Fundamentals of Analog VLSI Design
Exercise 7 - Problem

Gain and Bandwidth Enhancement

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1 Problem 1: Gain Enhancement

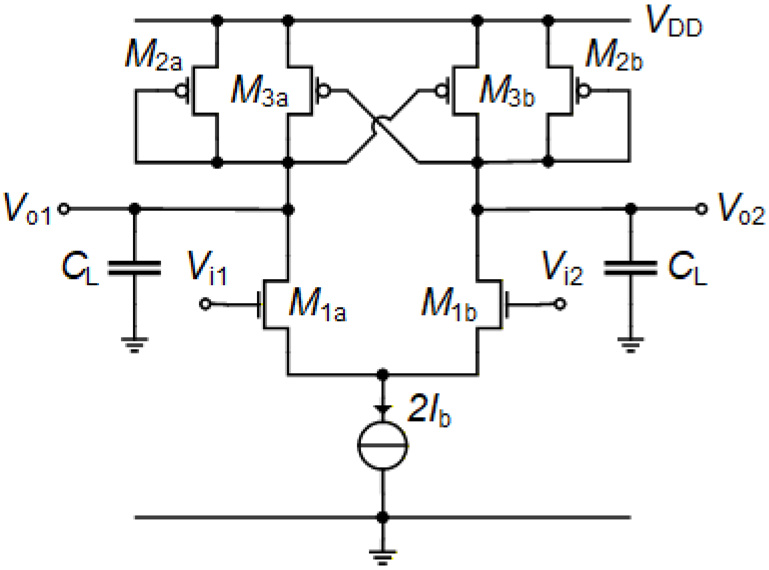

Figure 1.1: Schematic of gain enhancement amplifier.

The amplifier shown in Figure 1.1 allows to enhance the DC gain by reducing the load conductance seen by the differential pair M_{1a}-M_{1b} [1] [2].

1.1 Small-signal analysis

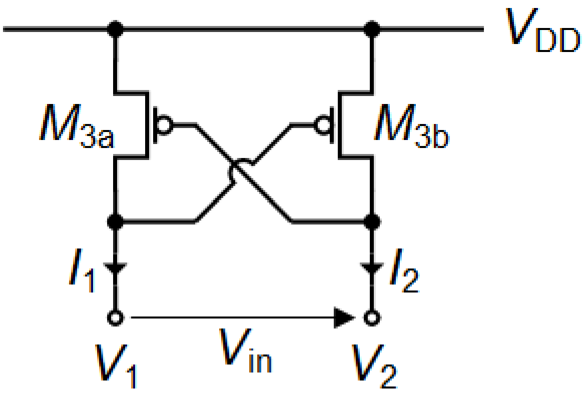

Figure 1.2: Negative conductance.

The cross-coupled pair M_{3a}-M_{3b} which is redrawn in Figure 1.2, allow to implement a negative conductance. When combined with transistors M_{2a}-M_{2b}, it allows to reduce the conductance seen by the differential pair M_{1a}-M_{1b}.

- Draw the small-signal schematic of the circuit shown in Figure 1.2.

- Derive the small-signal differential conductance $G = \Delta I_{in}/\Delta V_{in}$, where $\Delta I_{in} \triangleq \Delta I_1 - \Delta I_2$ assuming a perfect matching between $\mathrm{M_{3a}}$ and $\mathrm{M_{3b}}$.
- Draw the small-signal circuit of the amplifier shown in Figure 1.1 including the output conductances in differential mode assuming a perfect matching between all the transistors in one current branch and the other. Hint: simplify the circuit by using the fact that the voltage at the common source node of $\mathrm{M_{1a}}$-$\mathrm{M_{1b}}$ does not change in differential mode with perfect matching.
- From the above analysis, derive the small-signal differential transfer function

  $$A_d(s) \triangleq \frac{V_{od}(s)}{V_{id}(s)}, \tag{1.1}$$

  where $V_{od} \triangleq V_{o1} - V_{o2}$ and $V_{id} \triangleq V_{i1} - V_{i2}$ are the output and input differential voltages, respectively.
- Deduce the expression of the DC gain and the dominant pole.
- What is the condition on G_{m3} for the pole to remain in the left half-plane?
- What is the gain-enhancement factor K compared to the voltage gain without the cross-coupled pair $\mathrm{M_{3a}}$-$\mathrm{M_{3b}}$?
- What is the value of the DC gain for $G_{m3} = G_{m2}$? How does it compare to a common-stage stage?
- Find the gain-bandwidth product ω_u. Is the gain-bandwidth product also enhanced like the DC gain?

