

Fundamentals of Analog VLSI Design

Exercise 13 - Solution

Design of Switched-capacitor Filters

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1 Problem 1: Design of a low-pass cascade switched-capacitor filter

1.1 Filter specifications

We want to design a low-pass (LP) switched-capacitor filter (SCF) that satisfies the specifications given in Table 1.1 using the Chebyshev approximation [1] and the cascade implementation. The filter mask corresponding to the specifications of Table 1.1 is shown in Figure 1.1.

Table 1.1: Filter specifications.

Specification	Symbol	Value	Unit
Cut-off frequency	f_c	20	kHz
Stop-band frequency	f_s	120	kHz
Pass-band gain	G_p	-1	dB
Stop-band gain	G_s	-40	dB
Clock frequency	f_{ck}	2	MHz

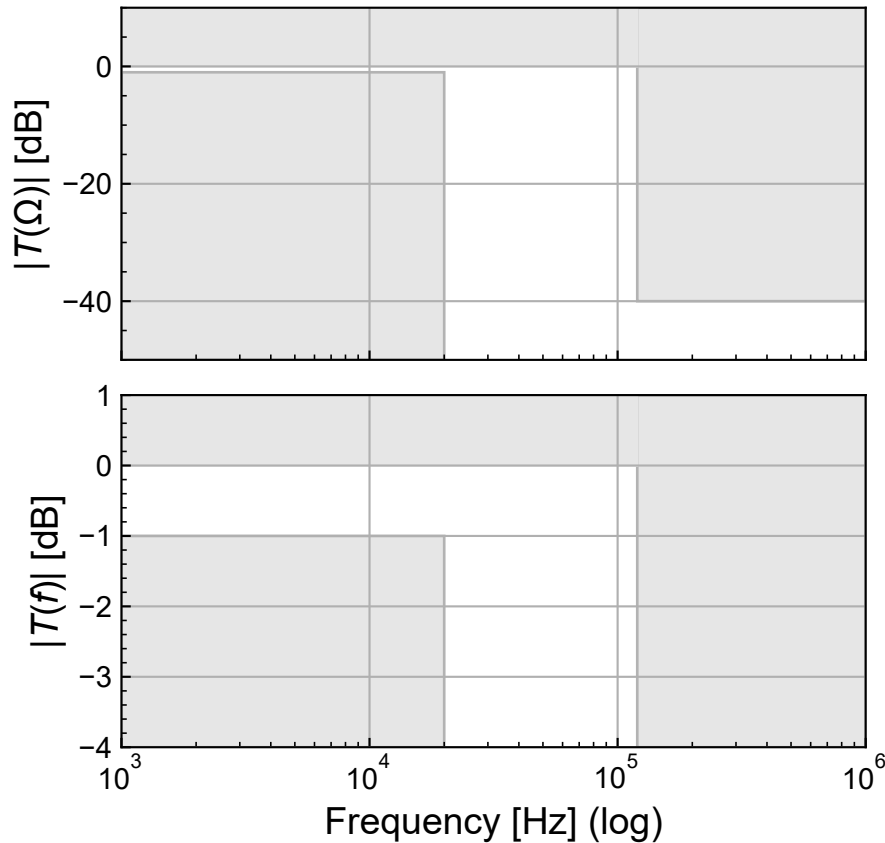


Figure 1.1: Filter mask.

1.2 Order estimation

We first need to evaluate the required order N in order to satisfy the specifications given in Table 1.1 and the mask shown in Figure 1.1 for a Chebyshev approximation. We can determine N with the help of the plot shown in Figure 1.2, where $\Omega_s \triangleq \omega_s/\omega_p$ is the ratio between the edge of the stop-band and the pass-band, $A_s = -G_s$ is the attenuation in the stop-band and ε determines the ripple allowed in the passband which is related to the attenuation in the passband $A_p = -G_p$ according to

$$\varepsilon = \sqrt{10^{\frac{A_p}{10}} - 1}. \quad (1.1)$$

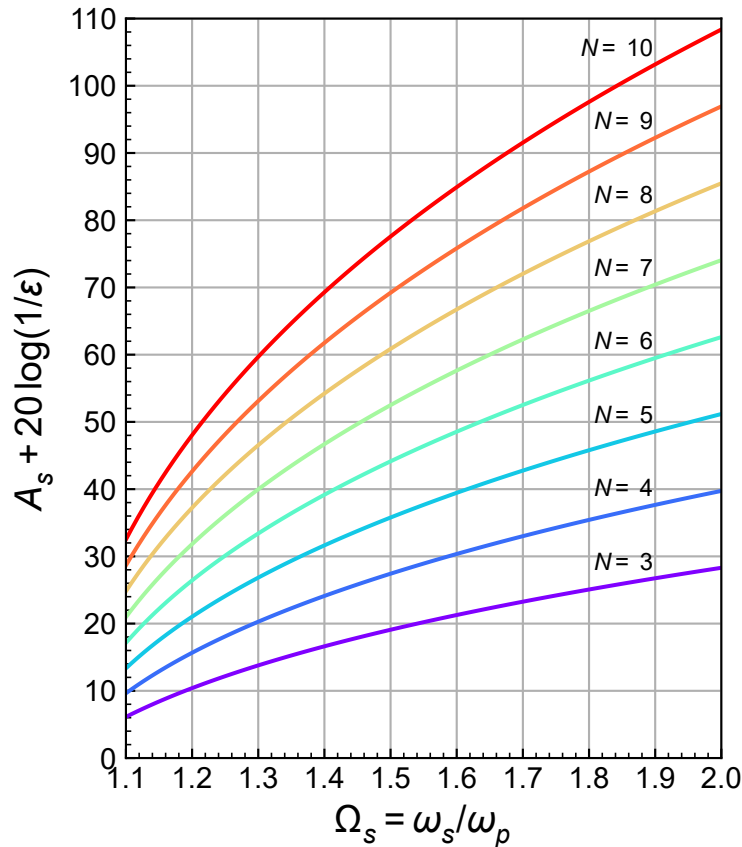


Figure 1.2: Order estimation for the Chebyshev approximation.

Knowing A_p and hence ε , A_s and Ω_s we can find a point in the above plot and take the order immediately on the curve above this point.

We can also use the approximate estimation of the order in terms of Ω_s , A_s and ε given by [1]

$$N \geq \frac{A_s + 20 \log(1/\varepsilon) + 6}{8.68 \cdot \operatorname{acosh}(\Omega_s)}. \quad (1.2)$$

which is plotted versus Ω_s in Figure 1.3 for various A_s and ε .

For the specifications given in Table 1.1, we have $A_p = 1 \text{ dB}$, $A_s = 40 \text{ dB}$, $\varepsilon = 0.508847$, $\Omega_s = 6$, $N_{est} = 2.412$ and hence the required order is $N = 3$. The magnitude of the corresponding transfer function is plotted in Figure 1.4.

