Fundamentals of Analog VLSI Design
Exercise 9 - Problem

Fully-differential OTAs and Common-mode Feedback (CMFB)

Christian Enz (christian.enz@epfl.ch)

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1 Problem 1: Fully-differential Simple OTA

In many mixed-signal circuits, the amplifiers are fully-differential amplifiers. This means that not only the input is differential but they also have a differential output. There are many advantages of using fully-differential amplifiers, including:

- Reject any common-mode disturbances,
- Provide larger output swing,
- Avoid poles from current mirrors, thus achieving higher closed-loop speed.

However, the DC common-mode output voltage is not defined and needs to be stabilized and set at a well-defined common-mode voltage. This requires an additional common-mode feedback (CMFB) circuit and hence leads to additional power consumption. Therefore, the extra power consumption needed for the CMFB is an important specification.

Depending on the application, the bandwidth of the closed-loop CMFB circuit may need to be sufficiently large to avoid both outputs to saturate for a long-time making the amplifier out-of-order for the differential signal. This means that the open-loop gain-bandwidth product of the CMFB circuit GBW_{cm} needs to be at least as high as the gain-bandwidth product in differential mode GBW_{dm}, i.e. $GBW_{dm} \leq GBW_{cm}$. Indeed, if the common-mode gain-bandwidth product GBW_{cm} is much smaller than GBW_{dm}, a fast and large common-mode signal at the amplifier input could saturate the amplifier outputs. Since the CMFB is slow, it takes time to recover, making the amplifier useless during this time. Requiring the GBW_{cm} to be as large as the GBW_{dm} will require a lot of power. Of course it depends on how the CMFB is implemented but usually fully-differential amplifiers consume about the double than their equivalent single-ended amplifiers.

For differential-in and differential-out amplifiers, we can define four gains according to

$$V_{od} = A_d \cdot V_{id} + A_{cd} \cdot V_{ic}, \tag{1.1}$$
$$V_{oc} = A_{dc} \cdot V_{id} + A_c \cdot V_{ic}, \tag{1.2}$$

where

$$V_{id} = V_{in+} - V_{in-}, \tag{1.3}$$
$$V_{ic} = \frac{V_{in+} + V_{in-}}{2}, \tag{1.4}$$
$$V_{od} = V_{out+} - V_{out-}, \tag{1.5}$$
$$V_{oc} = \frac{V_{out+} + V_{out-}}{2}. \tag{1.6}$$

A_d is the differential input to differential output voltage gain, A_{cd} is the common-mode input to differential output voltage gain, A_{dc} is the differential input to common-mode output voltage gain and finally A_c the common-mode input to common-mode output voltage gain. Of course we are mostly interested in A_d, which should be made larger than the other gains A_{cd}, A_{dc} and A_c. Note that if the amplifier can be assumed to be perfectly symmetrical, then $A_{cd} = A_{dc} = 0$. If there is any mismatch between the positive and negative signal path or in the CMFB circuit, $A_{cd} \neq 0$. This means that a change in the common-mode input voltage may not only change the output common-mode voltage but also generate a differential output voltage which translates into distortion. Even with perfect symmetry, $A_c \neq 0$ is possible, but not desired since the common-mode voltage then propagates along the circuit. We therefore want the common-mode gain A_c to be smaller than the differential gain

A_d. This is measured by the common-mode rejection ratio $CMRR$ defined as the ratio of the desired differential voltage gain A_d to the undesired common-mode voltage gain A_c

$$CMRR \triangleq \left|\frac{A_d}{A_c}\right|. \tag{1.7}$$

In a differential amplifier which is not perfectly symmetrical, we can also define $|A_d/A_{cd}|$ and $|A_d/A_{dc}|$. Of these $|A_d/A_{cd}|$ is particularly important since it measures to what extend the differential output voltage is actually produced by the desired differential input voltage instead of the undesired common-mode input voltage. This ratio is important because once the common-mode input voltage is converted into a differential output voltage it is then treated as the desired differential voltage by the subsequent amplifiers.

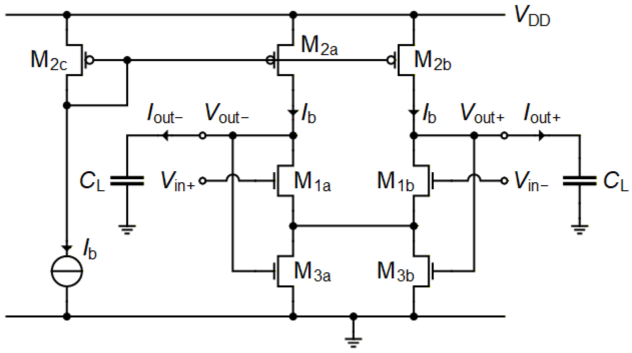

Figure 1.1: Schematic of the fully differential simple OTA.

Figure 1.1 presents the schematic of a fully-differential simple OTA which does not need any additional current for the CMFB. Transistors M_{3a} and M_{3b} are connected to the positive and negative outputs to establish the output common-mode voltage. The output common-mode voltage is extracted from M_{3a} and M_{3b} which operate in the linear region. They change the voltage at the common-source node and therefore also at the output through the action of M_{1a} and M_{1b}.

