Fundamentals of Analog VLSI Design
Exercise 12 - Problem

Design of Continuous-time Filters

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1 Problem 1: Design of a cascade G_m-C continuous-time filter

1.1 Filter specifications

In this problem, we want to design a low-pass filter that satisfies the specifications given in Table 1.1 using a Butterworth approximation. The filter mask corresponding to the specifications of Table 1.1 is shown in Figure 1.1.

Table 1.1: Filter specifications.

Specification	Symbol	Value	Unit
Cut-off frequency	f_c	20	kHz
Stop-band frequency	f_s	120	kHz
Pass-band gain	G_p	-1	dB
Stop-band gain	G_s	-40	dB

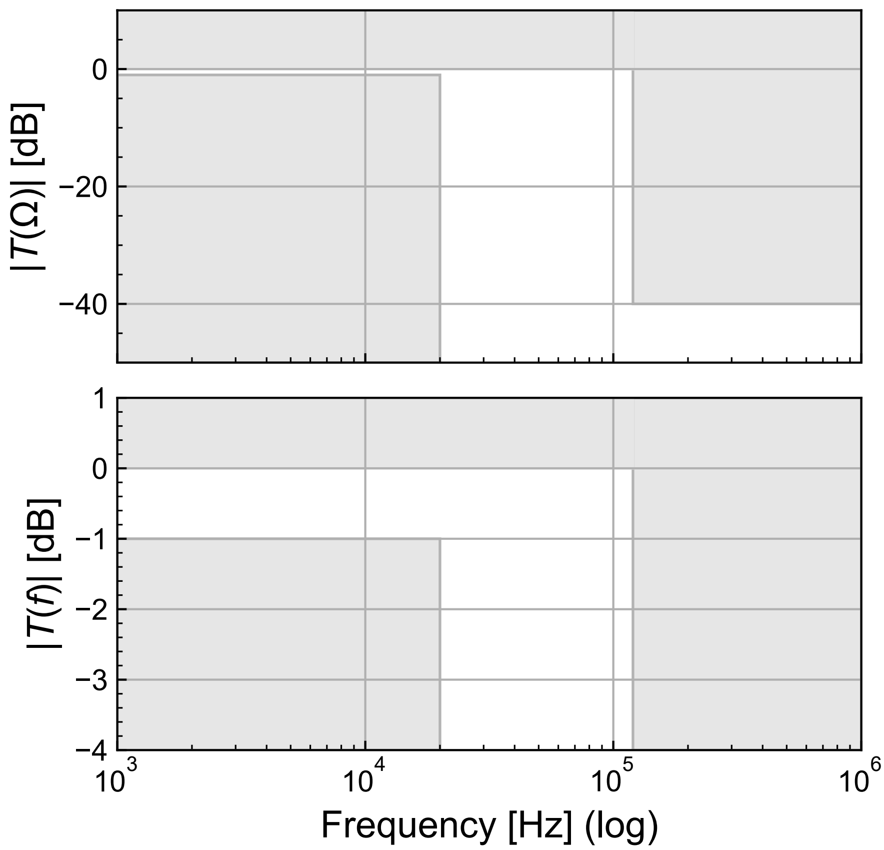

Figure 1.1: Filter mask.

We can show that if we choose a Butterworth approximation for the specifications shown in Table 1.1, then the minimum required order is $N = 3$. The filter transfer function is then given by

$$T(s) = \frac{\omega_0}{s + \omega_0} \cdot \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}\,s + \omega_0^2}. \tag{1.1}$$

where $\omega_0 = 2\pi\, f_0 = 2\pi\ 25.051528\ krad/s$ is the resonance frequency and $Q = 1$ the quality factor of the 2nd-order section. The magnitude of the corresponding transfer function is plotted in Figure 1.2.