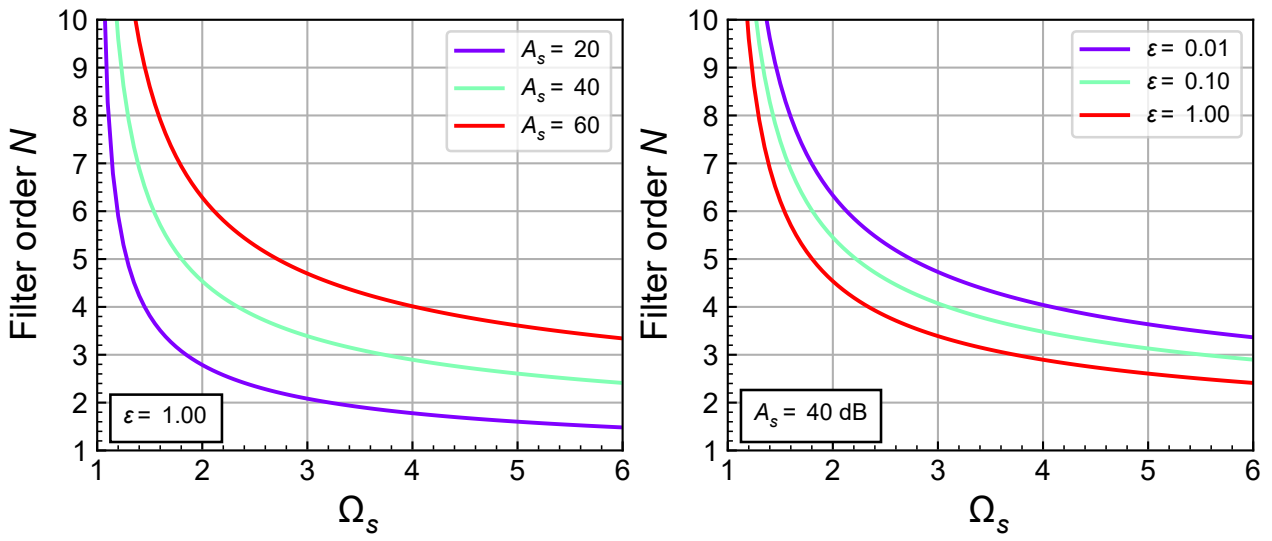


Figure 1.3: Chebyshev order estimation using (1.2).

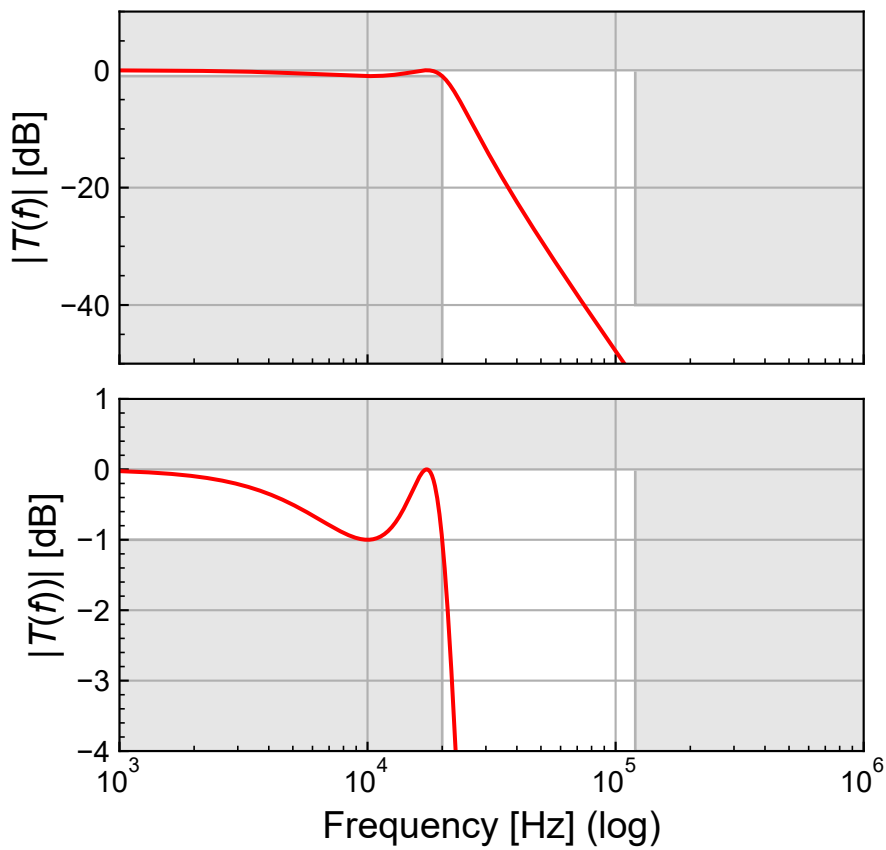


Figure 1.4: Magnitude of the Chebyshev transfer function for $N = 3$.

1.3 Transfer function factorization

It can be shown that the factorized transfer function is then given by

$$T(s) = \frac{\omega_{p1}}{s + \omega_{p1}} \cdot \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}, \tag{1.3}$$

where $\omega_{p1} = 2\pi \cdot 9.883412 \text{ krad/s}$, $\omega_0 = 2\pi \cdot 19.941962 \text{ krad/s}$ and $Q = 2.017720$. Figure 1.5 shows that the magnitude of the factorized transfer function actually corresponds to that of the Chebyshev approximation.

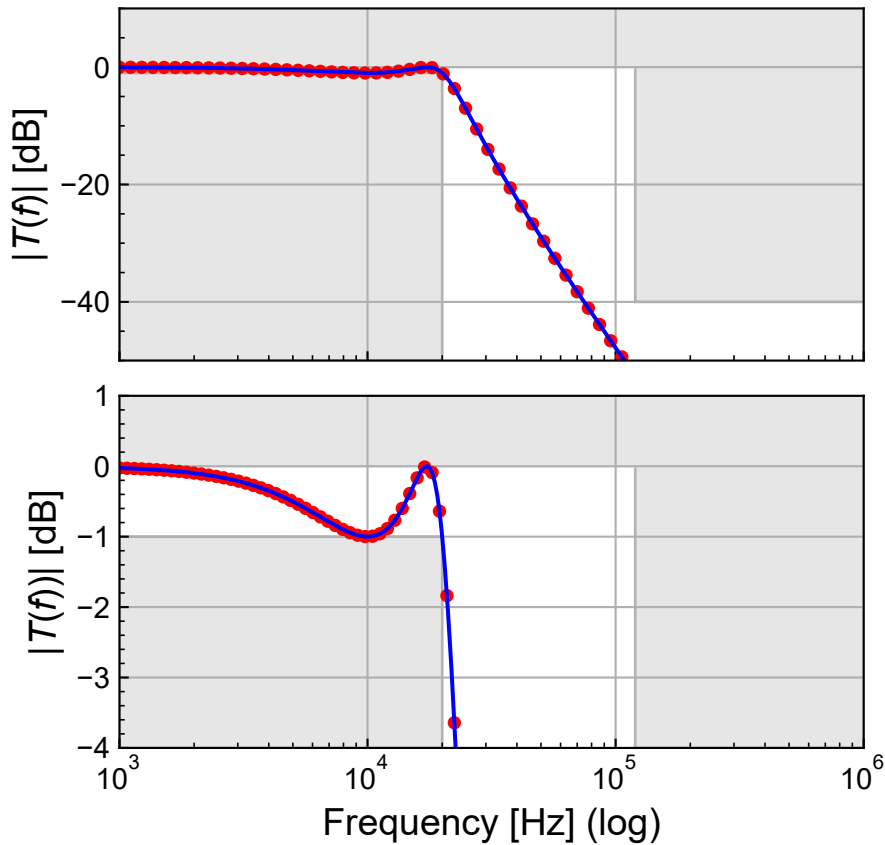


Figure 1.5: Magnitude of the Chebyshev transfer function for $N = 3$.

1.4 Cascade implementation

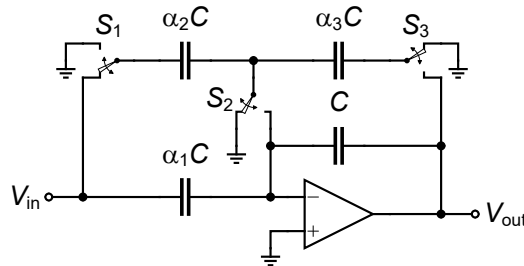


Figure 1.6: SC 1st-order section [2].

