Fundamentals of Analog & Mixed Signal VLSI Design

Single-ended Differential Amplifiers
Part 2

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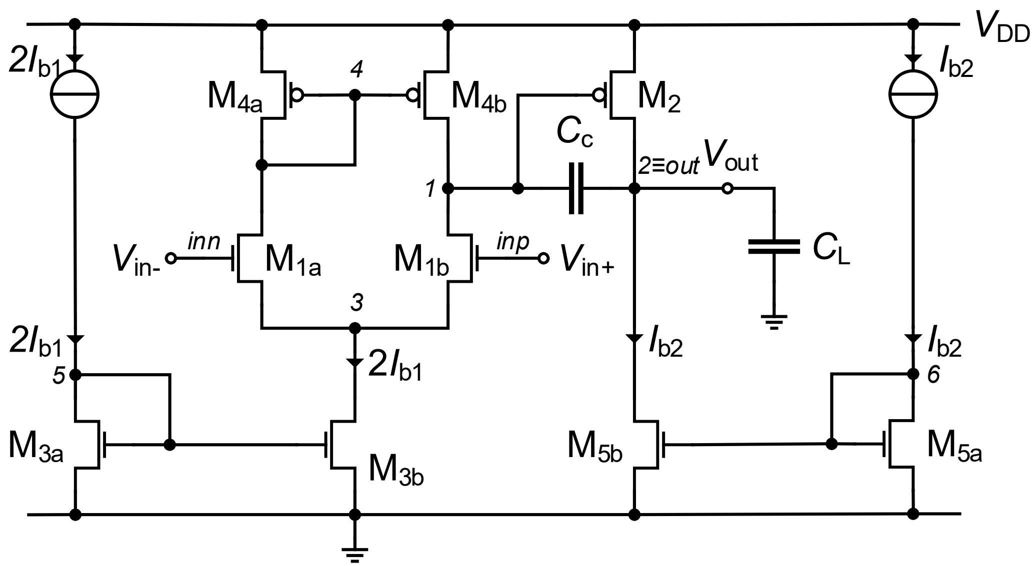
Outline

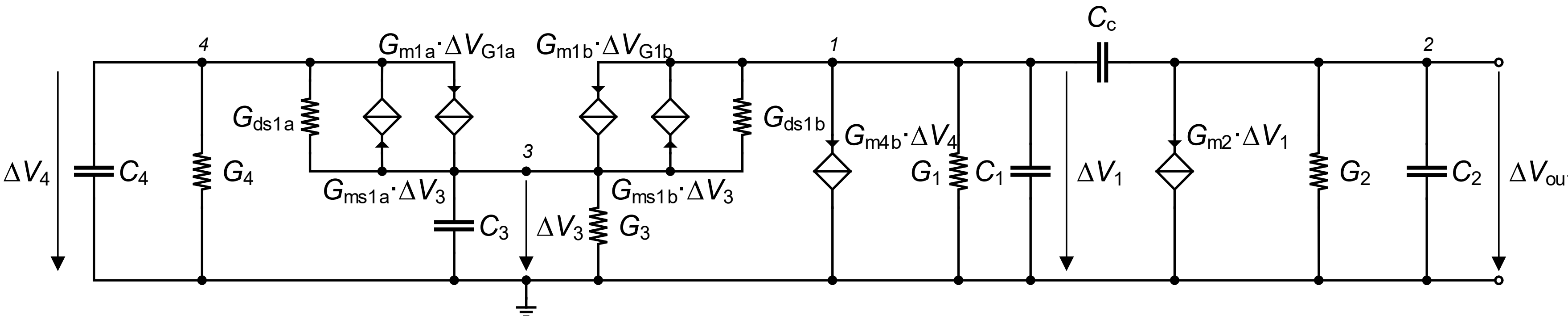
- **The two-stage OTA or Miller OTA**
- The telescopic OTA
- The folded cascode OTA

The Two-stage OTA or Miller OTA

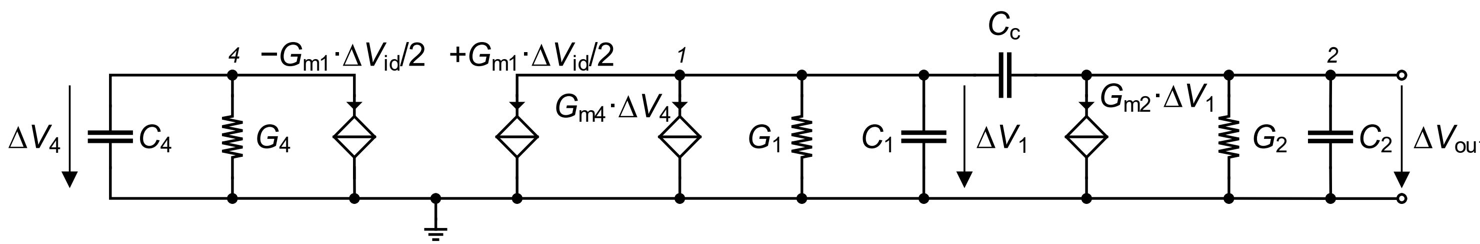


- The gain of the simple OTA can be enhanced by adding a 2nd gain stage implemented by M_2
- Because of the higher gain, the OTA needs to be **frequency compensated** to ensure that it will remain stable in all feedback configurations
- The compensation is achieved by adding capacitor C_c which takes advantage of the Miller effect hence its name of Miller compensation and Miller OTA
- The amplifier small-signal transfer function is analyzed next

The Miller OTA – Small-signal Analysis

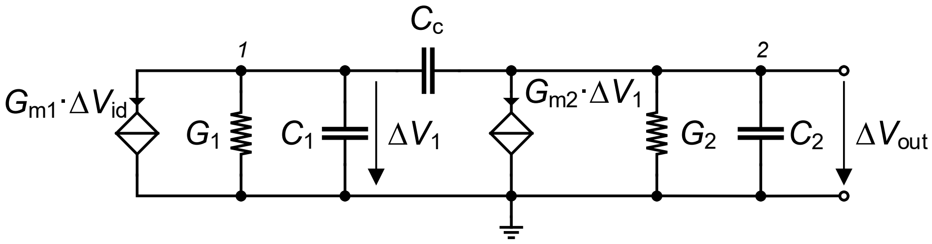


Differential mode



- The OTA small-signal circuit is shown in the top circuit
- Assuming perfect matching and differential operation $\Delta V_{G1b} = -\Delta V_{G1a} = V_{id}/2$, the voltage at node 3 remains unchanged and hence $\Delta V_3 = 0$. The source transconductances can then be omitted leading to the simplified circuit
- If the capacitance C_4 at the current mirror node 4 is neglected and we assume perfect matching, then $\Delta V_4 = -G_{m1}/G_{m4} \cdot (-V_{id}/2)$
- The two transconductances connected to node 1 can then be replaced by a single transconductance resulting in the schematic shown in the next slide

The Miller OTA – Small-signal Analysis



- The small-signal differential gain is then given by

$$A_d(s) \triangleq \frac{\Delta V_{out}}{\Delta V_{id}} = A_{dc} \cdot \frac{1-\frac{s}{z_1}}{\left(1-\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)} = A_{dc} \cdot \frac{1+n_1 s}{1+d_1 s+d_2 s^2} = A_{dc} \cdot \frac{1-\frac{s}{z_1}}{1-s\left(\frac{1}{p_1}+\frac{1}{p_2}\right)+\frac{s^2}{p_1 p_2}}$$

- with

$$A_{dc} = \frac{G_{m1}}{G_1} \cdot \frac{G_{m2}}{G_2}$$

$$n_1 = -\frac{1}{z_1} = -\frac{C_c}{G_{m2}}$$

$$d_1 = -\left(\frac{1}{p_1}+\frac{1}{p_2}\right) = \frac{C_1}{G_1}+\frac{C_2}{G_2}+\frac{C_c}{G_1}\left(1+\frac{G_1}{G_2}+\frac{G_{m2}}{G_2}\right)$$

$$d_2 = \frac{1}{p_1 p_2} = \frac{C_c C_2 + C_c C_1 + C_1 C_2}{G_1 G_2}$$

The Miller OTA – Small-signal Analysis

- Let's first ignore the compensation capacitor ($C_c = 0$)
- The zero disappears and the two poles are simply given by

$$p_1' = -\frac{G_1}{C_1}$$
$$p_2' = -\frac{G_2}{C_2}$$

- We see that the poles are actually associated to the nodes 1 and 2 (output)

The Miller OTA – Dominant-Pole

- The compensation capacitor introduces a **right half-plane (RHP) zero** $z_1 = G_{m2}/C_c$ and has two real poles p_1 and p_2
- Assuming that the poles are widely separated $|p_1| \ll |p_2|$ then

$$d_1 \cong -\frac{1}{p_1} = \frac{C_1}{G_1} + \frac{C_2}{G_2} + \frac{C_c}{G_1}\left(1 + \frac{G_1}{G_2} + \frac{G_{m2}}{G_2}\right)$$

- We can further assume that $G_{m2}/G_2 \gg 1$ and the **dominant pole** p_1 is approximately given by

$$\omega_{p1} \triangleq |p_1| \cong \frac{G_1 G_2}{G_{m2} C_c}$$

- The **gain-bandwidth product** GBW is then approximately given by

$$GBW = \omega_{p1} \cdot A_{dc} \cong \frac{G_{m1}}{C_c}$$

- Note that $\omega_{p2} \triangleq |p_2|$ must be at least equal to GBW for the above approximation to hold

$$GBW < \frac{G_{m2}}{C_2}$$

The Miller OTA – Non-dominant Pole

- The **non-dominant pole** p_2 is then approximately given by

$$\omega_{p2} \triangleq |p_2| = -\frac{1}{p_1 d_2} \cong \frac{G_{m2} C_c}{C_c C_2 + C_c C_1 + C_1 C_2}$$

- We see that the dominant pole magnitude ω_{p1} decreases as C_c increases, whereas ω_{p2} increases as C_c increases
- Thus, increasing C_c causes the poles to **split apart** as illustrated in the next slide
- If $C_2, C_c \gg C_1$ the non-dominant pole is approximately set by the output capacitance

$$\omega_{p2} \cong \frac{G_{m2}}{C_2}$$

The Miller OTA – Setting the Zero and Non-dominant Pole

- The ratio of the non-dominant pole ω_{p2} and the RHP zero ω_z to the unity-gain frequency ω_u are given by

$$\frac{\omega_{p2}}{\omega_u} = \frac{G_{m2}}{G_{m1}} \cdot \frac{C_c}{C_L} \text{ and } \frac{\omega_z}{\omega_u} = \frac{G_{m2}}{G_{m1}}$$

- and hence

$$\frac{\omega_{p2}}{\omega_u} = \frac{\omega_z}{\omega_u} \cdot \frac{C_c}{C_L} \text{ and } \frac{\omega_z}{\omega_{p2}} = \frac{C_c}{C_L}$$

- The unity gain frequency ω_u, non-dominant pole ω_{p2} and zero ω_z need to satisfy the following inequality

$$\omega_u < \omega_{p2} < \omega_z \text{ or } 1 < \frac{\omega_{p2}}{\omega_u} < \frac{\omega_z}{\omega_u}$$

- This translates to the following inequality

$$1 < \frac{C_L}{C_c} < \frac{G_{m2}}{G_{m1}}$$

- This means that the compensation capacitance C_c should stay smaller than the load capacitance C_L and that the ratio of the transconductance of M_2 to that of M_{1a}-M_{1b} should be larger than C_L/C_c

The Miller OTA – Choosing the Compensation Cap

- Usually the compensation capacitance C_c is a fraction of the load capacitance C_L which can be determined from the specified phase margin PM which is given by

$$PM = \arctan\left(\frac{\omega_u}{\omega_z}\right) + \arctan\left(\frac{\omega_u}{\omega_{p2}}\right) - \frac{\pi}{2}$$

- For example if $\omega_{p2}/\omega_u = 4$ and $\omega_z/\omega_{p2} = 2$, then $\omega_z/\omega_u = 8$ and $PM = 68.8°$ which is usually more than sufficient. However we need to account for parasitic capacitances which add to the load capacitance and reduce the non-dominant pole. Therefore a good trade-off to start the design and achieve a sufficient PM (typically larger than $45°$) is to choose $\omega_{p2} = 4\omega_u$ and $\omega_z = 2\omega_{p2} = 8\omega_u$
- This results in choosing $C_c = C_L/2$

The Miller OTA – A Note on Power Consumption

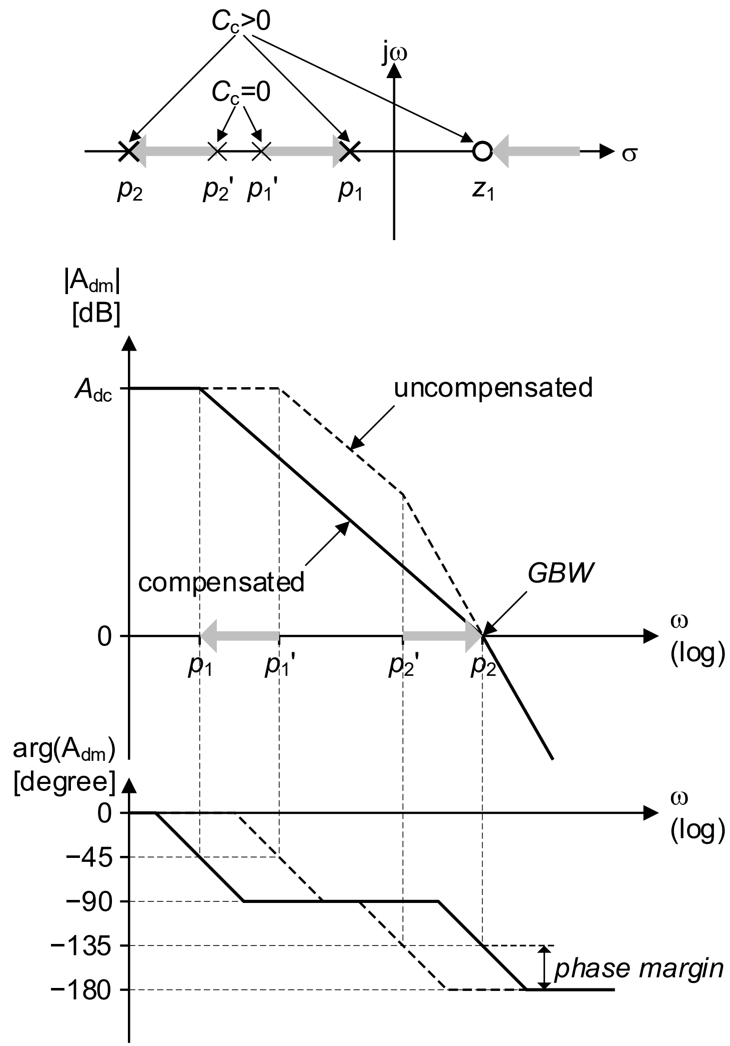
- It is important to note that choosing $\omega_z/\omega_u = G_{m2}/G_{m1} = 8$ for securing enough phase margin has a direct impact on the **power consumption**
- Indeed, if we assume that both M_{1a}-M_{1b} and M_2 are all biased in weak inversion for maximizing the current efficiency, then $G_{m1} = I_{b1}/(n_n\, U_T)$ and $G_{m2} = I_{b2}/(n_p\, U_T)$
- Assuming that $n_n = n_p$, $G_{m2}/G_{m1} = I_{b2}/I_{b1} = 8$
- This means that the bias current of M_2 is 8 times larger than that of M_{1a}-M_{1b}!
- The total current consumption, without accounting for the current flowing in M_{3a} and M_{5a}, is then $I_{tot} = 2I_{b1} + I_{b2} = 10\, I_{b1}$
- We can express the minimum total current consumption in terms of the gain-bandwidth product GBW as $I_{tot} \cong 10\, n\, U_T \cdot C_c \cdot GBW = 5\, n\, U_T \cdot C_L \cdot GBW$
- This can be compared to the total current consumption of the symmetrical cascode OTA $I_{tot} = 4I_b = 4\, n\, U_T \cdot C_L \cdot GBW$
- We deduce that for the same gain-bandwidth product GBW and load capacitance C_L, the Miller OTA consumes about 25% more current than the symmetrical cascode OTA

The Miller OTA – Pole Splitting

Uncompensated

$$p_1' = -\frac{G_2}{C_2}$$

$$p_2' = -\frac{G_1}{C_1}$$



Compensated

$$p_1 \cong -\frac{G_1 G_2}{G_{m2} C_c}$$

$$p_2 \cong -\frac{G_{m2} C_c}{C_c C_2 + C_c C_1 + C_1 C_2}$$

$$\cong -\frac{G_{m2}}{C_2} \text{ for } C_2, C_c \gg C_1$$

P. E. Allen and D. R. Holberg, *CMOS Analog Circuit Design*, 3rd-ed., Oxford University Press, 2012.

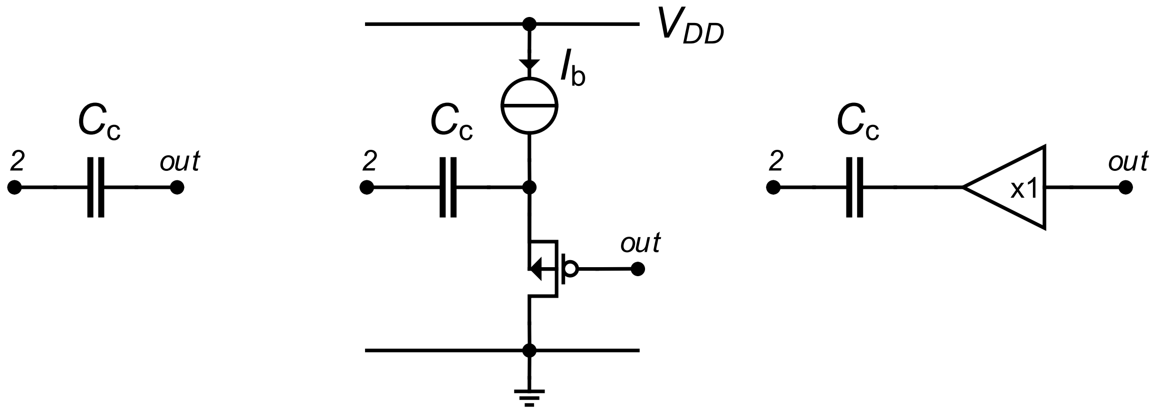
The Miller OTA – Miller Approximation

- The dominant-pole is often called a **Miller pole** because it takes advantage of the Miller effect
- The dominant-pole can actually be found by using the **Miller approximation**
- Using the result obtained earlier without the compensation capacitor and replacing C_1 by the Miller capacitance $C_M \cong G_m C_c / G_2$ results in

$$\omega_{p1} \cong \frac{G_1 G_2}{G_{m2} C_c}$$

- which is identical to the earlier result
- However, the Miller approximation does account for the RHP zero
- The later introduces very undesirable effects with regards to stability: it increases the phase shift and at the same time increases the magnitude
- The effects of the RHP zero can be mitigated by different means

