EE-465 - W13
GRID-CONNECTED VSI SMALL SIGNAL DYNAMIC MODELLING

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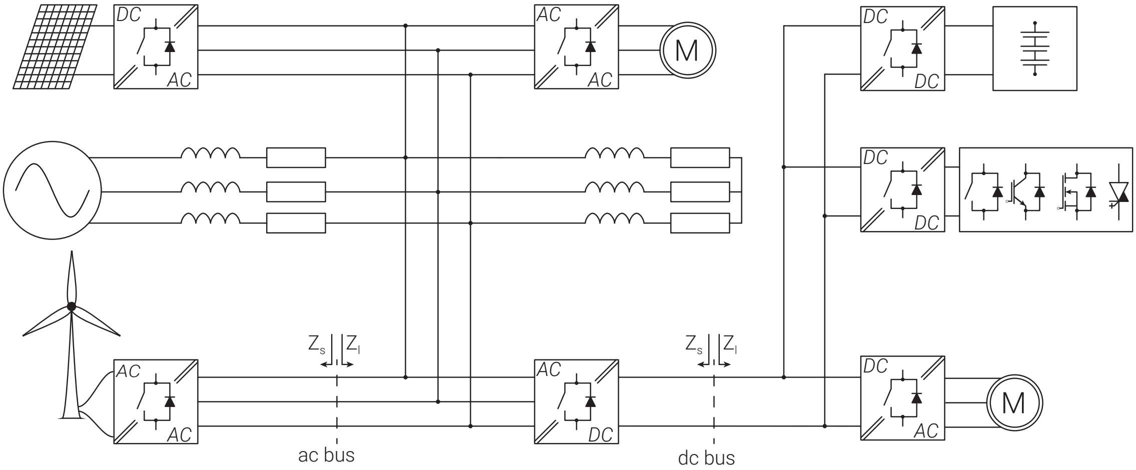

Figure 1 AD/DC power electronics dominated and distributed power system

- Both source and load power electronic converters are active, closed loop controlled devices with certain control bandwidth
- Understanding their interactions is important for the understanding of the overal system stability
- Yet, first step is to understand input/output characteristics of every device on its own

We can consider two generic cases of Grid Connected 2L 3-phase converter (VSI):

- Voltage Fed VSI: some other input converter manages (control) DC-link (supply) voltage
- Current Fed VSI: some other input converter behaves as current source (e.g. MPPT Boost in our course)

Voltage-Fed Grid-Connected Inverter

- Input voltage controlled by a stiff source, e.g. battery storage
- Input: non-controllable, variables — v_{dc} and v_g (stiff sources)
- Output: controllable, variables — i_g (i_{dc})
- Control: switch duty cycle d
- System state is affected by change of input variables v_{dc}, v_g

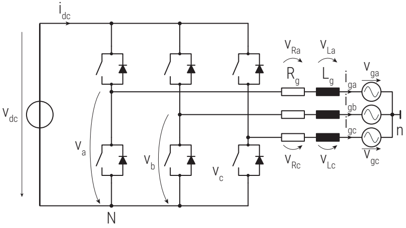

Figure 2 Three-phase grid connected inverter

Current-Fed Grid-Connected Inverter

- Input current controlled by a stiff source, e.g. boost converter
- Input: non-controllable, variables — i_{dc} and v_g (stiff sources)
- Output: controllable, variables — v_{dc} and i_g
- Control: switch duty cycle d
- System state is affected by change of input variables i_{dc}, v_g

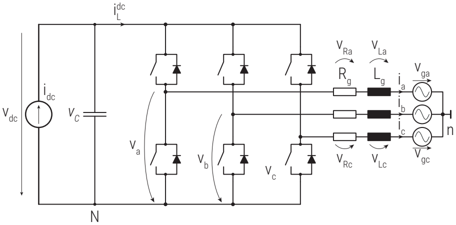

Figure 3 Three-phase grid connected inverter

DYNAMIC MODELLING - KCL AND KVL EQUATIONS

The open-loop model of the current-fed three-phase grid connected converter can be derived by applying Kirchoff's voltage and current laws to the circuit in **Fig.** 4.
We consider:

- **on-time:** upper switch in the leg turned ON (bottom switch if OFF)
- **off-time:** bottom switch in the leg turned ON (upper switch is OFF)

The equations are given as

- During the switch **on-time**

$$\begin{aligned} v_{\mathrm{La}} &= v_{\mathrm{dc}} - v_{\mathrm{ga}} - v_{\mathrm{Ra}} - v_{\mathrm{nN}} \\ v_{\mathrm{Lb}} &= v_{\mathrm{dc}} - v_{\mathrm{gb}} - v_{\mathrm{Rb}} - v_{\mathrm{nN}} \\ v_{\mathrm{Lc}} &= v_{\mathrm{dc}} - v_{\mathrm{gc}} - v_{\mathrm{Rc}} - v_{\mathrm{nN}} \\ v_{\mathrm{dc}} &= v_C \end{aligned} \quad (1)$$

$$\begin{aligned} i_L^{\mathrm{dc}} &= i_{\mathrm{ga}} + i_{\mathrm{gb}} + i_{\mathrm{gc}} \\ i_C &= i_{\mathrm{dc}} - i_L^{\mathrm{dc}} \end{aligned} \quad (2)$$

- During switch **off-time**

$$\begin{aligned} v_{\mathrm{La}} &= -v_{\mathrm{ga}} - v_{\mathrm{Ra}} - v_{\mathrm{nN}} \\ v_{\mathrm{Lb}} &= -v_{\mathrm{gb}} - v_{\mathrm{Rb}} - v_{\mathrm{nN}} \\ v_{\mathrm{Lc}} &= -v_{\mathrm{gc}} - v_{\mathrm{Rc}} - v_{\mathrm{nN}} \\ v_{\mathrm{dc}} &= v_C \end{aligned} \quad (3)$$

$$\begin{aligned} i_L^{\mathrm{dc}} &= 0 \\ i_C &= i_{\mathrm{dc}} \end{aligned} \quad (4)$$

Equations are simple application of KCL and KVL for loops created depending on the state of switch in each converter leg.