The 1st-order section can be implemented with the SC schematic shown in Figure 1.6, which has a transfer function given by [2]

$$H(z) = -\frac{\alpha_1 \cdot z - (\alpha_1 + \alpha_2)}{(1 + \alpha_3) \cdot z - 1} \tag{1.4}$$

In the case of the LP filter we can remove capacitor $\alpha_1 C$ leading to the schematic shown in Figure 1.7.

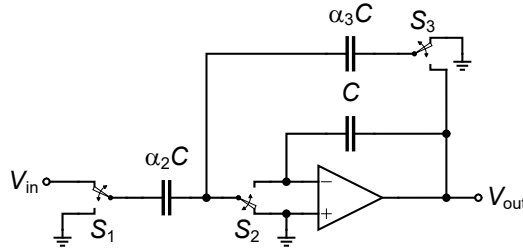


Figure 1.7: SC 1st-order LP section.

The transfer function then simplifies to

$$H_1(z) = \frac{\alpha_2}{(1 + \alpha_3) \cdot z - 1} \tag{1.5}$$

The design equations assuming that $\omega_z T \ll 1$ and $\omega_{p1} T \ll 1$ are simply

$$\alpha_2 = \alpha_3 = \omega_{p1} \cdot T. \tag{1.6}$$

where $T = 1/f_{ck}$. For our design this leads to $\alpha_2 = \alpha_3 = 0.031050$. The transfer function of the 1st-order section is plotted in Figure 1.8.

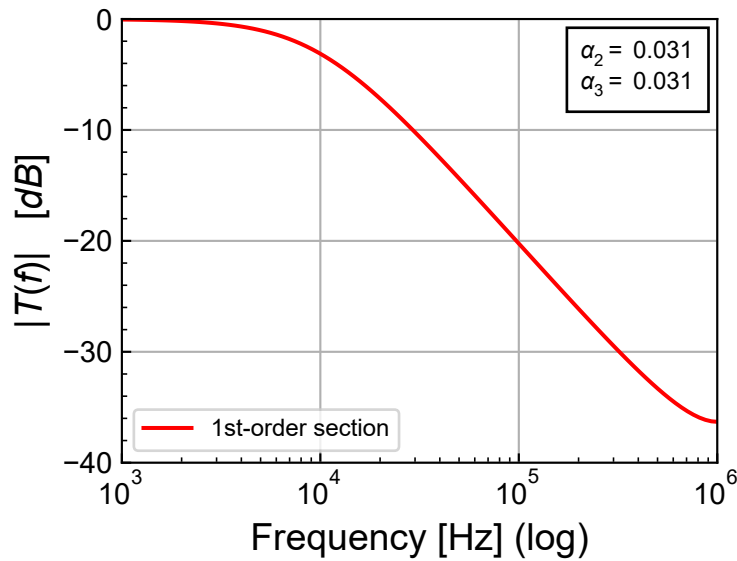


Figure 1.8: Transfer function magnitude of the 1st-order section.

The 2nd-order section can be implemented by the circuit shown in Figure 1.9, which has the following z transfer function [3]

$$H(z) = -\frac{\beta_1 z^2 - (2\beta_1 - \alpha_1 \cdot \beta_2) \cdot z + \beta_1}{(1 + \beta_3) \cdot z^2 - (2 - \alpha_2 \cdot \beta_2 + \beta_3) \cdot z + 1} \tag{1.7}$$

For the implementation of LP filters we don't need capacitance $\beta_1 C_2$ and the circuit simplifies to the one shown in Figure 1.10, which transfer function is given by [3]

$$H_2(z) = -\frac{\alpha_1 \cdot \beta_2 \cdot z}{(1 + \beta_3) \cdot z^2 - (2 - \alpha_2 \cdot \beta_2 + \beta_3) \cdot z + 1} \tag{1.8}$$

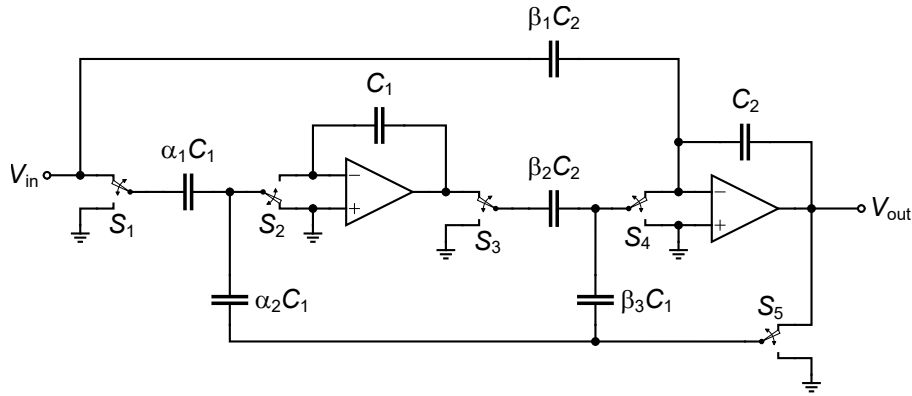


Figure 1.9: General SC 2nd-order section [3].

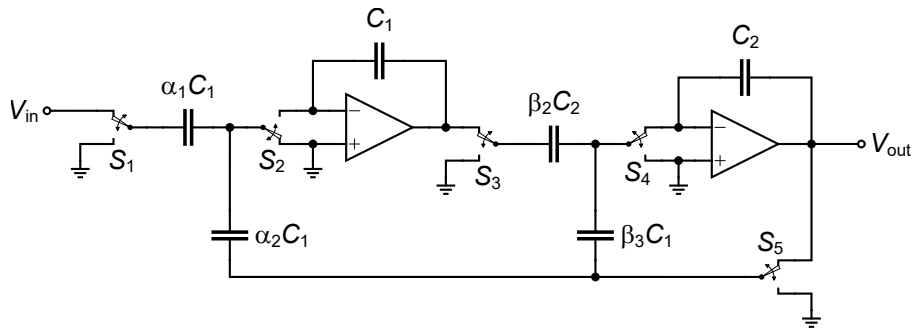


Figure 1.10: LP 2nd-order section.

with

$$\alpha_1 = \alpha_2 = \beta_2 = \omega_0 T, \tag{1.9}$$

$$\beta_3 = \frac{\omega_0 T}{Q}. \tag{1.10}$$

For the given specification we get $\alpha_1 = \alpha_2 = \beta_2 = 0.062650$ and $\beta_3 = 0.031050$. The transfer function of the 2nd-order section is shown in Figure 1.11.

The schematic of the complete filter is shown in Figure 1.12 with the capacitance ratios summarized in Table 1.2.

Table 1.2: Capacitance ratios.

Symbol	Value
α_{12}	0.03105
α_{13}	0.03105
α_{21}	0.06265
α_{22}	0.06265
β_{22}	0.06265
β_{23}	0.03105

The transfer function of the complete filter is then simply given by

$$H(z) = H_1(z) \cdot H_2(z), \tag{1.11}$$

where $H_1(z)$ and $H_2(z)$ are given by (1.5) and (1.8), respectively. The magnitude of the transfer functions are plotted in Figure 1.13.

The resulting transfer function is compared to the theoretical transfer function in Figure 1.14. We see a perfect match between the cascade and theoretical Chebyshev transfer functions.

