

# EE-465 - W12

# LCL FILTER

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Grid connected converters employ some kind of filter at their output, with the role:

- ▶ to provide inductive behavior and allow for control of active and reactive power exchange with the grid
- ▶ to provide attenuation of PWM caused harmonics

We have considered simple L filter, which is not always sufficient. High order filters, such as LCL are often used, as they offer:

- ▶ better attenuation of switching harmonics
- ▶ smaller filter size (physically)

There also some issues related to resonances, that must be considered during implementation

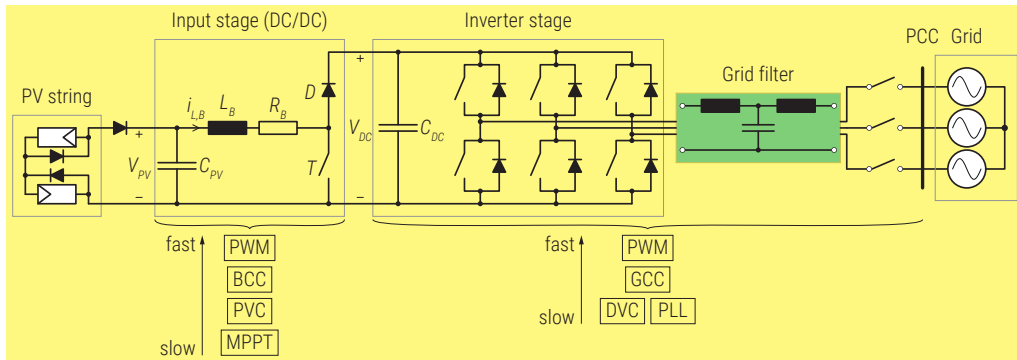


Figure 1 PV double-stage grid connected converter.

# POWER EXCHANGE WITH THE GRID

Power exchange between point A and B (grid connected converter):

$$S = \underline{V}_A \underline{I}^* = V_A \left[ \frac{V_A - V_B \angle -\delta}{Z \angle \theta} \right]^* = \frac{V_A^2}{Z} \angle \theta - \frac{V_A V_B}{Z} \angle (\theta + \delta)$$

Active and Reactive powers are ( $\delta$  is the power angle and  $\theta$  is the power factor angle):

$$P_A = \frac{V_A^2}{Z} \cos \theta - \frac{V_A V_B}{Z} \cos(\theta + \delta), \quad Q_A = \frac{V_A^2}{Z} \sin \theta - \frac{V_A V_B}{Z} \sin(\theta + \delta)$$

With  $R = Z \cos(\theta)$  and  $X = Z \sin(\theta)$  we have:

$$P_A = \frac{V_A}{R^2 + X^2} \left[ R(V_A - V_B \cos \delta) + X V_B \sin \delta \right]$$

$$Q_A = \frac{V_A}{R^2 + X^2} \left[ -R V_B \sin \delta + X(V_A - V_B \cos \delta) \right]$$

Neglecting  $R$  due to  $X \gg R$ , and considering small power angle  $\delta$ ,  $\sin \delta \approx \delta$ ,  $\cos \delta \approx 1$ :

$$P_A = \frac{V_A V_B}{X} \delta, \quad Q_A = \frac{V_A - V_B}{X} V_A$$

- ▶ The Active power is controlled by power angle  $\delta$
- ▶ The Reactive power is controlled by voltage difference

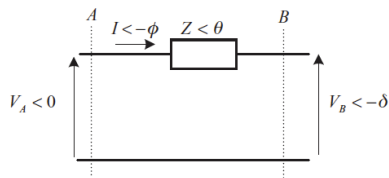


Figure 2 Converter - Impedance - Grid.

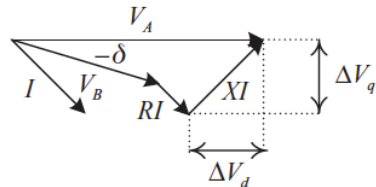


Figure 3 Phasor diagram.