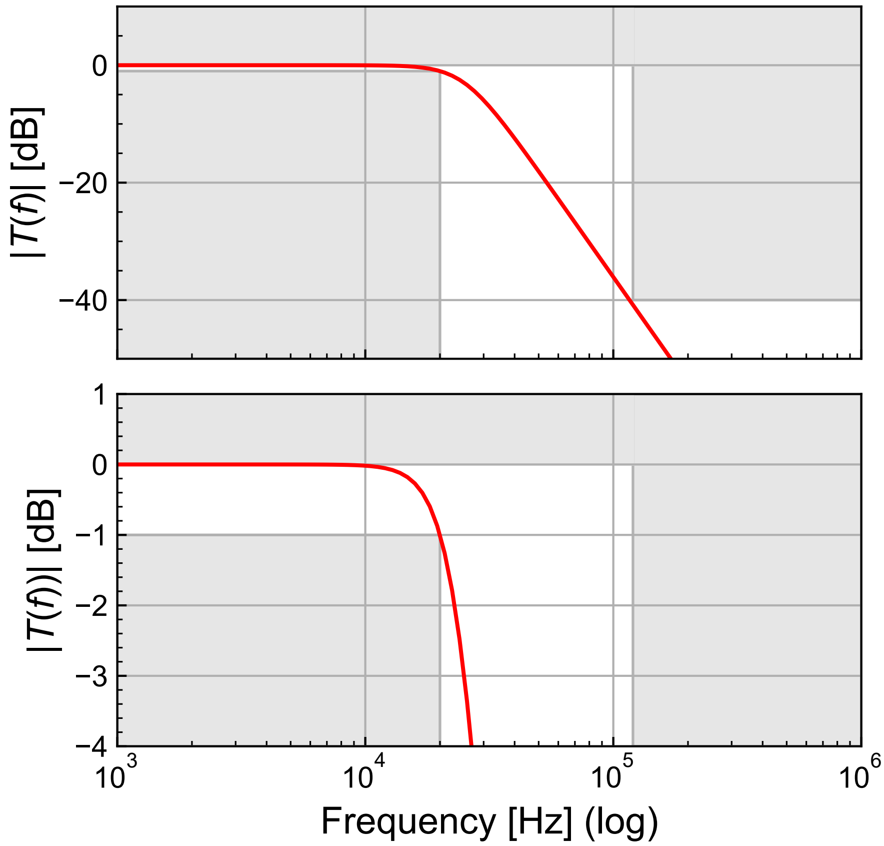

Figure 1.2: Filter transfer function.

Design a single-ended G_m-C cascade implementation that realizes the transfer function given by (1.1). To choose the transconductance or capacitance value (this is called defining the impedance level), we set the integrated kT/C thermal noise at the output of the filter to remain smaller than $V_{nout} = 120$ μV_{rms}. We assume that the transconductors have an equivalent thermal noise excess factor $\gamma_n = 2$ and contribute equally to the integrated white noise at the filter output. Of course that is not exactly true but allows to get an initial capacitance value close to the final value.

Check the designed filter with ngspice simulations in qucs-s.

2 Problem 2: Design of a G_m-C continuous-time filter using the LC ladder approach

2.1 The LC ladder approach

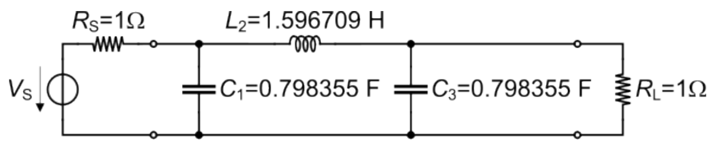

Figure 2.1: Low-pass prototype filer (LPPF) for a Butterworth approximation.

In this problem, we want to design a single-ended G_m-C filter that satisfies the same specifications given in Table 1.1 and used for Problem 1 and the same filter mask shown in Figure 1.1. We also will use a Butterworth approximation with the transfer function shown in Figure 1.2. The difference is that for this G_m-C implementation, we propose to use the indirect simulation of an LC ladder approach. To this purpose we start from the low-pass prototype filter (LPPF) shown in Figure 2.1 which implements a Butterworth approximation with $\Omega_s = \omega_s/\omega_p = 6$. The normalized components values for a Butterworth approximation are given in Table 2.1. The normalized transfer function corresponding to the LPPF is shown in Figure 2.2.

Table 2.1: Values of the LPPF components for a Butterworth approximation.