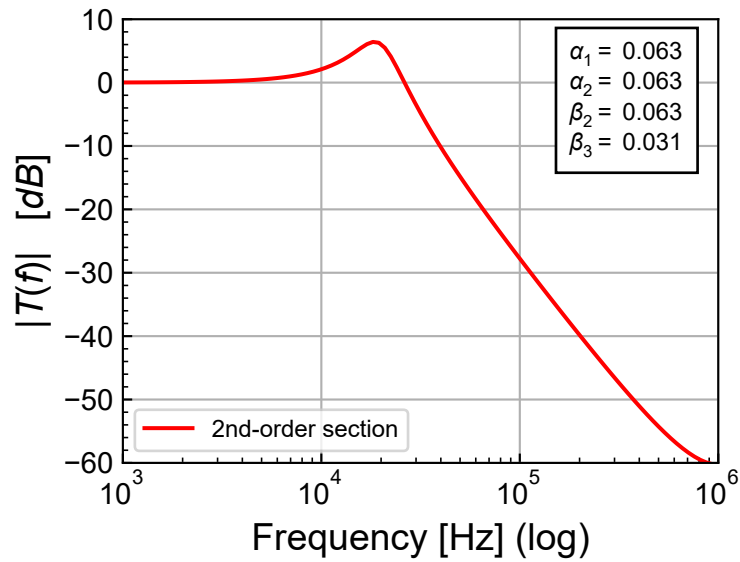


Figure 1.11: Transfer function magnitude of the 2nd-order section.

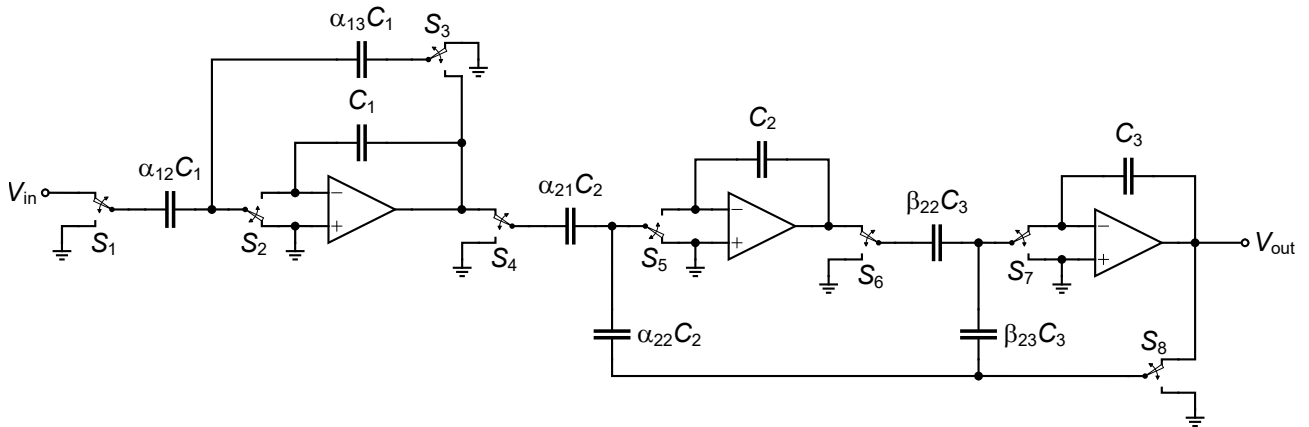


Figure 1.12: Schematic of the complete filter.

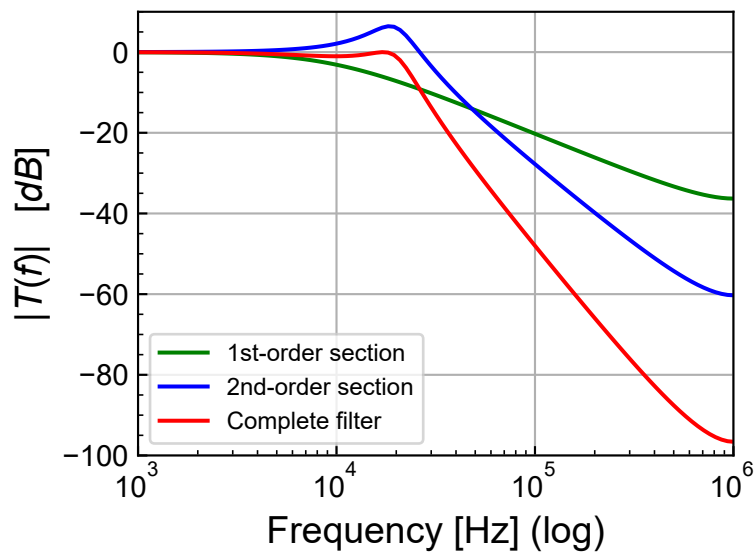


Figure 1.13: Transfer function magnitude of the 1st- and 2nd-order sections and of the complete filter.

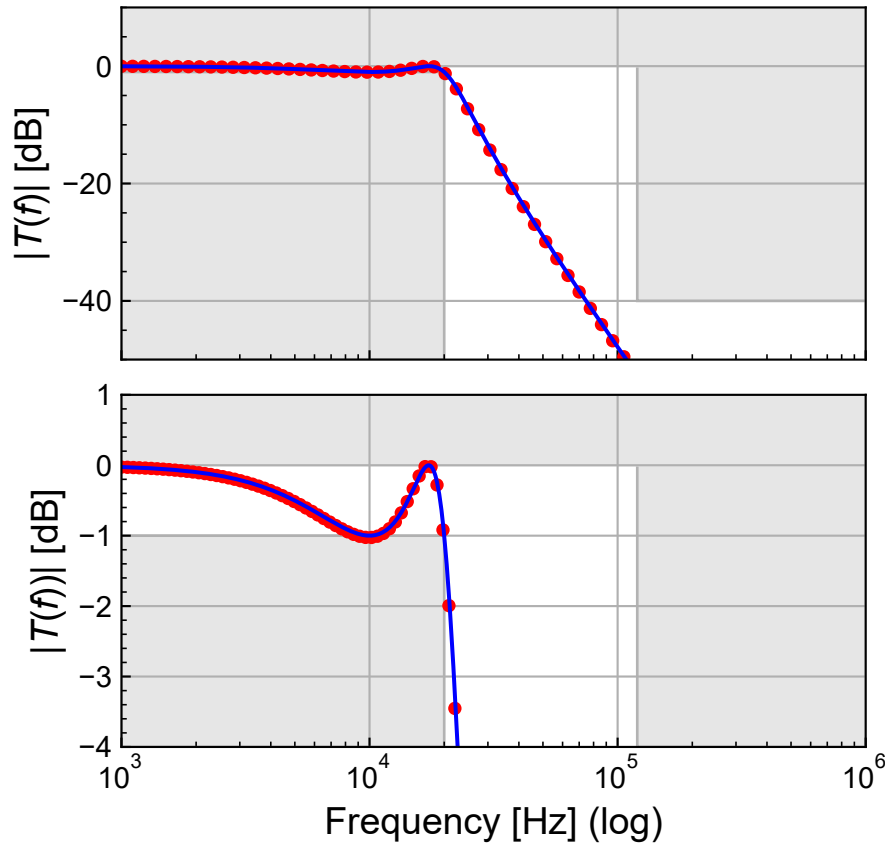


Figure 1.14: Transfer function magnitude of the complete filter.

We still need to choose the impedance level. Since we have very small capacitance ratios (because the sampling frequency was set much above the cut-off frequency), the switched-capacitors will be very small. However they need to be large enough in order to ensure a good matching and therefore a good control on the capacitance ratios. We can choose $C_{min} = 100 \text{ fF}$. This capacitance corresponds to capacitors $\alpha_{12}C_1$ and $\alpha_{13}C_1$ of the 1st-order section and $\beta_{23}C_3$ of the 2nd-order section. From there we get the values of the integrating capacitances $C_1 = 3.221 \text{ pF}$ and $C_3 = 3.221 \text{ pF}$.

We finally choose $C_1 = C_2 = C_3 = 3.2 \text{ pF}$.