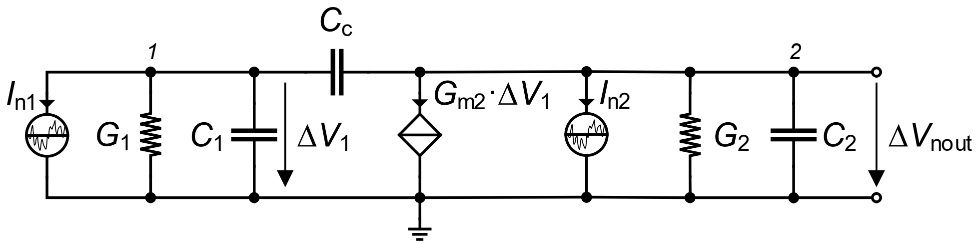
Mitigating the Effect of the RHP Zero



- The effect of the RHP can be eliminated by introducing a **voltage follower** in series with compensation capacitor which blocks the feedforward path through C_c
- The RHP zero has now disappeared while the dc gain, dominant-pole and non-dominant remain the same

$$A_{dc} = \frac{G_{m1}G_{m2}}{G_1 G_2}, \omega_{p1} \cong \frac{G_1 G_2}{G_{m2} C_c}, \omega_{p2} \cong \frac{G_{m2} C_c}{C_2 (C_c + C_1)} \cong \frac{G_{m2}}{C_2} \text{ for } C_c \gg C_1$$

The Miller OTA – Noise Analysis



- We can reuse the noise analysis performed for the simple OTA
- If we neglect the capacitances at the 1st-stage current mirror node and assume a perfect matching, the noise coming from the first stage can be modelled by the noisy current source I_{n1}, whereas I_{n2} models the noise coming from transistors M_2 and M_{5b}
- The input-referred equivalent noise is then given by

$$V_{neq} = \frac{I_{n1}}{G_{m1}} - \frac{G_1}{G_{m1} G_{m2}} \cdot \frac{1 + s(C_1 + C_c)/G_1}{1 - sC_c/G_{m2}} \cdot I_{n2} \cong \frac{I_{n1}}{G_{m1}} - \frac{G_1}{G_{m1} G_{m2}} \cdot I_{n2}$$

- For $\omega \ll G_1/(C_1 + C_c) < G_{m2}/C_c$
- The input-referred PSD is then given by

$$S_{V_{neq}} \cong \frac{S_{I_{n1}}}{G_{m1}^2} + \left(\frac{G_1}{G_{m1} G_{m2}}\right)^2 \cdot S_{I_{n2}}$$

The Miller OTA – Thermal Noise

- For thermal noise we have

$$S_{I_{n1}} = 4kT \cdot 2 \cdot (\gamma_{n1} \cdot G_{m1} + \gamma_{n4} \cdot G_{m4})$$
$$S_{I_{n2}} = 4kT \cdot (\gamma_{n2} \cdot G_{m2} + 2\gamma_{n5} \cdot G_{m5})$$

- The input-referred thermal noise resistance is then given by

$$R_{nth} = \frac{2\gamma_{n1}}{G_{m1}} \cdot (1 + \eta_{th}) = \frac{\gamma_{neq}}{G_{m1}}$$

- where $$\eta_{th} = \frac{\gamma_{n4}}{\gamma_{n1}} \cdot \frac{G_{m4}}{G_{m1}} + \frac{G_1^2}{2G_{m1}G_{m2}} \cdot \left(\frac{\gamma_{n2}}{\gamma_{n1}} + \frac{2\gamma_{n5}}{\gamma_{n1}} \cdot \frac{G_{m5}}{G_{m2}}\right)$$
- is the contribution to the input-referred thermal noise of the current mirror M_{4a}-M_{4b}, the 2nd-stage M_2 and the current mirror M_{5a}-M_{5b} relative to that of the differential pair and

$$\gamma_{neq} = 2\gamma_{n1} \cdot (1 + \eta_{th})$$

- is the OTA equivalent thermal noise excess factor
- The contribution of the current mirror M_{4a}-M_{4b} can be minimized by choosing $G_{m1} \gg G_{m4}$
- The contribution of M_2 and M_{5a}-M_{5b} are small thanks to the factor

$$\frac{G_1^2}{2G_{m1}G_{m2}} = \frac{(G_{ds1} + G_{ds4})^2}{2G_{m1}G_{m2}} \gg 1$$

- which is in the order of the dc gain
- The contribution of the current source M_{5a}-M_{5b} can be made negligible by choosing $G_{m2} \gg G_{m5}$

The Miller OTA – Flicker Noise

- For flicker noise we have

$$S_{I_{n1}} = \frac{4kT}{f} \cdot 2 \cdot \left(G_{m1}^2 \frac{\rho_n}{W_1 L_1} + G_{m4}^2 \frac{\rho_p}{W_4 L_4}\right)$$

$$S_{I_{n2}} = \frac{4kT}{f} \cdot \left(G_{m2}^2 \frac{\rho_p}{W_2 L_2} + 2G_{m5}^2 \frac{\rho_n}{W_5 L_5}\right)$$

- The input-referred flicker noise resistance is then given by

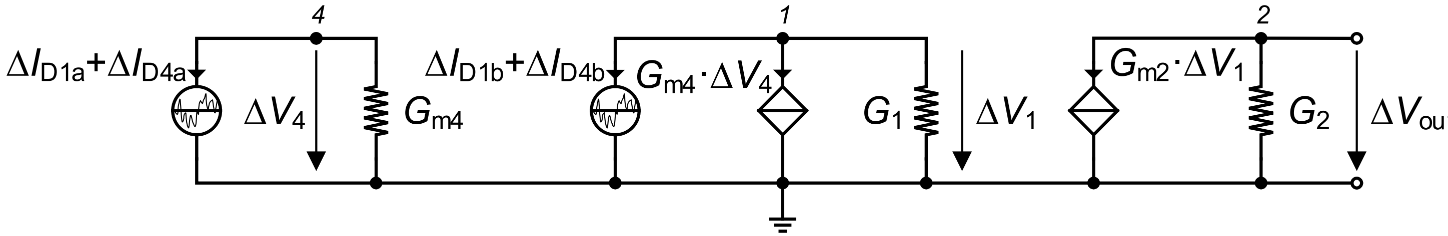
$$R_{nfl} = \frac{2\rho_n}{W_1 L_1 f} \cdot (1 + \eta_{fl})$$

- where $\eta_{fl} = \left(\frac{G_{m4}}{G_{m1}}\right)^2 \frac{\rho_p}{\rho_n} \frac{W_1 L_1}{W_4 L_4} + \frac{1}{2} \left(\frac{G_1}{G_{m1}}\right)^2 \left(\frac{\rho_p}{\rho_n} \frac{W_1 L_1}{W_2 L_2} + 2 \left(\frac{G_{m5}}{G_{m2}}\right)^2 \frac{W_1 L_1}{W_5 L_5}\right)$
- is the contribution to the input-referred flicker noise of the current mirror M_{4a}-M_{4b}, the 2nd-stage M_2 and the current mirror M_{5a}-M_{5b} relative to that of the differential pair
- We see that the contribution of the current mirror M_{4a}-M_{4b} can be minimized by choosing $G_{m1} \gg G_{m4}$ (same as for the simple OTA)
- The contributions of M_2 and M_{5a}-M_{5b} are small thanks to the first stage gain

$$\left(\frac{G_1}{G_{m1}}\right)^2 = \left(\frac{G_{ds1} + G_{ds4}}{G_{m1}}\right)^2 \gg 1$$

- The contribution of the current source M_{5a}-M_{5b} can be made negligible by choosing $G_{m2} \gg G_{m5}$

The Miller OTA – Input-referred Offset Voltage



- The estimation of the offset voltage can be handled similarly to the noise
- It is essentially due to the first stage and is therefore similar to what was done for the simple OTA
- The input-referred offset voltage standard deviation is then given by

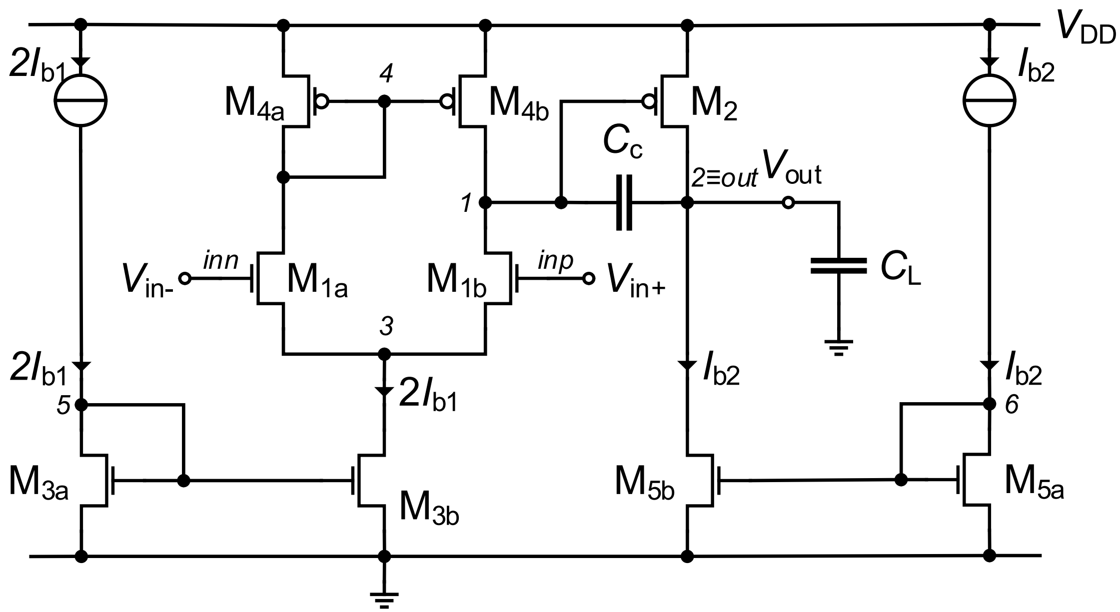
$$\sigma_{V_{os}} = \sqrt{\left(\frac{I_{b1}}{G_{m1}}\right)^2 \left(\sigma_{\beta_1}^2 + \sigma_{\beta_4}^2\right) + \left(\frac{G_{m4}}{G_{m1}}\right)^2 \sigma_{V_{T4}}^2 + \sigma_{V_{T1}}^2}$$

- with

$$\sigma_{\beta_1} = \frac{A_{\beta n}}{\sqrt{W_1 L_1}} \text{ and } \sigma_{V_{T1}} = \frac{A_{V_{Tn}}}{\sqrt{W_1 L_1}}$$

$$\sigma_{\beta_4} = \frac{A_{\beta p}}{\sqrt{W_4 L_4}} \text{ and } \sigma_{V_{T4}} = \frac{A_{V_{Tp}}}{\sqrt{W_4 L_4}}$$

The Miller OTA – Design Example



Specification	Symbol	Value	Unit
Minimum DC gain	A_{dc}	100	dB
Minimum gain-bandwidth product	GBW	1	MHz
Load capacitance	C_L	1	pF
Maximum input-referred random offset voltage	V_{os}	10	mV
Phase margin	PM	60	$\circ$

The Miller OTA – Technology Parameters

Parameter	Value	Unit
T	300	K
U_T	25.875	mV

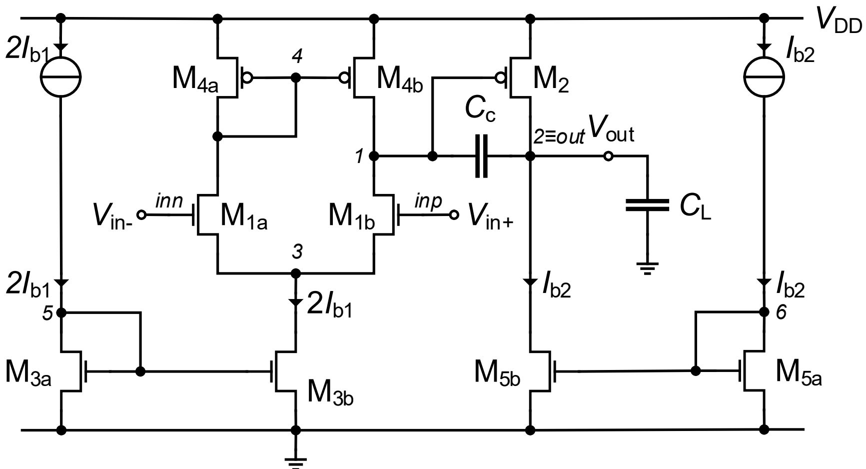
Parameter	Value	Unit
V_{DD}	1.8	V
C_{ox}	8.443	$\frac{fF}{\mu m^2}$
W_{min}	200	nm
L_{min}	180	nm

Parameter	NMOS	PMOS	Unit
sEKV parameters			
n	1.27	1.31	-
$I_{spec\square}$	715	173	nA
V_{T0}	0.455	0.445	V
L_{sat}	26	36	nm
λ	20	20	$\frac{V}{\mu m}$
Overlap capacitances parameters			
C_{GDo}	0.366	0.329	$\frac{fF}{\mu m}$
C_{GSo}	0.366	0.329	$\frac{fF}{\mu m}$
C_{GBo}	0	0	$\frac{fF}{\mu m}$
Junction capacitances parameters			
C_J	1	1.121	$\frac{fF}{\mu m^2}$
C_{JSW}	0.2	0.248	$\frac{fF}{\mu m}$
Flicker noise parameters			
K_F	8.1e-24	6.8e-23	J
AF	1	1	-
ρ	0.05794	0.4828	$\frac{V \cdot m^2}{A \cdot s}$
Matching parameters			
A_{VT}	5	5	$mV \cdot \mu m$
A_β	1	1	$\% \cdot \mu m$
Source and drain sheet resistance parameter			
R_{sh}	600	2386	$\frac{\Omega}{\mu m}$
Width and length parameters			
ΔW	39	54	nm
ΔL	-76	-72	nm

The Miller OTA – Design Equations

- Slew-rate: $SR = \frac{2I_{b1}}{C_c}$
- First-stage dc gain: $A_{v1} = -\frac{G_{m1}}{G_1}$ with $G_1 = G_{ds1} + G_{ds3}$
- Second-stage dc gain: $A_{v2} = -\frac{G_{m2}}{G_2}$ with $G_2 = G_{ds2} + G_{ds4}$
- Gain-bandwidth: $GBW = \frac{G_{m1}}{C_c}$
- Dominant pole: $p_1 \cong -\frac{G_1 G_2}{G_{m2} C_c}$
- Non-dominant pole: $p_2 \cong -\frac{G_{m2} C_c}{C_c C_o + C_c C_1 + C_1 C_2} \cong -\frac{G_{m2}}{C_o}$ for $C_2, C_c \gg C_1$
- RHP zero: $z_1 = \frac{G_{m2}}{C_c}$
- In addition to the above design equations, the specifications may also include the input common-mode range and the output swing, the power dissipation, the white noise and the flicker noise (or corner frequency)

