Algorithms for Wireless Communications

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Lecture 3/4: Diversity
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Non Coherent Detection (chapter 3.1)

$$y(m) = h(m)x(m) + w(m)$$

$$h(m) \sim \mathcal{CN}(0,1)$$

$$w(m) \sim \mathcal{CN}(0,N_0)$$

Orthogonal sequence

$$\mathbf{x}_A = \begin{bmatrix} a \\ 0 \end{bmatrix} = \begin{bmatrix} x(0) \\ x(1) \end{bmatrix} \; ; \mathbf{x}_B = \begin{bmatrix} 0 \\ a \end{bmatrix} = \begin{bmatrix} x(1) \\ x(0) \end{bmatrix}$$

- Note: x(0) = ±a does not work. The phase of y(m) is uniformly distributed [0, 2π] regardless whether ±a is transmitted. The received amplitude is independent of the transmitted symbol

ML-Detection

Note for x_A: $y[0] \sim CN(0, a^2 + N_0)$ and $y[1] \sim CN(0, N_0)$

$$\Lambda(\mathbf{y}) = \ln\left\{\frac{f(\mathbf{y}|x_A)}{f(\mathbf{y}|x_B)}\right\} \underset{x_B}{\overset{x_A}{\gtrless}} 0$$

It can be solved

$$\Lambda(\mathbf{y}) = \ln\left\{\frac{(|y(0)|^2 - |y(1)|^2)a^2}{(a^2+N_0)N_0}\right\}$$

Non Coherent Detection

Error probability

$$p_e = Pr\left[|y(1)|^2 > |y(0)|^2 \middle| x_A \right] = [2 + \frac{a^2}{N_0}]^{-1}$$

SNR definition [Tse]

$$SNR = \frac{\text{average received signal energy per complex symbol time}}{\text{noise energy per (complex) symbol time}}$$

For the orthogonal modulation scheme we have

$$\frac{a^2}{2} : \text{average received signal energy}$$

$$N_0 : \text{Noise energy}$$

$$SNR = \frac{a^2}{N_0}$$

and

$$p_e = \frac{1}{2(1 + SNR)}$$

Very discouraging result!

To get $p_e = 10^{-3}$ requires SNR≈500 (27dB)

Coherent Detection

BER for transmission over AWGN

$$p_e = Q\left(\frac{a}{\sqrt{N_0/2}}\right) = Q(\sqrt{2SNR}) \quad \text{with} \quad SNR = \frac{a^2}{N_0}$$

Approximation of

$$Q(x) > \frac{1}{\sqrt{2\pi x}}(1 - \frac{1}{x^2})e^{-x^2/2}, \; x > 1$$

We see that p_e *decreases exponentially* in SNR while it decreases with only 1/SNR in the fading channel

If the channel h(m) is known, we achieve

$$p_e = Q(\sqrt{2|h|^2 SNR})$$

Bit Error Rate in Rayleigh Fading Channels

For a given channel, the BER can be computed easily (QPSK)

$$P(e|h) = Q\left(\sqrt{2|h|^2 SNR}\right)$$

We are interested in the average bit error rate over many channel realizations

$$\left.\begin{aligned} \mathcal{E}_h\{P(e|h)\} &= \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{1+SNR}}\right) \\ \sqrt{\frac{SNR}{1+SNR}} &= 1 - \frac{1}{2SNR} + O\left(\frac{1}{4SNR^2}\right) \end{aligned}\right\} P(e) \sim \frac{1}{SNR}$$

- Error probability decays only very slowly with increasing SNR

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Error Probability

What is the reason for this poor performance?

- $|h|^2 SNR$ is the instantaneous SNR. Under typical channel conditions $|h|^2 SNR >> 1$ and the probability of error is very small.

