Fundamentals of Analog & Mixed Signal VLSI Design
Exercise 1 (17.09.2025)

Christian Enz

Swiss Federal Institute of Technology (EPFL), Lausanne, Switzerland

Problem 1 Stacked transistors

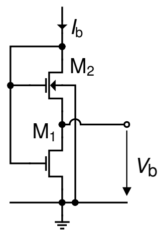

Figure 1: Stacked transistors.

1.1 Voltage V_b as a function of transistor's β_1 and β_2

Calculate the voltage V_b in circuit shown in Fig. 1 as a function of the betas (β_1 and β_2) of transistors and of the thermodynamic voltage (in weak inversion) or the pinch-off voltage (in strong inversion), assuming:

- Both transistors are biased in weak inversion.
- Both transistors are biased in strong inversion.

1.2 Channel voltage

If the transistors have the same width W, then

$$\frac{\beta_1}{\beta_2} = \frac{L_2}{L_1}, \tag{1}$$

and the circuit can be considered as a single transistor with length $L = L_1 + L_2$. The channel voltage can then be extracted at a fraction ξ along the channel by the common source and drain diffusion of M_2 and M_1 respectively

$$\xi = \frac{L_1}{L_1 + L_2} = \frac{\beta_2}{\beta_1 + \beta_2}, \tag{2}$$

Knowing that the current I_b is due to diffusion of minority carrier, it is given by,

$$I_b = -\mu \cdot \frac{W}{L} \cdot Q_i \cdot \frac{\mathrm{d}V}{\mathrm{d}\xi} \tag{3}$$

with $\xi = x/L$. Show that V_b is actually equal to the channel voltage along the channel $V(x)$.

Problem 2 The Vittoz current reference [1]

Fig. 2 shows the Vittoz current reference generating a current I_b which for M_1 and M_3 biased in weak inversion is proportional to absolute temperature (PTAT) [1]. The bias current I_b is available as a source current from M_6 or a sink current from M_5. Transistors M_2 and M_4 are assumed to be identical whereas M_3 is made K-times larger than M_1 (i.e. $\beta_3 = K \cdot \beta_1$). All transistors are biased in saturation.

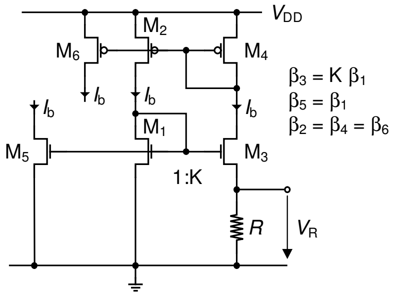

Figure 2: The Vittoz current reference.

2.1 Weak inversion

Calculate the value of the current I_b as a function of the transistor ratio K assuming that both M_1 and M_3 are in weak inversion. Do the transistor M_2 and M_4 need to be in weak inversion as well?

2.2 Strong inversion

Calculate the value of the current I_b as a function of the transistor ratio K assuming that both M_1 and M_3 are in strong inversion. Do the transistor M_2 and M_4 need to be in strong inversion as well?

Problem 3 The Oguey current reference [2]

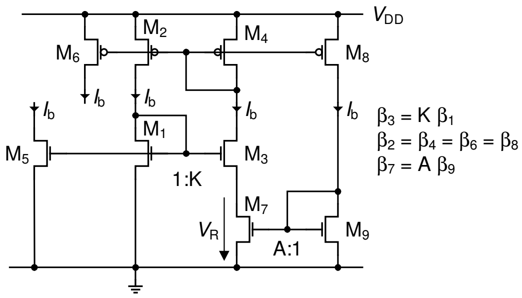

Figure 3: The Oguey current reference.

The resistor in the Vittoz current reference of Fig. 2 can actually be replaced by a transistor M_7 which is biased in the linear region by making it A-times larger than M_9 [2]. All other transistors are assumed to be biased in saturation. M_1 and M_3 are biased in weak inversion, whereas M_7 and M_9 are biased in strong inversion. Find the expression of the bias current I_b in terms of ratios K and A and the specific current I_{spec9} of M_9 and I_{spec7} of M_7.