# LC TRAP FILTERS

In case of low switching frequencies (e.g. few hundreds of Hz), LC trap filters are used:

- ▶ series connected LC elements are tuned to a specific resonance frequency
- ▶ at series resonance frequency impedance of the filter is at minimum (zero)
- ▶ this represents low impedance path for current harmonic at that frequency

Resonant frequency is calculated as:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Several series  $LC$  elements, tuned at different frequencies can be connected in parallel

Impedance of an LC circuit, ignoring resistances and damping, is:

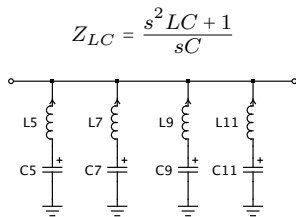


Figure 4 LC trap filters

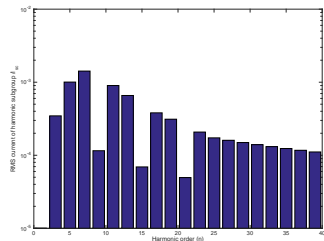


Figure 5 Example: BDEW grid codes.

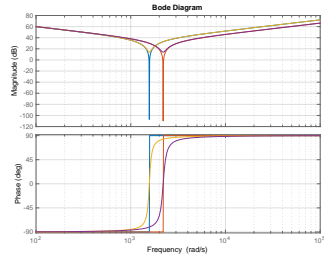


Figure 6 LC trap filter impedance (250Hz and 350Hz).

# LCL FILTER - IDEAL

LCL filter is a low pass filter, offering good high frequency attenuation.

Basic set of expressions related to ideal LCL filter (resistances are neglected):

$$v_c(s) = sL_c i_c(s) + v_f(s)$$

$$v_g(s) = v_f(s) - sL_g i_g(s)$$

$$v_f(s) = i_f(s)/sC_f$$

$$i_c(s) = i_f(s) + i_g(s)$$

Transfer function of interest is  $i_g(s)/v_c(s)$ , and after setting  $v_g(s) = 0$  we have:

$$H_{LCL}(s) = \frac{1}{L_c C_f L_g s^3 + (L_c + L_g)s}$$

Resonant frequency is defined with:

$$\omega_r = \sqrt{\frac{1}{L_{eq} C_f}} \Rightarrow L_{eq} = \frac{L_c L_g}{L_c + L_g} \Rightarrow \omega_r = \sqrt{\frac{L_c + L_g}{L_c C_f L_g}}$$

- ▶ Frequency response show high peak at resonant frequency
- ▶  $-60dB$  attenuation slope at higher frequencies (above resonant frequency)

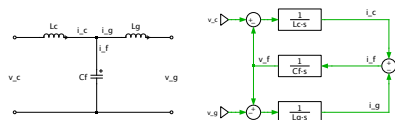


Figure 7 Single-phase ideal LCL filter and model.

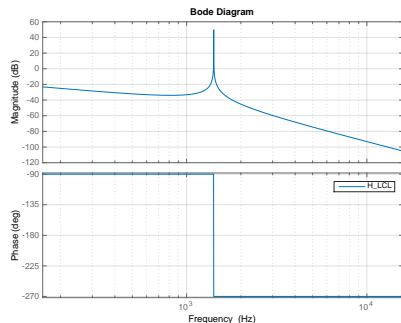


Figure 8 Frequency response - ideal LCL filter.

# LCL FILTER - DESIGN CONSIDERATIONS

Certain limits are also usually applied during the design:

- ▶ capacitor value is limited by decrease of power factor at rated power (typically less than 5%)
- ▶ total value of inductance should be less than 0.1 p.u. to limit AC voltage drop
- ▶ the resonant frequency should be in the range between tens times the grid frequency and one half the switching frequency
- ▶ resonance problems should be avoided by some means
- ▶ **Passive Damping** is simple approach but introduces additional losses
- ▶ **Active Damping** solves the problem through a control modifications (lossless)

Design of an LCL filter must consider:

- ▶ desired or allowed current ripple on the converter side of the filter
- ▶  $L_c$  is designed to limit converter output current ripple
- ▶ harmonic limits imposed on the grid side (standards, grid codes)
- ▶  $L_g$  is designed considering high frequency harmonics
- ▶ the amount of installed reactive power of the LCL filter should be minimized
- ▶ the resonant frequency should be carefully selected, considering control bandwidth
- ▶  $C_f$  is designed considering these conditions

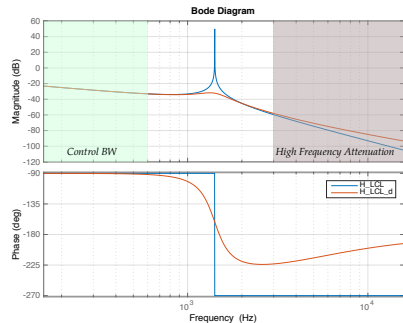


Figure 9 Frequency response - ideal and damped LCL filter.

## LCL FILTER - DESIGN I: $C_f$

To design  $C_f$  we have to consider reactive power variations due to LCL filter

We can define base values as:

$$Z_b = \frac{V_{LL}^2}{P_n}, \quad C_b = \frac{1}{\omega_g Z_b}, \quad L_b = \frac{Z_b}{\omega_g}$$

where:

- ▶  $V_{LL}$  is grid line-to-line RMS voltage
- ▶  $P_n$  is the rated active power
- ▶  $\omega_g = 2\pi f_g$  is the grid angular frequency

Filter capacitance is related to the base value as percentage of it:

$$C_f = x C_b$$

Setting maximum power variation seen by the grid, e.g.  $x = 5\%$ , we have:

$$C_f = 0.05 C_b$$

Different percentage values can be used as well ( $x \in (1 \div 5)\%$ ).

## LCL FILTER - DESIGN I: $L_c$

To design  $L_c$  we have to consider worst case current ripple

We will assume short circuit condition for filter capacitor  $C_f$  at ripple frequency

Maximum current ripple at the output of a 3-phase 2-level VSI can be determined as:

$$\Delta i_{L_c-MAX} = \frac{V_{DC}}{6L_c f_{sw}}$$

Setting, for example, desired current ripple as 10%:

$$\Delta i_{L_c-MAX} = 0.1 I_{L_c-peak}$$

where peak and rms current values are determined from rated voltage and power:

$$I_{L_c-MAX} = \frac{P_n}{\sqrt{3}V_{ll}} \Rightarrow I_{L_c-peak} = \sqrt{2} I_{L_c-MAX}$$

Converter side LCL filter inductance minimum  $L_c$  value can be calculated as:

$$L_{c-MIN} = \frac{V_{DC}}{6\Delta i_{L_c-MAX} f_{sw}}$$

Selected  $L_c$  value can be related to grid side  $L_g$  through scaling factor  $r$  as:

$$L_g = r L_c$$

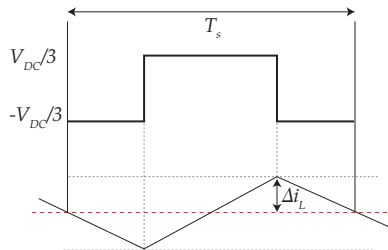


Figure 10 VSI output current ripple.

## LCL FILTER - DESIGN I: $L_g$

To design  $L_g$  we have to consider:

- ▶ harmonic attenuation
- ▶ choice of the resonant frequency

Attenuation of high frequency components of interest can be checked against the standards

Neglecting losses, grid current harmonic attenuation can be defined as:

$$\frac{i_g(h_{sw})}{i_c(h_{sw})} = \frac{1}{|1 + r(1 - L_g x C_b \omega_{sw}^2)|} = k_a$$

With  $L_c$  already selected,  $k_a$  represents desired attenuation at switching frequency  $\omega_{sw}$ :

Considering desired attenuation  $k_a$ , grid side inductance  $L_g$  can be calculated as

$$L_g = \frac{\sqrt{\frac{1}{k_a^2}} + 1}{C_f \omega_{sw}^2}$$

where  $C_f = x C_b$ , and it may have to be varied to adjust resonant frequency

In practice, grid inductance constitutes the part of  $L_g$

Also choice of  $L_c = L_g$  provides good compromise (as discussed on the next slide)

## LCL FILTER - DESIGN II

Alternative way to consider problem of design of inductances  $L_c$  and  $L_g$  ( $C_f$  is designed as in Design I).