1.5 Simulation with LTSpice

The above design can unfortunately not be verified by a simple SPICE simulator. However, we have designed a special library that allows for the simulation of two non-overlapping phases switched-capacitor circuits like the one we have designed. Without going into the details of the implementation of this library, what it does is to simulate the circuits in phase Φ_1 and Φ_2 concurrently and then insure the charge conservation at the virtual ground nodes between the two phases [4]. The library was implemented for LTSpice taking advantage of the Laplace instruction. Unfortunately the latter is not available in ngspice, reason for which we have done the simulation in LTSpice.

The SC components are part of my analog.lib library. To install the library follow the instructions given in the moodle site. To be able to run the LTSpice simulation from the Jupyter Notebook, you need to install the PyLTSpice package. Refer to the instructions given in PyLTSpice site

The simulations are done with the schematic shown in Figure 1.15 with the component values given in Table 1.3.

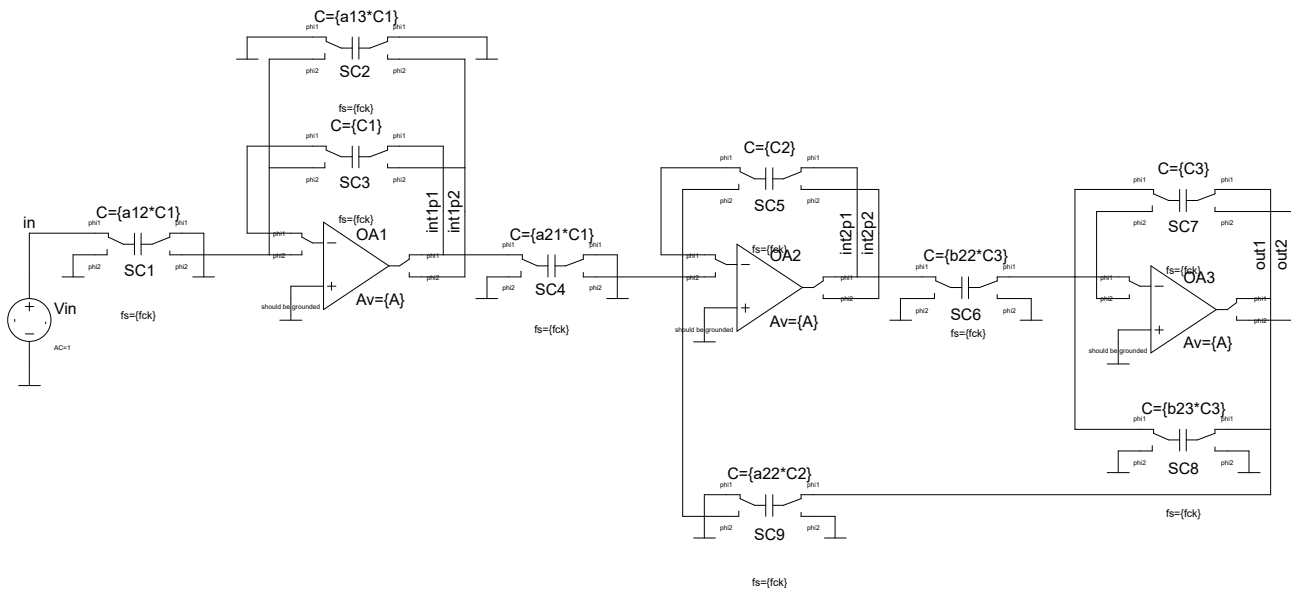


Figure 1.15: Schematic used for the LTSpice simulation.

Table 1.3: Component values of the 3rd-order cascade low-pass SC filter of Figure 1.15.

Symbol	Value	Unit
A	100	dB
f_{ck}	2	MHz
C_1	3.2	pF
C_2	3.2	pF
C_3	3.2	pF
α_{12}	0.03105	-
α_{13}	0.03105	-
α_{21}	0.06265	-
α_{22}	0.06265	-
β_{22}	0.06265	-
β_{23}	0.03105	-
$\alpha_{12} C_1$	99	fF
$\alpha_{13} C_1$	99	fF
$\alpha_{21} C_2$	200	fF
$\alpha_{22} C_2$	200	fF
$\beta_{22} C_3$	200	fF
$\beta_{23} C_3$	99	fF

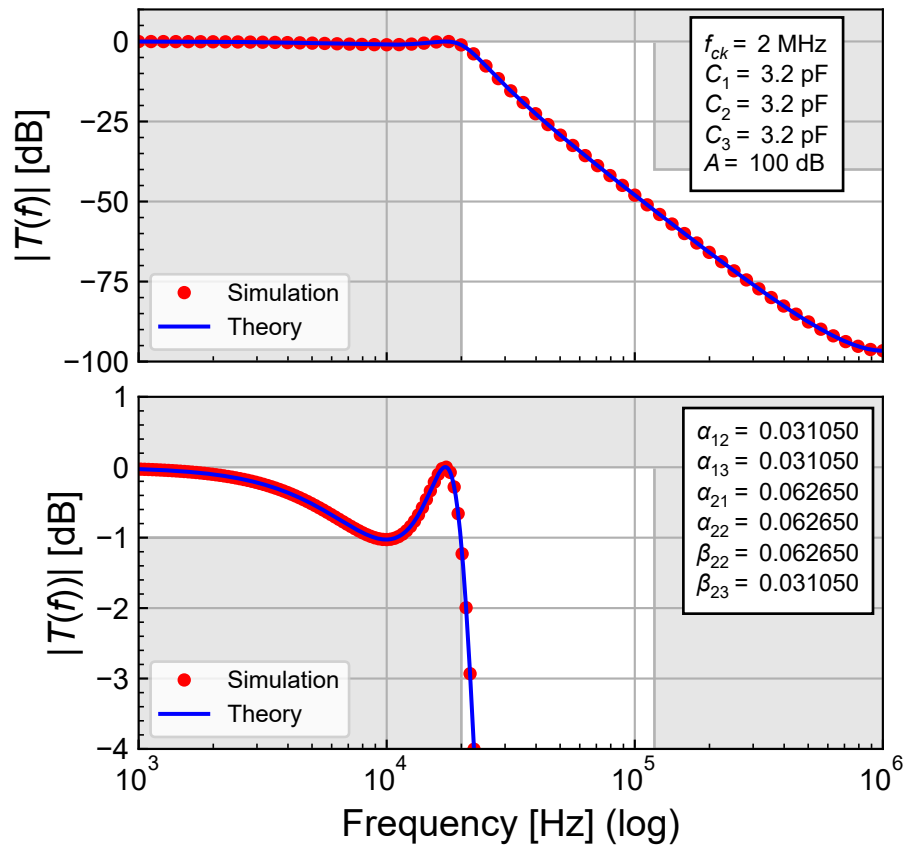


Figure 1.16: Simulated transfer function of the 3rd-order low-pass cascade SC filter of Figure 1.15.

From Figure 1.16, we can observe a perfect fit between the simulated and the theoretical transfer function.

2 Problem 2: Design of a low-pass switched-capacitor filter from an LC ladder prototype (indirect simulation)

In this problem we want to design a low-pass switched-capacitor filter that satisfies the same specifications given in Table 1.1 and filter mask shown in Figure 1.1 than in Problem 1, also with a Chebyshev approximation, but using the indirect simulation of an LC passive ladder filter.

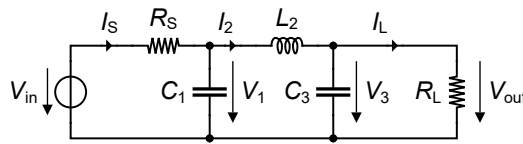


Figure 2.1: Low-pass prototype filter (LPPF).