Solutions to Exercise 1 (17.09.2025)

Problem 1 Stacked Transistor

1.1 Voltage V as a function of transistor's β_1 and β_2

1.1.1 Both transistors in weak inversion

Both transistors M_1 and M_2 share the same gate and have therefore the same gate voltage $V_{G1} = V_{G2} = V_G$. They have therefore also the same pinch-off voltage $V_{P1} = V_{P2} = V_P$ and slope factors $n_1 = n_2 = n$. Transistor M_2 is in saturation because $V_{D2} = V_G \cong V_{T0} \gg U_T$. However, M_2 is in the linear region because $V_{D1} = V_b \cong V_G - V_{T0}$ which is only slightly larger than zero. If both transistors are in weak inversion ($I_{D1} = I_{D2} \ll I_{spec}$), the current going through both transistors is then given by

$$I_{D1} = I_{spec1} \cdot e^{\frac{V_P}{U_T}} \cdot \left(1 - e^{\frac{-V_b}{U_T}}\right), \tag{4a}$$

$$I_{D2} = I_{spec2} \cdot e^{\frac{V_P}{U_T}} \cdot \left(e^{\frac{-V_b}{U_T}} - e^{\frac{-V_{D2}}{U_T}}\right), \tag{4b}$$

where $I_{spec1} \triangleq 2n\beta_1 U_T^2$ and $I_{spec2} \triangleq 2n\beta_2 U_T^2$ with $\beta_1 \triangleq \mu C_{ox} W/L_1$ and $\beta_2 \triangleq \mu C_{ox} W/L_2$. In saturation $V_{D2} \gg U_T$ and we have

$$I_{D1} = I_{spec1} \cdot e^{\frac{V_P}{U_T}} \cdot \left(1 - e^{\frac{-V_b}{U_T}}\right), \tag{5a}$$

$$I_{D2} = I_{spec2} \cdot e^{\frac{V_P}{U_T}} \cdot e^{\frac{-V_b}{U_T}}. \tag{5b}$$

Since $I_{D1} = I_{D2}$, we have

$$I_{spec1} \cdot \left(1 - e^{\frac{-V_b}{U_T}}\right) = I_{spec2} \cdot e^{\frac{-V_b}{U_T}}, \tag{6}$$

from which we obtain the bias voltage V_b

$$V_b = U_T \cdot ln\left(1 + \frac{I_{spec2}}{I_{spec1}}\right). \tag{7}$$

Since both transistor have the same pinch-off voltage, they also have the same slope factor n. (7) can then be simplified as

$$V_b = U_T \cdot \ln\left(1 + \frac{\beta_2}{\beta_1}\right) = U_T \cdot \ln\left(1 + \frac{L_1}{L_2}\right), \tag{8}$$

From (8), we see that the bias voltage V_b does not depend on the current I_D, but only depends on the channel length ratio.

1.1.2 Both transistors in strong inversion

Again, both transistors M_1 and M_2 share the same gate and have therefore the same gate, pinch-off voltages and slope factors $V_{G1} = V_{G2} = V_G$, $V_{P1} = V_{P2} = V_P$ and $n_1 = n_2 = n$. Transistor M_2 is in saturation because $V_{D2} = V_G = n \cdot V_P + V_{T0} > V_P$ since $V_{T0} > 0$ and $n > 1$. However, transistor M_1 cannot be biased in saturation. If it would be in saturation then $V_{D1} = V_{S2} > V_P$, which is contradiction with the fact that M_2 is in strong inversion and in saturation and hence $V_{S2} < V_P$. The drain currents of transistors M_1 and M_2 are then given by

$$I_{D1} = \frac{n\beta_1}{2} \cdot \left[V_P^2 - (V_P - V_b)^2\right], \tag{9a}$$

$$I_{D2} = \frac{n\beta_2}{2} \cdot (V_P - V_b)^2. \tag{9b}$$

By equating the currents $I_{D1} = I_{D2} = I_b$, we obtain

$$\beta_1 \cdot \left[V_P^2 - (V_P - V_b)^2\right] = \beta_2 \cdot (V_P - V_b)^2, \tag{10}$$

or

$$\beta_1 \cdot V_P^2 = (\beta_1 + \beta_2) \cdot (V_P - V_b)^2\,, \tag{11}$$

and

$$\frac{V_P - V_b}{V_P} = 1 - \frac{V_b}{V_P} = \sqrt{\frac{\beta_1}{\beta_1 + \beta_2}}. \tag{12}$$

Finally we obtain the channel voltage as

$$V_b = V_P \cdot \left(1 - \sqrt{\frac{\beta_1}{\beta_1 + \beta_2}}\right) = V_P \cdot \left(1 - \sqrt{\frac{L_2}{L_1 + L_2}}\right). \tag{13}$$

1.2 Channel voltage in weak inversion

The current in weak inversion is a diffusion current given by

$$I_D = W \cdot \mu_n \cdot U_T \cdot \frac{\mathrm{d}Q_i}{\mathrm{d}x} \tag{14}$$

