Exercises Antennas

Series 4

Exercise 1

Design a rectangular microstrip antenna so that it will resonate at 1.6 GHz. The idealistic lossless substrate (RT/Duroid 6010.2) has a dielectric constant of 10.2 and a height of 0.05 in. (0.127 cm).

a) Determine the physical dimensions (width and length) of the patch (in cm).

b) Approximate range of lengths (in cm) between the two radiating slots of the rectangular patch, if we want the input impedance (taking into account both radiating slots) to be real.

c) What is the real input impedance of Part b? Neglect coupling.

d) Location (in cm from the leading radiating slot) of a coaxial feed so that the total input impedance is 75 ohms.

Solution :

Exercice 1

a) For an efficient radiator, a practical width that leads to good radiation efficiencies is:

$$W = \frac{1}{2f_r\sqrt{\mu_0\varepsilon_0}}\sqrt{\frac{2}{\varepsilon_r+1}} = \frac{c}{2f_r}\sqrt{\frac{2}{\varepsilon_r+1}} \rightarrow W = \frac{3\cdot 10^8}{2\cdot 1.6\cdot 10^9}\sqrt{\frac{2}{10.2+1}} = 0.03962\ m = \mathbf{3.962\ cm}$$

Determine the effective dielectric constant of the microstrip antenna using:

$$\varepsilon_{reff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left[1+12\frac{h}{W}\right]^{-1/2} \quad for \quad \frac{W}{h} > 1$$

$$\frac{W}{h} = \frac{3.962}{0.127} = 31.19 > 1 \quad \rightarrow \quad \varepsilon_{reff} = \frac{10.2+1}{2} + \frac{10.2-1}{2}\left[1+12\frac{0.127}{3.962}\right]^{-1/2} = \mathbf{9.51}$$

The extended incremental length of the patch ΔL is:

$$\Delta L = h \cdot 0.412 \cdot \frac{(\varepsilon_{reff} + 0.3)\left(\frac{W}{h} + 0.264\right)}{(\varepsilon_{reff} - 0.258)\left(\frac{W}{h} + 0.8\right)} \rightarrow \Delta L = 0.127 \cdot 0.412 \cdot \frac{(9.51 + 0.3)(31.19 + 0.264)}{(9.51 - 0.258)(31.19 + 0.8)} = \mathbf{0.054\ cm}$$

The actual length L of the patch is found using:

$$L = \frac{c}{2f_r\sqrt{\varepsilon_{reff}}} - 2\Delta L = \frac{3 \cdot 10^{10}}{2 \cdot 1.6 \cdot 10^9\sqrt{9.51}} - 2 \cdot 0.054 = \mathbf{2.93\ cm}$$

Finally, the effective length is:

$$L_e = L + 2\Delta L = 2.93 + 2 \cdot 0.054 = \mathbf{3.038\ cm}$$

In summary:

$$\boldsymbol{W = 3.962\ cm}$$

$$\boldsymbol{L = 2.93\ cm}$$

$$\boldsymbol{\varepsilon_{reff} = 9.51}$$

b) Typical lengths of microstrip patches vary between:

$$L \approx (0.47 - 0.49)\lambda_g = (0.47 - 0.49)\frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

where λ_g is the wavelength in the dielectric.

$$L_{min} = 0.47\lambda_g = 0.47\frac{\lambda_0}{\sqrt{\varepsilon_r}} = 0.47\frac{3 \cdot 10^8}{1.6 \cdot 10^9\sqrt{10.2}} = 0.0276\ m = \mathbf{2.76\ cm}$$

$$L_{max} = 0.49\lambda_g = 0.47\frac{\lambda_0}{\sqrt{\varepsilon_r}} = 0.49\frac{3 \cdot 10^8}{1.6 \cdot 10^9\sqrt{10.2}} = 0.0288\ m = \mathbf{2.88\ cm}$$

The smaller the dielectric constant of the substrate, the larger is the fringing; thus, the length of the microstrip patch is smaller. In contrast, the larger the dielectric constant, the more tightly the fields are held within the substrate; thus, the fringing is smaller and the length is longer and closer to half-wavelength in the dielectric.

c)

$$\lambda_0 = \frac{c}{f_r} = \frac{3 \cdot 10^8}{1.6 \cdot 10^9} = 0.1875\ m = \mathbf{18.75\ cm}$$

$$k_0 = \frac{2\pi}{\lambda_0} = \frac{2\pi}{18.75} = 0.335\, rad/cm$$

$$G_1 = \frac{W}{120\lambda_0}\left[1 - \frac{1}{24}(k_0 h)^2\right] \qquad \frac{h}{\lambda_0} < \frac{1}{10}$$

$$\frac{h}{\lambda_0} = \frac{0.127 \cdot 10^{-2}}{0.1875} = 0.00847 < 0.1$$

$$G_1 = \frac{W}{120\lambda_0}\left[1 - \frac{1}{24}(k_0 h)^2\right] = \frac{3.962 \cdot 10^{-2}}{120 \cdot 0.1875}\left[1 - \frac{1}{24}(0.1875 \cdot 0.127 \cdot 10^{-2})^2\right] = 1.76 \cdot 10^{-3} S$$

Taking into account both radiating slots and neglecting coupling:

$$R_{in} = \frac{1}{2G_1} = \frac{1}{2 \cdot 1.76 \cdot 10^{-3}} = \mathbf{284\ \Omega}$$

d) The impedance at a specific point of the patch can be expressed as:

$$R_{in}(y = y_0) = \frac{1}{2(G_1 \pm G_{12})}\cos^2\left(\frac{\pi}{L}y_0\right) = R_{in}(y = 0)\cos^2\left(\frac{\pi}{L}y_0\right)$$

Since we are neglecting the coupling:

$$R_{in}(y = 0) = \frac{1}{2G_1}$$

$R_{in}(y = 0)$ is the impedance at the edge of the patch of the patch and $R_{in}(y = y_0)$ is the impedance at the desired point, this is 75 Ω. Therefore:

$$R_{in}(y = y_0) = R_{in}(y = 0)\cos^2\left(\frac{\pi}{L}y_0\right) \rightarrow 75 = 284 \cdot \cos^2\left(\frac{\pi}{2.93}y_0\right)$$

$$y_0 = \frac{2.93}{\pi}\cos^{-1}\left(\sqrt{\frac{75}{284}}\right) = \mathbf{0.964\ cm}$$