1.2 Noise analysis

- Draw the small-signal schematic of Figure 1.1 including all the noise sources but with $\Delta V_{id} = 0$. Hint: simplify the circuit by using the fact that the voltage at the common source node of $\mathrm{M_{1a}}$-$\mathrm{M_{1b}}$ does not change in differential mode with perfect matching.
- Derive the output noise resistance R_{nout}.
- Deduce the input-referred noise resistance R_{nin}.
- Calculate the input-referred thermal noise resistance R_{nt}. How does it compare to the noise of the same amplifier without $\mathrm{M_{3a}}$-$\mathrm{M_{3b}}$?
- Calculate the input-referred flicker noise resistance R_{nf}. How does it compare to the noise of the same amplifier without $\mathrm{M_{3a}}$-$\mathrm{M_{3b}}$?

2 Problem 2: Bandwidth enhancement

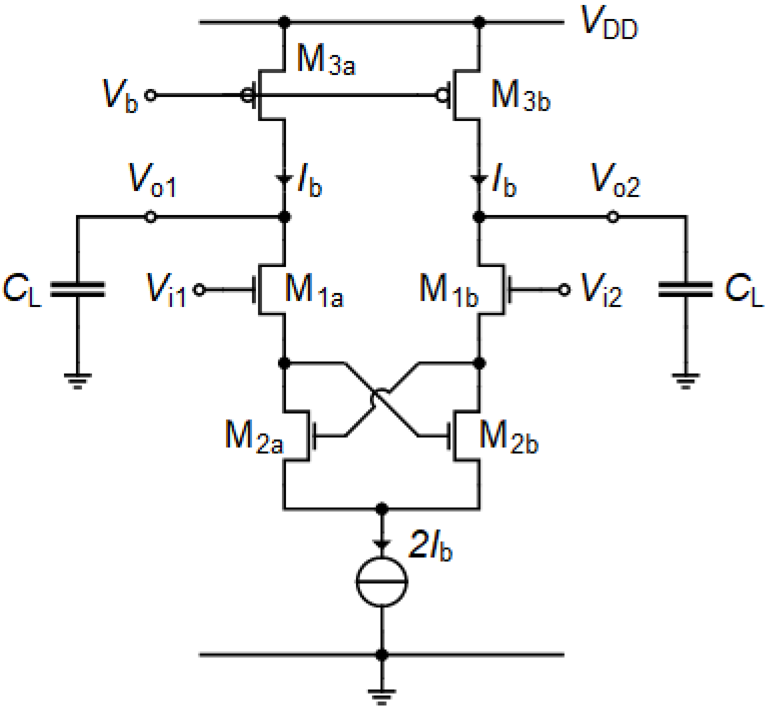

Figure 2.1: Enhanced G_m differential OTA

The differential OTA shown in Figure 2.1 shows a larger gain-bandwidth product compared to the case without the cross-coupled transistors M_{2a}-M_{2b} [1] [3].

2.1 Small-signal analysis

The OTA of Figure 2.1 uses the differential transconductor shown in Figure 2.2 which allows to increase the equivalent transconductance compared to the case of a simple differential pair.

- Draw the small-signal schematic of Figure 2.2. Hint: simplify the circuit by using the fact that the voltage at the common source node of M_{2a}-M_{2b} does not change in differential mode with perfect matching.
- Derive the small-signal equivalent transconductance

$$G_{meq} \triangleq \frac{\Delta I_{od}}{\Delta V_{id}}, \tag{2.1}$$

  where $\Delta I_{od} \triangleq \Delta I_1 - \Delta I_2$ is the small-signal differential output current and $\Delta V_{id} \triangleq \Delta V_{i1} - \Delta V_{i2}$ is the small-signal differential input voltage. How does it compare to the transconductance of a simple differential pair?
- Derive the small-signal equivalent transadmittance