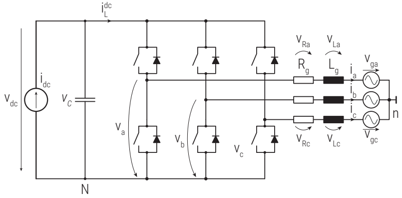

Figure 4 Three-phase grid connected inverter

Multiplying (1) by d and (3) by $1 - d$ and summing them together one obtains time-averaged equations as

$$\begin{aligned}
\langle v_{\mathrm{La}} \rangle &= d_a \langle v_{\mathrm{dc}} \rangle - \langle v_{\mathrm{ga}} \rangle - \langle v_{\mathrm{Ra}} \rangle - \langle v_{\mathrm{nN}} \rangle \\
\langle v_{\mathrm{Lb}} \rangle &= d_b \langle v_{\mathrm{dc}} \rangle - \langle v_{\mathrm{gb}} \rangle - \langle v_{\mathrm{Rb}} \rangle - \langle v_{\mathrm{nN}} \rangle \\
\langle v_{\mathrm{Lc}} \rangle &= d_c \langle v_{\mathrm{dc}} \rangle - \langle v_{\mathrm{gc}} \rangle - \langle v_{\mathrm{Rc}} \rangle - \langle v_{\mathrm{nN}} \rangle \\
\langle v_{\mathrm{dc}} \rangle &= \langle v_C \rangle
\end{aligned} \tag{5}$$

Same holds for the current equations (2) and (4) where time-averaged equations are obtained as

$$\begin{aligned}
\langle i_L^{\mathrm{dc}} \rangle &= d_a \langle i_{\mathrm{ga}} \rangle + d_b \langle i_{\mathrm{gb}} \rangle + d_c \langle i_{\mathrm{gc}} \rangle \\
\langle i_C \rangle &= \langle i_{\mathrm{dc}} \rangle - \langle i_L^{\mathrm{dc}} \rangle
\end{aligned} \tag{6}$$

Time domain derivatives of equations (5) and (6) can be obtained remembering that

$$\begin{aligned}
\frac{\mathrm{d}i_{\mathrm{L}}}{\mathrm{d}t} &= \frac{1}{L} v_{\mathrm{L}} \\
\frac{\mathrm{d}i_{\mathrm{C}}}{\mathrm{d}t} &= \frac{1}{C} i_{\mathrm{C}}
\end{aligned} \tag{7}$$

Applying (7) to (5) and (6) gives

$$
\begin{aligned}
\frac{\mathrm{d}\langle i_{\mathrm{ga}}\rangle}{\mathrm{d}t} &= \frac{d_a\langle v_{\mathrm{dc}}\rangle - \langle v_{\mathrm{ga}}\rangle - \langle v_{\mathrm{Ra}}\rangle - \langle v_{\mathrm{nN}}\rangle}{L} \\
\frac{\mathrm{d}\langle i_{\mathrm{gb}}\rangle}{\mathrm{d}t} &= \frac{d_b\langle v_{\mathrm{dc}}\rangle - \langle v_{\mathrm{gb}}\rangle - \langle v_{\mathrm{Rb}}\rangle - \langle v_{\mathrm{nN}}\rangle}{L} \\
\frac{\mathrm{d}\langle i_{\mathrm{gc}}\rangle}{\mathrm{d}t} &= \frac{d_c\langle v_{\mathrm{dc}}\rangle - \langle v_{\mathrm{gc}}\rangle - \langle v_{\mathrm{Rc}}\rangle - \langle v_{\mathrm{nN}}\rangle}{L} \\
\frac{\mathrm{d}\langle v_{\mathrm{dc}}\rangle}{\mathrm{d}t} &= \frac{\langle i_{\mathrm{dc}}\rangle - d_a\langle i_{\mathrm{ga}}\rangle - d_b\langle i_{\mathrm{gb}}\rangle - d_c\langle i_{\mathrm{gc}}\rangle}{C}
\end{aligned}
\tag{8}
$$

Same equations can be represented in their matrix form for easier manipulation. Transforming the equations to this form, and taking into consideration that the average value of v_{nN} is zero, gives

$$
\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\langle i_{\mathrm{ga}}\rangle \\ \langle i_{\mathrm{gb}}\rangle \\ \langle i_{\mathrm{gc}}\rangle\end{bmatrix} = \frac{1}{L}\begin{bmatrix}d_a \\ d_b \\ d_c\end{bmatrix}\langle v_{\mathrm{dc}}\rangle - \frac{1}{L}\begin{bmatrix}\langle v_{\mathrm{Ra}}\rangle \\ \langle v_{\mathrm{Rb}}\rangle \\ \langle v_{\mathrm{Rc}}\rangle\end{bmatrix} - \frac{1}{L}\begin{bmatrix}\langle v_{\mathrm{ga}}\rangle \\ \langle v_{\mathrm{gb}}\rangle \\ \langle v_{\mathrm{gc}}\rangle\end{bmatrix} \tag{9}
$$

$$
\frac{\mathrm{d}\langle v_{\mathrm{dc}}\rangle}{\mathrm{d}t} = \frac{\langle i_{\mathrm{dc}}\rangle}{C} - \frac{1}{C}\begin{bmatrix}d_a \\ d_b \\ d_c\end{bmatrix}\begin{bmatrix}\langle i_{\mathrm{ga}}\rangle \\ \langle i_{\mathrm{gb}}\rangle \\ \langle i_{\mathrm{gc}}\rangle\end{bmatrix} \tag{10}
$$

Applying Clarke-Transform, three-phase variables can be represented in the stationary reference frame (or in $\alpha - \beta$-domain) as

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{T_{\alpha\beta}\begin{bmatrix}\langle i_{\mathrm{ga}}\rangle\\ \langle i_{\mathrm{gb}}\rangle\\ \langle i_{\mathrm{gc}}\rangle\end{bmatrix}}_{\mathbf{i}_g^{\alpha\beta}} = \frac{1}{L}\underbrace{T_{\alpha\beta}\begin{bmatrix}d_a\\ d_b\\ d_c\end{bmatrix}}_{\mathbf{d}^{\alpha\beta}}\langle v_{\mathrm{dc}}\rangle - \frac{1}{L}\underbrace{T_{\alpha\beta}\begin{bmatrix}\langle v_{\mathrm{Ra}}\rangle\\ \langle v_{\mathrm{Rb}}\rangle\\ \langle v_{\mathrm{Rc}}\rangle\end{bmatrix}}_{\mathbf{v}_R^{\alpha\beta}} - \frac{1}{L}\underbrace{T_{\alpha\beta}\begin{bmatrix}\langle v_{\mathrm{ga}}\rangle\\ \langle v_{\mathrm{gb}}\rangle\\ \langle v_{\mathrm{gc}}\rangle\end{bmatrix}}_{\mathbf{v}_g^{\alpha\beta}} \tag{11}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \mathbf{i}_g^{\alpha\beta}\rangle = \frac{\mathbf{d}^{\alpha\beta}\langle v_{\mathrm{dc}}\rangle}{L} - \frac{\langle \mathbf{v}_R^{\alpha\beta}\rangle}{L} - \frac{\langle \mathbf{v}_g^{\alpha\beta}\rangle}{L}$$

The same can be applied for the dc side (K is Clarke-Transformer scaling adjustment, if needed)

$$\frac{\mathrm{d}\langle v_{\mathrm{dc}}\rangle}{\mathrm{d}t} = \frac{\langle i_{\mathrm{dc}}\rangle}{C} - \frac{3}{2K^2}\frac{d_\alpha\langle i_{\mathrm{g}\alpha}\rangle + d_\beta\langle i_{\mathrm{g}\beta}\rangle}{C}$$

$$\frac{\mathrm{d}\langle v_{\mathrm{dc}}\rangle}{\mathrm{d}t} = \frac{\langle i_{\mathrm{dc}}\rangle}{C} - \frac{3}{2K^2}\frac{\Re\left(\langle \mathbf{d}^{\alpha\beta}\rangle\langle \mathbf{i}_g^{\alpha\beta}\rangle^*\right)}{C} \tag{12}$$