- If $|h|^2 SNR \sim 1$, then the error probability becomes significant. This event is called a *<u>deep fade</u>*

- We are interested in

$$P(|h|^2 SNR < 1)$$

The pdf of $|h|^2$ decides how likely it is to end up in a deep fade

If h is circular symmetric complex Gaussian random variable, then

$$P(|h|^2 SNR < 1) = \int_0^{1/SNR} e^{-x} dx = \frac{1}{SNR} + O\left(\frac{1}{SNR^2}\right)$$

Performance of BPSK over Rayleigh Fading Channel

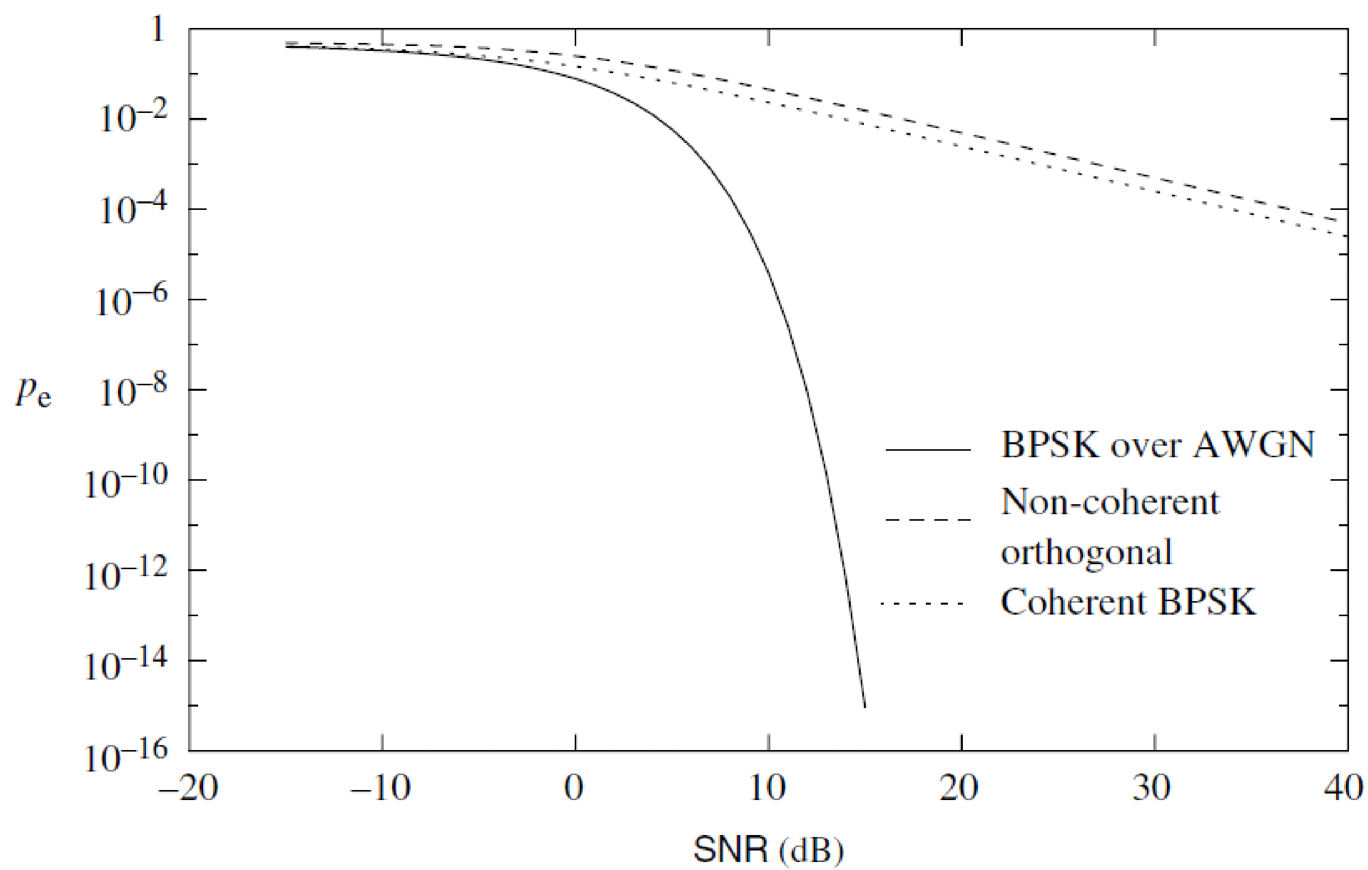


Figure 3.2 Performance of coherent BPSK vs. non-coherent orthogonal signaling over Rayleigh fading channel vs. BPSK over AWGN schannel.

