1 Orthogonal Frequency Division Multiplexing

Wideband single carrier systems require very complicated equalization and channel estimation schemes due to frequency selective fading. Orthogonal frequency division multiplexing (OFDM) is a way to convert a wideband channel into many intersymbol interference free narrowband channels, for which equalization and channel estimation are straightforward.

1.1 The Concept of OFDM

Imagine that we want to transmit a data stream, consisting of complex symbols, at a symbol rate R. In a single carrier system (as we have discussed up to now), the data symbols would simply be transmitted one after the other, using the whole available bandwidth B, which is approximately equal to the desired symbol rate R.
In a multicarrier system, the data stream is split into N parallel streams. Each stream is transmitted over a narrow frequency band of bandwidth B/N, at a rate R/N. Figure 1 illustrates the different concepts in the time-frequency plane. The left subfigure shows a single-carrier system, where the symbol duration is $T = 1/R$, i.e., shorter than $1\,\mu s$ in modern high-speed systems. Since the length of the channel impulse response in typical outdoor scenarios is in the order of several microseconds, intersymbol interference between many consecutive symbols arises. The right subfigure shows an OFDM transmission with $N = 8$ subcarriers.[1] The data symbols of the OFDM system are denoted as $a[n, m]$, where n is the OFDM symbol index, and m is the subcarrier index. The symbol duration is now $T = N/R$, i.e. N times longer than in the singlecarrier system. The OFDM symbols are separated by a guard interval. If this interval is longer than the channel impulse response (due to the longer symbol duration, this is now possible without introducing too much overhead), the system is free of intersymbol interference.

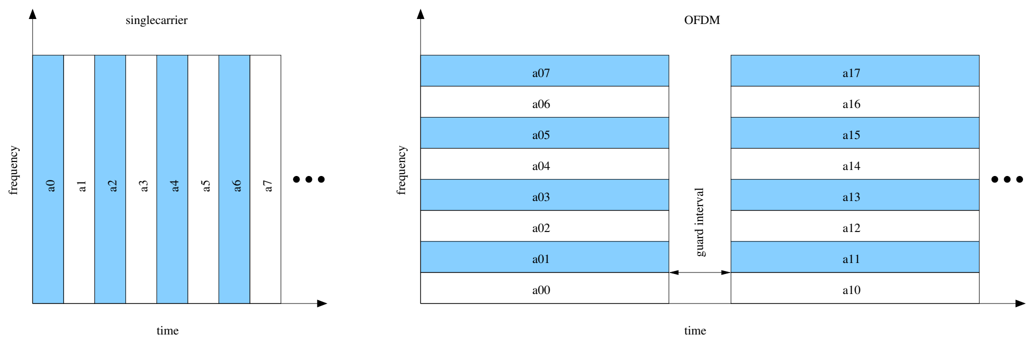


Figure 1: Comparison of singlecarrier and multicarrier (OFDM) transmission

[1]In practice, the number of subcarriers is usually between 128 and 8192.

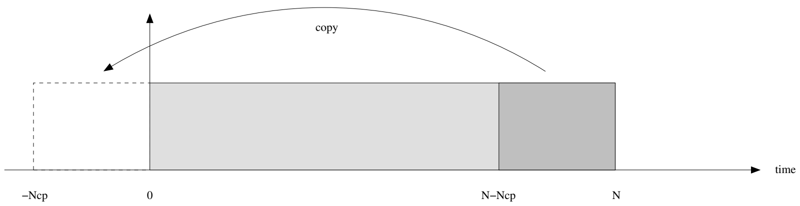


Figure 2: Cyclic prefix

1.2 Cyclic Prefix

What do we transmit during this guard interval? One option is to simply transmit nothing. Most systems, however, use a so-called *cyclic prefix*, which means that the last N_{cp} samples of each OFDM symbol are copied and inserted in front of the symbol, as illustrated in Figure 2. The reason for this cyclic extension has to do with the properties of the discrete Fourier transform (DFT), which is used in digital processors. Remember that our aim is to transform the received signal, which is a convolution of the transmitted signal with the channel impulse response, into the frequency domain, and then compensate for the influence of the channel by dividing the received signal spectrum by the channel transfer function.
Now imagine that we have two signals $a[k]$ and $b[k]$, with $k = 0, \ldots, N-1$. Their N-point-DFTs are denoted $A[k]$ and $B[k]$. Now define the two sequences $c[k] = a[k] * b[k]$ and $C[k] = A[k]B[k]$. The sequence $C[k]$ is the DFT of $c[k]$ only if $c[k]$ is the *circular* convolution of $a[k]$ and $b[k]$,