$$Y_{meq}(s) \triangleq \frac{\Delta I_{od}(s)}{\Delta V_{id}(s)}, \tag{2.2}$$

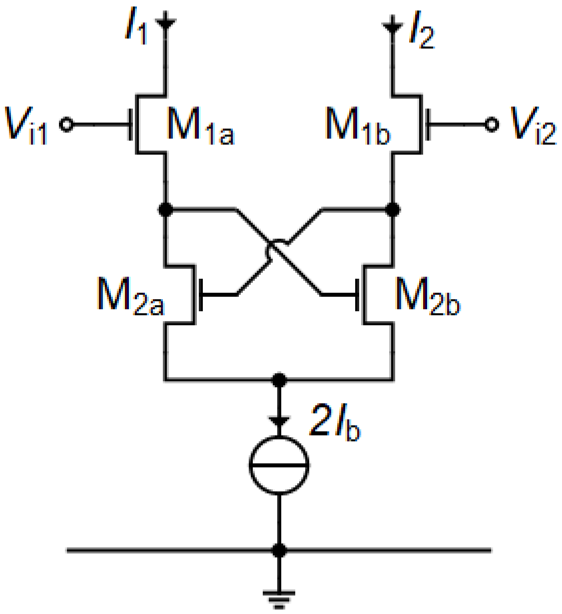

Figure 2.2: Enhanced G_m differential transconductor

accounting for the the parasitic capacitance C_p at the source node of $\mathrm{M_{1a}}$-$\mathrm{M_{1b}}$. Hint: add a parasitic capacitance C_p connected between the source node of $\mathrm{M_{1a}}$ and ground to the small-signal half-circuit used above.

- Deduce the expression of the dominant pole ω_p.
- What is the condition on G_{m2} for the pole ω_p to remain in the left half-plane? Hint: express the condition in terms of the transconductance ratio $\alpha \triangleq G_{ms1}/G_{m2}$.
- Express the G_m-enhancement factor K compared to the transconductance of the differential pair $\mathrm{M_{1a}}$-$\mathrm{M_{1b}}$ without the cross-coupled transistors $\mathrm{M_{2a}}$-$\mathrm{M_{2b}}$. Hint: express K in terms of the transconductance ratio α.
- What is the value of K when $\mathrm{M_{1a}}$-$\mathrm{M_{1b}}$ and $\mathrm{M_{2a}}$-$\mathrm{M_{2b}}$ are both biased in weak inversion? Hint: $\mathrm{M_{1a}}$-$\mathrm{M_{1b}}$ and $\mathrm{M_{2a}}$-$\mathrm{M_{2b}}$ share the same bias current I_b.
- Derive the small-signal differential transfer function

$$A_d \triangleq \frac{V_{od}}{V_{id}}. \tag{2.3}$$

- What is the value of the DC gain for $G_{ms1} = G_{m2}$? How does it compare to a common-stage stage?
- Find the gain-bandwidth product ω_u. Is the gain-bandwidth product also enhanced like the DC gain?

2.2 Noise analysis

- Draw the small-signal schematic of Figure 2.2 including all the noise sources and with $V_{id} = 0$ and neglecting all output conductances. Hint: simplify the circuit by using the fact that the voltage at the common source node of $\mathrm{M_{2a}}$-$\mathrm{M_{2b}}$ does not change in differential mode with perfect matching.
- Derive the output noise conductance G_{nout}.
- Deduce the input-referred noise resistance R_{nin}.
- Calculate the input-referred thermal noise resistance R_{nt}. How does it compare to the noise of a simple differential pair?
- Calculate the input-referred flicker noise resistance R_{nf}. How does it compare to the noise of a simple differential pair?

References

[1] W. Sansen, "Opamps, gm-blocks or inverters?" in *Efficient sensor interfaces, advanced amplifiers and low power RF systems: Advances in analog circuit design 2015*, Springer, 2016, pp. 125–138.

[2] K. B. Ohri and M. J. Callahan, "Integrated PCM codec," *IEEE Journal of Solid-State Circuits*, vol. 14, no. 1, pp. 38–46, 1979, doi: 10.1109/JSSC.1979.1051139.

[3] R. Castello, A. G. Grassi, and S. Donati, "A 500-nA sixth-order bandpass SC filter," *IEEE Journal of Solid-State Circuits*, vol. 25, no. 3, pp. 669–676, 1990, doi: 10.1109/4.102659.