From the order estimation made in Problem 1, we have found that we need an order $N = 3$ to meet the specifications. We can therefore start with the 3rd-order low-pass prototype filter (LPPF) shown in Figure 2.1. It can be shown that the component values of the LPPF for a Chebyshev approximation can be calculated directly from the order N and ε assuming that $R_S = 1 \Omega$ [1] [5] [6]. The values are given in Table 2.1.

Table 2.1: LPPF component values for a Chebyshev approximation.

Symbol	Value	Unit
R_S	1	Ω
$C_{1,norm}$	2.02359	F
$L_{2,norm}$	0.994102	H
$C_{3,norm}$	2.02359	F
R_L	1	Ω

In the indirect simulation or operational simulation approach we start writing the state equations of the LC ladder filter shown in Figure 2.1 as

$$I_S = \frac{V_{in} - V_1}{R_S}, \quad (2.1)$$

$$V_1 = \frac{I_S - I_2}{s C_1}, \quad (2.2)$$

$$I_2 = \frac{V_1 - V_3}{s L_2}, \quad (2.3)$$

$$V_{out} = \frac{I_2 - I_L}{s C_3}, \quad (2.4)$$

$$I_L = \frac{V_{out}}{R_L}. \quad (2.5)$$

We then multiply all the currents by a normalization impedance Z_0 to only have voltages

$$V_S \triangleq Z_0 \cdot I_S = \frac{Z_0}{R_S} \cdot (V_{in} - V_1), \tag{2.6}$$

$$V_1 = \frac{I_S - I_2}{s Z_0 C_1}, \tag{2.7}$$

$$V_2 \triangleq Z_0 \cdot I_2 = \frac{Z_0}{s L_2} \cdot (V_1 - V_3), \tag{2.8}$$

$$V_{out} = \frac{I_2 - I_L}{s Z_0 C_3}, \tag{2.9}$$

$$V_L \triangleq Z_0 \cdot I_L = \frac{Z_0}{R_L} \cdot V_{out}. \tag{2.10}$$

In the above equations we can identify 3 integration operators. It is then easy to draw the signal flow graph (SFG) corresponding to the above equations, which is shown in Figure 2.2.

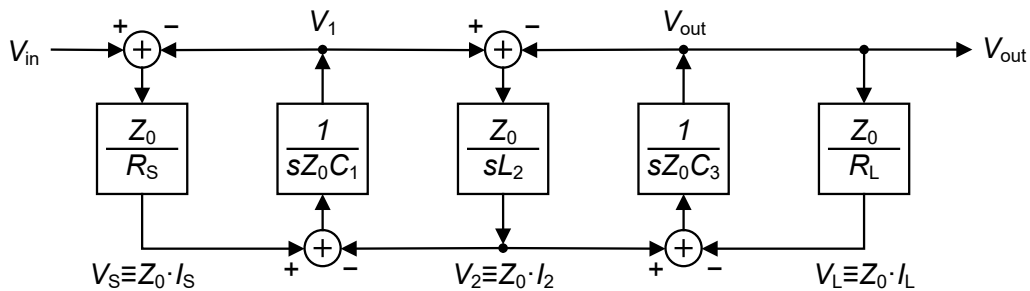


Figure 2.2: Signal-flow diagram corresponding to the LC ladder of Figure 2.1.

We can simplify the SFG of Figure 2.2 by choosing $Z_0 = R_L$. The last box can then be replaced by a wire. We can also get rid of the first summation operator by adding the weighted input voltage to the second summation operator. This results in the SFG shown in Figure 2.3.

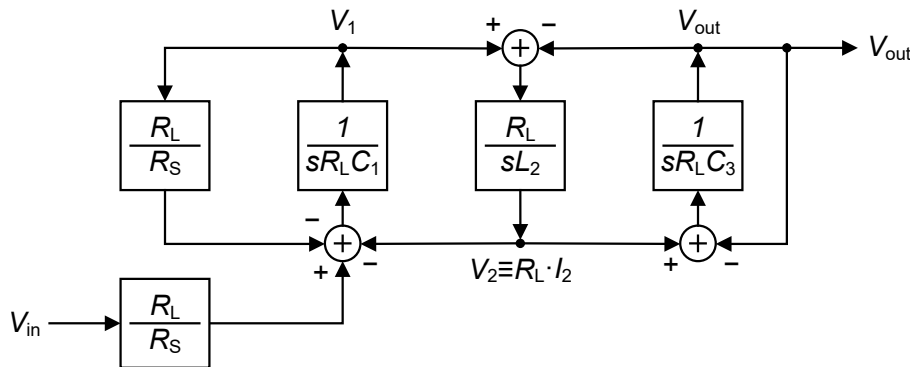


Figure 2.3: Simplification the SFG of Figure 2.2 after setting $Z_0 = R_L$.

The integration operators can be implemented by summing integrators with time constants given by

$$\tau_1 \triangleq R_L C_1, \tag{2.11}$$

$$\tau_2 \triangleq \frac{L_2}{R_L}, \tag{2.12}$$

$$\tau_3 \triangleq R_L C_3. \tag{2.13}$$

This results in the SFG shown in Figure 2.4.

Since for odd order Chebyshev approximations the termination resistances are equal $R_L = R_S$, the gain boxes $R_L/R_S = 1$ can also be replaced by a wire resulting in the SFG of Figure 2.5.

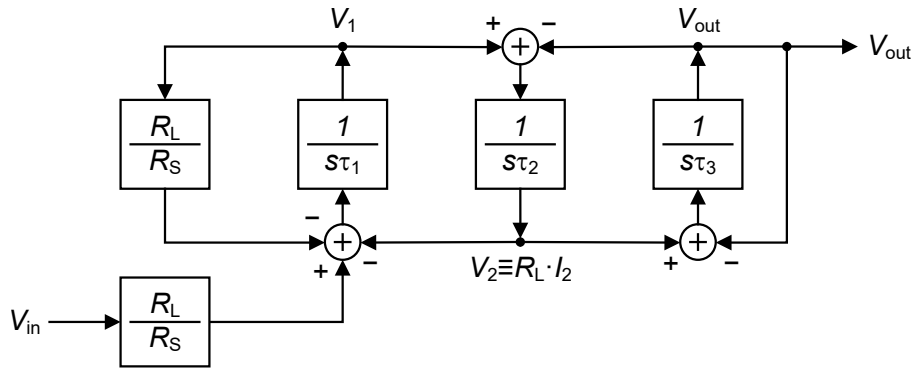


Figure 2.4: SFG of Figure 2.3 with 3 summing integrators.

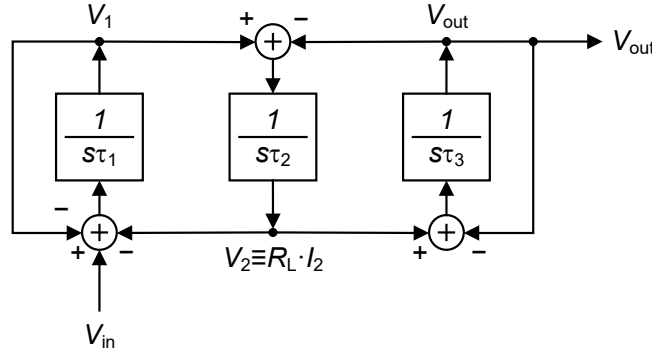


Figure 2.5: Final SFG corresponding to the LPPF of Figure 2.1.

The integrator time constants for achieving the specifications of Table 1.1 can be obtained by denormalizing the time constants of the LPPF according to

$$\tau_1 = \frac{R_{norm} C_{1,norm}}{\omega_p}, \tag{2.14}$$

$$\tau_2 = \frac{L_{2,norm}}{R_{norm} \omega_p}, \tag{2.15}$$

$$\tau_3 = \frac{R_{norm} C_{3,norm}}{\omega_p}, \tag{2.16}$$

where $R_{norm} = R_S = R_L = 1 \Omega$. The denormalized time constants are presented in Table 2.2.