Considering that at low frequencies, the LCL filter behaves as an inductance, defined by:

$$L_{dc} = L_c + L_g$$

There is a relation between  $L_c$  and  $L_g$  that minimizes the voltage drop at fundamental frequency and maximizes filtering ability.

If we express  $L_c$  and  $L_g$ , as percentage of  $L_{dc}$  with  $\alpha \in (0 \div 1)$

$$L_c = \alpha L_{dc} \quad L_g = (1 - \alpha)L_{dc}$$

The  $L_{eq}$  that sets the resonant frequency can be redefined as:

$$L_{eq} = \frac{L_c L_g}{L_c + L_g} \Rightarrow L_{eq} = \alpha(1 - \alpha)L_{dc}$$

Minimum of  $L_{eq}$  is achieved for  $\alpha = 1/2$  or both inductances being equal  $L_c = L_g$ :

$$\frac{dL_{eq}}{d\alpha} = 0 \Rightarrow \alpha = 1/2 \Rightarrow L_c = L_g$$

By choosing  $L_{dc}$  to be 0.1 p.u. of the base impedance  $L_b$  at fundamental frequency  $\omega_g$ ,  $L_c$  and  $L_g$  are easily determined

# LCL FILTER - PASSIVE DAMPING

Zero impedance at the LCL filter resonant frequency may lead to control instability

To reduce high peak gain at resonant frequency, damping resistance  $R_f$  is included.

Basic set of equations is now modified as:

$$\begin{aligned}v_c(s) &= sL_c i_c(s) + v_f(s) \\v_g(s) &= v_f(s) - sL_g i_g(s) \\v_f(s) &= (R_f + 1/sC_f) i_f(s) \\i_c(s) &= i_f(s) + i_g(s)\end{aligned}$$

Transfer function linking filter output  $i_g(s)$  and filter input  $v_c(s)$  is:

$$H_{LCL-d}(s) = \frac{C_f R_f s + 1}{L_c C_f L_g s^3 + C_f (L_c + L_g) R_f s^2 + (L_c + L_g) s}$$

Damping resistance  $R_f$  attenuates the high peak gain, but also reduces filter effectiveness ( $-40dB$  instead of  $-60dB$  of ideal LCL) and increases losses.

The value of resistor  $R_f$  is often selected as one third of the impedance of the filter capacitor  $C_f$  at the resonant frequency:

$$R_f = \frac{1}{3\omega_{res} C_f}$$

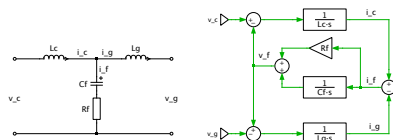


Figure 11 Single-phase damped LCL filter and model.

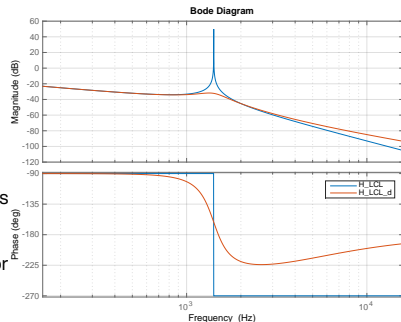


Figure 12 Frequency response - damped LCL filter.

# LCL FILTER - DESIGN COMPARISON

Input data are:

- ▶  $V_{LL} = 400$  - line to line RMS voltage [V]
- ▶  $P_n = 3500$  - rated active power [W]
- ▶  $V_{DC} = 650$  - rated dc-link voltage [V]
- ▶  $\omega_g = 2\pi \cdot 50$  - grid angular frequency [Hz]
- ▶  $f_{sw} = 10000$  - switching frequency [Hz]
- ▶  $x = 0.05$  - maximum power factor variation seen by the grid [%] (design 1)
- ▶  $k_a = 0.2$  - desired harmonic current attenuation [%] (design 1)
- ▶  $\Delta i = 0.1$  - allowed current ripple [A] (design 1)
- ▶  $k_l = 0.1$  - total DC inductance as percentage of base value [%] (design 2)