Diversity

The concept of diversity is to stabilize the link by providing multiple signal paths from transmitter to the receiver, so that when one path fails, other paths are still likely to provide a "good connection"

(Time) Diversity

Idea:

Transmit a codeword $\mathbf{x} = [x_1, \ldots x_L]^T$ over L (nearly) independent fading gains by interleaving

$$y_l = h_l x_l + w_l, \qquad l = 1, \ldots, L$$

L : number of *diversity branches*

Simplest code: Repetition code

$$x_l = x_1, \quad \text{for } l = 1, \ldots, L$$

$$\mathbf{y} = \mathbf{h} x_1 + \mathbf{w}$$

where

$$\mathbf{y} = [y_1, \ldots, y_L]^T, \mathbf{h} = [h_1, \ldots, h_L]^T, \mathbf{w} = [w_1, \ldots, w_L]^T$$

This is a vector Gaussian detection problem.

We assume perfect knowledge of h => *coherent detection*

(Time) Diversity

The scalar

$$\frac{\mathbf{h}^*}{\|\mathbf{h}\|}\mathbf{y} = \|\mathbf{h}\|x_1 + \frac{\mathbf{h}^*}{\|\mathbf{h}\|}\mathbf{w}$$

with

$$\frac{\mathbf{h}^*}{\|\mathbf{h}\|}\mathbf{w} \sim \mathcal{CN}(0, N_0)$$

is sufficient statistics to decide for x_1 .

Receiver structure is a *matched filter* also called a *maximum ratio combiner*.

$$\|\mathbf{h}\|^2 = \sum_{l=1}^{L} |h_l|^2$$

weighs received signal in each branch and aligns phase

We get an equivalent channel gain of $\|\boldsymbol{h}\|^2$.

The fluctuation of $\|\boldsymbol{h}\|^2$ becomes smaller because we average over *L*- independent values.

Given $\|\boldsymbol{h}\|^2$, the error probability for a given channel equals (AWGN)

$$P_e(\|\mathbf{h}\|^2) = Q\left(\sqrt{2\|\mathbf{h}\|^2 SNR}\right)$$

Probability for a Deep Fade

We need the distribution of $\|\mathbf{h}\|^2$, since we are interested in the average BER over all possible channel realizations obtain p_e

$$\chi^2_{2L}(\|\mathbf{h}\|^2) = \frac{1}{(L-1)!}\|\mathbf{h}\|^{2^{L-1}} e^{-\|\mathbf{h}\|^2}$$

$$P\left(\|\mathbf{h}\|^2 < \epsilon\right) \approx \frac{1}{L!}\epsilon^L$$

$$p_e \approx P\left(\|\mathbf{h}\|^2 < \frac{1}{\mathsf{SNR}}\right) \approx \frac{1}{L!}\frac{1}{\mathsf{SNR}^L}$$