Next step is Park-Transform. An arbitrary vector in the $\alpha - \beta$-frame can be represented in the *d*-*q*-frame as

$$\underline{x}^{dq} = \underline{x}^{\alpha\beta}\mathrm{e}^{-j\omega_g t}$$

$$\underline{x}^{\alpha\beta} = \underline{x}^{dq}\mathrm{e}^{j\omega_g t} \tag{13}$$

Thus, the averaged state-space model in *d*-*q*-frame can be given as:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\left(\langle \mathbf{i}_g^{\mathrm{dq}}\rangle \mathrm{e}^{j\omega_g t}\right) &= \frac{\mathbf{d}^{\mathrm{dq}}\langle v_{\mathrm{dc}}\rangle \mathrm{e}^{j\omega_g t}}{L} - \frac{\langle \mathbf{v}_R^{\mathrm{dq}}\rangle \mathrm{e}^{j\omega_g t}}{L} - \frac{\langle \mathbf{v}_g^{\mathrm{dq}}\rangle \mathrm{e}^{j\omega_g t}}{L} \\ \frac{\mathrm{d}\langle v_{\mathrm{dc}}\rangle}{\mathrm{d}t} &= \frac{\langle i_{\mathrm{dc}}\rangle}{C} - \frac{3}{2K^2}\frac{d_d\langle i_{\mathrm{gd}}\rangle + d_q\langle i_{\mathrm{gq}}\rangle}{C} \end{aligned} \tag{14}$$

After the derivation of $\frac{\mathrm{d}}{\mathrm{d}t}\left(\langle \mathbf{i}_g^{\mathrm{dq}}\rangle \mathrm{e}^{j\omega_g t}\right)$ and separating the real and imaginary parts of the equations gives the averaged state-space model in the *d*-*q*-frame as

$$\begin{aligned} \frac{\mathrm{d}\langle i_{\mathrm{gd}}\rangle}{\mathrm{d}t} &= \omega_g\langle i_{\mathrm{gq}}\rangle + \frac{d_d\langle v_{\mathrm{dc}}\rangle}{L} - \frac{R}{L}\langle i_{\mathrm{gd}}\rangle - \frac{\langle v_{\mathrm{gd}}\rangle}{L} \\ \frac{\mathrm{d}\langle i_{\mathrm{gq}}\rangle}{\mathrm{d}t} &= -\omega_g\langle i_{\mathrm{gd}}\rangle + \frac{d_q\langle v_{\mathrm{dc}}\rangle}{L} - \frac{R}{L}\langle i_{\mathrm{gq}}\rangle - \frac{\langle v_{\mathrm{gq}}\rangle}{L} \\ \frac{\mathrm{d}\langle v_{\mathrm{dc}}\rangle}{\mathrm{d}t} &= \frac{\langle i_{\mathrm{dc}}\rangle}{C} - \frac{3}{2K^2}\frac{d_d\langle i_{\mathrm{gd}}\rangle + d_q\langle i_{\mathrm{gq}}\rangle}{C} \end{aligned} \tag{15}$$

This model allows us to find the steady-state operating point of the converter (all derivatives equal to zero) as

$$\omega_g L I_{\mathrm{gq}} + D_d V_{\mathrm{dc}} - R I_{\mathrm{gd}} - V_{\mathrm{gd}} = 0 \tag{16}$$

$$-\omega_g L I_{\mathrm{gd}} + D_q V_{\mathrm{dc}} - R I_{\mathrm{gq}} - V_{\mathrm{gq}} = 0 \tag{17}$$

$$I_{\mathrm{dc}} - \frac{3}{2}\left(D_d I_{\mathrm{gd}} + D_q I_{\mathrm{gq}}\right) = 0 \tag{18}$$

More precisely, for a unity power factor operation, *q*-axis voltage and current are set to zero, i.e. $V_{\mathrm{gq}} = 0$ and $I_{\mathrm{gq}} = 0$ which allows us to find the duty cycle and grid current steady state values as

$$D_d = \frac{V_{\mathrm{gd}} + \sqrt{V_{\mathrm{gd}}^2 + (8/3)\, V_{\mathrm{dc}} I_{\mathrm{dc}} R}}{2V_{\mathrm{dc}}} \tag{19}$$

$$I_{\mathrm{gd}} = \frac{2}{3}\frac{I_{\mathrm{dc}}}{D_d} \tag{20}$$

$$D_q = \frac{\omega_g L I_{\mathrm{gd}}}{V_{\mathrm{dc}}} \tag{21}$$

Presence of the derivative terms in the equation (14) means that the model is nonlinear and it is required to linearize it around a small-signal operation point (K = 1).

$$\begin{aligned}
\frac{\mathrm{d}\tilde{i}_{\mathrm{gd}}}{\mathrm{d}t} &= \omega_g \tilde{i}_{\mathrm{gq}} + \frac{D_d}{L}\tilde{v}_{\mathrm{dc}} + \frac{V_{\mathrm{dc}}}{L}\tilde{d}_d - \frac{R}{L}\tilde{i}_{\mathrm{gd}} - \frac{1}{L}\tilde{v}_{\mathrm{gd}} \\
\frac{\mathrm{d}\tilde{i}_{\mathrm{gq}}}{\mathrm{d}t} &= -\omega_g \tilde{i}_{\mathrm{gd}} + \frac{D_q}{L}\tilde{v}_{\mathrm{dc}} + \frac{V_{\mathrm{dc}}}{L}\tilde{d}_q - \frac{R}{L}\tilde{i}_{\mathrm{gq}} - \frac{1}{L}\tilde{v}_{\mathrm{gq}} \\
\frac{\mathrm{d}\tilde{v}_{\mathrm{dc}}}{\mathrm{d}t} &= \frac{1}{C}\tilde{i}_{\mathrm{dc}} - \frac{3}{2}\frac{d_d}{C}\tilde{i}_{\mathrm{gd}} - \frac{3}{2}\frac{d_q}{C}\tilde{i}_{\mathrm{gq}} - \frac{3}{2}\frac{i_{\mathrm{gd}}}{C}\tilde{d}_d - \frac{3}{2}\frac{i_{\mathrm{gq}}}{C}\tilde{d}_q
\end{aligned} \tag{22}$$