Resulting designs can be summarized as:

LCL	Design 1	Design 2
$L_c$	15.16 mH	7.28 mH
$L_g$	0.44 mH	7.28 mH
$C_f$	3.48 $\mu F$	3.48 $\mu F$
$f_r$	4.14 kHz	1.41 kHz
$R_f$	3.68 $\Omega$	10.76 $\Omega$

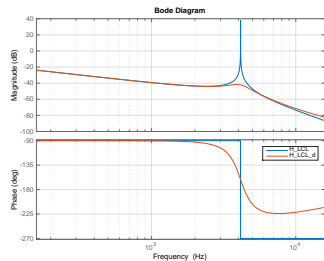


Figure 13 Frequency response - Design 1.

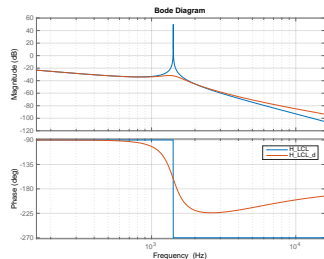


Figure 14 Frequency response - Design 2.

# LCL FILTER - ACTIVE DAMPING (I)

Rather than using passive damping with real resistor, it is possible to achieve similar effect by means of control and use of *virtual resistor*

Control structure can be modified, taking into account:

- ▶  $v_f(s)$  - capacitor voltage as feedback variable
- ▶  $i_f(s)$  - capacitor current as feedback variable

From the basic set of expressions related to ideal LCL filter (resistances are neglected) and after setting  $v_g(s) = 0$ :

$$v_c(s) = sL_c i_c(s) + v_f(s) \Rightarrow v_c(s) = sL_c(i_f(s) + i_g(s)) + v_f(s)$$

$$v_g(s) = v_f(s) - sL_g i_g(s) \Rightarrow i_g(s) = v_f(s)/sL_g$$

$$v_f(s) = i_f(s)/sC_f \Rightarrow i_f(s) = sC_f v_f(s)$$

$$v_f^d(s) = i_f(s)(R_f + 1/sC_f) \Rightarrow i_f(s) = v_f^d(s)/(R_f + 1/sC_f)$$

$$i_c(s) = i_f(s) + i_g(s)$$

We can derive two transfer functions of interest:  $v_f(s)/v_c(s)$ , and  $i_f(s)/v_c(s)$  for ideal LCL case as:

$$H_{v_f}(s) = \frac{v_f(s)}{v_c(s)} = \frac{1}{L_c C_f} \frac{1}{s^2 + \omega_r^2}, \quad H_{i_f}(s) = \frac{i_f(s)}{v_c(s)} = \frac{i_f(s)}{v_f(s)} \frac{v_f(s)}{v_c(s)} = \frac{1}{L_c} \frac{s}{s^2 + \omega_r^2}$$

For LCL case with damping resistor we have:

$$H_{v_f}^d(s) = \frac{v_f^d(s)}{v_c(s)} = \frac{1}{L_c C_f} \frac{1 + sR_f C_f}{s^2 + \omega_r^2(1 + sR_f C_f)}, \quad H_{i_f}^d(s) = \frac{i_f(s)}{v_c(s)} = \frac{i_f(s)}{v_f^d(s)} \frac{v_f^d(s)}{v_c(s)} = \frac{1}{L_c} \frac{s}{s^2 + \omega_r^2(1 + sR_f C_f)}$$

## LCL FILTER - ACTIVE DAMPING (II)

*Virtual resistor* can be placed inside the control structure with ideal LCL filter, to emulate presence of a real resistor

We will consider capacitor voltage  $v_f(s)$  as feedback variable, and act on the converter output voltage:

In an ideal LCL case, without damping resistor we have:

$$H_{v_f}(s) = \frac{v_f(s)}{v_c(s)} = \frac{1}{L_c C_f} \frac{1}{s^2 + \omega_r^2} = \frac{L_g}{s^2 L_c C_f L_g + L_c + L_g} = H_1(s), \quad \frac{i_f(s)}{v_f(s)} = s C_f = H_2(s)$$