χ^2_{2L}

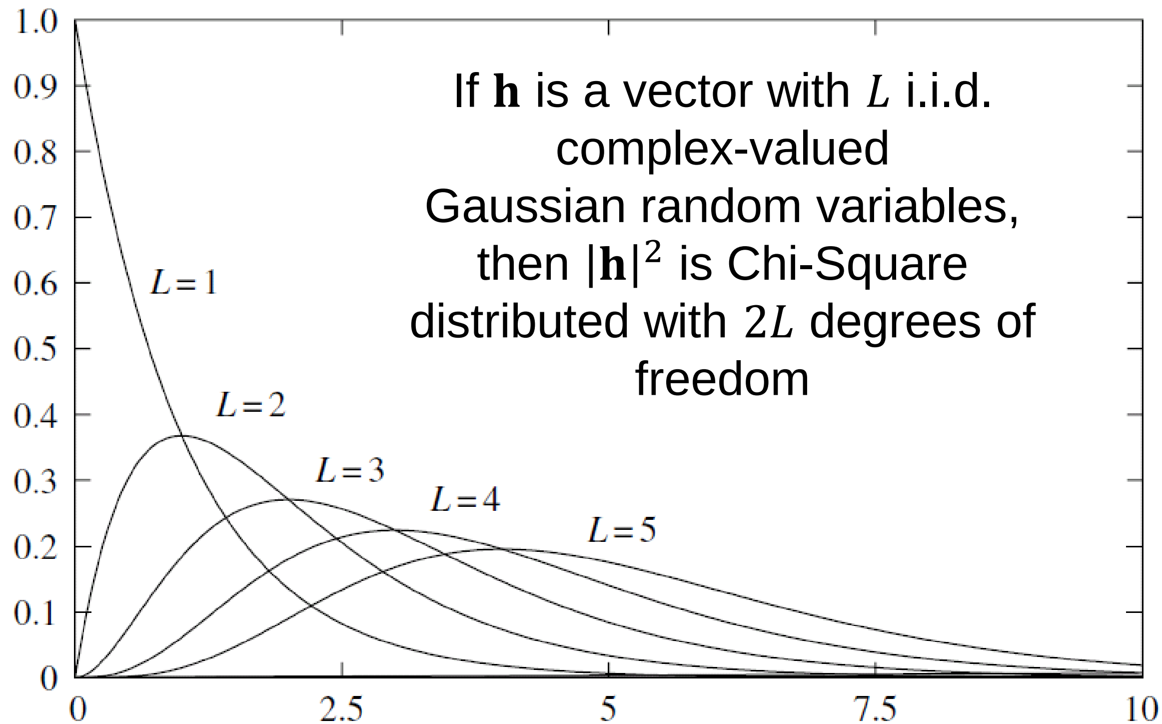


Figure 3.7 The probability density function of $\|\mathbf{h}\|^2$ for different values of L. The larger the L, the faster the probability density function drops off around 0.

The factor of 2 in the degrees of freedom of the Chi-Square distribution comes from the complex channel coefficients, which all comprise a real- and an imaginary part that are both independent (circular symmetric) Gaussians