Sizing Summary



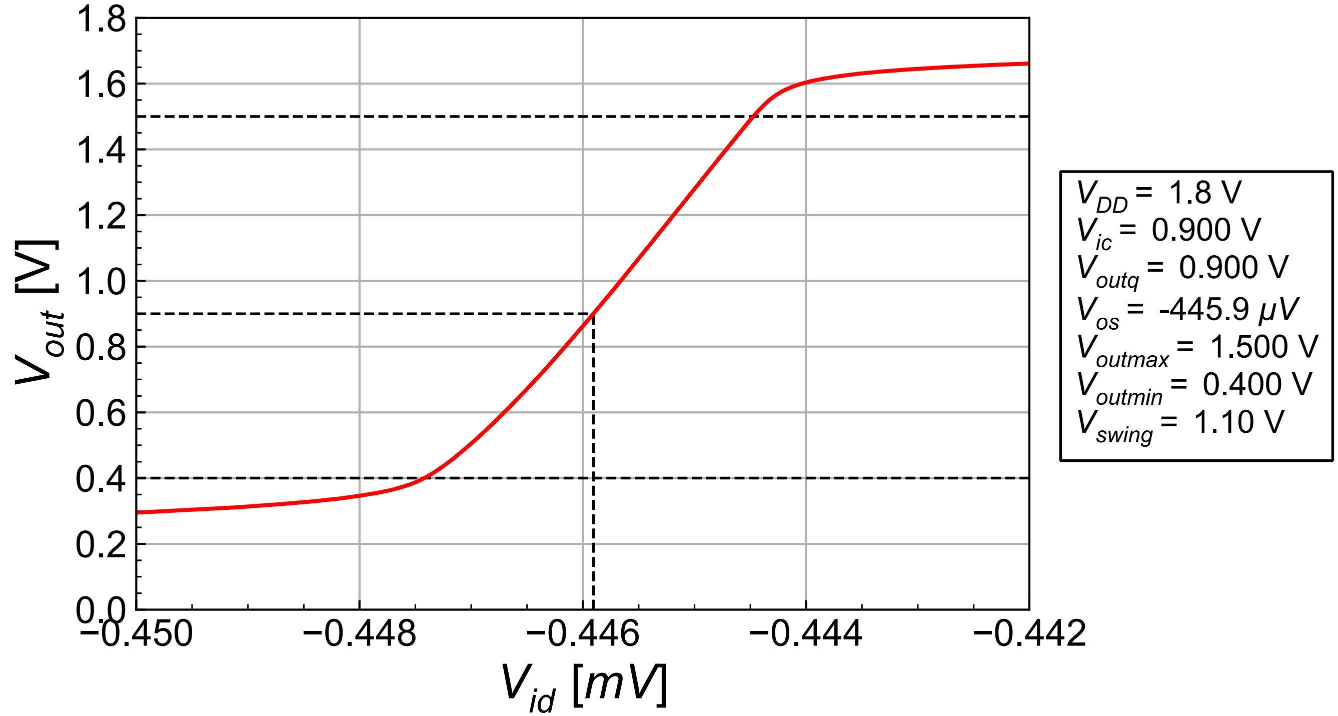
Bias

Name	Value
VDD	1.8
Ib1	130.0E-9
Ib2	2.0E-6
Cc	500.0E-15

Transistor sizes

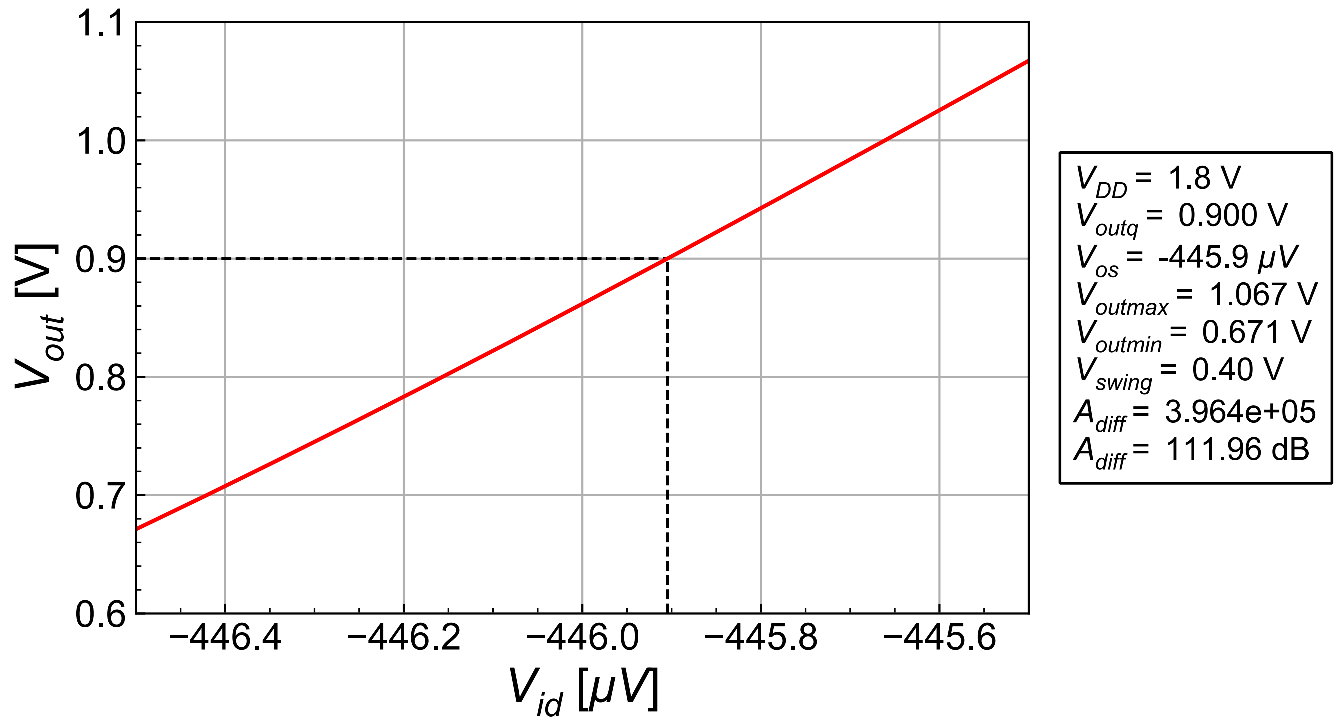
	Type	Function	W	L	ID	W/L	Ispec	IC	VP-VS	n	VG-VT0	VDSsat	Gspec	Gms	Gm	Gds	gamman
M1a	n	DP	2.1E-6	1.1E-6	130.0E-9	1.816	1.3E-6	0.10	-0.057	1.27	-0.045	0.105	50.2E-6	4.6E-6	3.6E-6	5.7E-9	0.65
M1b	n	DP	2.1E-6	1.1E-6	130.0E-9	1.816	1.3E-6	0.10	-0.057	1.27	-0.045	0.105	50.2E-6	4.6E-6	3.6E-6	5.7E-9	0.65
M2	p	CS	12.0E-6	2.2E-6	2.0E-6	5.530	957.4E-9	2.09	0.054	1.31	0.041	0.128	37.0E-6	38.1E-6	29.2E-6	46.1E-9	0.76
M3a	n	CM	200.0E-9	17.1E-6	260.0E-9	0.012	8.4E-9	31.07	0.306	1.27	0.241	0.306	323.4E-9	1.6E-6	1.3E-6	760.7E-12	0.81
M3b	n	CM	200.0E-9	17.1E-6	260.0E-9	0.012	8.4E-9	31.07	0.306	1.27	0.241	0.306	323.4E-9	1.6E-6	1.3E-6	760.7E-12	0.81
M4a	p	CM	200.0E-9	6.9E-6	130.0E-9	0.029	5.0E-9	25.94	0.279	1.31	0.213	0.283	193.7E-9	894.3E-9	684.8E-9	940.7E-12	0.83
M4b	p	CM	200.0E-9	6.9E-6	130.0E-9	0.029	5.0E-9	25.94	0.279	1.31	0.213	0.283	193.7E-9	894.3E-9	684.8E-9	940.7E-12	0.83
M5a	n	CM	210.0E-9	2.2E-6	2.0E-6	0.097	69.2E-9	28.90	0.295	1.27	0.232	0.297	2.7E-6	13.1E-6	10.3E-6	46.1E-9	0.81
M5b	n	CM	210.0E-9	2.2E-6	2.0E-6	0.097	69.2E-9	28.90	0.295	1.27	0.232	0.297	2.7E-6	13.1E-6	10.3E-6	46.1E-9	0.81

Large-signal Transfer Characteristic – Simulations

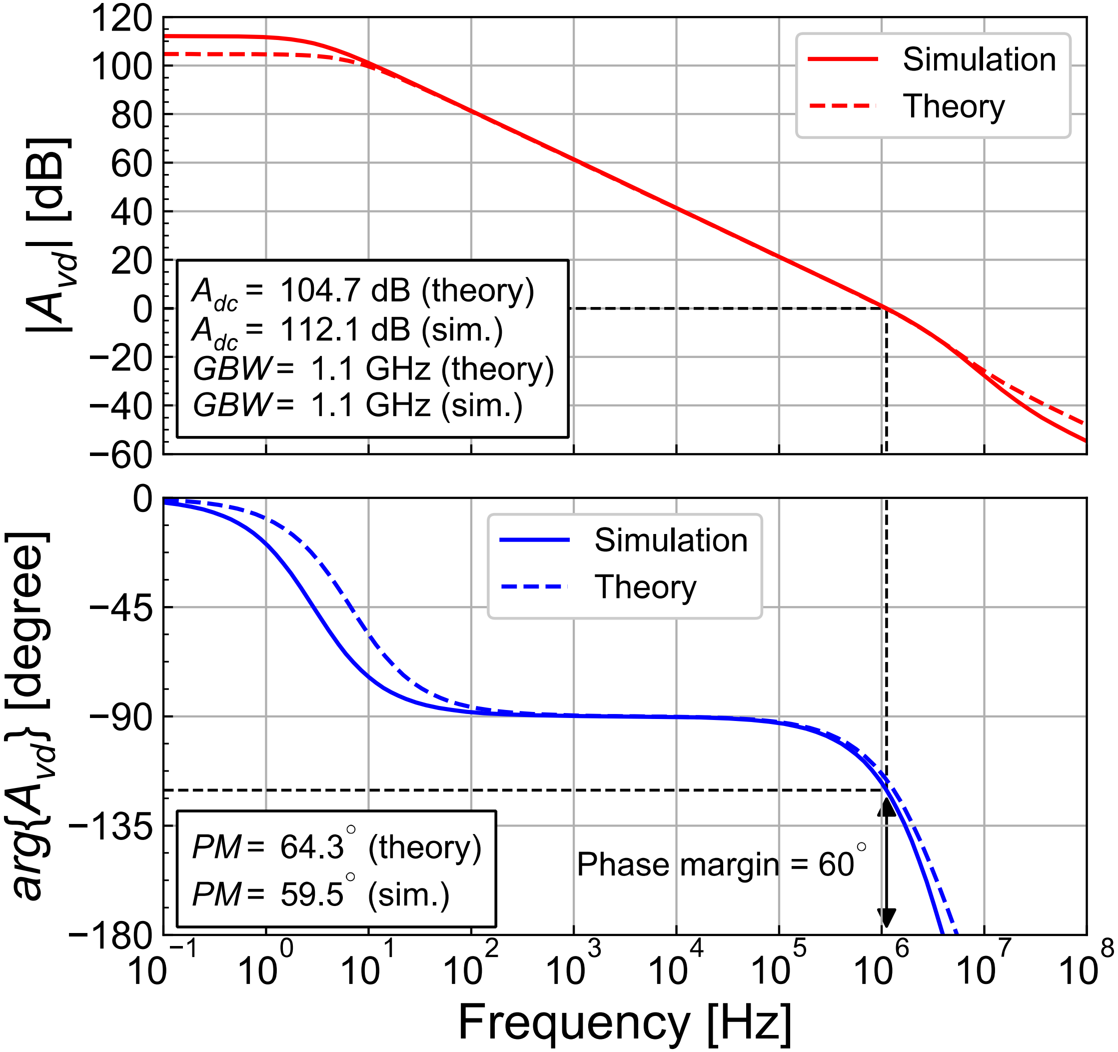


- There is an important systematic offset voltage due to the output conductances and the fact that M_2 imposes a DC voltage at node 1 that is about $V_{DD} - V_{T0p}$ which is different than the voltage at node 4

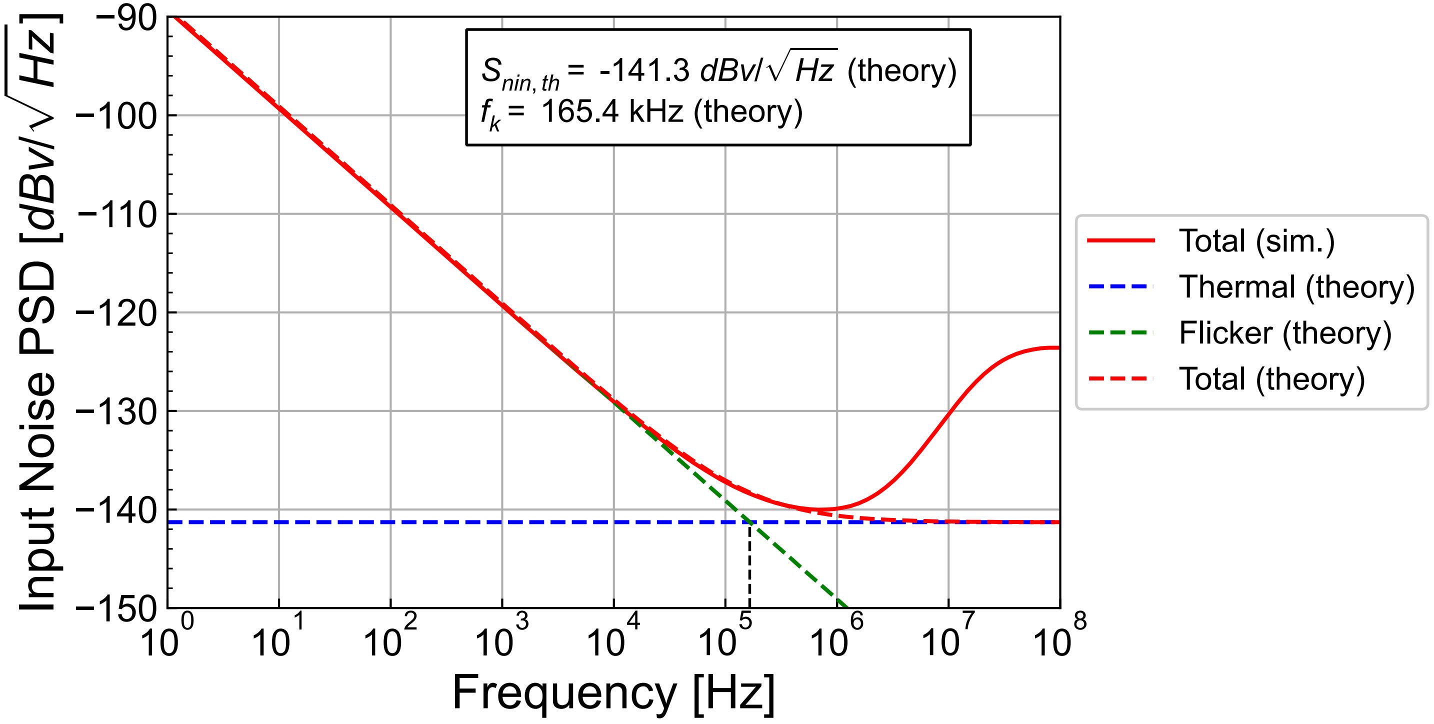
Large-signal Transfer Characteristic – Simulations



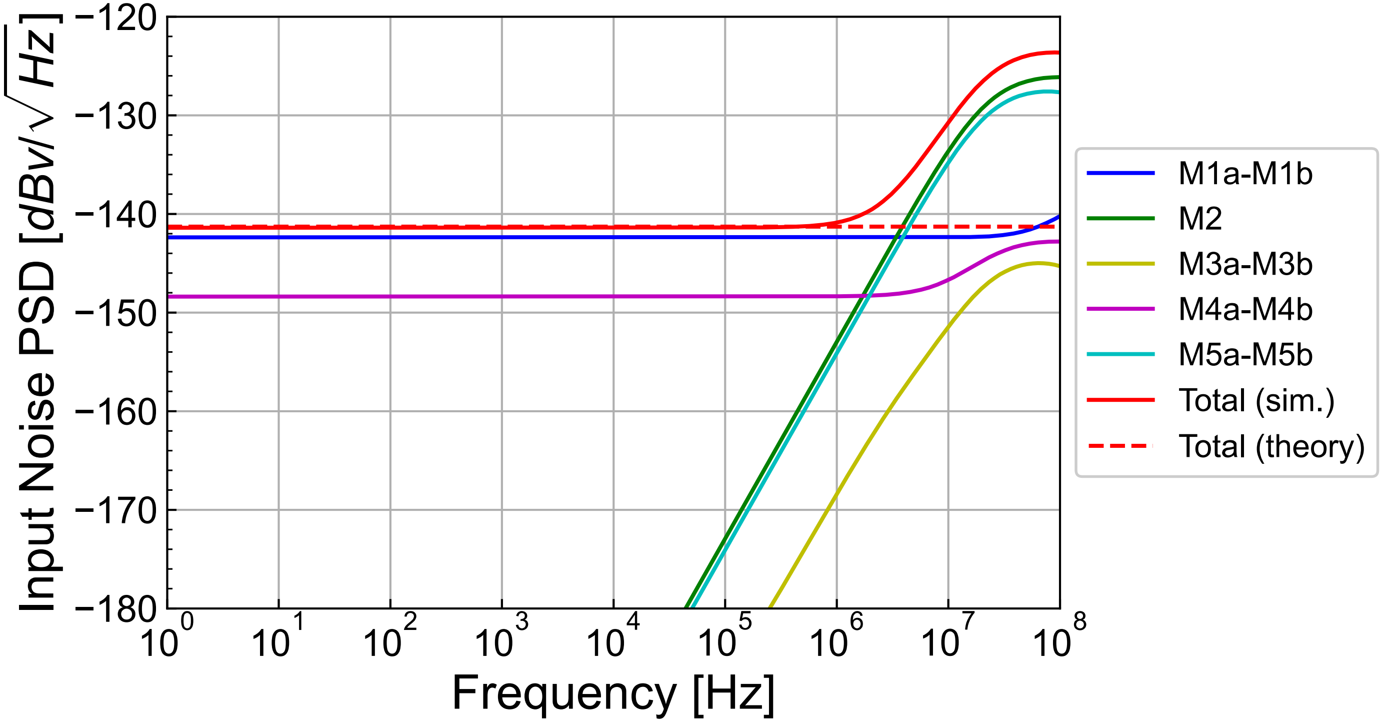
Open-loop Gain Response



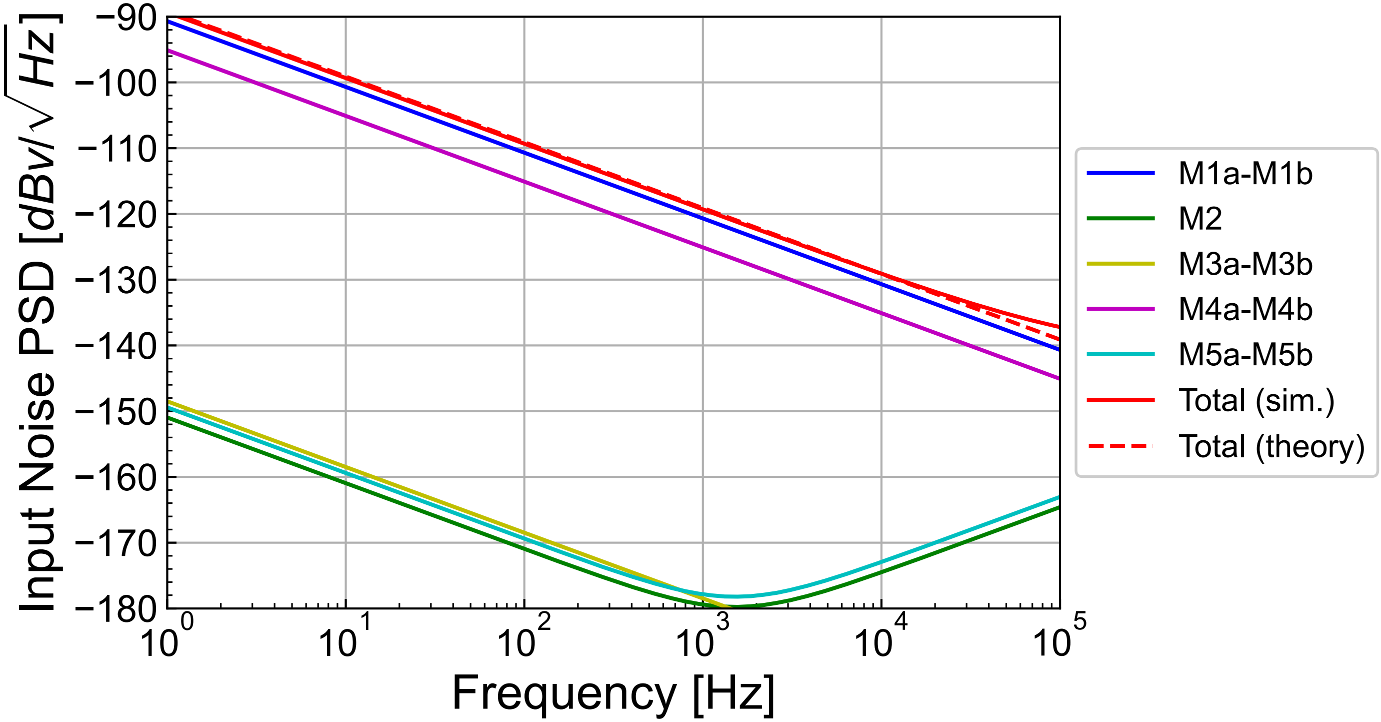
Input-referred Noise PSD



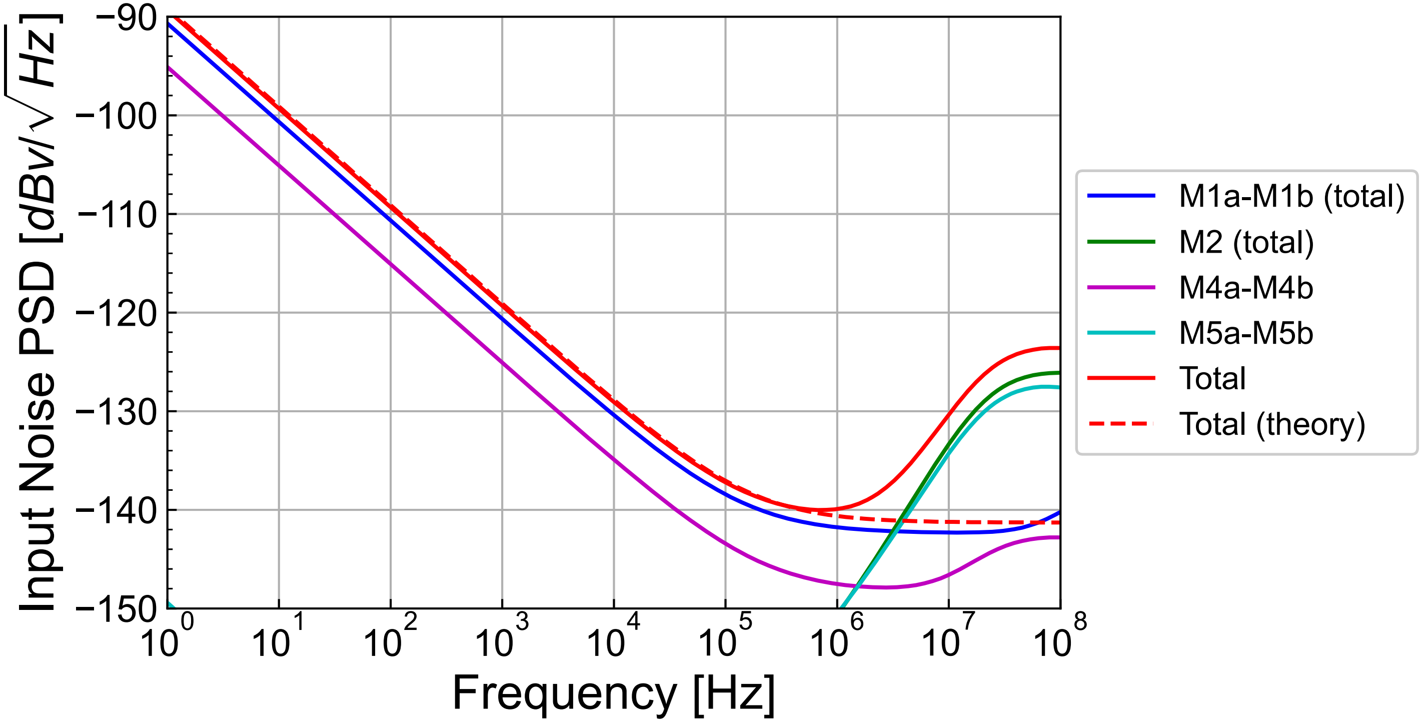
Input-referred Noise PSD – White Noise



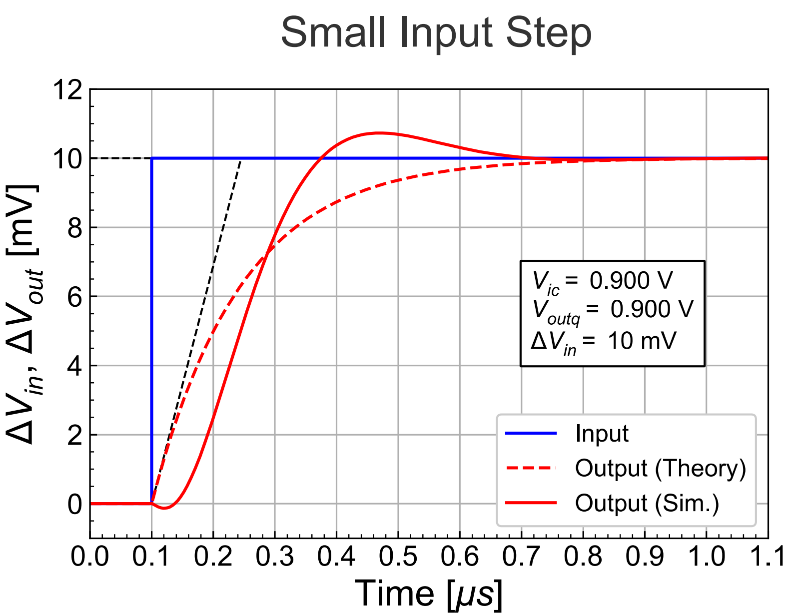
Input-referred Noise PSD – Flicker Noise