The same equation in matrix form can be represented in s-domain by replacing the time derivative $\mathrm{d}/\mathrm{d}t$ by the operator s:

$$s\begin{bmatrix}\tilde{i}_{\mathrm{gd}}\\ \tilde{i}_{\mathrm{gq}}\\ \tilde{v}_{\mathrm{dc}}\end{bmatrix}^s = \underbrace{\begin{bmatrix}-\frac{R}{L} & \omega_g & \frac{D_d}{L}\\ -\omega_g & -\frac{R}{L} & \frac{D_q}{L}\\ -\frac{3}{2}\frac{D_d}{C} & -\frac{3}{2}\frac{D_q}{C} & 0\end{bmatrix}}_{\mathbf{A}}\begin{bmatrix}\tilde{i}_{\mathrm{gd}}\\ \tilde{i}_{\mathrm{gq}}\\ \tilde{v}_{\mathrm{dc}}\end{bmatrix}^s + \underbrace{\begin{bmatrix}-\frac{1}{L} & 0 & 0 & \frac{V_{\mathrm{dc}}}{L} & 0\\ 0 & -\frac{1}{L} & 0 & 0 & \frac{V_{\mathrm{dc}}}{L}\\ 0 & 0 & \frac{1}{C} & -\frac{3}{2}\frac{I_{\mathrm{gd}}}{C} & -\frac{3}{2}\frac{I_{\mathrm{gq}}}{C}\end{bmatrix}}_{\mathbf{B}}\begin{bmatrix}\tilde{v}_{\mathrm{gd}}\\ \tilde{v}_{\mathrm{gq}}\\ \tilde{i}_{\mathrm{dc}}\\ \tilde{d}_d\\ \tilde{d}_q\end{bmatrix}^s \quad (23)$$

$$\begin{bmatrix}\tilde{i}_{\mathrm{gd}}\\ \tilde{i}_{\mathrm{gq}}\\ \tilde{v}_{\mathrm{dc}}\end{bmatrix}^s = \underbrace{\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}}_{\mathbf{C}}\begin{bmatrix}\tilde{i}_{\mathrm{gd}}\\ \tilde{i}_{\mathrm{gq}}\\ \tilde{v}_{\mathrm{dc}}\end{bmatrix}^s + \underbrace{\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}}_{\mathbf{D}}\begin{bmatrix}\tilde{v}_{\mathrm{gd}}\\ \tilde{v}_{\mathrm{gq}}\\ \tilde{i}_{\mathrm{dc}}\\ \tilde{d}_d\\ \tilde{d}_q\end{bmatrix}^s \quad (24)$$

State-space model variables are

- state and output variables — $\tilde{i}_{\mathrm{gd}}, \tilde{i}_{\mathrm{gq}}, \tilde{v}_{\mathrm{dc}}$
- input variables — $\tilde{v}_{\mathrm{gd}}, \tilde{v}_{\mathrm{gq}}, \tilde{i}_{\mathrm{dc}}, \tilde{d}_d, \tilde{d}_q$

With two previous equations the open-loop dynamics can be solved as

$$\mathbf{G} = \frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$$

$$\begin{bmatrix} \tilde{i}_{gd} \\ \tilde{i}_{gq} \\ \tilde{v}_{dc} \end{bmatrix}^s = \begin{bmatrix} -Y_{indd,o} & -Y_{indq,o} & G_{iod,o} & G_{cidd,o} & G_{cidq,o} \\ -Y_{inqd,o} & -Y_{inqq,o} & G_{ioq,o} & G_{ciqd,o} & G_{ciqq,o} \\ T_{oid,o} & T_{oiq,o} & Z_{out,o} & G_{cod,o} & G_{coq,o} \end{bmatrix} \begin{bmatrix} \tilde{v}_{gd} \\ \tilde{v}_{gq} \\ \tilde{i}_{dc} \\ \tilde{d}_d \\ \tilde{d}_q \end{bmatrix}^s \quad (25)$$

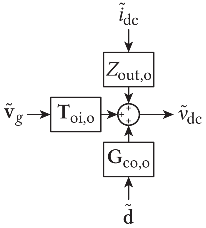

Figure 5 DC side open-loop dynamics

Where the transfer function in matrix $\mathbf{G}$ links the output and input variables and depict the physical relation between the variables as follows:

- $\mathbf{Y}_{in,o}$ - input admittance matrix
- $\mathbf{G}_{io,o}$ - forward transfer function matrix
- $\mathbf{G}_{co,o}$ - control-to-output transfer function matrix
- $\mathbf{T}_{oi,o}$ - input-to-output reverse transfer function matrix
- $Z_{out,o}$ - output impedance
- $\mathbf{G}_{ci,o}$ - control-to-input transfer function matrix

The open-loop dynamics can also be represented graphically with **Figs.** 5 and 6

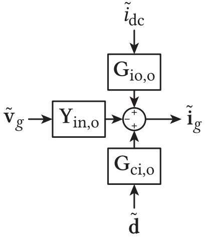

Figure 6 AC side open-loop dynamics

MEASURING SOURCE - LOAD INTERACTIONS

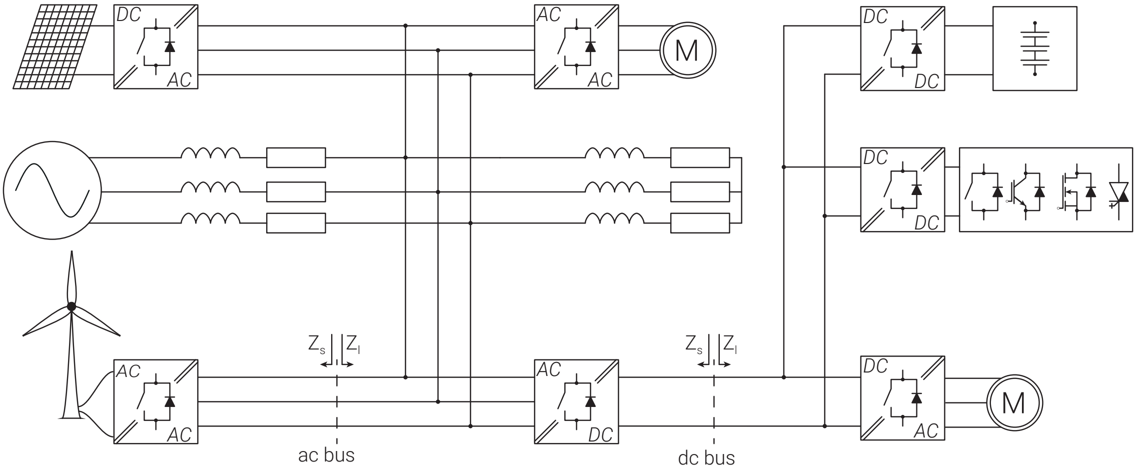

Figure 7 AD/DC power electronics dominated and distributed power system

- Stability can be analysed on the complex system level
- Every new device is connected into system that can be aggregated as source and have its own characteristics
- Every device can be also characterized on its own

MEASURING THE OPEN-LOOP DYNAMICS

In order to measure one of the characteristics of the system of the system one needs to:

- Perturb a variable that can change the system state
- Measure the resulting response from the output variable
- Perform some kind of identification algorithm, e.g. a DFT to obtain the response in the frequency domain
- Identify a characteristics by comparing the response to the perturbation

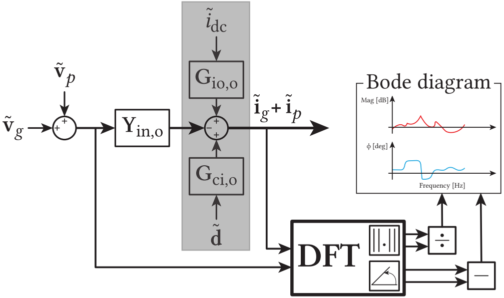

Figure 8 Input admittance measurement principle

Perturbation signals can be classified into:

Narrowband signals

- Single (few) frequency components(s)
- Signal energy concentrated around its frequency, easy to identify the response
- Measurement time could be long, e.g. sine sweep

Wideband signals

- Several (many) frequency components
- Signal energy spread over the signal spectrum, some frequencies might be harder to identify
- Shorter measurement time, e.g. Pseudo Random Binary Sequence (PRBS)

Sine-Sweep

- Perturb the system with a single frequency at the time
- Signal energy concentrated at a single frequency
- Strong system response but the measurement time might be an issue

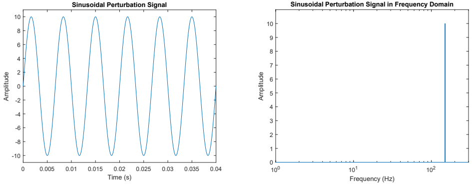

Figure 9 Sine signal in time and frequency domain

Pseudo Random Binary Signal

- Perturb the system with a large set of frequencies, e.g. 4095 frequencies in **Fig.** 10
- Signal energy is spread over the spectrum and drops to zero at the signal generation frequency, e.g. 5kHz in **Fig.** 10
- Measurement time is reduced and we can perform more measurements and average them

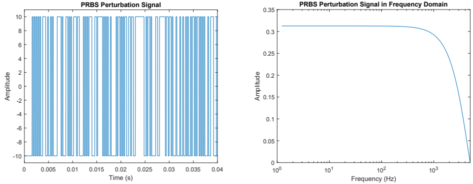

Figure 10 PRBS signal in time and frequency domain

For the same number of frequencies, sine sweep and PRBS measurement times compare as

$$t_{\text{meas}} = \sum_{i=1}^{N_{\text{freq}}=4095} \frac{1}{f_i} \approx 7.3s \quad f_i = i\,\frac{f_{\text{gen}}}{N_{\text{freq}}} \tag{26}$$

$$t_{\text{meas}} = \frac{N_{\text{freq}}}{f_{\text{gen}}} \approx \frac{4095}{5kHz} = 0.82s \tag{27}$$

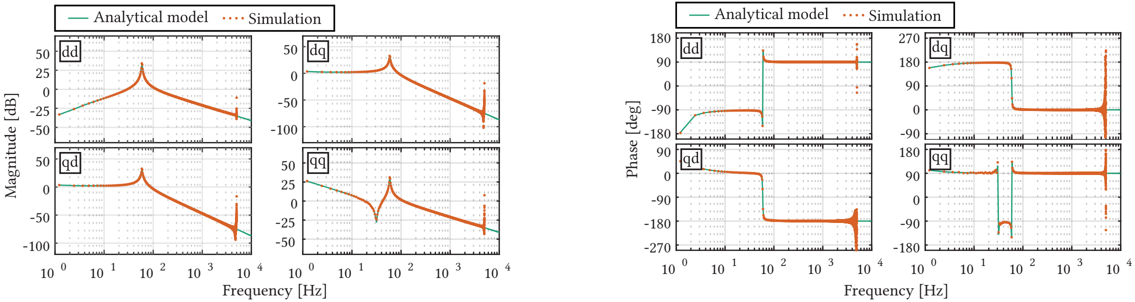

Figure 11 Grid-connected inverter AC side admittance - open loop

Table 1 System parameters.

v_{gd}	580 V	v_{gq}	0 V
i_{gd}	41 A	i_{gq}	0 A
L_g	1.8 mH	R_g	10 mΩ
C_{DC}	5 mF	V_{DC}	1200 V
I_{DC}	30 A	v_p	±30 V
f_{sw}	10 kHz	f_g	50 Hz

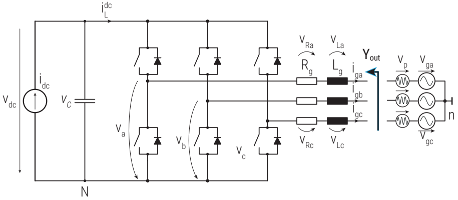

Figure 12 Three-phase grid connected inverter with a small-signal perturbation injected in series with the grid voltage

OPEN-LOOP DC-SIDE IMPEDANCE MEASUREMENT

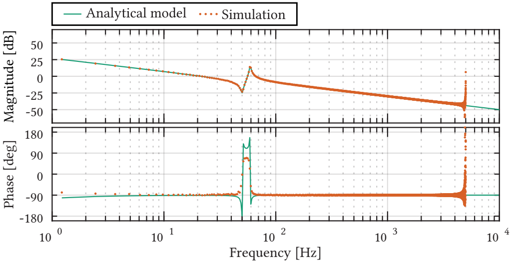

Figure 13 Grid-connected inverter DC side impedance - open loop

Table 2 System parameters.

v_{gd}	580 V	v_{gq}	0 V
i_{gd}	41 A	i_{gq}	0 A
L_g	1.8 mH	R_g	10 mΩ
C_{DC}	5 mF	V_{DC}	1200 V
I_{DC}	30 A	v_p	±30 V
f_{sw}	10 kHz	f_g	50 Hz

Please note that in this case perturbation is injected into DC side supply current. See **Fig.** 5

C_{dc} ND L_g INFLUENCE ON THE AC SIDE IMPEDANCE SHAPE

- From (23) and (24) it is obvious that any of the model parameters can influence the output admittance and input impedance.

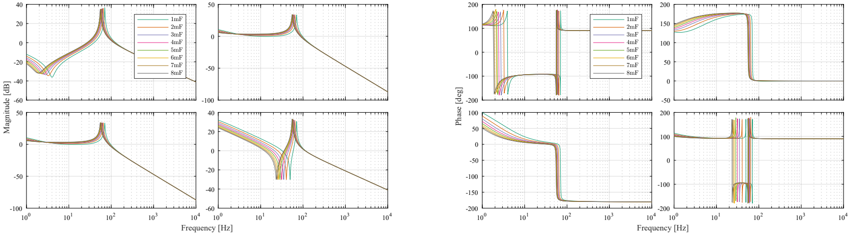

Figure 14 Dependence of the a) magnitude and b) phase of the output admittance of the inverter on the change of dc-link capacitance C_{dc}.

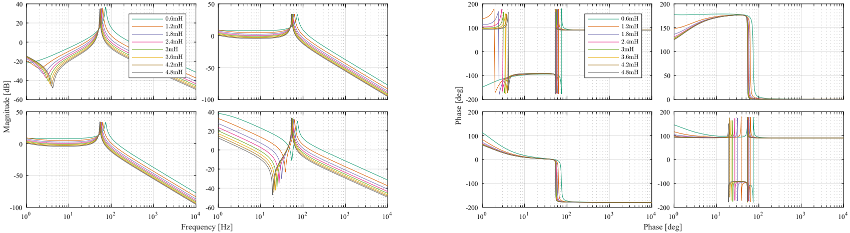

Figure 15 Dependence of the a) magnitude and b) phase of the output admittance of the inverter on the change of grid filter inductance L_g.