Table 2.2: Integrator time constants.

Symbol	Value	Unit
τ_1	16.1032	μs
τ_2	7.91082	μs
τ_3	16.1032	μs

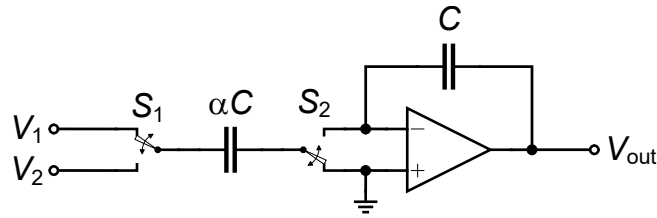


Figure 2.6: Basic summing SC integrator.

Each summing integrator can be implemented by the SC circuit shown in Figure 2.6 having the following z-transfer function

$$V_{out}(z) = \frac{\alpha}{1 - z^{-1}} \cdot \left[z^{-\frac{1}{2}} \cdot V_1(z) - V_2(z) \right]. \quad (2.17)$$

The resulting SC filter is shown in Figure 2.7 with the capacitance ratios given by

$$\alpha_{11} = \frac{1}{\tau_1 f_{ck}}, \quad (2.18)$$

$$\alpha_{12} = \alpha_{11}, \quad (2.19)$$

$$\alpha_2 = \frac{1}{\tau_2 f_{ck}}, \quad (2.20)$$

$$\alpha_3 = \frac{1}{\tau_3 f_{ck}}, \quad (2.21)$$

where $f_{ck} = 2.0 \text{ MHz}$. The capacitance ratios are given in Table 2.3. Notice that we added capacitor $\alpha_{12} C_1$ which is equal to $\alpha_{11} C_1$ in order to correct for the -6 dB loss of the passive LC ladder filter of Figure 2.1.

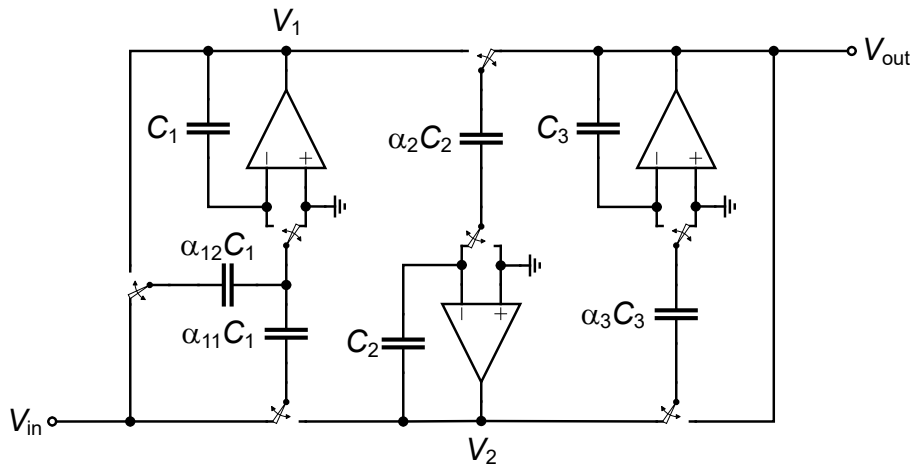


Figure 2.7: 3rd-order SC filter schematic.

Table 2.3: Capacitance ratios of the 3rd-order SC filter shown in Figure 2.7.

Symbol	Value	Unit
α_{11}	0.03105	-
α_{12}	0.03105	-
α_2	0.063205	-
α_3	0.03105	-

From the analysis of the SC circuit shown in Figure 2.7 using charge conservation at the virtual grounds of each integrator leads to the following z-transfer function

$$H(z) = \frac{a_2 z^2}{b_3 z^3 + b_2 z^2 + b_1 z + b_0}, \quad (2.22)$$

where

$$a_2 = (\alpha_{11} + \alpha_{12}) \alpha_2 \alpha_3, \quad (2.23)$$

$$b_0 = -1, \quad (2.24)$$

$$b_1 = 3 + \alpha_{12} - \alpha_{11} \alpha_2 + \alpha_3 - \alpha_2 \alpha_3, \quad (2.25)$$

$$b_2 = -3 - 2 \alpha_{12} + \alpha_{11} \alpha_2 - 2 \alpha_3 - \alpha_{12} \alpha_3 + \alpha_2 \alpha_3 + \alpha_{11} \alpha_2 \alpha_3 + \alpha_{12} \alpha_2 \alpha_3, \quad (2.26)$$

$$b_3 = (1 + \alpha_{12})(1 + \alpha_3). \quad (2.27)$$

The transfer function of (2.22) is compared to the theoretical transfer function in Figure 2.8. We see an almost perfect match except at high frequency where the transfer function given by (2.22) deviates because of the sampled-data nature of the SC filter.

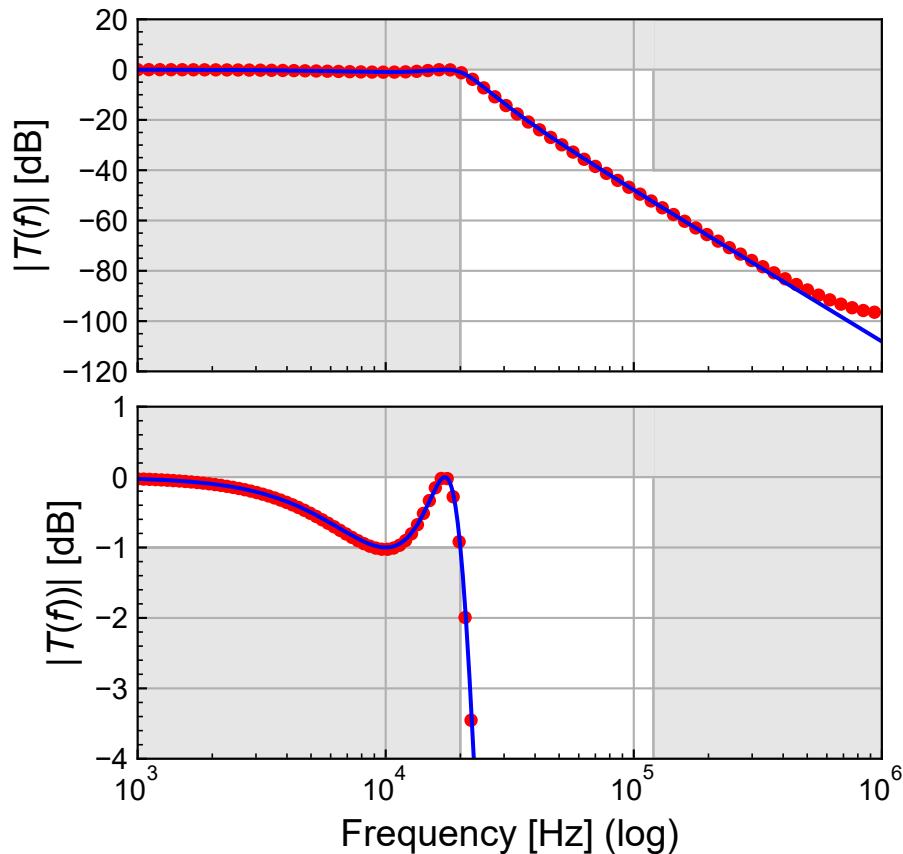


Figure 2.8: Comparison between the theoretical and z-transfer function..

We still need to choose the impedance level. Since we have very small capacitance ratios (because the clock frequency was set much above the cut-off frequency for the approximation to be valid), the switched-capacitors will be very small. However they need to be large enough in order to ensure a good matching and therefore a good control on the capacitance ratios.