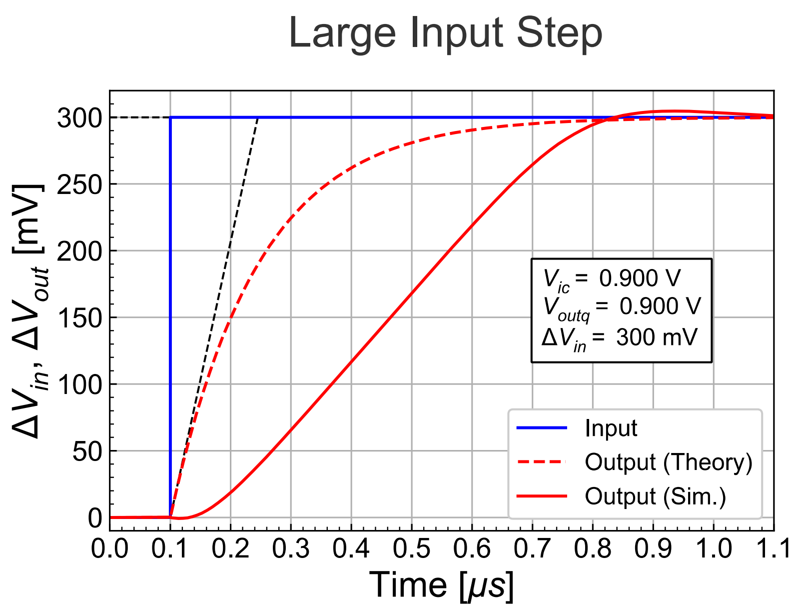


Input-referred Noise PSD – Total Noise

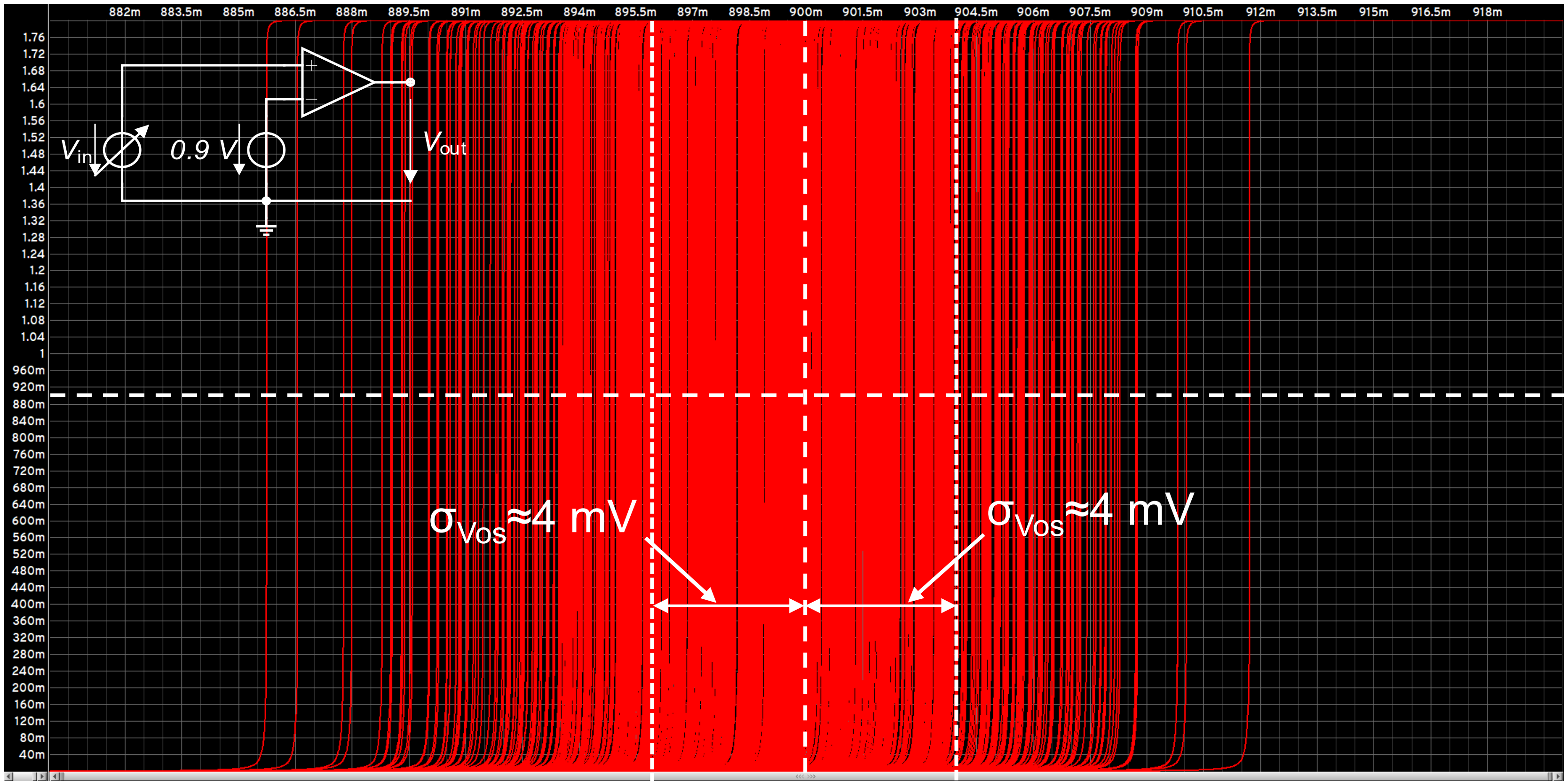


Step-response



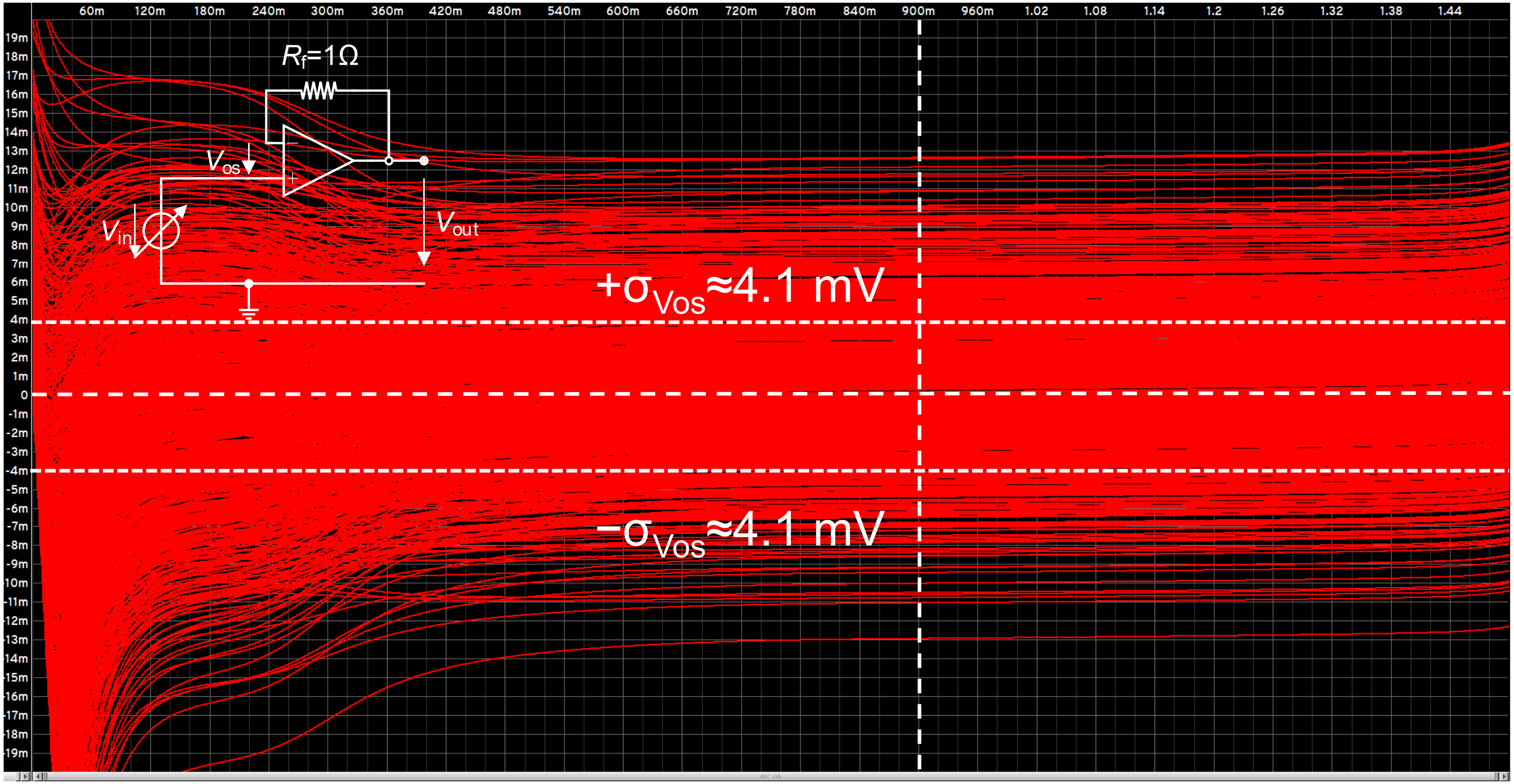


Monte Carlo Simulation of Offset Voltage (open-loop)



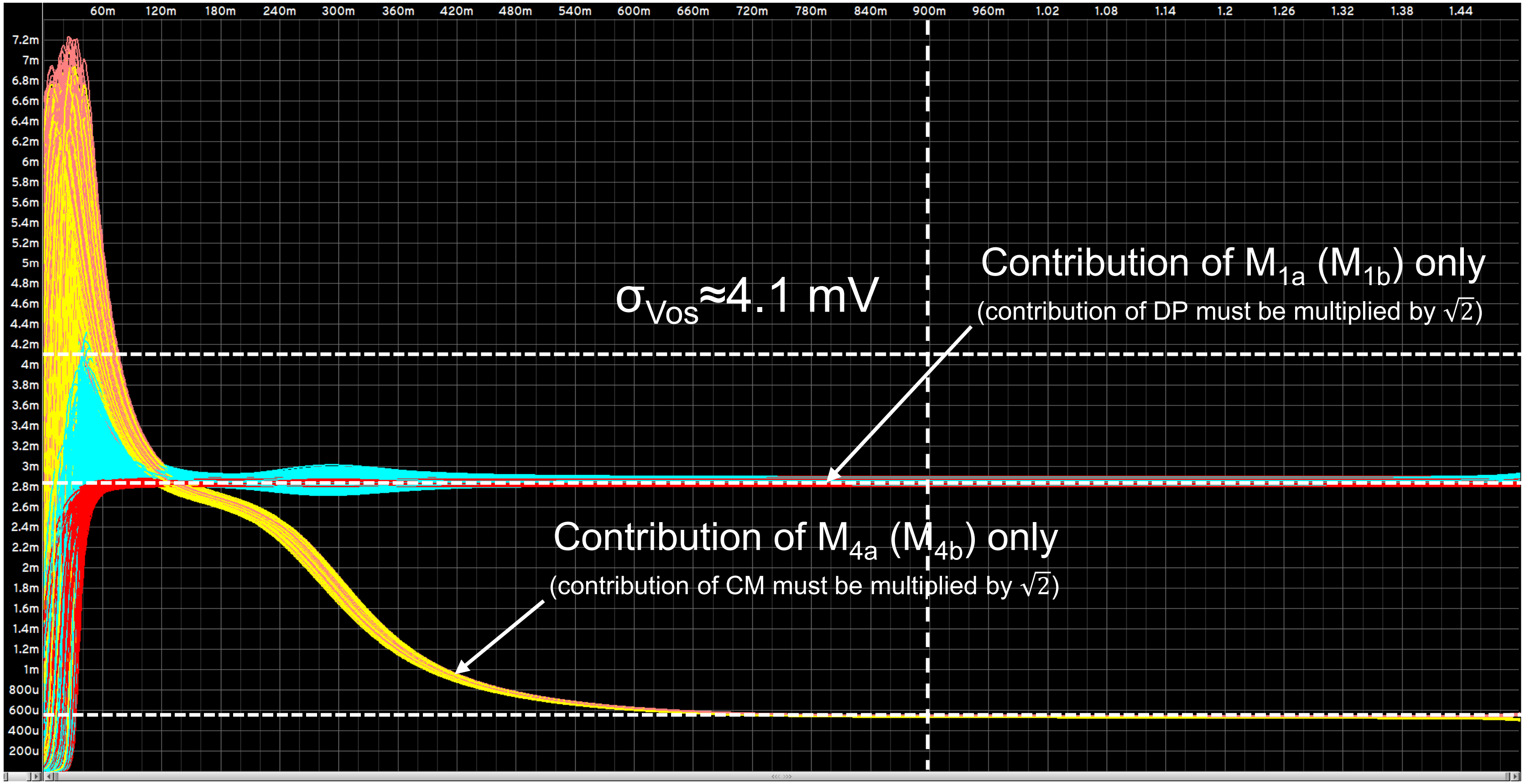
- Monte Carlo simulation of V_{os} versus V_{in} for 1000 runs in open-loop mode
- The standard deviation of V_{os} is about 4.09 mV which is consistent with the dispersion simulation giving 4.1 mV and equal to the 4 mV theoretical prediction

Monte Carlo Simulation of Offset Voltage (closed-loop)



- Monte Carlo simulation of V_{os} versus V_{in} for 1000 runs in voltage follower mode
- The standard deviation of V_{os} is about 4.07 mV which is consistent with the dispersion simulation giving 4.1 mV and close to the 4 mV theoretical prediction

Monte Carlo Simulation of Offset Voltage

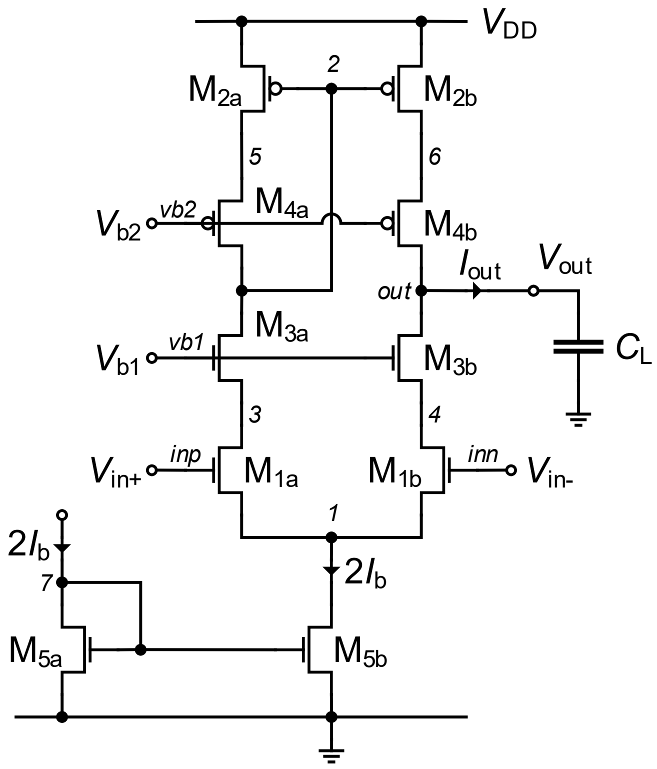


- Offset simulation using Monte Carlo simulations with 1000 runs
- As expected the contribution of the differential pair (M_{1a}-M_{1b}) dominates within the linear range

Outline

- The two-stage OTA or Miller OTA
- **The telescopic OTA**
- The folded cascode OTA

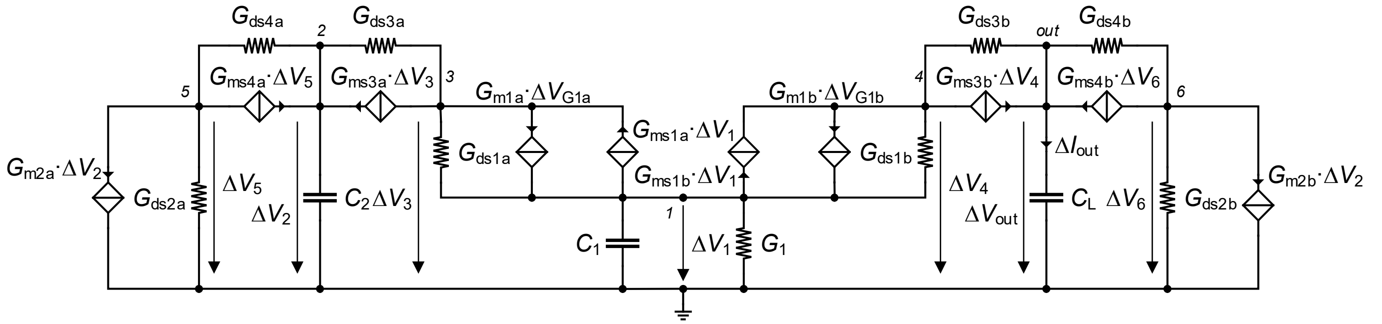
Telescopic Differential OTA



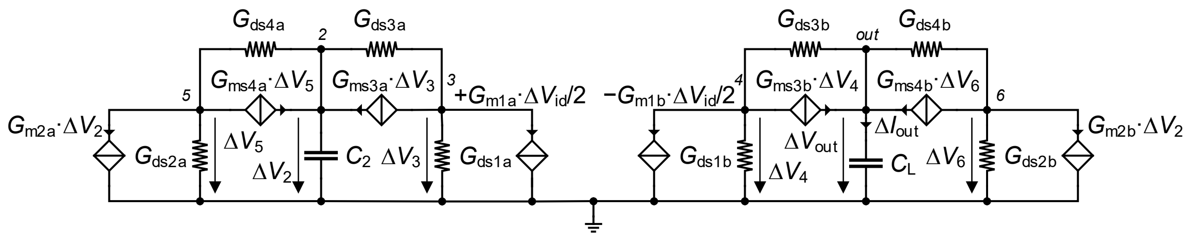
- \+ Current reuse
- \+ Self-compensation
- \+ High DC gain