$$c[k] = \sum_{i=0}^{N-1} a[i]b[(k-i) \bmod N]\,, \quad k = 0, \ldots, N-1\,, \tag{1}$$

instead of the *linear* convolution, which would be given as

$$c[k] = \sum_{i=0}^{N-1} a[i]b[k-i]\,, \quad k = 0, \ldots, 2N-2\,. \tag{2}$$

We therefore want the convolution of the transmitted signal with the CIR to be circular, but unfortunately it is linear. However, from (1) and (2) we see that the circular and linear convolution are identical if one of the input sequences is periodic with period N. This is where the cyclic prefix comes into play. By prepending each signal block of length N with a copy of the last N_{cp} samples of that block, and under the condition that $N_{\text{cp}} \geq L$ (the length of the discrete-time CIR), the convolution of the transmitted signal with the CIR is effectively transformed into a circular one.

1.3 OFDM Transmission

In this section, we will give a step-by-step description of an OFDM system. A block diagram of a baseband transmitter is shown in Figure 3.

1. The complex data symbols are generated as usual and converted into N parallel streams, as illustrated in Figure 1. We denote the symbols as $a[n, m]$, where $m = -N/2, \ldots, N/2-1$ indicates the stream, and n is the time index. For simpler notation, we use the vector $\mathbf{a}[n] = (a[n, -N/2], \ldots, a[n, N/2-1])$.

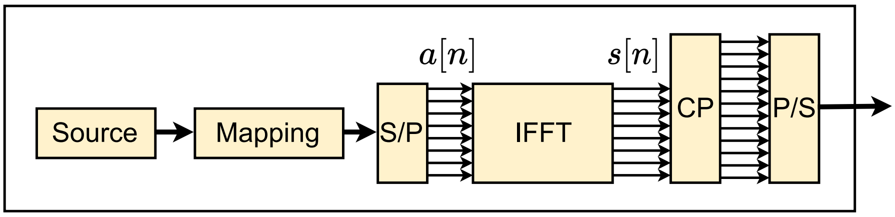


Figure 3: OFDM baseband transmitter

2. The data symbols are transformed into the time domain via an IDFT operation:

$$\mathbf{s}[n] = \text{IDFT}\{\mathbf{a}[n]\}. \tag{3}$$

3. The cyclic prefix is inserted.

4. The receiver must synchronize itself to the OFDM symbols, i.e. it must be aware at which sample a new OFDM symbol is starting. This is done by the usual frame synchronization.

5. For each time step n, the receiver collects the received samples that belong to the nth OFDM symbol into the vector $\mathbf{r}[n]$. This vector is then converted back into the frequency domain: $\mathbf{z}[n] = \text{DFT}\{\mathbf{r}[n]\}$.

6. The received symbols in the frequency domain can be written as

$$z[n,m] = H[n,m]\,a[n,m] + W[n,m]\,, \tag{4}$$

   where $H[n,m]$ is the channel transfer function at time instant n and at the frequency of the subcarrier m, and $W[n,m]$ is additive white Gaussian noise.

   In (4) we see that in an OFDM system, the data symbols $a[n,m]$ do not interfere with each other.[2] Rather, there is a one-to-one correspondence between transmitted and received symbols. The whole OFDM system, consisting of the IDFT, the cyclic prefix insertion, the channel, the CP removal and the DFT can thus be modeled as N parallel flat fading channels. The receiver removes the influence of the channel by dividing the received symbol by an estimate of the channel.

1.4 Synchronization in Single Carrier and OFDM Receivers

In this section, we highlight some differences in the synchronization tasks between the single carrier receiver, which we have discussed in the previous labs, and an OFDM receiver.

1. In an OFDM system, we do not have a matched filter. Instead, the receiver uses a simple lowpass filter. This is because, from the receiver's perspective, the combination of the transmit pulse $g(t)$ and the channel $h_c(t)$ looks like an effective transmit pulse

$$g_{\text{eff}}(t) = g(t) * h_c(t) \tag{5}$$

[2] This statement assumes some idealizations: Perfect frequency synchronization, perfect analog components, and a sufficiently long cyclic prefix. Furthermore, the channel variations must be so slow that the channel can be regarded as constant during one OFDM symbol period.

and hence, a receiver filter $g^*(-t)$ would not be matched to the effective transmit pulse anymore. Instead, the frequency domain equalizer acts as a combined matched filter and channel equalizer.