For case with damping resistor we have:

$$H_{v_f}^d(s) = \frac{v_f^d(s)}{v_c(s)} = \frac{1}{L_c C_f} \frac{1 + s R_f C_f}{s^2 + \omega_r^2 (1 + s R_f C_f)} = \frac{L_g (1 + s R_f C_f)}{s^2 L_c C_f L_g + (L_c + L_g) (1 + s R_f C_f)} = H_3(s)$$

Ideally, we would like to achieve  $H_3(s)$  without real  $R_f$  but with  $R_v$  (virtual)

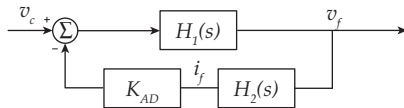


Figure 15 Active Damping control Loop with capacitor voltage/current as feedback.

## LCL FILTER - ACTIVE DAMPING (III)

If we measure capacitor voltage  $v_f(s)$ , calculate capacitor current  $i_f(s)$  and add suitable gain as feedback  $K_{AD}$  we have:

$$H_{CL}(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)K_{AD}} = \frac{\frac{L_g}{s^2 L_c C_f L_g + L_c + L_g}}{1 + \frac{L_g}{s^2 L_c C_f L_g + L_c + L_g} s C_f K_{AD}} = \frac{L_g}{s^2 L_c C_f L_g + L_c + L_g + s L_g C_f K_{AD}}$$

With  $K_{AD}$  selected as:

$$K_{AD} = \frac{L_c + L_g}{L_g} R_v$$

We have

$$H_{CL}(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)K_{AD}} = \frac{L_g}{s^2 L_c C_f L_g + L_c + L_g + s L_g C_f \frac{L_c + L_g}{L_g} R_v} = \frac{L_g}{s^2 L_c C_f L_g + (L_c + L_g)(1 + s R_v C_f)}$$

Control modification achieves closed-loop transfer function similar to targeted  $H_3(s)$

Term  $R_f C_f$  found in numerator of  $H_3(s)$  neglected, as it is much smaller than 1

Implementation must consider both axis of regulator ( $v_d, v_q$ )

# THE P-Q THEORY

Recalling definitions for instantaneous active and reactive powers (Clarke with  $K = 2/3$ ):

- ▶  $p_{dq} = \frac{3}{2}(v_\alpha i_\alpha + v_\beta i_\beta) = \frac{3}{2}(v_d i_d + v_q i_q)$  - (instantaneous active power)
- ▶  $q_{dq} = \frac{3}{2}(v_\alpha i_\beta - v_\beta i_\alpha) = \frac{3}{2}(v_d i_q - v_q i_d)$  - (instantaneous reactive power)

Current references can be calculated from power references as:

$$\mathbf{i}_{\alpha\beta}^* = \begin{bmatrix} i_\alpha^* \\ i_\beta^* \end{bmatrix} = \frac{1}{v_\alpha^2 + v_\beta^2} \begin{bmatrix} v_\alpha & -v_\beta \\ v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} p_{\alpha\beta}^* \\ q_{\alpha\beta}^* \end{bmatrix}$$
$$\mathbf{i}_{dq}^* = \begin{bmatrix} i_d^* \\ i_q^* \end{bmatrix} = \frac{1}{v_d^2 + v_q^2} \begin{bmatrix} v_d & -v_q \\ v_q & v_d \end{bmatrix} \begin{bmatrix} p_{dq}^* \\ q_{dq}^* \end{bmatrix}$$

With  $d$ -axis perfectly aligned with grid voltage space vector ( $v_q = 0$ ), we have:

- ▶  $p_{dq} = \frac{3}{2}v_d i_d$  - instantaneous active power is proportional to  $i_d$
- ▶  $q_{dq} = -\frac{3}{2}v_d i_q$  - instantaneous reactive power is proportional to  $i_q$

Orientation is achieved with help of PLL

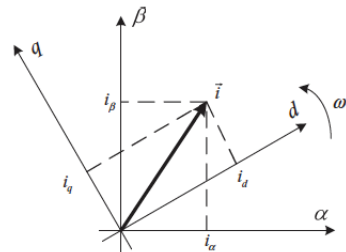


Figure 16  $\alpha\beta$  and  $dq$  reference frames.