- – Limited output swing
- – Limited input common-mode range
- – Not appropriate for voltage follower

Telescopic OTA – Small-signal Analysis



Simplified schematic for differential input



- Accounting only for the capacitances at high impedance nodes (those at low impedance cascode nodes 3, 4, 5 and 6 can be neglected in 1st-order analysis)
- Analysis becomes tedious, but can be simplified for differential input voltage using the simplified schematic and assuming perfect matching (i.e. $G_{mia} = G_{mib} = G_{mi}$ and $G_{dsia} = G_{dsib} = G_{dsi}$ for $i = 1{,}2 \ldots 4$)

Telescopic OTA – Differential Open-loop Gain

- Similarly to the simple OTA, the telescopic OTA has its dominant pole ω_0 at the output and non-dominant pole ω_p at the current mirror node 2 and a pole-zero doublet

$$A_{dm}(s) \triangleq \frac{\Delta V_{out}}{\Delta V_{id}} \cong A_{dc} \cdot \frac{1+\frac{s}{\omega_z}}{\left(1+\frac{s}{\omega_0}\right)\left(1+\frac{s}{\omega_p}\right)} \cong \frac{A_{dc}}{1+\frac{s}{\omega_0}} \cong \frac{\omega_u}{s}$$

- where

$$A_{dc} \cong \frac{G_{m1}}{G_o} \text{ with } G_o = \frac{G_{ds2} G_{ds4}}{G_{ms4}} + \frac{G_{ds1} G_{ds3}}{G_{ms3}}$$

$$\omega_0 \cong \frac{G_o}{C_L}$$

$$\omega_p \cong \frac{G_{m2}}{C_2}$$

$$\omega_z = 2\omega_p$$

$$\omega_u = A_{dc} \cdot \omega_0 \cong \frac{G_{m1}}{C_L}$$

Noise Analysis

- At low-frequency the noise of the cascode transistors M_{3a}-M_{3b} and M_{4a}-M_{4b} can be neglected and the noise analysis is then identical to that of the simple OTA
- The PSD of the output noise current is given by

$$S_{nout} \cong 2\left(S_{I_{n1}} + S_{I_{n2}}\right)$$

- or if we express the output PSD in terms of the output noise conductance

$$S_{nout} = 4kT \cdot G_{nout} \text{ where } G_{nout} \cong 2(G_{n1} + G_{n2})$$

- with $$G_{ni} = \gamma_{ni} G_{mi} + G_{mi}^2 \frac{\rho_i}{W_i L_i f} \quad \text{for } i = 1,2$$
- The input-referred noise resistance is then given by

$$R_{nin} \triangleq \frac{G_{nout}}{G_{m1}^2} = \frac{2(G_{n1} + G_{n2})}{G_{m1}^2} = \frac{2G_{n1}}{G_{m1}^2} \cdot \left(1 + \frac{G_{n2}}{G_{n1}}\right)$$

- which can be written as $R_{nin} = \frac{2G_{n1}}{G_{m1}^2} \cdot (1 + \eta)$ with $\eta = \frac{G_{n2}}{G_{n1}}$
- represents the contributions of the current mirrors referred to the input and normalized to the contribution of the differential pair

Input-referred Thermal Noise

- The input-referred thermal noise resistance is given by

$$R_{nth} = \frac{2\gamma_{n1}}{G_{m1}} \cdot (1 + \eta_{th})$$

- where

$$\eta_{th} = \frac{\gamma_{n2}}{\gamma_{n1}} \frac{G_{m2}}{G_{m1}}$$

- represents the contributions to the input-referred thermal noise of the current mirrors relative to that of the differential pair
- In case $G_{m1} \gg G_{m2}$, the thermal noise is dominated by the input differential pair and the previous expression can be simplified

$$R_{nth} \cong \frac{2\gamma_{n1}}{G_{m1}}$$

Thermal Noise Excess Factor

- To compare with other OTA it is useful to derive the thermal noise excess factor

$$\gamma_{ota} \triangleq G_m \cdot R_{nth}$$

- where $G_m = G_{m1}$ is the OTA transconductance
- This results in

$$\gamma_{ota} = 2\gamma_{n1} \cdot (1 + \eta_{th})$$

- In case $G_{m1} \gg G_{m2}$, then $\eta_{th} \ll 1$ and the noise is dominated by the input differential pair and the previous expression can be simplified

$$\gamma_{ota} \cong 2\gamma_{n1}$$

Input-referred Flicker Noise

- The input-referred flicker noise is given by

$$R_{nfl} = \frac{2}{f}\left[\frac{\rho_n}{W_1 L_1} + \left(\frac{G_{m2}}{G_{m1}}\right)^2 \frac{\rho_p}{W_2 L_2}\right]$$

- which can be written as

$$R_{nfl} = \frac{2\rho_n}{W_1 L_1 f} \cdot \left(1 + \eta_{fl}\right)$$

- where

$$\eta_{fl} = \frac{\rho_p}{\rho_n}\left(\frac{G_{m2}}{G_{m1}}\right)^2 \frac{W_1 L_1}{W_2 L_2}$$

- represents the contributions to the input-referred flicker noise of the current mirror relative to that of the differential pair

Output Offset Current

- The offset analysis is identical to that of the simple OTA because the contribution of the mismatch of the cascode transistors can be neglected
- The random offset current is then mainly due to the mismatch between M_{1a}-M_{1b} and M_{2a}-M_{2b}
- The variance of the output offset current is then given by

$$\sigma_{I_{os}}^2 = \sigma_{\Delta I_{D1}}^2 + \sigma_{\Delta I_{D2}}^2 = I_b^2 \cdot \left(\sigma_{\Delta I_{D1}/I_{D1}}^2 + \sigma_{\Delta I_{D2}/I_{D2}}^2\right)$$

- where $$\sigma_{\Delta I_{Di}/I_{Di}}^2 = \sigma_{\beta_i}^2 + \left(\frac{G_{mi}}{I_b}\right)^2 \sigma_{V_{Ti}}^2 \quad \text{for } i = 1,2$$
- with $$\sigma_{\beta_i}^2 = \frac{A_\beta^2}{W_i L_i} \text{ and } \sigma_{V_{Ti}}^2 = \frac{A_{V_T}^2}{W_i L_i} \quad \text{for } i = 1,2$$
- The variance of the output offset current then becomes

$$\sigma_{I_{os}}^2 = I_b^2 \cdot \left(\sigma_{\beta_1}^2 + \sigma_{\beta_2}^2\right) + G_{m1}^2 \cdot \sigma_{V_{T1}}^2 + G_{m2}^2 \cdot \sigma_{V_{T2}}^2$$

Input-referred Offset Voltage

- The variance of the input-referred offset voltage is obtained by dividing the variance of the output offset current by G_{m1}^2 resulting in

$$\sigma_{V_{os}}^2 = \left(\frac{I_b}{G_{m1}}\right)^2 \left(\sigma_{\beta_1}^2 + \sigma_{\beta_2}^2\right) + \sigma_{V_{T1}}^2 + \left(\frac{G_{m2}}{G_{m1}}\right)^2 \sigma_{V_{T2}}^2$$

- which can be written as

$$\sigma_{V_{os}}^2 = \sigma_{V_{T1}}^2 \cdot \left(1 + \xi_{V_T}\right) + \left(\frac{I_b}{G_{m1}}\right)^2 \cdot \sigma_{\beta_1}^2 \cdot \left(1 + \xi_\beta\right)$$

- where ξ_{V_T} represents the V_T mismatch contributions to the input-referred offset of the current mirror relative to that of the differential pair

$$\xi_{V_T} = \left(\frac{G_{m2}}{G_{m1}}\right)^2 \frac{\sigma_{V_{T2}}^2}{\sigma_{V_{T1}}^2}$$

- and ξ_β represents the β mismatch contributions to the input-referred offset of the current mirror relative to that of the differential pair

$$\xi_\beta = \frac{\sigma_{\beta_2}^2}{\sigma_{\beta_1}^2}$$

Input-referred Offset Voltage

- with

$$\sigma^2_{V_{T1}} = \frac{A^2_{V_{Tn}}}{W_1 L_1} \qquad \sigma^2_{V_{T2}} = \frac{A^2_{V_{Tp}}}{W_2 L_2}$$

$$\sigma^2_{\beta_1} = \frac{A^2_{\beta n}}{W_1 L_1} \qquad \sigma^2_{\beta_2} = \frac{A^2_{\beta p}}{W_2 L_2}$$

- Replacing results in

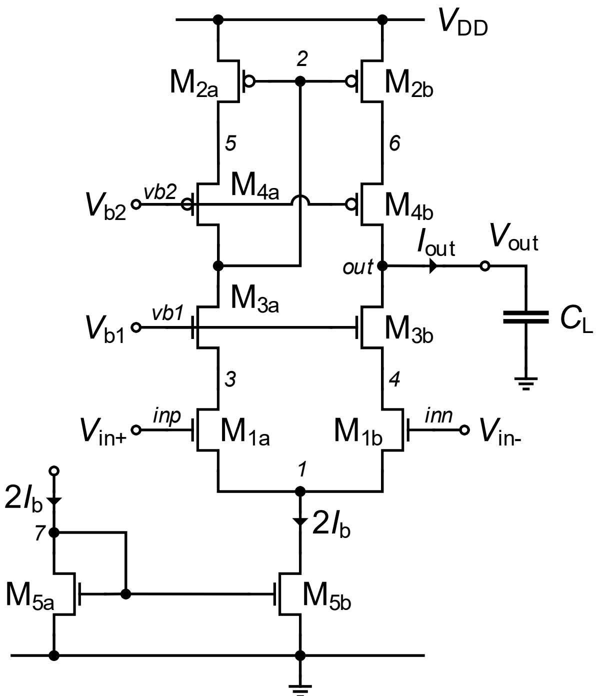
$$\xi_{V_T} = \left(\frac{G_{m2}}{G_{m1}}\right)^2 \cdot \left(\frac{A_{V_{Tp}}}{A_{V_{Tn}}}\right)^2 \cdot \frac{W_1 L_1}{W_2 L_2}$$

- And

$$\xi_{\beta} = \left(\frac{A_{\beta_p}}{A_{\beta_n}}\right)^2 \frac{W_1 L_1}{W_2 L_2}$$

- Similarly to the flicker noise, the input-referred offset (variance or standard deviation) can be reduced by increasing the M_{1a}-M_{1b} area $W_1 L_1$ but at the same time also increasing the area $W_2 L_2$ of the current mirror M_{2a}-M_{2b}

The Telescopic OTA – Design Example



Specification	Symbol	Value	Unit
Minimum DC gain	A_{dc}	100	dB
Minimum gain-bandwidth product	GBW	1	MHz
Load capacitance	C_L	1	pF
Maximum input-referred random offset voltage	V_{os}	10	mV
Phase margin	PM	60	$\circ$
Corner frequency	f_k	100	kHz

The Telescopic OTA – Technology Parameters

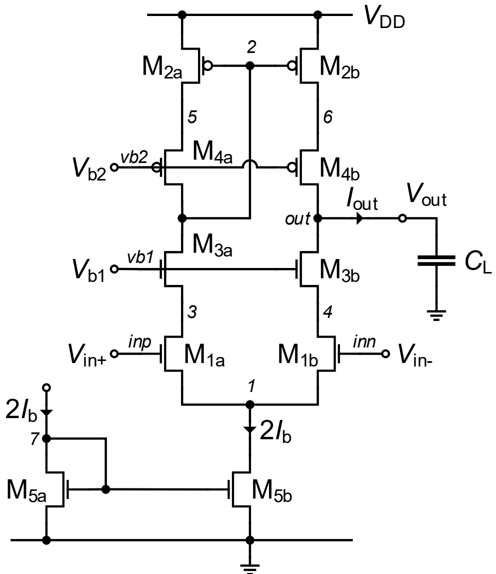
Parameter	Value	Unit
T	300	K
U_T	25.875	mV

Parameter	Value	Unit
V_{DD}	1.8	V
C_{ox}	8.443	$\frac{fF}{\mu m^2}$
W_{min}	200	nm
L_{min}	180	nm

Parameter	NMOS	PMOS	Unit
sEKV parameters			
n	1.27	1.31	-
$I_{spec\square}$	715	173	nA
V_{T0}	0.455	0.445	V
L_{sat}	26	36	nm
λ	20	20	$\frac{V}{\mu m}$
Overlap capacitances parameters			
C_{GDo}	0.366	0.329	$\frac{fF}{\mu m}$
C_{GSo}	0.366	0.329	$\frac{fF}{\mu m}$
C_{GBo}	0	0	$\frac{fF}{\mu m}$
Junction capacitances parameters			
C_J	1	1.121	$\frac{fF}{\mu m^2}$
C_{JSW}	0.2	0.248	$\frac{fF}{\mu m}$
Flicker noise parameters			
K_F	8.1e-24	6.8e-23	J
AF	1	1	-
ρ	0.05794	0.4828	$\frac{V \cdot m^2}{A \cdot s}$
Matching parameters			
A_{VT}	5	5	$mV \cdot \mu m$
A_β	1	1	$\% \cdot \mu m$
Source and drain sheet resistance parameter			
R_{sh}	600	2386	$\frac{\Omega}{\mu m}$
Width and length parameters			
ΔW	39	54	nm
ΔL	-76	-72	nm

