + (\Phi - \Phi')) \\ &= V\sin(\theta - \theta')\end{aligned}$$

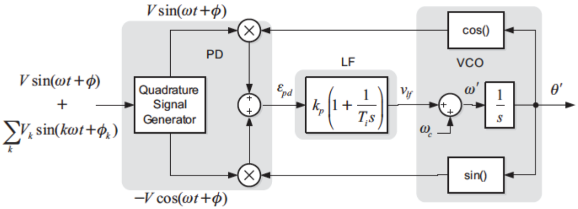


**Figure 20** PLL with an ideal in-quadrature PD.

The same tuning as before:

- $t_s = 100ms, \xi = 1/\sqrt{2}$
- $\Delta\omega_L \approx \pm 92 rad/s, T_L \approx 96ms$

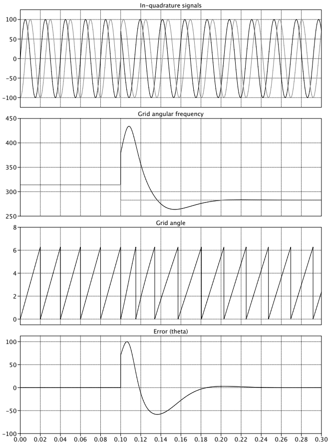


**Figure 21** PLL response with in-quadrature PD ($\omega_c = 2\pi 50$)

If we recall Park Transform ($\theta_k$ is replaced with $\theta'$):

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} \cos\theta' & \sin\theta' \\ -\sin\theta' & \cos\theta' \end{bmatrix} \cdot \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}$$

and QSG output provides $v_\alpha$ and $v_\beta$ from input signal $v(t)$ as:

$$v_{\alpha\beta} = \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = V \begin{bmatrix} \sin(\theta) \\ -\cos(\theta) \end{bmatrix}$$

then output of PD based on Park Transform will be:

$$v_{dq} = \begin{bmatrix} v_d \\ v_q \end{bmatrix} = V \begin{bmatrix} \sin(\theta - \theta') \\ -\cos(\theta - \theta') \end{bmatrix}$$

Even though we are dealing with single-phase grid, space vector notion can be used:

- LF will drive its input to zero
- if LF is placed on $d$-axis, grid voltage space vector will be aligned with $q$-axis
- if LF is placed on $q$-axis, grid voltage space vector will be aligned with $d$-axis
- once PLL is locked, grid voltage is real variable in $dq$ frame

We will work with LF in $q$ axis, so that $i_d$ control active, and $i_q$ reactive power

VCO block is replaced with Frequency/Phase Generator (FPG) block

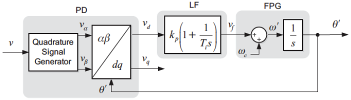


Figure 22 PLL with the LF on the $d$ axis of the QSG.

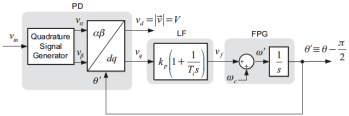


Figure 23 PLL with the LF on the $q$ axis of the QSG.

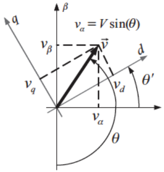


Figure 24 Vector representation of the QSG output signals.

# PLL BASED ON QUADRATURE SIGNAL GENERATOR (III)

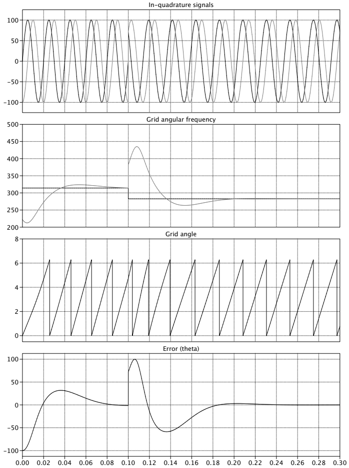


**Figure 25** PLL response with the LF on the $q$ axis of the QSG.

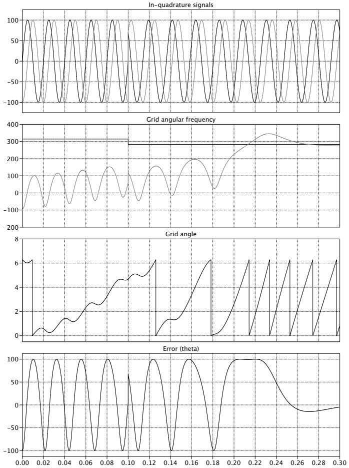


**Figure 26** PLL pull-in with the LF on the $q$ axis of the QSG.

# PLL BASED ON QUADRATURE SIGNAL GENERATOR (IV)

There are several other ways, how QSG can be implemented for single-phase grids

**PLL based on a $T/4$ transport delay**

- with $T$ being the grid fundamental period, this is the simplest QSG
- if grid frequency changes, fixed transport delay will result in errors
- if grid harmonic components are present, fixed transport delay is wrong

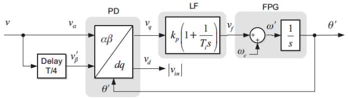


Figure 27 PLL based on a T/4 transport delay.