Symbol	Value	Unit
C_1	0.798355	F
L_2	1.59671	H
C_3	0.798355	F

Start from the LPPF given in Figure 2.1 and follow these steps:

- Sketch the signal-flow graph corresponding to the LC ladder prototype filter, correcting for the DC gain to match the mask.
- Calculate the denormalized integration time constants.
- Draw the resulting single-ended G_m-C filter accounting for the $-6\,dB$ DC gain correction.
- Denormalize the impedances by choosing the minimum capacitance such that the integrated kT/C thermal noise at the output of the filter remains smaller than $V_{nout} = 120\ \mu V_{rms}$. It is assumed that all the transconductors have the same thermal noise excess factor $\gamma_n = 2$ and only the integrators with the smallest capacitance contribute to the integrated thermal noise at the filter output.
- Calculate the component values of the filter.
- Check the designed filter with ngspice simulations in qucs-s.

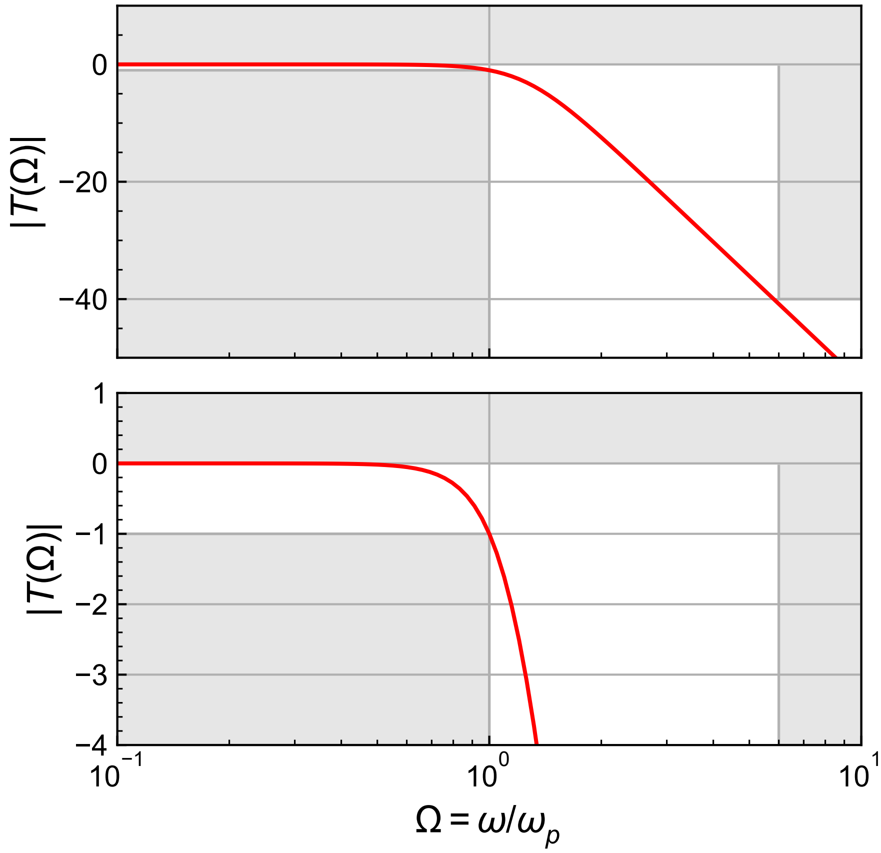

Figure 2.2: Normalized transfer function.

3 Problem 3: Design of a MOSFET-C continuous-time filter

In this problem, we want to design a fully differential MOSFET-C filter with the indirect simulation of an LC ladder approach already used in Problem 2. The first steps are the same than in Problem 2.

- Denormalize the impedances by choosing the minimum capacitance such that the integrated kT/C thermal noise at the output of the filter remains smaller than $V_{nout} = 120\ \mu V_{rms}$. It is assumed that the OPAMP noise can be neglected compared to the contribution of the resistances and all the integrators contribute equally to the total output kT/C integrated white noise.
- Calculate the component values of the filter.
- Check the designed filter with ngspice simulations in qucs-s.