- What is approximately the level of the common-mode output voltage $V_{oc} \triangleq (V_{out+} + V_{out-})/2$?
- Derive the small-signal differential-mode transconductance

$$G_{md} = \frac{\Delta I_{out+} - \Delta I_{out-}}{\Delta V_{id}} \tag{1.8}$$

  and the differential gain-bandwidth product GBW_{dm} assuming a perfectly symmetrical circuit.
- Calculate the small-signal common-mode transconductance G_{mc} of the CMFB circuit in open-loop

$$G_{mc} = \frac{\Delta I_{out-}}{\Delta V_{G3a}} = \frac{\Delta I_{out+}}{\Delta V_{G3b}}. \tag{1.9}$$

  To do this you need to disconnect the gates of M_{3a} and M_{3b} from the outputs. The open-loop common-mode transconductance is then obtained by applying a common-mode voltage at the gates of M_{3a} and M_{3b} and measuring the common-mode output current. Hint: Assuming a perfectly symmetrical circuit, you can use the half-circuit in common-mode operation.
- How do they compare?
- Calculate the small-signal differential voltage gain $A_d \triangleq \Delta V_{od}/\Delta V_{id}$ assuming a perfectly symmetrical circuit where $\Delta V_{id} \triangleq \Delta V_{in+} - \Delta V_{in-}$ and $\Delta V_{od} \triangleq \Delta V_{out+} - \Delta V_{out-}$ are the input and output small-signal differential voltages. Deduce the differential DC voltage gain A_{d0}, the cut-off frequency $\omega_{c,dm}$ and the differential mode gain-bandwidth product GBW_{dm}.

- Calculate the small-signal common-mode voltage gain $A_c \triangleq \Delta V_{oc}/\Delta V_{ic}$ assuming a perfectly symmetrical circuit where $\Delta V_{ic} \triangleq (\Delta V_{in+} + \Delta V_{in-})/2$ and $\Delta V_{oc} \triangleq (\Delta V_{out+} + \Delta V_{out-})/2$ are the input and output common-mode voltages. Deduce the common-mode DC voltage gain A_{c0}, the cut-off frequency $\omega_{c,cm}$ and the common-mode gain-bandwidth product GBW_{cm}.
- Calculate the corresponding DC $CMRR = A_{d0}/A_{c0}$.
- Calculate the common-mode to differential-mode voltage gain $A_{cd} \triangleq \Delta V_{od}/\Delta V_{ic}$ assuming there is a G_m-mismatch ΔG_{m1} between M_{1a} and M_{1b}.
- Propose a way to improve the CMFB offering a better control on the output common-mode voltage without increasing the current consumption.

2 Problem 2: Fully-differential Folded-cascode OTA

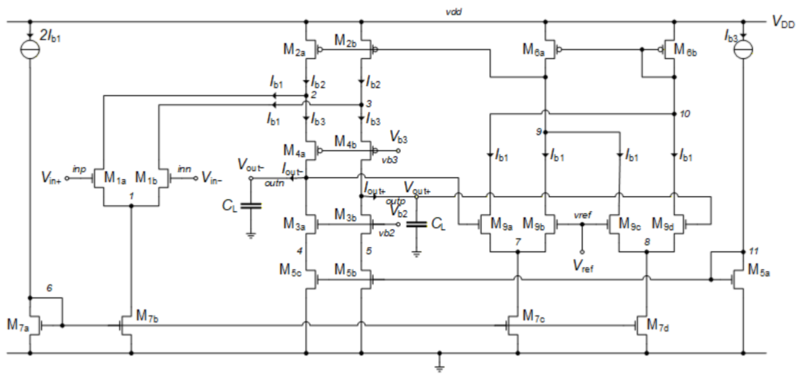

Figure 2.1: Schematic of the fully-differential folded-cascode OTA including the CMFB circuit.

Figure 2.1 presents a fully differential folded-cascode OTA. The common-mode voltage is extracted by differential pairs M_{9a}-M_{9b} and M_{9c}-M_{9d} and compared to the common-mode reference voltage V_{ref}. If the output common-mode voltage is different from V_{ref}, it is then adjusted by means of the current sources M_{2a}-M_{2b} to bring it back to V_{ref}. To analyze the CMFB, we will derive the open-loop gain (actually transconductance) by opening the loop at the input of the CMFB circuit (i.e. disconnecting the gates of M_{9a} and M_{9d}).

- What is the level of the common-mode output voltage $V_{oc} \triangleq (V_{out+} + V_{out-})/2$?
- Derive the differential-mode transconductance G_{md}

$$G_{md} = \frac{\Delta I_{out+} - \Delta I_{out-}}{\Delta V_{id}} \tag{2.1}$$

  and the differential gain-bandwidth product GBW_{dm} assuming a perfectly symmetrical circuit.
- Calculate the DC open-loop common-mode transconductance G_{mc}

$$G_{mc} = \frac{\Delta I_{out-}}{\Delta V_{G9a}} = \frac{\Delta I_{out+}}{\Delta V_{G9d}}. \tag{2.2}$$

  and gain bandwidth product GBW_{cm} assuming again a perfectly symmetrical circuit.
- How do they compare?
- What is the total current consumption for $GBW_{cm} > GBW_{dm}$? How much is the current penalty compared to the single-ended case?