Error Probability for BPSK

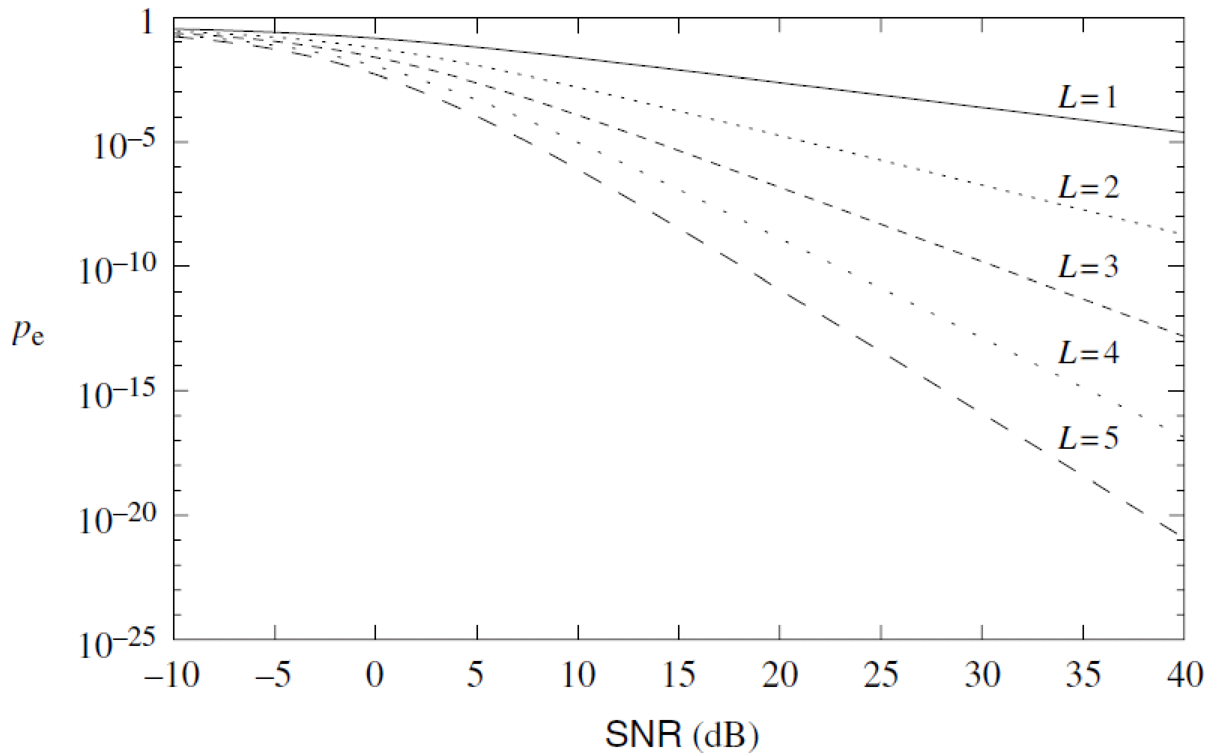


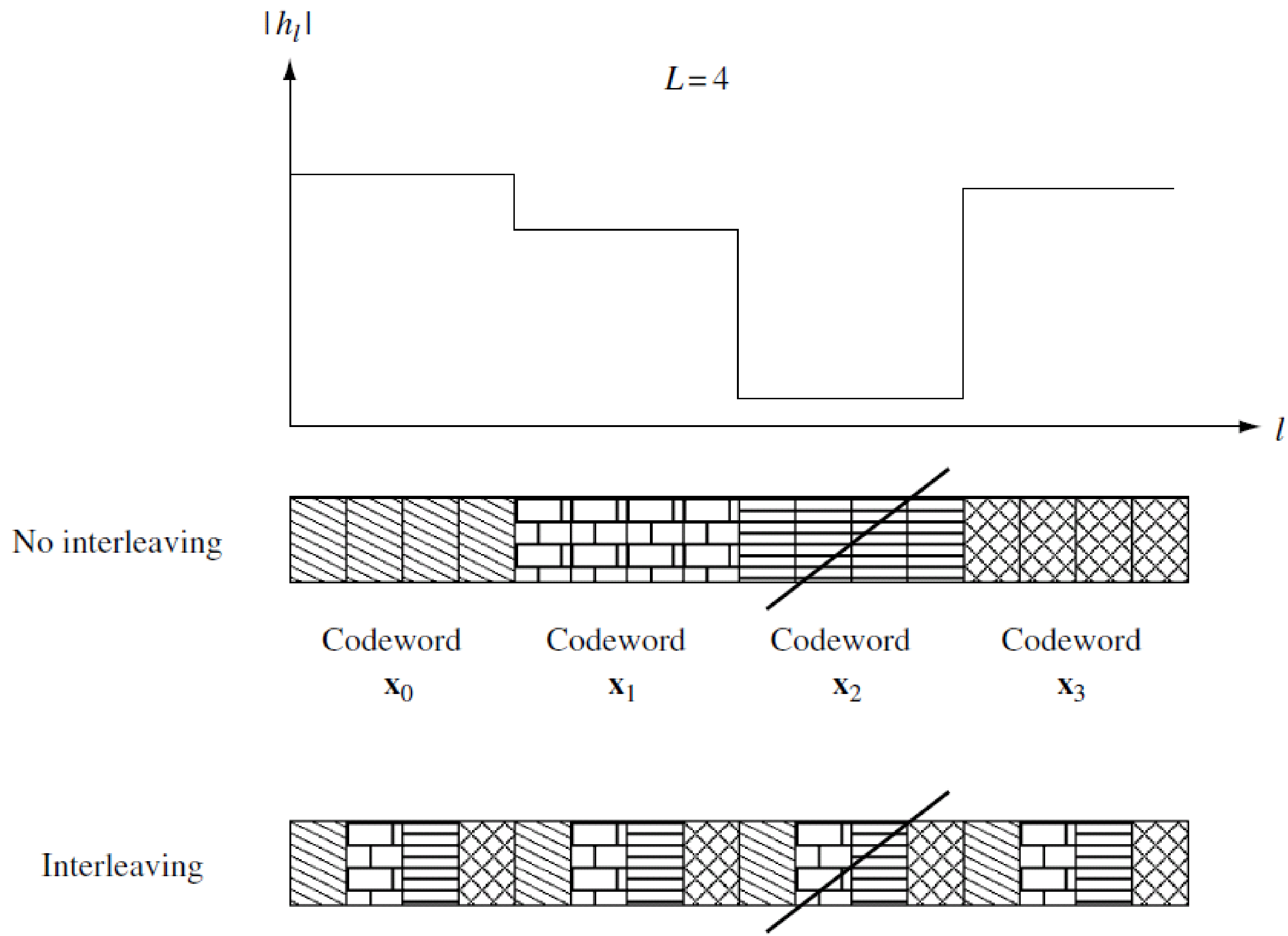
Figure 3.6 Error probability as a function of SNR for different numbers of diversity branches *L*.