# SENSOR LOCATION

As filter has reactive elements, it may influence desired power factor

Equivalent impedance at the PCC (point of common coupling) will depend on position of voltage and current sensors

With LCL filter installed there are several options regarding sensor location

- ▶ a)  $V_f$  and  $i_c$  sensed
  - ▶  $Z_g = R + j(X_2 + X_g - X_c)$
  - ▶  $Z_c = R + jX_1$
- ▶ b)  $V_f$  and  $i_g$  sensed
  - ▶  $Z_g = R + j(X_2 + X_g)$
  - ▶  $Z_c = R + j(X_1 - X_c)$
- ▶ c)  $V_g$  and  $i_g$  sensed
  - ▶  $Z_g = R + jX_g$
  - ▶  $Z_c = R + j(X_1 - X_2 - X_g - X_c)$
- ▶ d)  $V_g$  and  $i_c$  sensed
  - ▶  $Z_g = R + j(X_g - X_c)$
  - ▶  $Z_c = R - j(X_1 + X_2)$

Impedance seen at PCC is different for each case

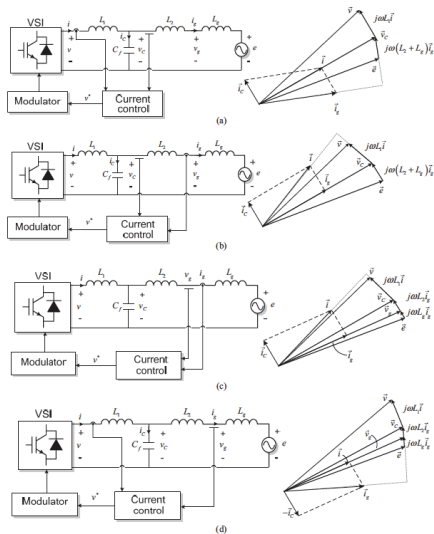


Figure 17 Vector diagram depending on sensor location.

# PQ OPEN-LOOP / CLOSED-LOOP CONTROL IN $dq$ FRAME

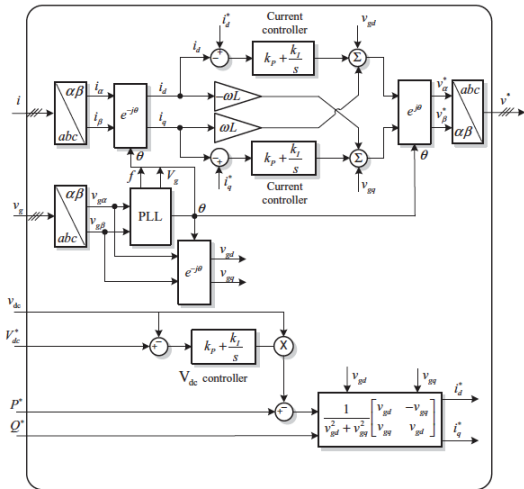


Figure 18 PQ open-loop voltage oriented control in  $dq$  frame.

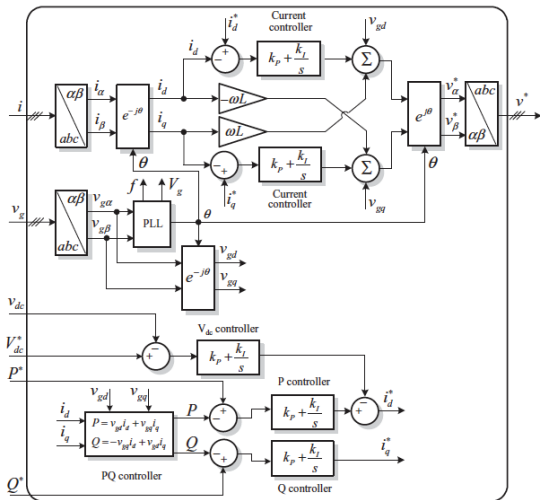


Figure 19 PQ closed-loop voltage oriented control in  $dq$  frame.