Since the current is constant along the channel, the inversion charge $Q_i(x)$ is a linear function of the position and since transistor M_2 is in saturation we have $Q_i(x = L) = Q_{iD} = 0$. It follows that

$$Q_i(x) = Q_{iS} \cdot \left(1 - \frac{x}{L}\right) = Q_{iS} \cdot (1 - \xi), \tag{15}$$

where $Q_{iS} = Q_i(x = 0)$ and $\xi = x/L$. The drain current can then be written as

$$I_D = -W \cdot \mu_n \cdot U_T \cdot \frac{Q_{iS}}{L}. \tag{16}$$

The current is also given as a function of the channel voltage V by

$$I_D = \mu_n \cdot W \cdot (-Q_i) \cdot \frac{\mathrm{d}V}{\mathrm{d}x} = \mu_n \cdot \frac{W}{L} \cdot (-Q_i) \cdot \frac{\mathrm{d}V}{\mathrm{d}\xi} \tag{17}$$

Replacing Q_i in (17) by (15), results in

$$I_D = \mu_n \cdot \frac{W}{L} \cdot (-Q_{iS}) \cdot (1 - \xi) \cdot \frac{\mathrm{d}V}{\mathrm{d}\xi} \tag{18}$$

Equating (18) to (16), leads to

$$(1 - \xi) \cdot \frac{\mathrm{d}V}{\mathrm{d}\xi} = U_T, \tag{19}$$

and

$$\frac{\mathrm{d}V}{\mathrm{d}\xi} = \frac{U_T}{1 - \xi}, \tag{20}$$

which can be integrated from $\xi = 0$ ($x = 0$) to ξ (x), leading to

$$V(\xi) - V(\xi = 0) = -U_T \cdot ln(1 - \xi). \tag{21}$$

But $V(\xi = 0) = V_S = 0$ and hence

$$V(\xi) = -U_T \cdot ln(1 - \xi) = -U_T \cdot ln\left(1 - \frac{x}{L}\right). \tag{22}$$

Now,

$$\frac{x}{L} = \frac{L_1}{L_1 + L_2} = \frac{\beta_2}{\beta_1 + \beta_2}, \tag{23}$$

and therefore

$$V_b = V(x = L_1) = -U_T \cdot ln\left(1 - \frac{L_1}{L_1 + L_2}\right) = -U_T \cdot ln\left(\frac{L_2}{L_1 + L_2}\right) = U_T \cdot ln\left(1 + \frac{L_1}{L_2}\right) = U_T \cdot ln\left(1 + \frac{\beta_2}{\beta_1}\right), \tag{24}$$

which is identical to (8). Therefore, in weak inversion, the voltage V_b at the intermediate node between M_1 and M_2 is equal to the channel voltage V at position $x = L_1$.

Problem 2 The Vittoz current reference [1]

2.1 Weak inversion

Both transistors M_1 and M_3 share the same gate and have therefore the same gate voltage $V_{G1} = V_{G3} = V_G$. They have therefore also the same pinch-off voltage $V_{P1} = V_{P3} = V_P$ and the same slope factor $n_1 = n_3 = n$. Assuming that transistors M_1 and M_3 are biased in weak inversion and in saturation, we have

$$I_{D1} = I_{spec1} \cdot e^{\frac{V_P}{U_T}}, \tag{25a}$$

$$I_{D3} = I_{spec3} \cdot e^{\frac{V_P - V_R}{U_T}}, \tag{25b}$$

where $I_{spec1} \triangleq 2n\beta_1 U_T^2$ and $I_{spec3} \triangleq 2n\beta_3 U_T^2$ with $\beta_3 = K \cdot \beta_1$. Assuming a perfect matching between transistors M_1 and M_3 and between M_2 and M_4, we have $I_b = I_{D1} = I_{D3}$, resulting in

$$e^{\frac{V_R}{U_T}} = \frac{I_{spec3}}{I_{spec1}} = \frac{\beta_3}{\beta_1} = K. \tag{26}$$

and hence

$$V_R = U_T \cdot \ln(K) \tag{27}$$

and

$$I_b = \frac{U_T}{R} \cdot \ln(K). \tag{28}$$

This circuit provides a reference voltage V_R proportional to absolute temperature or PTAT. The temperature dependence of the reference current I_b depends on the temperature dependence of the resistance R. So the reference current I_b is PTAT only if the resistance R can be considered as temperature-independent.

2.2 Strong inversion

As above, both transistors M_1 and M_3 share the same gate and have therefore the same gate voltage $V_{G1} = V_{G3} = V_G$, pinch-off voltage $V_{P1} = V_{P3} = V_P$ and slope factor $n_1 = n_3 = n$. Assuming that transistors M_1 and M_3 are long-channel transistors biased in strong inversion and in saturation, we have

$$I_{D1} = \frac{n\beta_1}{2} \cdot V_P^2, \tag{29a}$$

$$I_{D3} = \frac{n\beta_3}{2} \cdot (V_P - V_R)^2, \tag{29b}$$

since $V_{S1} = 0$ and $V_{S3} = V_R$. Assuming a perfect matching between transistors M_1 and M_3 and between M_2 and M_4, we have $I_b = I_{D1} = I_{D3}$, resulting in

$$V_P = \sqrt{K} \cdot (V_P - V_R) \tag{30}$$

and

$$V_P = \frac{\sqrt{K}}{\sqrt{K} - 1} \cdot V_R = \frac{\sqrt{K}}{\sqrt{K} - 1} \cdot R \cdot I_b. \tag{31}$$