- From (23) and (24) it is obvious that any of the model parameters can influence the output admittance and input impedance.

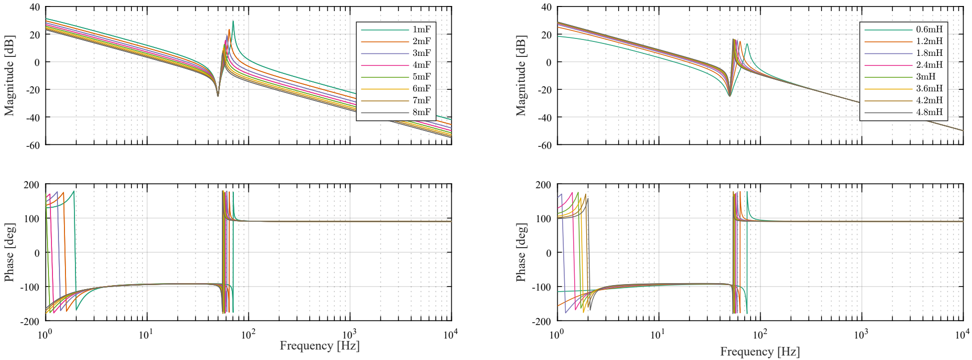

Figure 16 Dependence of the a) magnitude and b) phase of the input impedance of the inverter on the change of (left) dc-link capacitance C_{dc} and (right) grid filter inductance L_g.

Closed-loop modelling of grid-connected converter is performed in three stages. Each of these stages is

- Modelling the Phase-Locked Loop
- Modelling the Grid Current Control
- Modelling the dc-link Voltage Control

Phase Locked Loop Modelling

Ideally, the angle detected by the PLL should be identical to the grid angle, but due to the limited PLL bandwidth there will exist a small deviation $\tilde{\Theta}$ between the two angles. Thus, an arbitrary signal in *d*-*q*-frame affected by the misalignment can be given as:

$$\begin{bmatrix} X_d^c + \tilde{x}_d^c \\ X_q^c + \tilde{x}_q^c \end{bmatrix} = \begin{bmatrix} \cos\tilde{\theta} & \sin\tilde{\theta} \\ -\sin\tilde{\theta} & \cos\tilde{\theta} \end{bmatrix} \begin{bmatrix} X_d + \tilde{x}_d \\ X_q + \tilde{x}_q \end{bmatrix} \tag{28}$$

By solving the (28) and assuming that for very small values of $\tilde{\theta}$, $\cos\tilde{\theta} = 1$ and $\sin\tilde{\theta} = \tilde{\theta}$ and that the steady-state values are equal, i.e. $\mathbf{X}^c = \mathbf{X}$ one obtains the relation revealing the effect the grid synchronization has on a system variable:

$$\begin{aligned} \tilde{x}_d^c &= \tilde{x}_d + X_q\tilde{\theta} \\ \tilde{x}_q^c &= \tilde{x}_q - X_d\tilde{\theta} \end{aligned} \tag{29}$$

From the PLL control loop (c.f. **Fig.** 17), the relation between the output angle and input q-axis voltage is given as

$$\tilde{\theta} = \tilde{v}_q^c \underbrace{\left(K_{p,PLL} + \frac{K_{i,PLL}}{s}\right)}_{G_{\mathrm{PI}}^{\mathrm{PLL}}(s)} \frac{1}{s} = \tilde{v}_q^c \underbrace{\frac{K_{p,PLL}s + K_{i,PLL}}{s^2}}_{H_{\mathrm{PLL,ol}}(s)} \tag{30}$$

Where $H_{\mathrm{PLL,ol}}(s)$ is the PLL open-loop transfer function. From (29), replacing the q-axis signal by voltage and plugging in (30) gives the relation between the angle $\tilde{\theta}$ and unaffected voltage as

$$\tilde{\theta} = \frac{H_{\mathrm{PLL,ol}}}{1 + V_d H_{\mathrm{PLL,ol}}} \tilde{v}_q \tag{31}$$

When this equation is replaced in (28), it gives the effect of the PLL dynamics on the small-signal variables $\tilde{x}_d^c$ and $\tilde{x}_q^c$ as

$$\begin{aligned} \tilde{x}_d^c &= \tilde{x}_d + X_q \frac{H_{\mathrm{PLL,ol}}}{1 + V_d H_{\mathrm{PLL,ol}}} \tilde{v}_q \\ \tilde{x}_q^c &= \tilde{x}_q - X_d \underbrace{\frac{H_{\mathrm{PLL,ol}}}{1 + V_d H_{\mathrm{PLL,ol}}}}_{G_{\mathrm{PLL}}(s)} \tilde{v}_q \end{aligned} \tag{32}$$

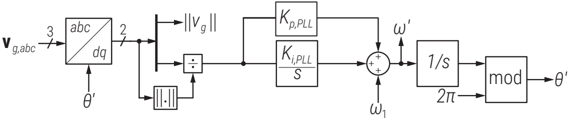

Figure 17 Control block diagram of SRF-PLL

Grid Current Control Modelling

The controller action, affected by the PLL dynamics and with the cross coupling between axes and input voltage feed-forward can be formulated as:

$$\mathbf{d}_{dq}^{\star} = \frac{1}{V_{\mathrm{dc}}}\left[\underbrace{\left(K_{p,gcc} + \frac{K_{i,gcc}}{s}\right)}_{G_{\mathrm{PI}}^{\mathrm{i}}(s)}\left(\mathbf{i}_{g,dq}^{\star} - \mathbf{H}_{\mathrm{i}}\tilde{\mathbf{i}}_{g,dq}^{c}\right) + \mathbf{G}_{\mathrm{dec}}\mathbf{H}_{\mathrm{i}}\tilde{\mathbf{i}}_{g,dq}^{c} + \tilde{\mathbf{v}}_{g,dq}^{c}\right] \quad (33)$$

$\mathbf{G}_{\mathrm{dec}}$ is the decoupling matrix given as

$$\mathbf{G}_{\mathrm{dec}} = \begin{bmatrix} 0 & -\omega_g L_g \\ \omega_g L_g & 0 \end{bmatrix} \quad (34)$$

$\mathbf{H}_i$ is the matrix representing the current transducer and

$$\mathbf{H}_{\mathrm{i}} = \begin{bmatrix} H_{i,d} & 0 \\ 0 & H_{i,q} \end{bmatrix} \quad (35)$$

The transfer function H_i can be associated with the delays caused by the ADC, measurement sampling through the ZOH or any other sampling type and with control action update. ADC and control action and ZOH can be modelled in the Laplace domain as

$$H_{\mathrm{ADC}}(s) = \frac{1 - \frac{T_{\mathrm{sw}}}{2}s}{1 + \frac{T_{\mathrm{sw}}}{2}s} \qquad H_{\mathrm{ZOH}}(s) = \frac{1}{1 + \frac{T_{\mathrm{sw}}}{2}s} \quad (36)$$