quarto jupyter

Sizing Summary



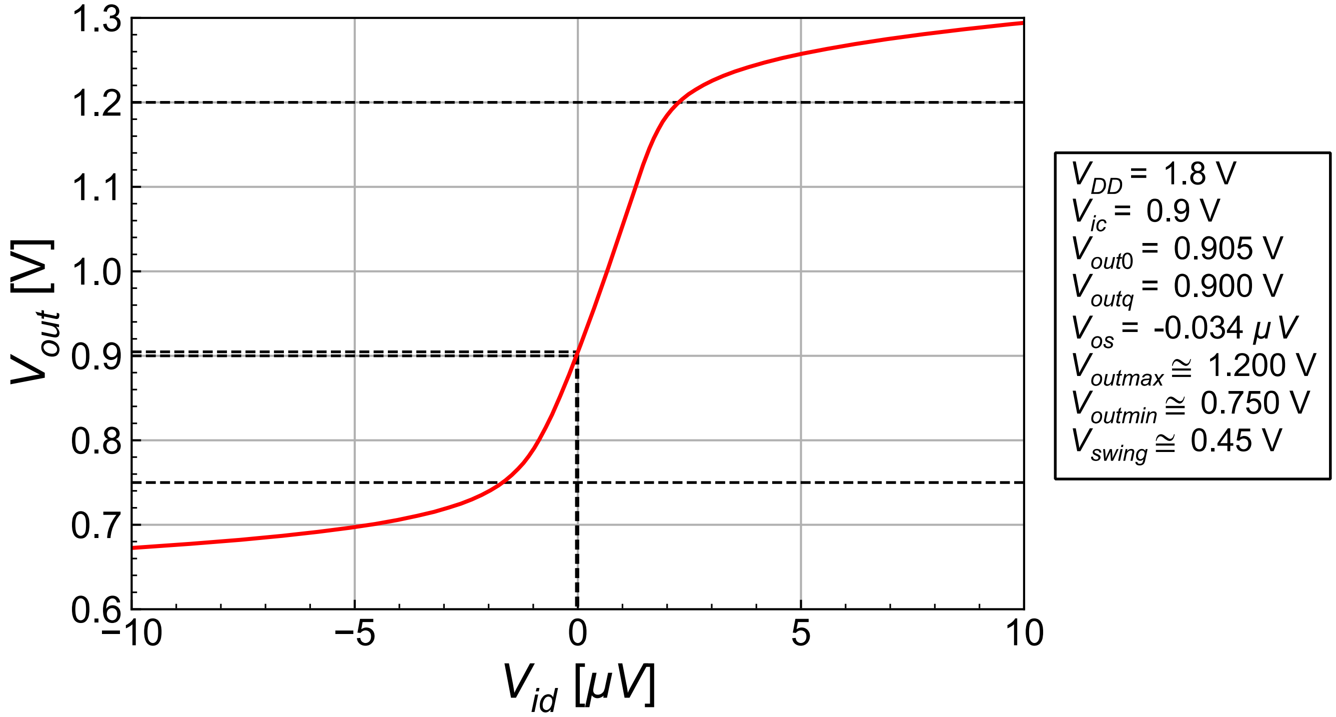
Bias

Name	Value
VDD	1.8
Ib	250.0E-9
Vb1	1.2
Vb2	0.7

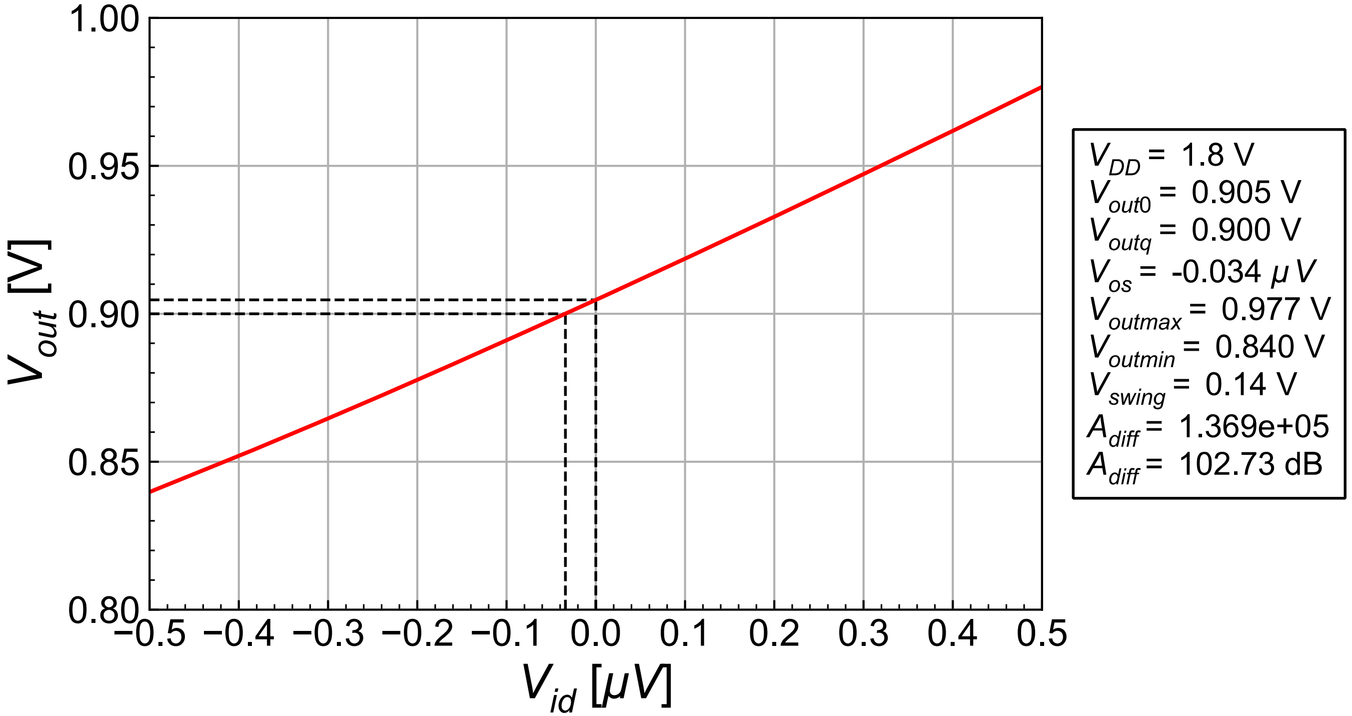
Transistor sizes

	Type	Function	W	L	ID	W/L	Ispec	IC	VP-VS	n	VG-VT0	VDSsat	Gspec	Gms	Gm	Gds	gamman
M1a	n	DP	7.1E-6	2.0E-6	250.0E-9	3.500	2.5E-6	0.10	-0.057	1.27	-0.045	0.105	96.7E-6	8.9E-6	7.0E-6	6.2E-9	0.65
M1b	n	DP	7.1E-6	2.0E-6	250.0E-9	3.500	2.5E-6	0.10	-0.057	1.27	-0.045	0.105	96.7E-6	8.9E-6	7.0E-6	6.2E-9	0.65
M2a	p	CM	220.0E-9	6.0E-6	250.0E-9	0.037	6.3E-9	39.38	0.345	1.31	0.264	0.341	245.3E-9	1.4E-6	1.1E-6	2.1E-9	0.84
M2b	p	CM	220.0E-9	6.0E-6	250.0E-9	0.037	6.3E-9	39.38	0.345	1.31	0.264	0.341	245.3E-9	1.4E-6	1.1E-6	2.1E-9	0.84
M3a	n	CA	1.9E-6	540.0E-9	250.0E-9	3.500	2.5E-6	0.10	-0.057	1.27	-0.045	0.105	96.7E-6	8.9E-6	7.0E-6	23.1E-9	0.65
M3b	n	CA	1.9E-6	540.0E-9	250.0E-9	3.500	2.5E-6	0.10	-0.057	1.27	-0.045	0.105	96.7E-6	8.9E-6	7.0E-6	23.1E-9	0.65
M4a	p	CA	7.8E-6	540.0E-9	250.0E-9	14.444	2.5E-6	0.10	-0.057	1.31	-0.044	0.105	96.6E-6	8.9E-6	6.8E-6	23.1E-9	0.67
M4b	p	CA	7.8E-6	540.0E-9	250.0E-9	14.444	2.5E-6	0.10	-0.057	1.31	-0.044	0.105	96.6E-6	8.9E-6	6.8E-6	23.1E-9	0.67
M5a	n	CM	200.0E-9	8.5E-6	500.0E-9	0.024	16.9E-9	29.62	0.298	1.27	0.235	0.300	652.5E-9	3.2E-6	2.5E-6	3.0E-9	0.81
M5b	n	CM	200.0E-9	8.5E-6	500.0E-9	0.024	16.9E-9	29.62	0.298	1.27	0.235	0.300	652.5E-9	3.2E-6	2.5E-6	3.0E-9	0.81

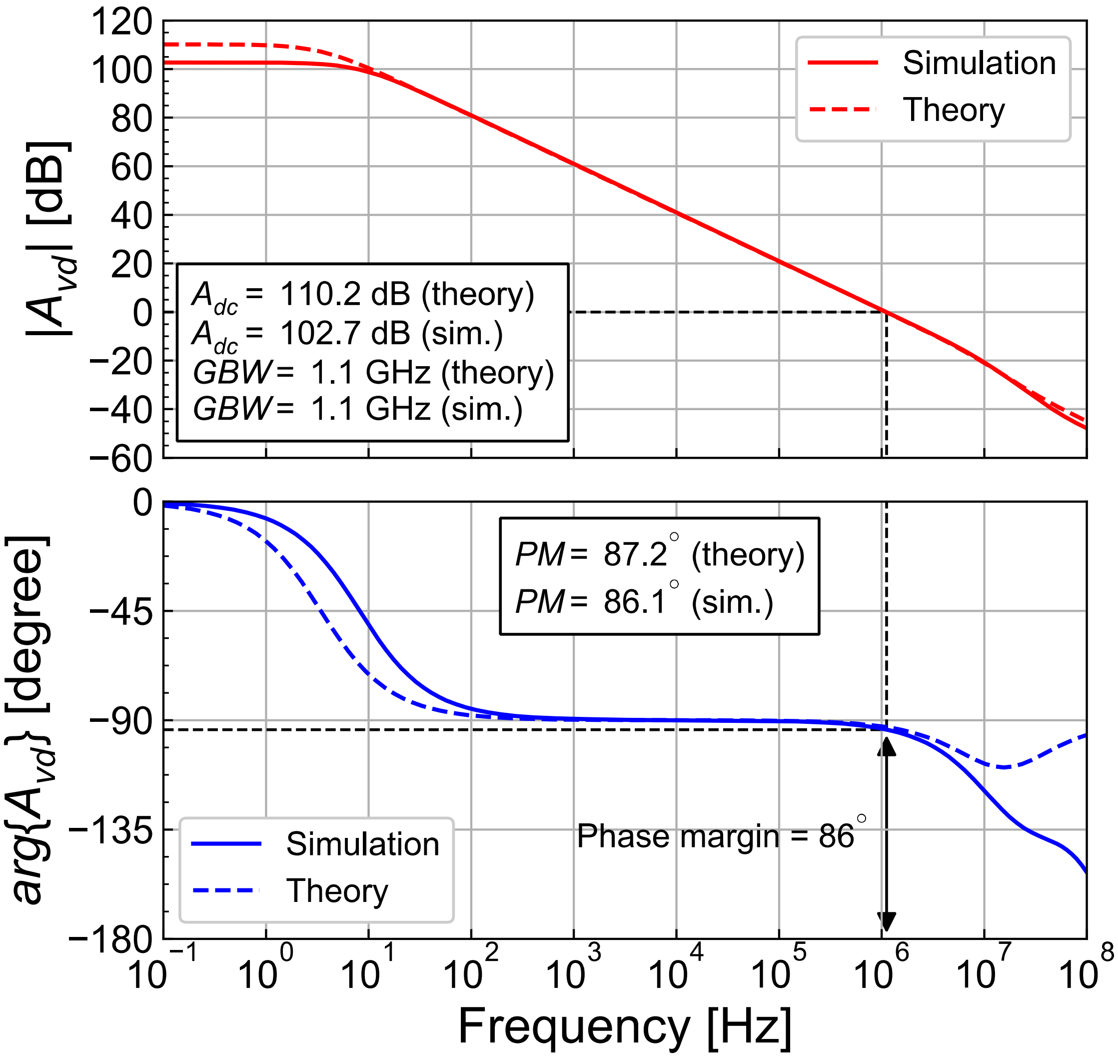
Large-signal Transfer Characteristic – Simulations



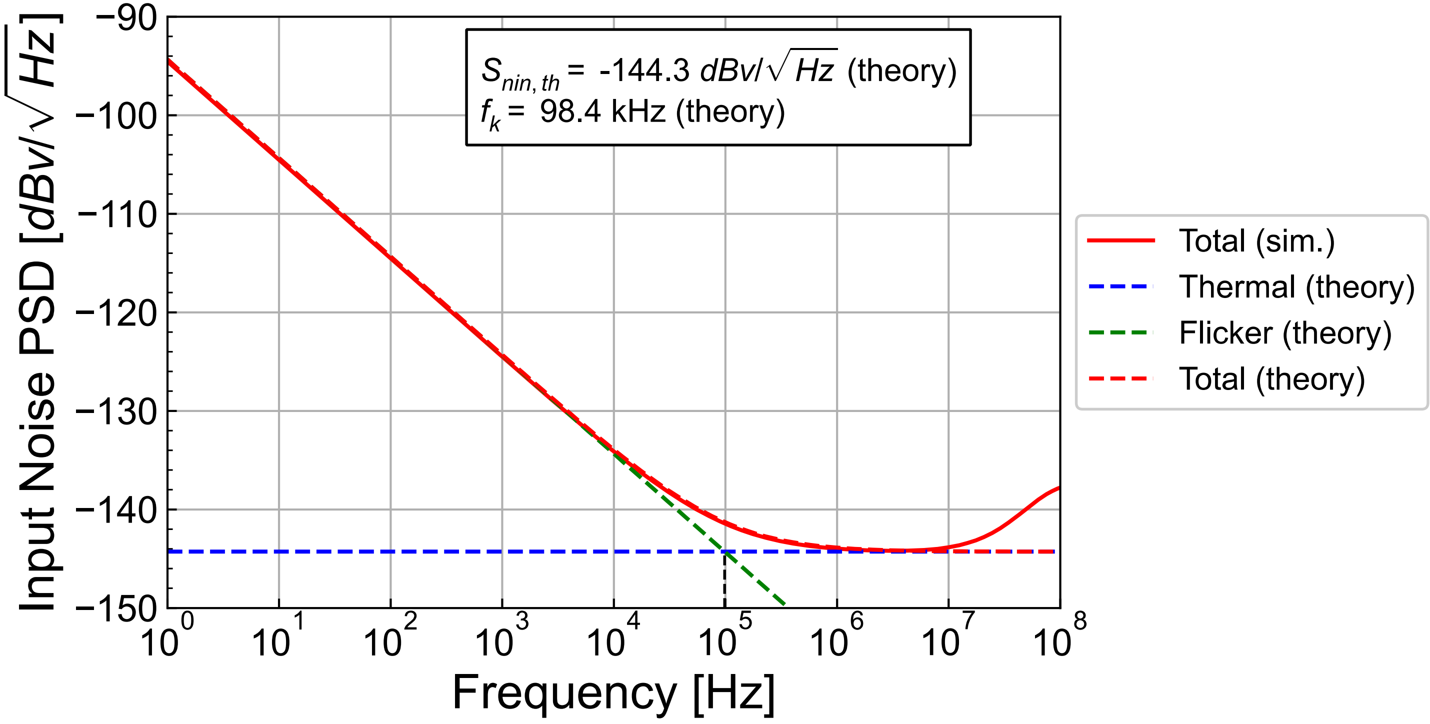
Large-signal Transfer Characteristic – Simulations



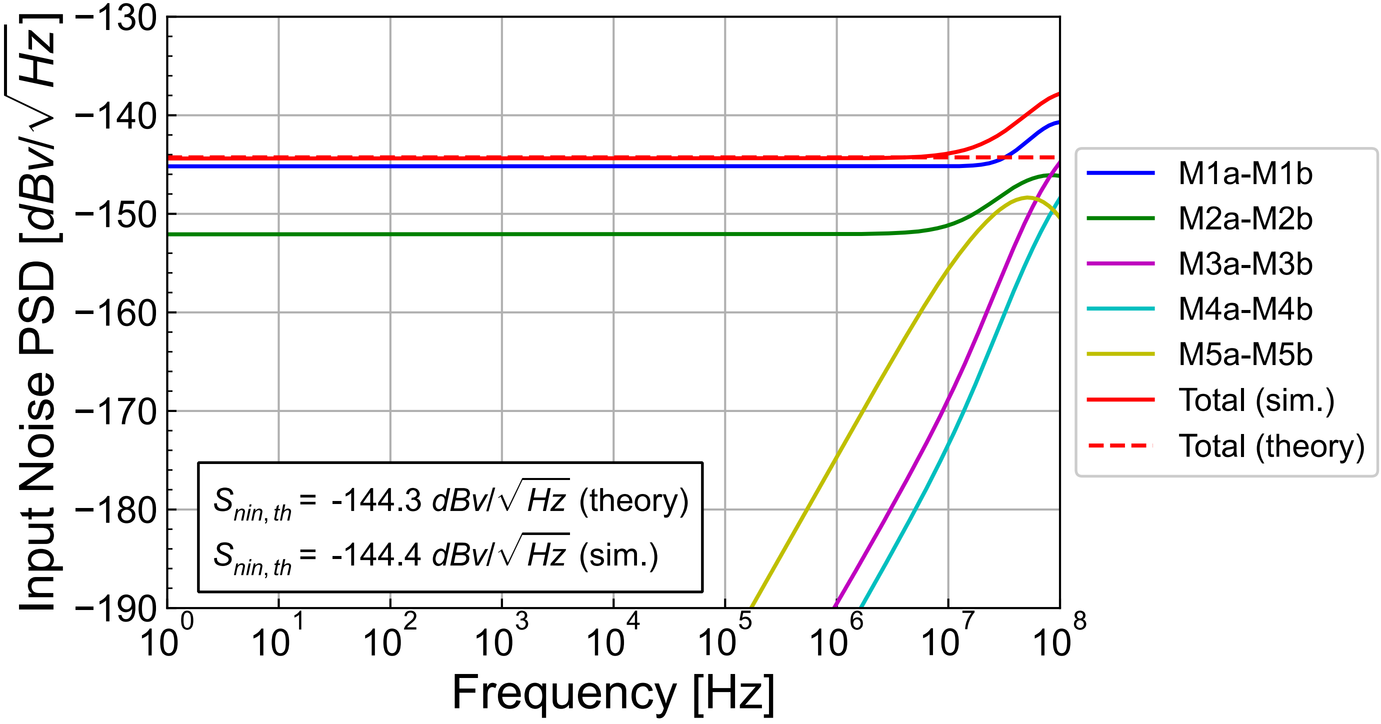
Open-loop Gain Response



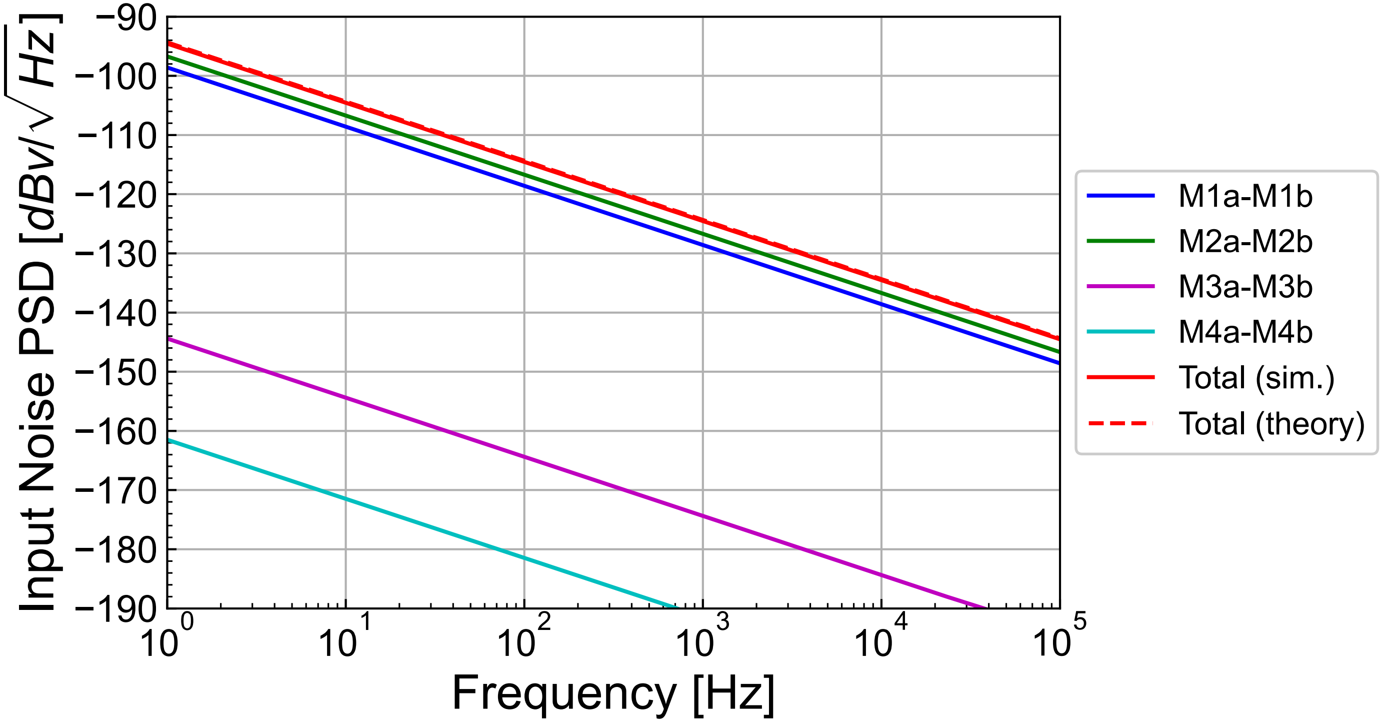
Input-referred Noise PSD



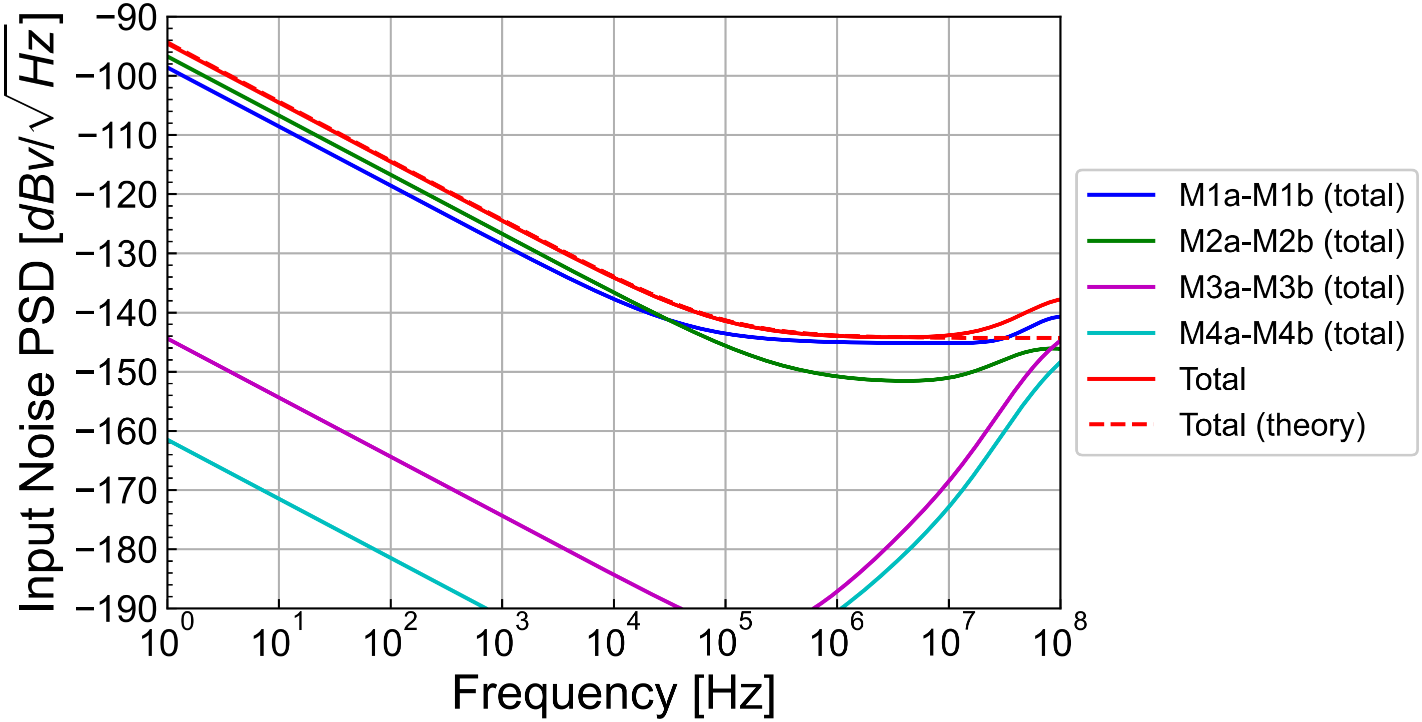
Input-referred Noise PSD – White Noise



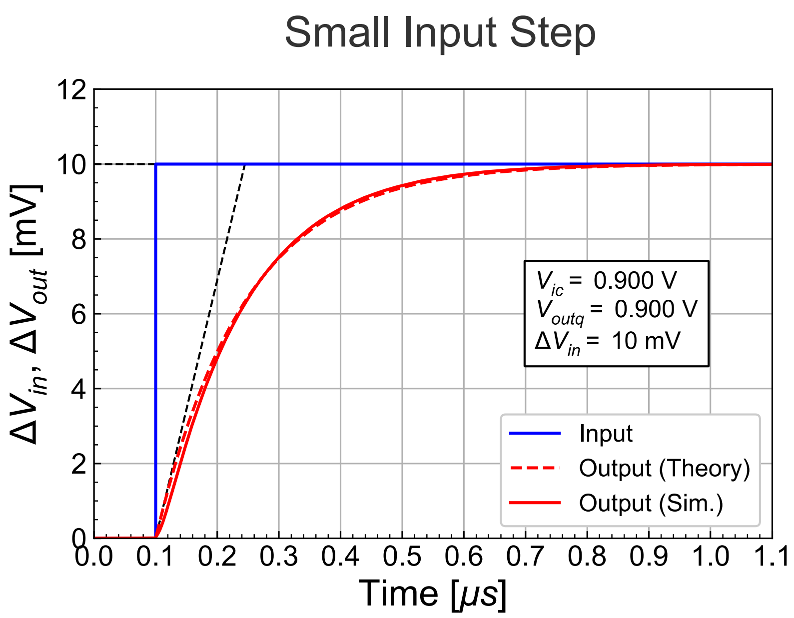
Input-referred Noise PSD – Flicker Noise