Introducing (31) into (29a) and solving for I_R results in

$$I_b = \frac{2}{n \cdot \beta_3 \cdot R^2} \cdot (\sqrt{K} - 1)^2 \tag{32}$$

and

$$V_R = \frac{2}{n \cdot \beta_3 \cdot R} \cdot (\sqrt{K} - 1)^2. \tag{33}$$

The current reference in strong inversion is actually set by $1/(\beta_3 \cdot R)$, which is strongly technology dependent.

2.3 General Remark

Note that if the current I_b is used to bias another n-channel transistor with factor β and operating also in saturation with the same level of inversion than transistor M_3, its source transconductance G_{ms} is then inversely proportional to R according to

$$G_{ms} = \frac{B}{R} \tag{34}$$

where factor B is given by

$$B = \begin{cases} \ln(K) & \text{in weak inversion} \\ 2 \cdot \sqrt{\frac{\beta}{\beta_3}} \cdot (\sqrt{K} - 1) & \text{in strong inversion.} \end{cases} \tag{35}$$

and is, to first-order, independent of the technology and of the temperature.

Problem 3 The Oguey current reference [2]

From Problem 2 we get $V_R = U_T \cdot \ln(K)$. Since M_7 and M_9 share the same gate voltage $V_{G7} = V_{G9} = V_G$ they also have the same pinch-off voltage $V_{P7} = V_{P9} = V_P$. Assuming M_7-M_9 are biased in strong inversion, for values of K such that $V_{D7} = V_R = U_T \ln(K) < V_P$, we can consider that M_7 is biased in the linear region. We can then write

$$I_b = I_{D7} = A \cdot I_{spec9} \cdot \left[\left(\frac{V_P}{2U_T} \right)^2 - \left(\frac{V_P - V_R}{2U_T} \right)^2 \right], \tag{36a}$$

$$I_b = I_{D9} = I_{spec9} \cdot \left(\frac{V_P}{2U_T} \right)^2. \tag{36b}$$

Solving (36a) and (36b) for I_b results in

$$I_b = I_{spec9} \cdot \left(\frac{A \cdot \ln K}{2} \right)^2 \cdot \left(1 + \sqrt{1 + \frac{1}{A}} \right)^2. \tag{37}$$

We see that the bias current I_b is now proportional to the specific current of M_9. If A can be made much larger than 1, (37) reduces to

$$I_b \cong I_{spec9} \cdot (A \cdot \ln K)^2 \quad \text{for } A \gg 1. \tag{38}$$

Since $I_{spec7} = A \cdot I_{spec9}$, I_b is also proportional to I_{spec7}

$$I_b \cong I_{spec7} \cdot A \cdot (\ln K)^2 \quad \text{for } A \gg 1. \tag{39}$$

This circuit provides a current which is proportionnal to the specific current I_{spec9} of M_9 or I_{spec7} of M_7. This allows to set the inversion coefficient of a given transistor. Say we want to set the inversion coefficient of transistor M_x to IC_x. We can then use a multiple N of the bias current I_b

$$I_{Dx} = I_{specx} \cdot IC_x = N \cdot I_b = N \cdot I_{spec7} \cdot IC_7 \tag{40}$$

which leads to

$$I_{specx} = N \cdot I_{spec7} \cdot \frac{IC_7}{IC_x}. \tag{41}$$

or

$$I_{specn\square} \cdot \frac{W_x}{L_x} = N \cdot I_{specn\square} \cdot \frac{W_7}{L_7} \cdot \frac{IC_7}{IC_x}. \tag{42}$$

The aspect ratio W/L of M_x is then given by

$$\frac{W_x}{L_x} = N \cdot \frac{W_7}{L_7} \cdot \frac{IC_7}{IC_x}. \tag{43}$$

Choosing the aspect ratio to W_x/L_x given by (43) and knowing IC_7 and W_7/L_7 will set the inversion coefficient to the desired IC_x.

Note that this biasing technique is totally agnostic of the threshold voltage as long as all transistors are properly biased in saturation.

References

[1] E. Vittoz and J. Fellrath, "CMOS analog integrated circuits based on weak inversion operations," *Solid-State Circuits, IEEE Journal of*, vol. 12, no. 3, pp. 224–231, June 1977.

[2] H. J. Oguey and D. Aebischer, "CMOS current reference without resistance," *Solid-State Circuits, IEEE Journal of*, vol. 32, no. 7, pp. 1132–1135, July 1997.