P_e **decreases rapidly with increasing** L **.**

L: is called the *diversity gain*

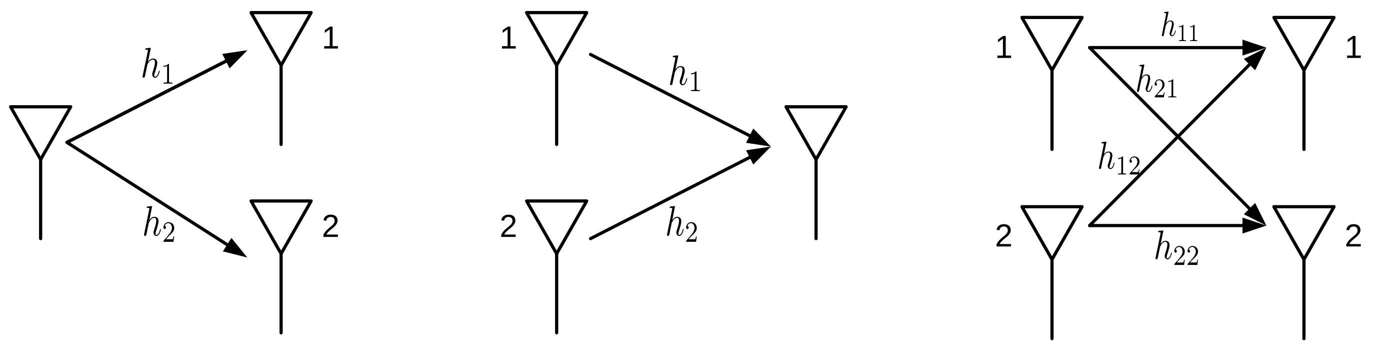
Interleaving

Figure 3.5 The codewords are transmitted over consecutive symbols (top) and interleaved (bottom). A deep fade will wipe out the entire codeword in the former case but only one coded symbol from each codeword in the latter. In the latter case, each codeword can still be recovered from the other three unfaded symbols.



Antenna diversity

We consider transmission using several antennas at the transmitter or/and at the receiver.



a) Receive diversity (SIMO)

b) Transmit diversity (MISO)

c) Transmit and receive diversity (MIMO)

If the antennas are placed sufficiently apart, the channel gains between different antenna pairs fade more or less independently and create independent signal paths.

Here we consider only the SIMO case (see Part II of the lecture for the MISO and MIMO case)

SIMO: Receive Diversity

Channel Model

$$y_l(m) = h_l(m)x(m) + w_l(m), \quad l = 1, \ldots, L$$

Exactly the same model as for time diversity. The L-diversity branches are now over space instead over time

The error probability for BPSK equals

$$Q\left(\sqrt{2||\mathbf{h}||^2 SNR}\right)$$

We can break up the total SNR conditioned on $||\boldsymbol{h}||^2$

$$||\mathbf{h}||^2 \cdot SNR = L \cdot SNR \cdot \frac{||\mathbf{h}||^2}{L}$$

$L \cdot SNR$: power gain (array gain)

$\frac{||\mathbf{h}||^2}{L}$: diversity gain $\rightarrow$ affects the exponent of $\left(\frac{1}{SNR}\right)^L$

for large $L \rightarrow$ the diversity gain tends to 1