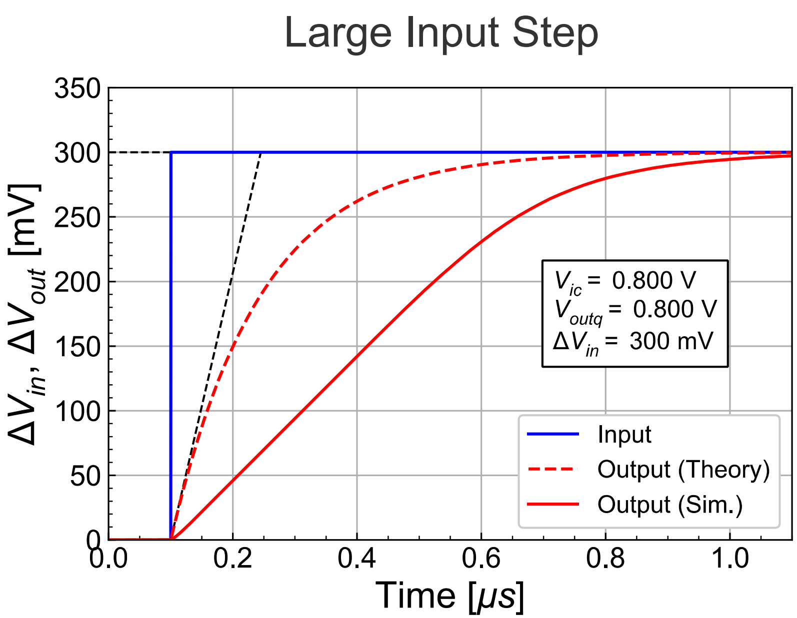


Input-referred Noise PSD – Total Noise

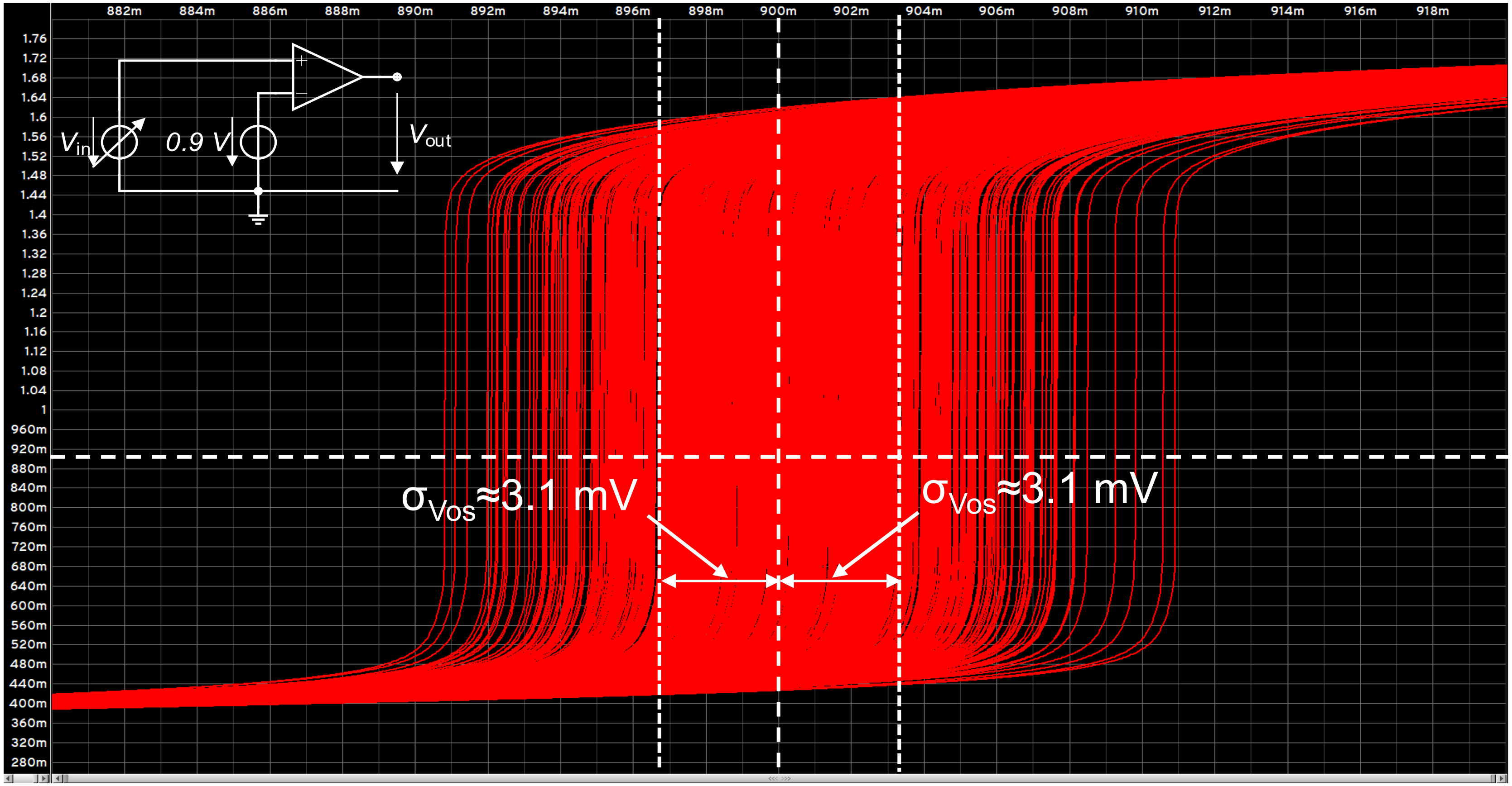


Step Response



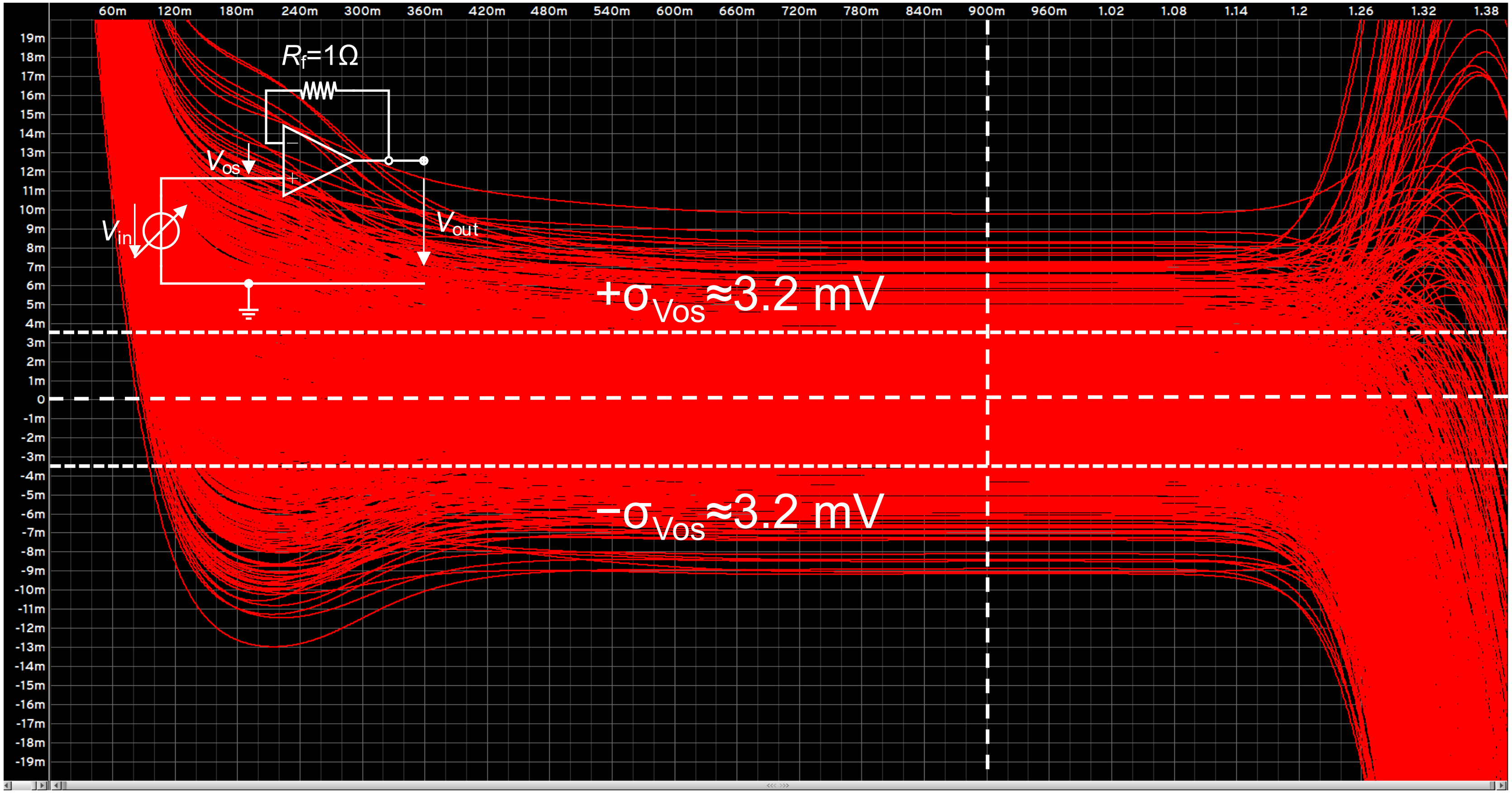


Monte Carlo Simulation of Offset Voltage (open-loop)



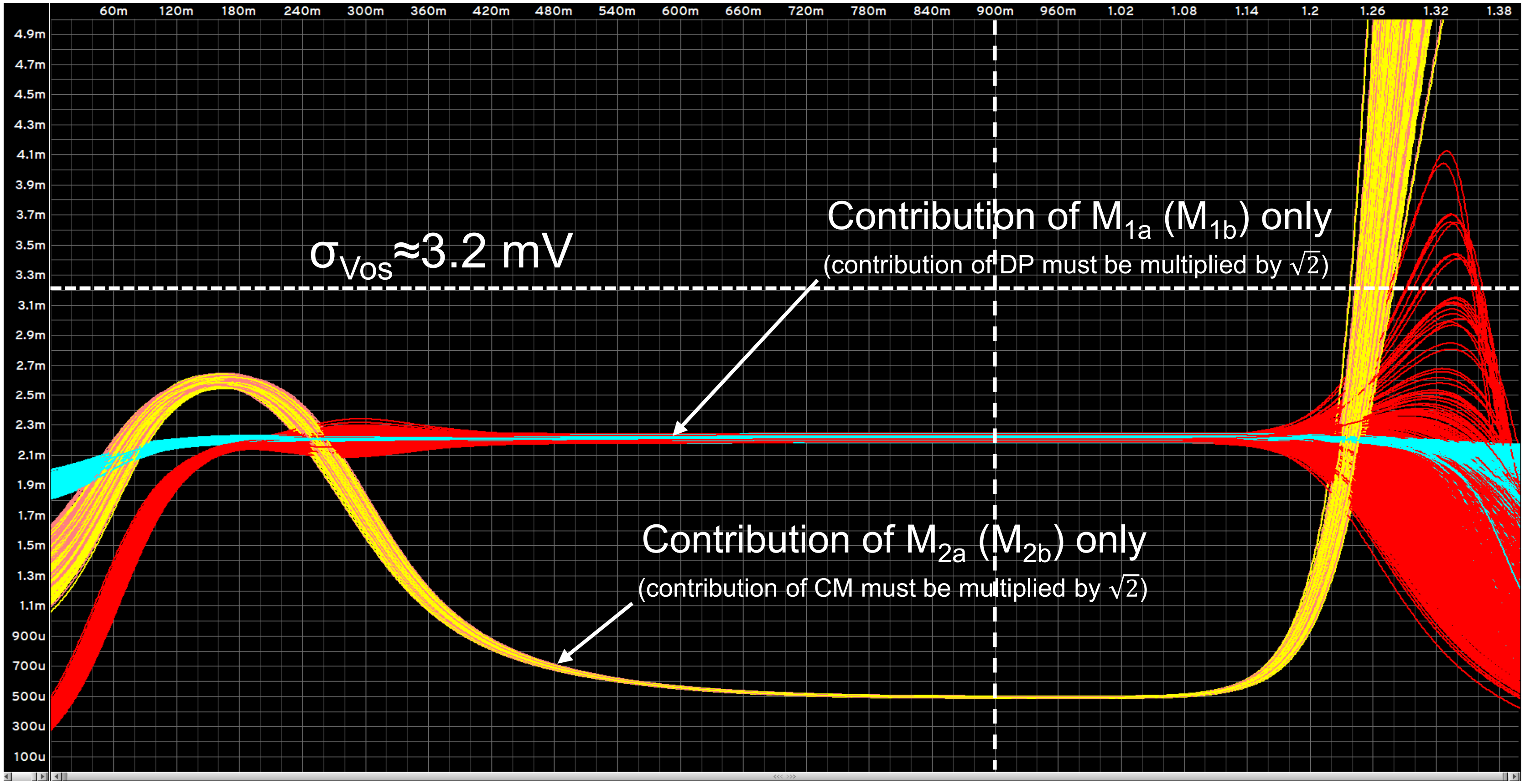
- Monte Carlo simulation of V_{os} versus V_{in} for 1000 runs in open-loop mode
- The standard deviation of V_{os} is about 3.56 mV which is consistent with the dispersion simulation giving 3.21 mV and equal to the 3.65 mV theoretical prediction

Monte Carlo Simulation of Offset Voltage (closed-loop)



- Monte Carlo simulation of V_{os} versus V_{in} for 1000 runs in voltage follower mode
- The standard deviation of V_{os} is about 3.17 mV which is consistent with the dispersion simulation giving 3.21 mV and close to the 3.12 mV theoretical prediction

Monte Carlo Simulation of Offset Voltage

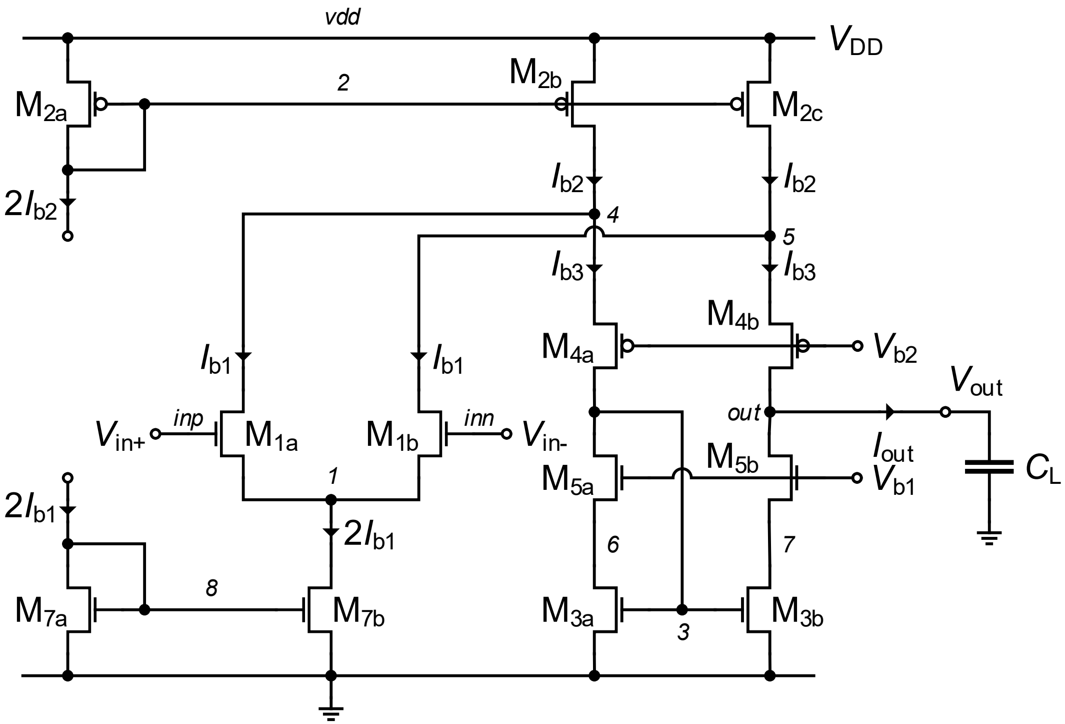


- Offset simulation using Monte Carlo simulations with 1000 runs
- As expected the contribution of the differential pair (M_{1a}-M_{1b}) dominates within the linear range

Outline

- The two-stage OTA or Miller OTA
- The telescopic OTA
- **The folded cascode OTA**

Folded-cascode Differential OTA



+ Self-compensation
+ High DC gain and GBW
+ Good input common-mode range
+ Improved output swing
+ Can work as voltage follower