# PQ OPEN-LOOP / CLOSED-LOOP CONTROL IN $\alpha\beta$ FRAME

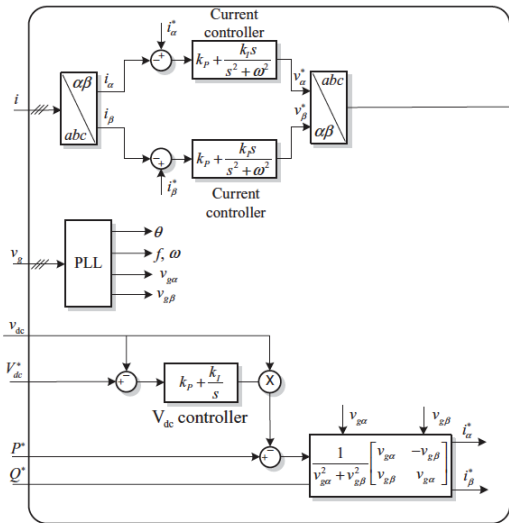


Figure 20 PQ open-loop voltage oriented control in  $\alpha\beta$  frame.

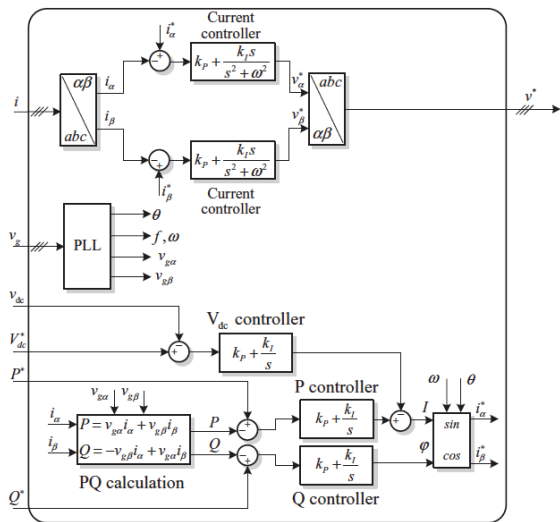


Figure 21 PQ closed-loop voltage oriented control in  $\alpha\beta$  frame.

# SUMMARY

LCL filter is commonly found in grid connected converters:

- ▶ it offers good attenuation of high frequencies
- ▶ often results in smaller physical size than simple L filter
- ▶ resonance can be damped either passively or actively
- ▶ modifications of the control structure are required
- ▶ power exchange with the grid can be performed in various ways (more to be covered next week - not PV related)

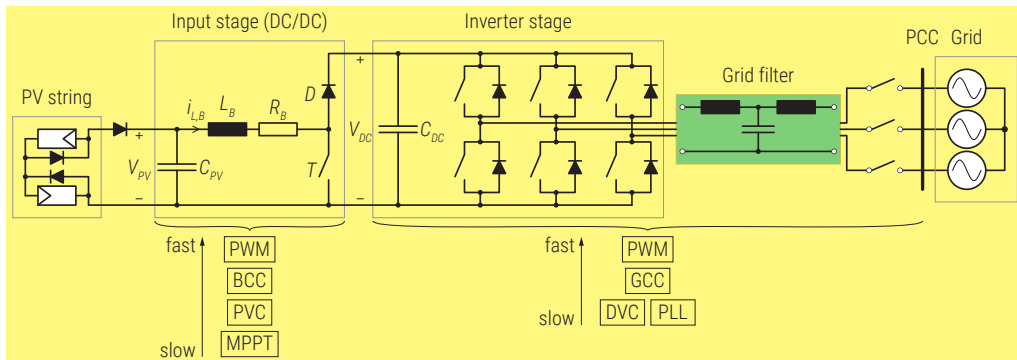


Figure 22 PV double-stage grid connected converter.