**PLL based on the Hilbert Transform**

- Hilbert Transform shift $\pm 90^\circ$ phase-angle of input signal, depending on its sign
- it only affects the phase and has no effect on the amplitude

$$H(v) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{v(\tau)}{t-\tau}d(\tau) = \frac{1}{\pi t} * v$$

where $*$ describes the convolution product of the function $h(t) = 1/\pi t$ with the signal $v(t)$

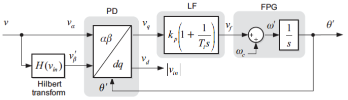


Figure 28 PLL based on Hilbert Transform.

**PLL based on the Inverse Park Transform**

- in-quadrature image of the input signal can be achieved
- filter is introduced in the loop of direct and inverse Park Transform

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & -\cos\theta' \end{bmatrix} \cdot \begin{bmatrix} v_d \\ v_q \end{bmatrix}$$

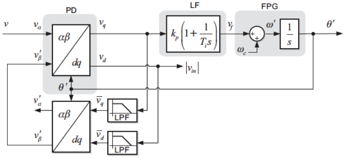


Figure 29 PLL based on the Inverse Park Transform.

# PLL IN 3-PHASE SYSTEMS

Based on the single-phase case PLL, there is a trivial solution for the QSG in three-phase system:

- Clarke Transform applied to balanced set of three-phase signals $(v_a, v_b, v_c)$ will result in-quadrature signals $(v_\alpha, v_\beta)$
- Park Transform can be used as before for PD
- LF and FPG can be placed in either of axes ($q$ in our case)
- we will work with grid voltage aligned with $d$-axis of RRF

$$\mathbf{v}_{\alpha\beta} = [T_{\alpha\beta}]\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

$$\mathbf{v}_{dq} = [T_{dq}]\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \underbrace{[T_{dq}][T_{\alpha\beta}]}_{[T_\theta]}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ -\sin(\theta) & -\sin(\theta - 2\pi/3) & -\sin(\theta + 2\pi/3) \end{bmatrix}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

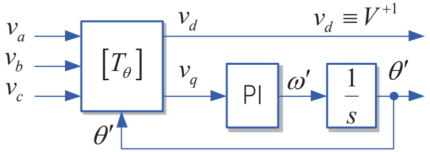


**Figure 30** 3-ph PLL block diagram (simple and suitable for balanced grid conditions).

# SUMMARY

Basic PLL principles and properties have been presented:

- basic functions: Phase Detector, Loop Filter, Voltage Controlled Oscilator. Frequency/Phase Generator
- importance of in-quadrature signal generation
- use of Park Transform in PLL implementation
- simple 3-phase PLL is derived

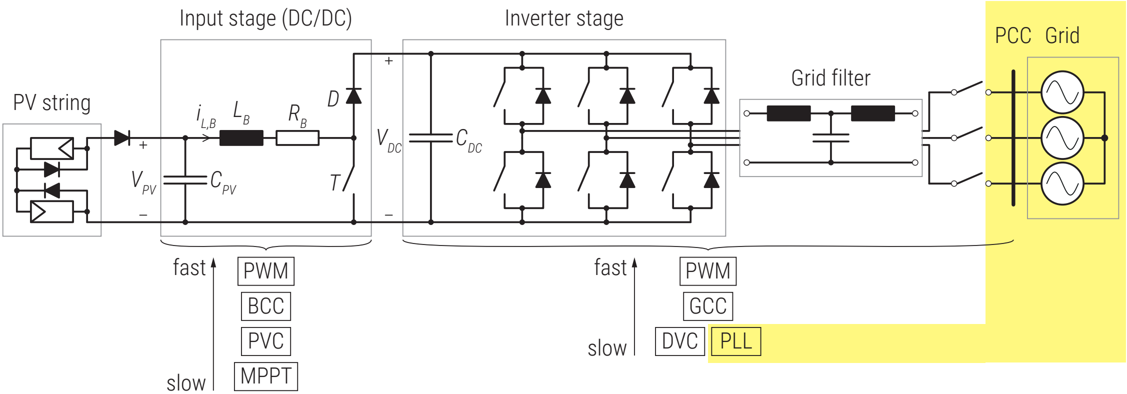


**Figure 31** PV double-stage grid connected converter.