We can choose $C_{min} = 100 \text{ fF}$. this leads to the following integration capacitances $C_1 = 3.221 \text{ pF}$, $C_2 = 1.582 \text{ pF}$ and $C_3 = 3.221 \text{ pF}$.

We finally choose $C_1 = C_2 = C_3 = 3.300 \text{ pF}$.

2.1 Simulation with LTSpice

We can now check the transfer function using LTSpice simulations which are done with the schematic shown in Figure 2.9 and the component values given in Table 2.4.

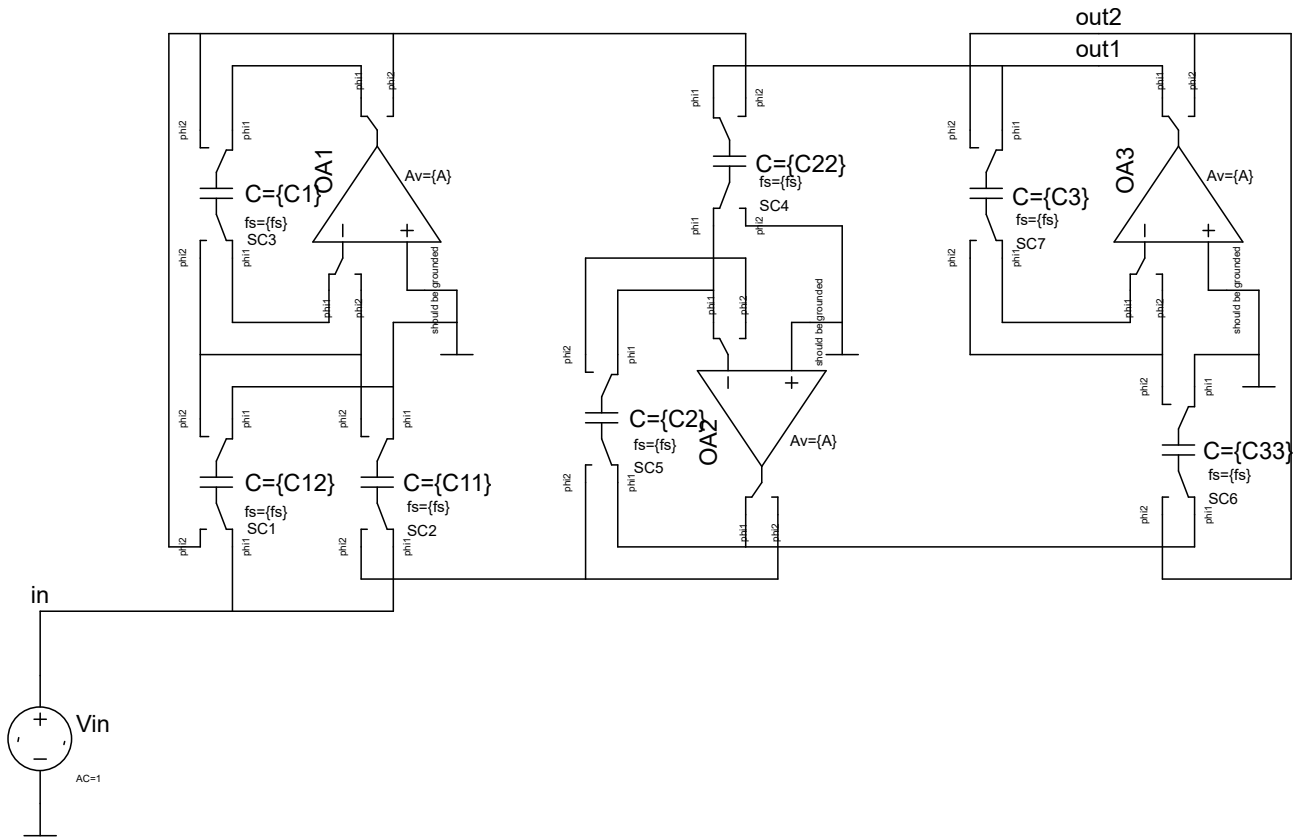


Figure 2.9: Schematic used for the LTSpice simulation.

Table 2.4: Component values of the 3rd-order cascade low-pass SC filter of Figure 2.9.

Symbol	Value	Unit
A	100	dB
f_{ck}	2	MHz
C_1	3.3	pF
C_2	3.3	pF
C_3	3.3	pF
α_{11}	0.03105	-
α_{12}	0.03105	-
α_2	0.063205	-
α_3	0.03105	-
$\alpha_{11} C_1$	102	fF
$\alpha_{12} C_1$	102	fF
$\alpha_2 C_2$	209	fF
$\alpha_3 C_3$	102	fF

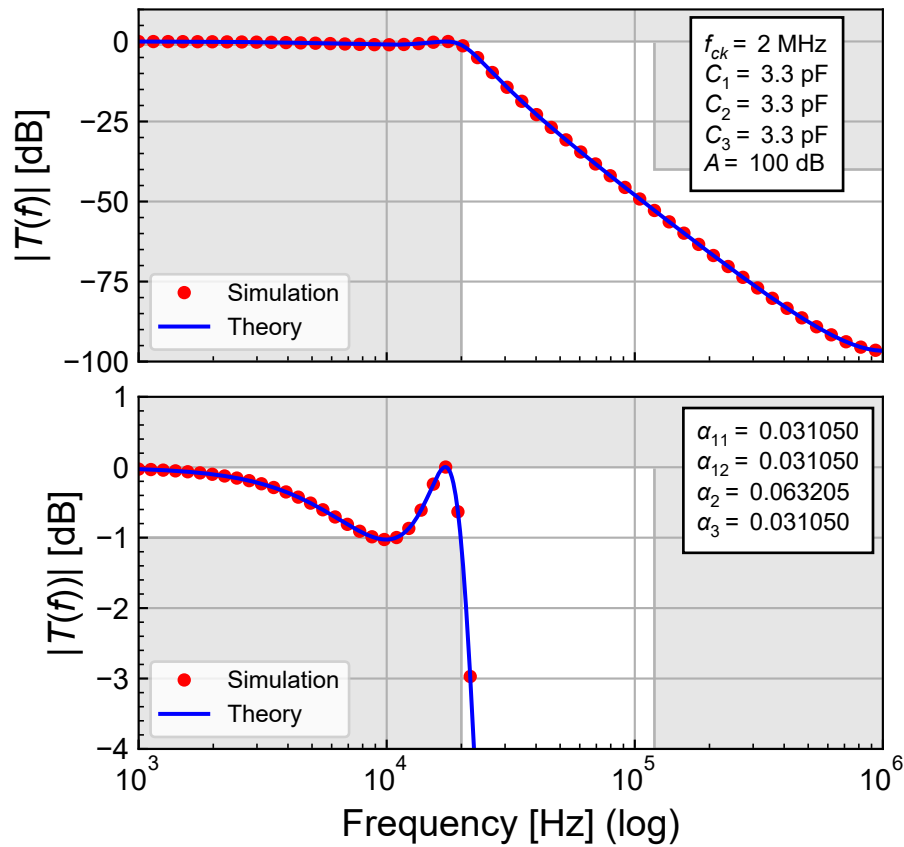


Figure 2.10: Simulated transfer function of the 3rd-order low-pass cascade SC filter of Figure 2.9.

From Figure 2.10 we see a perfect match between the simulations and the transfer function given by (2.22), even close to the clock frequency.

References

- [1] P. Brackett and A. Sedra, *Filter Theory and Design: Active and Passive*. Matrix Pub, 1984.
- [2] R. Gregorian and G. C. Temes, *Analog MOS Integrated Circuits for Signal Processing*. John Wiley & Sons, 2015.
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