The grid current, voltage and the duty cycle are given in the control system reference frame as

$$\begin{bmatrix}\tilde{i}^c_{g,d}\\ \tilde{i}^c_{g,q}\end{bmatrix} = \begin{bmatrix}\tilde{i}_{g,d}\\ \tilde{i}_{g,q}\end{bmatrix} + \underbrace{\begin{bmatrix}0 & I_{g,q}G_{\mathrm{PLL}}\\ 0 & -I_{g,d}G_{\mathrm{PLL}}\end{bmatrix}}_{\mathbf{G}^{\mathrm{i}}_{\mathrm{PLL}}} \cdot \begin{bmatrix}\tilde{v}_{g,d}\\ \tilde{v}_{g,q}\end{bmatrix} \quad (37)$$

$$\begin{bmatrix}\tilde{v}^c_{g,d}\\ \tilde{v}^c_{g,q}\end{bmatrix} = \begin{bmatrix}\tilde{v}_{g,d}\\ \tilde{v}_{g,q}\end{bmatrix} + \underbrace{\begin{bmatrix}0 & V_{g,q}G_{\mathrm{PLL}}\\ 0 & -V_{g,d}G_{\mathrm{PLL}}\end{bmatrix}}_{\mathbf{G}^{\mathrm{v}}_{\mathrm{PLL}}} \cdot \begin{bmatrix}\tilde{v}_{g,d}\\ \tilde{v}_{g,q}\end{bmatrix} \quad (38)$$

$$\begin{bmatrix}\tilde{d}_d\\ \tilde{d}_q\end{bmatrix} = \begin{bmatrix}\tilde{d}^c_d\\ \tilde{d}^c_q\end{bmatrix} + \underbrace{\begin{bmatrix}0 & -D_qG_{\mathrm{PLL}}\\ 0 & D_dG_{\mathrm{PLL}}\end{bmatrix}}_{\mathbf{G}^{\mathrm{d}}_{\mathrm{PLL}}} \cdot \begin{bmatrix}\tilde{v}_{g,d}\\ \tilde{v}_{g,q}\end{bmatrix} \quad (39)$$

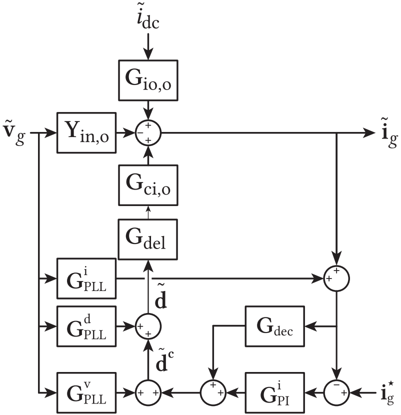

Figure 18 Small-signal closed loop model of the AFE with the influence of the PLL and under the current control

The converter duty cycle can be solved from **Fig.** 18 and is given as

$$\tilde{\mathbf{d}}_{dq} = \mathbf{G}^{\mathrm{i}}_{\mathrm{PI}}\mathbf{i}^{\star}_{g,dq} + \left(\mathbf{G}_{\mathrm{dec}} - \mathbf{G}^{\mathrm{i}}_{\mathrm{PI}}\right)\tilde{\mathbf{i}}_{g,dq} + \underbrace{\left[\mathbf{I} + \left(\mathbf{G}_{\mathrm{dec}} - \mathbf{G}^{\mathrm{i}}_{\mathrm{PI}}\right)\mathbf{G}^{\mathrm{i}}_{\mathrm{PLL}} + \mathbf{G}^{\mathrm{d}}_{\mathrm{PLL}} + \mathbf{G}^{\mathrm{v}}_{\mathrm{PLL}}\right]}_{\mathbf{G}^{\mathrm{vd}}_{\mathrm{PLL}}}\tilde{\mathbf{v}}_{g,dq} \quad (40)$$

With the duty cycle defined as in (40) the closed loop dynamics under current control can be solved. To solve the closed loop dynamics one has to take a look back to (25). The output side dynamics is given as

$$\mathbf{G}_{\mathrm{io,cl}}^{\mathrm{gcc}} = \left[\mathbf{I} + \mathbf{G}_{\mathrm{ci,o}}\mathbf{G}_{\mathrm{del}}\left(\mathbf{G}_{\mathrm{PI}}^{\mathrm{i}} - \mathbf{G}_{\mathrm{dec}}\right)\right]^{-1}\mathbf{G}_{\mathrm{io,o}} \tag{41}$$

$$\mathbf{Y}_{\mathrm{in,cl}}^{\mathrm{gcc}} = \left[\mathbf{I} + \mathbf{G}_{\mathrm{ci,o}}\mathbf{G}_{\mathrm{del}}\left(\mathbf{G}_{\mathrm{PI}}^{\mathrm{i}} - \mathbf{G}_{\mathrm{dec}}\right)\right]^{-1}\left[-\mathbf{Y}_{\mathrm{in,o}} + \mathbf{G}_{\mathrm{ci,o}}\mathbf{G}_{\mathrm{del}}\left(\mathbf{I} + \left(\mathbf{G}_{\mathrm{dec}} - \mathbf{G}_{\mathrm{PI}}^{\mathrm{i}}\right)\mathbf{G}_{\mathrm{PLL}}^{\mathrm{i}} + \mathbf{G}_{\mathrm{PLL}}^{\mathrm{d}} + \mathbf{G}_{\mathrm{PLL}}^{\mathrm{v}}\right)\right] \tag{42}$$

$$\mathbf{G}_{\mathrm{ci,cl}}^{\mathrm{gcc}} = \left[\mathbf{I} + \mathbf{G}_{\mathrm{ci,o}}\mathbf{G}_{\mathrm{del}}\left(\mathbf{G}_{\mathrm{PI}}^{\mathrm{i}} - \mathbf{G}_{\mathrm{dec}}\right)\right]^{-1}\mathbf{G}_{\mathrm{ci,o}}\mathbf{G}_{\mathrm{del}}\mathbf{G}_{\mathrm{PI}}^{\mathrm{i}} \tag{43}$$

While the input dynamics is given as

$$Z_{\mathrm{out,cl}}^{\mathrm{gcc}} = Z_{\mathrm{out,o}} + \mathbf{G}_{\mathrm{co,o}}\mathbf{G}_{\mathrm{del}}\left(\mathbf{G}_{\mathrm{dec}} - \mathbf{G}_{\mathrm{PI}}^{\mathrm{i}}\right)\mathbf{G}_{\mathrm{io,cl}}^{\mathrm{gcc}} \tag{44}$$

$$\mathbf{G}_{\mathrm{co,cl}}^{\mathrm{gcc}} = \mathbf{G}_{\mathrm{co,o}}\mathbf{G}_{\mathrm{del}}\mathbf{G}_{\mathrm{PI}}^{\mathrm{i}} + \mathbf{G}_{\mathrm{co,o}}\mathbf{G}_{\mathrm{del}}\left(\mathbf{G}_{\mathrm{dec}} - \mathbf{G}_{\mathrm{PI}}^{\mathrm{i}}\right)\mathbf{G}_{\mathrm{ci,cl}}^{\mathrm{gcc}} \tag{45}$$

$$\mathbf{T}_{\mathrm{oi,cl}}^{\mathrm{gcc}} = \mathbf{T}_{\mathrm{oi,o}} + \mathbf{G}_{\mathrm{co,o}}\mathbf{G}_{\mathrm{del}}\left[\left(\mathbf{G}_{\mathrm{dec}} - \mathbf{G}_{\mathrm{PI}}^{\mathrm{i}}\right)\mathbf{Y}_{\mathrm{in,cl}}^{\mathrm{gcc}} + \mathbf{G}_{\mathrm{PLL}}^{\mathrm{vd}}\right] \tag{46}$$