2. Timing synchronization is also not needed in an OFDM receiver. This is because, due to the multipath channel, the received signal is composed of many replicas of the transmitted waveform, and there is no longer an optimal sampling instant. Instead, a timing shift of a fraction of the sampling duration would simply translate to a phase shift in the frequency domain and would be compensated by the channel equalizer.

3. For frame synchronisation, you can use a BPSK encoded single carrier preamble. You may take the same sequence as in all previous exercises. As you work in an oversampled system, you need to apply pulse shaping to the preamble (but not to your OFDM symbols). Be careful that you do not destroy your OFDM symbols.

2 OFDM for Acoustic Transmission

To this point, we have discussed the baseband processing of an OFDM system. However, in a practical system, the baseband signal must be upconverted to a carrier frequency before transmission. In our acoustic transmission system, this upconversion is done in the digital domain. We will now briefly discuss the main details of the OFDM transmitter for acoustic transmission depicted in Fig. 4 .

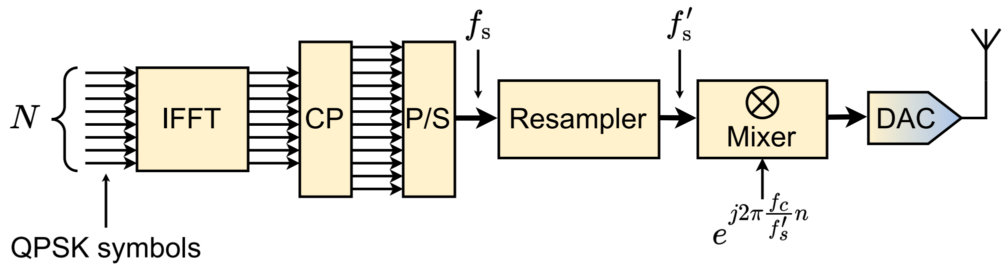


Figure 4: OFDM Transmitter for acoustic transmission

2.1 Sampling Rate

As discussed earlier, OFDM modulation can be performed without oversampling at a sampling rate f_s given by:

$$f_s = {}^{N}\!/_{T}, \tag{6}$$

where N is the number of carriers and where $T = {}^{1}\!/_{f_{\text{spacing}}}$ corresponds the OFDM symbol duration. However, in our acoustic transmission system, the sampling rate f'_s is fixed by the sampling rate of the sound card of the computer. Therefore, we have to resample the OFDM baseband signal from the sampling rate f_s to the fixed sampling rate f'_s.

We provide you with two MATLAB functions to perform this resampling operation at the transmitter and receiver side, respectively:

- `ofdm_tx_resample.m`: Resamples the OFDM baseband signal from f_s to f_s' at the transmitter side.
- `ofdm_rx_resample.m`: Resamples the received OFDM baseband signal from f_s' to f_s at the receiver side.

2.2 Channel Tracking

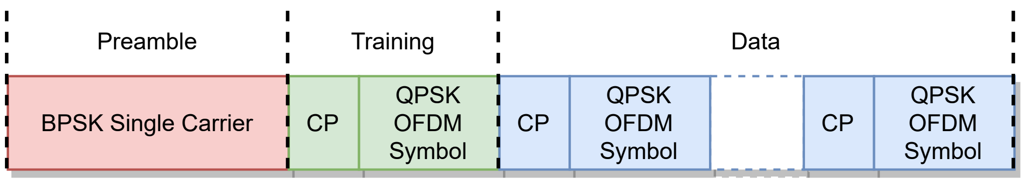
In OFDM each subcarrier has a different phase offset and attenuation. To be able to correctly detect the symbols, you need initial estimates of the channel phase and magnitude. You can obtain such estimates by transmitting a known OFDM symbol at the beginning of every frame. It is important to note that you can not use the initial phase estimate provided by the frame synchronization unit, since this is an estimate of the phase offset in the time domain, not in the frequency domain. Afterwards, you can track the phase deviation as in the single carrier system. Remember that you have to do this for each subcarrier independently. This set of phase offsets and magnitudes describes your channel. In other words, it is the estimate of the frequency response of the channel.