Where the transfer function matrix $\mathbf{G}_{\mathrm{del}}$ is associated with half-a-period update delay due to the PWM and is given as

$$\mathbf{G}_{\mathrm{del}} = \begin{bmatrix} G_{del,d} & 0 \\ 0 & G_{del,q} \end{bmatrix} \qquad G_{del}\left(s\right) = \frac{1 - \frac{T_{\mathrm{sw}}}{4}s}{1 + \frac{T_{\mathrm{sw}}}{4}s} \tag{47}$$

DC-link Voltage Control Modelling

The controller action can be formulated as

$$i^{\star}_{g,d} = \underbrace{\left(K_{p,dvc} + \frac{K_{i,dvc}}{s}\right)}_{G^{\mathrm{v}}_{\mathrm{PI}}(s)}\left(v^{\star}_{\mathrm{dc}} - \mathbf{H}_v \tilde{v}_{\mathrm{dc}}\right) \tag{48}$$

The closed loop dynamics under cascaded control and dc-link voltage control can be solved by first solving the dc-side (c.f. **Fig.** 20), and then solving the ac side dynamics (c.f. **Fig.** 19). The input side dynamics is given as

$$Z^{\mathrm{dvc}}_{\mathrm{out,cl}} = \left(\mathbf{I} + \mathbf{G}^{\mathrm{gcc}}_{\mathrm{co,cl}}\mathbf{G}^{\mathrm{v}}_{\mathrm{PI}}\right)^{-1} Z^{\mathrm{gcc}}_{\mathrm{out,cl}} \tag{49}$$

$$\mathbf{G}^{\mathrm{dvc}}_{\mathrm{co,cl}} = \left(\mathbf{I} + \mathbf{G}^{\mathrm{gcc}}_{\mathrm{co,cl}}\mathbf{G}^{\mathrm{v}}_{\mathrm{PI}}\right)^{-1} \mathbf{G}^{\mathrm{gcc}}_{\mathrm{co,cl}}\mathbf{G}^{\mathrm{v}}_{\mathrm{PI}} \tag{50}$$

$$\mathbf{T}^{\mathrm{dvc}}_{\mathrm{oi,cl}} = \left(\mathbf{I} + \mathbf{G}^{\mathrm{gcc}}_{\mathrm{co,cl}}\mathbf{G}^{\mathrm{v}}_{\mathrm{PI}}\right)^{-1} \mathbf{T}^{\mathrm{gcc}}_{\mathrm{oi,cl}} \tag{51}$$

While the output side dynamics is given as

$$\mathbf{G}^{\mathrm{dvc}}_{\mathrm{io,cl}} = \mathbf{G}^{\mathrm{gcc}}_{\mathrm{io,cl}} + \mathbf{G}^{\mathrm{gcc}}_{\mathrm{ci,cl}}\mathbf{G}^{\mathrm{v}}_{\mathrm{PI}} Z^{\mathrm{dvc}}_{\mathrm{out,cl}} \tag{52}$$

$$\mathbf{Y}^{\mathrm{dvc}}_{\mathrm{in,cl}} = \mathbf{Y}^{\mathrm{gcc}}_{\mathrm{in,cl}} + \mathbf{G}^{\mathrm{gcc}}_{\mathrm{ci,cl}}\mathbf{G}^{\mathrm{v}}_{\mathrm{PI}}\mathbf{T}^{\mathrm{dvc}}_{\mathrm{oi,cl}} \tag{53}$$

$$\mathbf{G}^{\mathrm{dvc}}_{\mathrm{ci,cl}} = \mathbf{G}^{\mathrm{gcc}}_{\mathrm{ci,cl}}\mathbf{G}^{\mathrm{v}}_{\mathrm{PI}} - \mathbf{G}^{\mathrm{gcc}}_{\mathrm{ci,cl}}\mathbf{G}^{\mathrm{v}}_{\mathrm{PI}}\mathbf{G}^{\mathrm{dvc}}_{\mathrm{co,cl}} \tag{54}$$

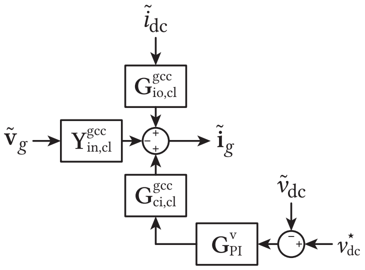

Figure 19 AC side closed-loop dynamics

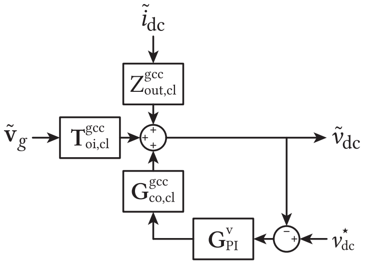

Figure 20 DC side closed-loop dynamics

Grid connected converters are essential elements of new "power electronics dominated power systems":

- PV energy generation (DC by nature) requires converter to connect to the AC grid
- Battery energy storage (DC by nature)
- Wind energy generation - from electrical machine to AC grid through a converters

We have covered various "technologies" needed to master to realize these grid connected system:

- HW - various converter topologies and role of passive elements
- SW - modulations, grid synchronization, current, voltage, power control in various reference frames

There also some issues related to resonances, that must be considered during implementation

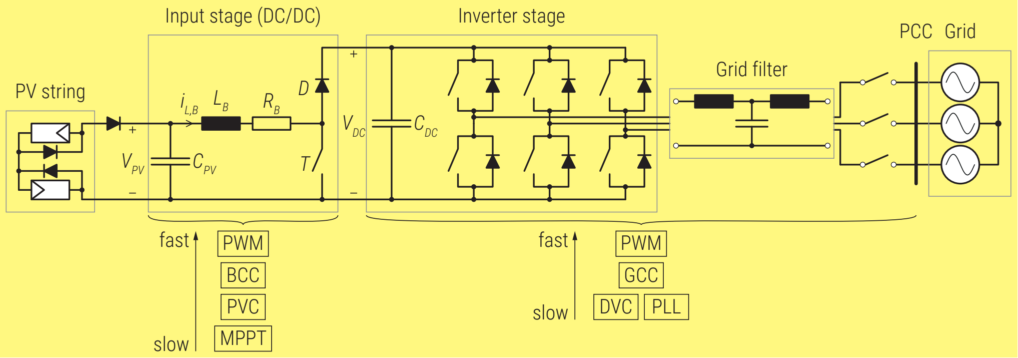

Figure 21 PV double-stage grid connected converter.