2.3 Down Conversion

As discussed in Section 1.4-1, the receiver uses a simple lowpass filter instead of a matched filter. We provide you with a lowpass filter function `ofdmlowpass.m` that you can use in your receiver. Note that the filter cut-off point should be slightly above the bandwidth of the baseband signal.

2.4 Recommended Frame Structure

The recommended starting frame structure, which we described previously, is illustrated bellow. It consists of a single carrier preamble for frame synchronization (identical to the previous assignment), a known OFDM symbol for initial channel estimation, and data OFDM symbols.



3 Default System Configuration

We recommend you to start with the following default configuration for your system:

Single Carrier Preamble		OFDM	
# Symbols	500	Bandwidth	2 kHz
Symbol rate	1 kHz	# subcarriers	512
		CP length	256
		# data symbols	50

Center frequency: 8 kHz

4 Your Tasks

———— Minimum Requirements ————

The following tasks are mandatory to reach a passing grade up to 4.5.

1. Implement an acoustic OFDM transmission system.
   - Provide a block diagram of your system.
   - Verify that your system works with a simple channel (i.e., emulated channel ID 1 and 100 dB SNR).
   - Up to which SNR can you reach a BER $< 1\,\%$, using the default configuration proposed.
2. Implement a phase tracking method for your system and use the emulated channel ID 2 to evaluate your implementation.
   - How many OFDM symbols can you receive successfully without and with the phase tracking method you implemented?
   - Can you explain why your phase tracking method fails after a certain number of OFDM symbols?
3. Each of the emulated channels (ID 2 to 5) has one or two special channel characteristics.
   - Using plots such as power delay profile, frequency response, channel evolution over time, etc., identify and describe the particularities of each channel.
4. Calculate the spectral efficiency of your system (in bit/s/Hz). Which parameters are involved? How can you improve the spectral efficiency of your system? Discuss possible improvements and their limitations.
5. Verify your system in a real acoustic channel (you can reduce the number of data bits to shorten the transmission duration).
   - Discuss the characteristics of the real channel using plots from your measurements.
   - What bit error rate are you able to achieve?

———— Extended Acoustic System ————

To reach a higher grade ($4.5 - 6.0$), you can extend your system and discuss some of the following aspects. The list is non-exhaustive, and you are encouraged to propose your own ideas.

1. **Channel estimation:** Implement and compare different channel estimation methods. What happens to your system when the microphone is moving? How can you improve the robustness of your system?
2. **Maximize throughput:** Improve your system to maximize the throughput. What change did you make to improve the throughput? What throughput can you reach while still achieving a BER $< 10\,\%$?

3. **Real data transmission:** Transmit images and discuss issues or particular care required compared to random bitstreams.

4. **Channel measurements:** Measure different channels and interpret the results. Be creative to observe interesting channel conditions and highlight specific effects.

5. **Personal ideas:** Any proposals you have to improve or evaluate your system are welcome and encouraged!

4.1 General Hints

- The single carrier preamble and OFDM data are processed very differently. Make sure that you separate the two processing chains well and not mix their respective parameters.
- Build the transmitter and receiver in parallel, to test that every part of the transmitter and receiver are working together correctly, e.g., test the mapping with the demapping, the up-conversion with the down-conversion, etc.
- OFDM can have a high peak-to-average power ratio (PAPR). Thus, the large dynamic range can pose problems, like sound clipping. To get best results, ensure that your preamble and the OFDM symbols have the same energy, and that the signal you send to the speaker has most of its absolute values ≤ 1.
- If you experience issues with the recording, e.g., very low SNR even with clearly audible signal, verify that your sound card is not using any automatic gain control (AGC) or noise cancellation.

Final Presentation and Report

Each group is required to do a 10-minutes final presentation of their audio transmission system during the last week of the semester. The presentation should essentially be a summary of the report providing the key answer and observations for each task. The duration of the presentation is quite short, so focus on the most important aspects of your work and plan a reasonable number of slides ($\sim$10).
Each group is required to submit a report of 10 to 20 pages after the final presentation. Your report should contain:

1. A brief description of your implementation with diagrams and any problems that you encountered.
2. Discussions of the different tasks, including plots and explanations.

The slides (as presented during the oral presentation), the report, and the code must be submitted through Moodle.