– Higher power consumption
– Higher noise
– Additional poles

Folded Cascode OTA – Differential Open-loop Gain

- Similarly to the simple OTA, the folded cascode OTA has its dominant pole ω_0 at the output and non-dominant pole ω_p at the current mirror node 2 and a pole-zero doublet

$$A_{dm}(s) \triangleq \frac{\Delta V_{out}}{\Delta V_{id}} \cong A_{dc} \cdot \frac{1+\frac{s}{\omega_z}}{\left(1+\frac{s}{\omega_0}\right)\left(1+\frac{s}{\omega_p}\right)} \cong \frac{A_{dc}}{1+\frac{s}{\omega_0}} \cong \frac{\omega_u}{s}$$

- where

$$A_{dc} \cong \frac{G_{m1}}{G_o} \text{ with } G_o = \frac{G_{ds3} G_{ds5}}{G_{ms5}} + \frac{(G_{ds1}+G_{ds2})\, G_{ds4}}{G_{ms4}}$$

$$\omega_0 \cong \frac{G_o}{C_L}$$

$$\omega_p \cong \frac{G_{m2}}{C_2}$$

$$\omega_z = 2\omega_p$$

$$\omega_u = A_{dc} \cdot \omega_0 \cong \frac{G_{m1}}{C_L}$$

Noise Analysis

- If M_{2b}-M_{2c} are assumed to be perfectly matched, the noise generated by M_{2a} cancels out at the output node and hence the noise coming from M_{2a} can be neglected
- Neglecting also the contribution of the cascode transistors M_{4a}-M_{4b} and M_{5a}-M_{5b} and assuming that M_{1a}-M_{1b} and M_{3a}-M_{3b} are perfectly matched, the PSD of the output noise current is given by

$$S_{nout} \cong 2\left(S_{I_{n1}} + S_{I_{n2}} + S_{I_{n3}}\right)$$

- or if we express the output PSD in terms of the output noise conductance

$$S_{nout} = 4kT \cdot G_{nout} \text{ where } G_{nout} \cong 2(G_{n1} + G_{n2} + G_{n3})$$

- with $\quad G_{ni} = \gamma_{ni} G_{mi} + G_{mi}^2 \frac{\rho_i}{W_i L_i f} \quad$ for all transistors
- The input-referred noise is then given by

$$R_{nin} \triangleq \frac{G_{nout}}{G_{m1}^2} = \frac{2(G_{n1} + G_{n2} + G_{n3})}{G_{m1}^2} = \frac{2G_{n1}}{G_{m1}^2} \cdot \left(1 + \frac{G_{n2}}{G_{n1}} + \frac{G_{n3}}{G_{n1}}\right)$$

- which can be written as $R_{nin} = \frac{2G_{n1}}{G_{m1}^2} \cdot (1 + \eta)$ with $\eta = \frac{G_{n2}}{G_{n1}} + \frac{G_{n3}}{G_{n1}}$
- represents the contributions of the current mirrors referred to the input and normalized to the contribution of the differential pair

Input-referred Thermal Noise

- The input-referred thermal noise resistance is given by

$$R_{nth} = \frac{2\gamma_{n1}}{G_{m1}} \cdot (1 + \eta_{th})$$

- where

$$\eta_{th} = \frac{\gamma_{n2}}{\gamma_{n1}} \frac{G_{m2}}{G_{m1}} + \frac{\gamma_{n3}}{\gamma_{n1}} \frac{G_{m3}}{G_{m1}}$$

- represents the contributions to the input-referred thermal noise of the current mirrors relative to that of the differential pair
- In case $G_{m1} \gg G_{m2}$ and $G_{m1} \gg G_{m3}$, the thermal noise is dominated by the input differential pair and the previous expression can be simplified

$$R_{nth} \cong \frac{2\gamma_{n1}}{G_{m1}}$$

Thermal Noise Excess Factor

- To compare with other OTA it is useful to derive the thermal noise excess factor

$$\gamma_{ota} \triangleq G_m \cdot R_{nth}$$

- where $G_m = G_{m1}$ is the OTA transconductance
- This results in

$$\gamma_{ota} = 2\gamma_{n1} \cdot (1 + \eta_{th})$$

- In case $G_{m1} \gg G_{m2}$ and $G_{m1} \gg G_{m3}$, then $\eta_{th} \ll 1$ and the noise is dominated by the input differential pair and the previous expression can be simplified

$$\gamma_{ota} \cong 2\gamma_{n1}$$

Input-referred Flicker Noise

- The input-referred flicker noise is given by

$$R_{nfl} = \frac{2}{f}\left[\frac{\rho_n}{W_1 L_1} + \left(\frac{G_{m2}}{G_{m1}}\right)^2 \frac{\rho_p}{W_2 L_2} + \left(\frac{G_{m3}}{G_{m1}}\right)^2 \frac{\rho_n}{W_3 L_3}\right]$$

- which can be written as

$$R_{nfl} = \frac{2\rho_n}{W_1 L_1 f} \cdot (1 + \eta_{fl})$$

- where

$$\eta_{fl} = \frac{\rho_p}{\rho_n}\left(\frac{G_{m2}}{G_{m1}}\right)^2 \frac{W_1 L_1}{W_2 L_2} + \left(\frac{G_{m3}}{G_{m1}}\right)^2 \frac{W_1 L_1}{W_3 L_3}$$

- represents the contributions to the input-referred flicker noise of the current mirrors relative to that of the differential pair

Output Offset Current

- The random offset current is mainly due to the mismatch between M_{1a}-M_{1b}, M_{2b}-M_{2c} and M_{3a}-M_{3b}
- The variance of the output offset current is then given by

$$\sigma_{I_{out}}^2 = \sigma_{\Delta I_{D1}}^2 + \sigma_{\Delta I_{D2}}^2 + \sigma_{\Delta I_{D3}}^2$$
$$= I_{b1}^2 \cdot \sigma_{\Delta I_{D1}/I_{D1}}^2 + I_{b2}^2 \cdot \sigma_{\Delta I_{D2}/I_{D2}}^2 + I_{b3}^2 \cdot \sigma_{\Delta I_{D3}/I_{D3}}^2$$

- where $\sigma_{\Delta I_{Di}/I_{Di}}^2 = \sigma_{\beta_i}^2 + \left(\frac{G_{mi}}{I_b}\right)^2 \sigma_{V_{Ti}}^2$ for $i = 1{,}2$
- with $\sigma_{\beta_i}^2 = \frac{A_\beta^2}{W_i L_i}$ and $\sigma_{V_{Ti}}^2 = \frac{A_{V_T}^2}{W_i L_i}$ for $i = 1{,}2$
- The variance of the output offset current then becomes

$$\sigma_{I_{out}}^2 = I_{b1}^2 \cdot \sigma_{\beta_1}^2 + I_{b2}^2 \cdot \sigma_{\beta_2}^2 + I_{b3}^2 \cdot \sigma_{\beta_3}^2$$
$$+ G_{m1}^2 \cdot \sigma_{V_{T1}}^2 + G_{m2}^2 \cdot \sigma_{V_{T2}}^2 + G_{m3}^2 \cdot \sigma_{V_{T3}}^2$$

Input-referred Offset Voltage

- The variance of the input-referred offset voltage is obtained by dividing the variance of the output offset current by G_{m1}^2 resulting in

$$\sigma_{V_{os}}^2 = \left(\frac{I_{b1}}{G_{m1}}\right)^2 \sigma_{\beta_1}^2 + \left(\frac{I_{b2}}{G_{m1}}\right)^2 \sigma_{\beta_2}^2 + \left(\frac{I_{b3}}{G_{m1}}\right)^2 \sigma_{\beta_3}^2$$
$$+\sigma_{V_{T1}}^2 + \left(\frac{G_{m2}}{G_{m1}}\right)^2 \sigma_{V_{T2}}^2 + \left(\frac{G_{m3}}{G_{m1}}\right)^2 \sigma_{V_{T3}}^2$$

- which can be written as

$$\sigma_{V_{os}}^2 = \sigma_{V_{T1}}^2 \cdot (1 + \xi_{V_T}) + \left(\frac{I_{b1}}{G_{m1}}\right)^2 \cdot \sigma_{\beta_1}^2 \cdot (1 + \xi_\beta)$$

- where ξ_{V_T} represents the V_T mismatch contributions to the input-referred offset of the current mirror relative to that of the differential pair

$$\xi_{V_T} = \left(\frac{G_{m2}}{G_{m1}}\right)^2 \frac{\sigma_{V_{T2}}^2}{\sigma_{V_{T1}}^2} + \left(\frac{G_{m3}}{G_{m1}}\right)^2 \frac{\sigma_{V_{T3}}^2}{\sigma_{V_{T1}}^2}$$

- and ξ_β represents the β mismatch contributions to the input-referred offset of the current mirror relative to that of the differential pair

$$\xi_\beta = \left(\frac{I_{b2}}{I_{b1}}\right)^2 \frac{\sigma_{\beta_2}^2}{\sigma_{\beta_1}^2} + \left(\frac{I_{b3}}{I_{b1}}\right)^2 \frac{\sigma_{\beta_3}^2}{\sigma_{\beta_1}^2}$$

Input-referred Offset Voltage

- with

$$\sigma^2_{V_{T1}} = \frac{A^2_{V_{Tn}}}{W_1 L_1} \quad \sigma^2_{V_{T2}} = \frac{A^2_{V_{Tp}}}{W_2 L_2} \quad \sigma^2_{V_{T3}} = \frac{A^2_{V_{Tn}}}{W_3 L_3}$$

$$\sigma^2_{\beta_1} = \frac{A^2_{\beta n}}{W_1 L_1} \quad \sigma^2_{\beta_2} = \frac{A^2_{\beta p}}{W_2 L_2} \quad \sigma^2_{\beta_3} = \frac{A^2_{\beta n}}{W_3 L_3}$$

- Replacing results in

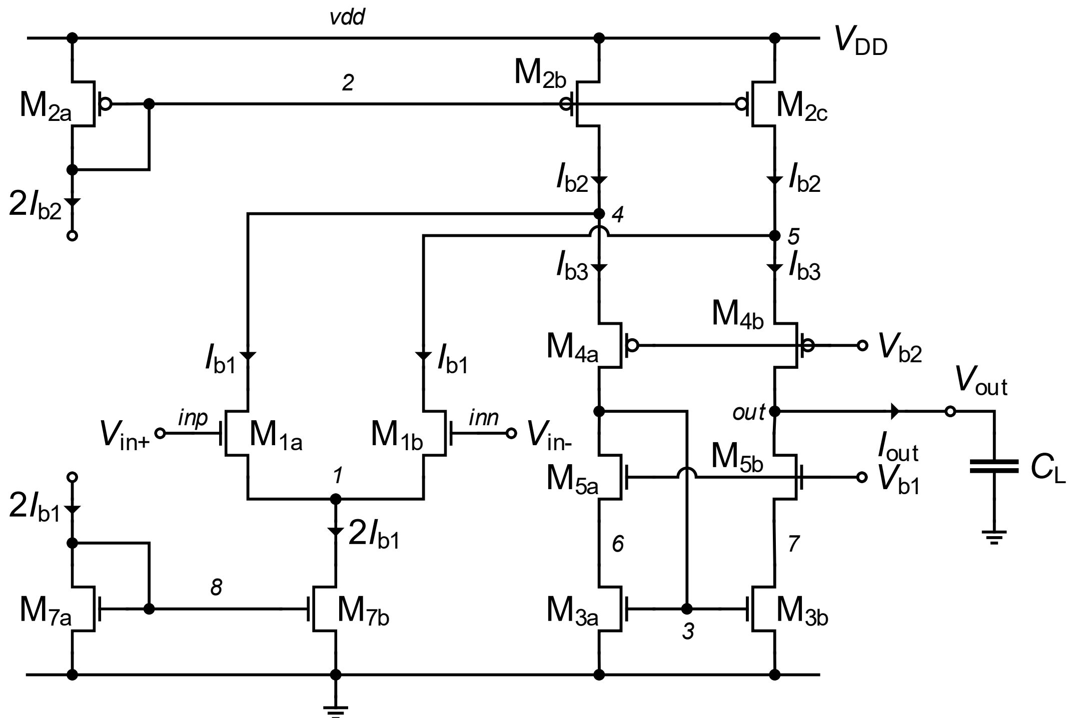
$$\xi_{V_T} = \left(\frac{G_{m2}}{G_{m1}}\right)^2 \cdot \left(\frac{A_{V_{Tp}}}{A_{V_{Tn}}}\right)^2 \cdot \frac{W_1 L_1}{W_2 L_2} + \left(\frac{G_{m3}}{G_{m1}}\right)^2 \cdot \frac{W_1 L_1}{W_3 L_3}$$

- And

$$\xi_{\beta} = \left(\frac{I_{b2}}{I_{b1}}\right)^2 \left(\frac{A_{\beta_p}}{A_{\beta_n}}\right)^2 \frac{W_1 L_1}{W_2 L_2} + \left(\frac{I_{b3}}{I_{b1}}\right)^2 \frac{W_1 L_1}{W_3 L_3}$$

- Similarly to the flicker noise, the input-referred offset (variance or standard deviation) can be reduced by increasing the M_{1a}-M_{1b} area $W_1 L_1$ but at the same time also increasing the area $W_2 L_2$ of the current sources M_{2b}-M_{2c} and also the area $W_3 L_3$ of the current mirror M_{3a}-M_{3b}

The Folded Cascode OTA – Design Example



Specification	Symbol	Value	Unit
Minimum DC gain	A_{dc}	100	dB
Minimum gain-bandwidth product	GBW	1	MHz
Load capacitance	C_L	1	pF
Maximum input-referred random offset voltage	V_{os}	10	mV
Phase margin	PM	60	$^\circ$

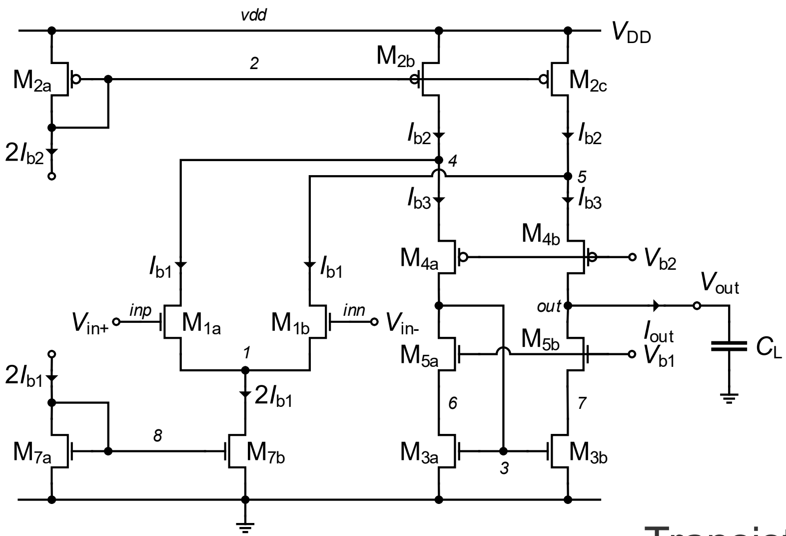
The Folded Cascode OTA – Technology Parameters

Parameter	Value	Unit
T	300	K
U_T	25.875	mV

Parameter	Value	Unit
V_{DD}	1.8	V
C_{ox}	8.443	$\frac{fF}{\mu m^2}$
W_{min}	200	nm
L_{min}	180	nm

Parameter	NMOS	PMOS	Unit
sEKV parameters			
n	1.27	1.31	-
$I_{spec\square}$	715	173	nA
V_{T0}	0.455	0.445	V
L_{sat}	26	36	nm
λ	20	20	$\frac{V}{\mu m}$
Overlap capacitances parameters			
C_{GDo}	0.366	0.329	$\frac{fF}{\mu m}$
C_{GSo}	0.366	0.329	$\frac{fF}{\mu m}$
C_{GBo}	0	0	$\frac{fF}{\mu m}$
Junction capacitances parameters			
C_J	1	1.121	$\frac{fF}{\mu m^2}$
C_{JSW}	0.2	0.248	$\frac{fF}{\mu m}$
Flicker noise parameters			
K_F	8.1e-24	6.8e-23	J
AF	1	1	-
ρ	0.05794	0.4828	$\frac{V \cdot m^2}{A \cdot s}$
Matching parameters			
A_{VT}	5	5	$mV \cdot \mu m$
A_β	1	1	$\% \cdot \mu m$
Source and drain sheet resistance parameter			
R_{sh}	600	2386	$\frac{\Omega}{\mu m}$
Width and length parameters			
ΔW	39	54	nm
ΔL	-76	-72	nm

Sizing Summary



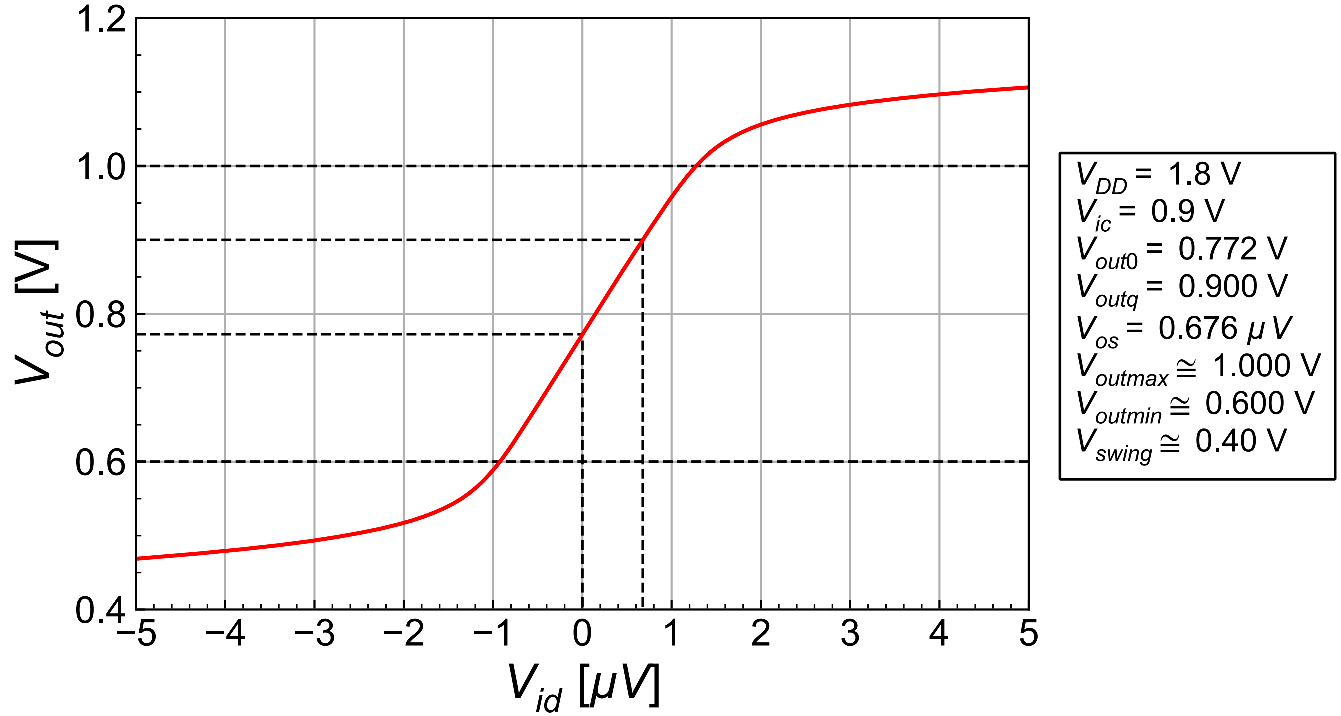
Bias

Name	Value
VDD	1.8
Ib1	250.0E-9
Ib2	550.0E-9
Vb1	1
Vb2	0.6

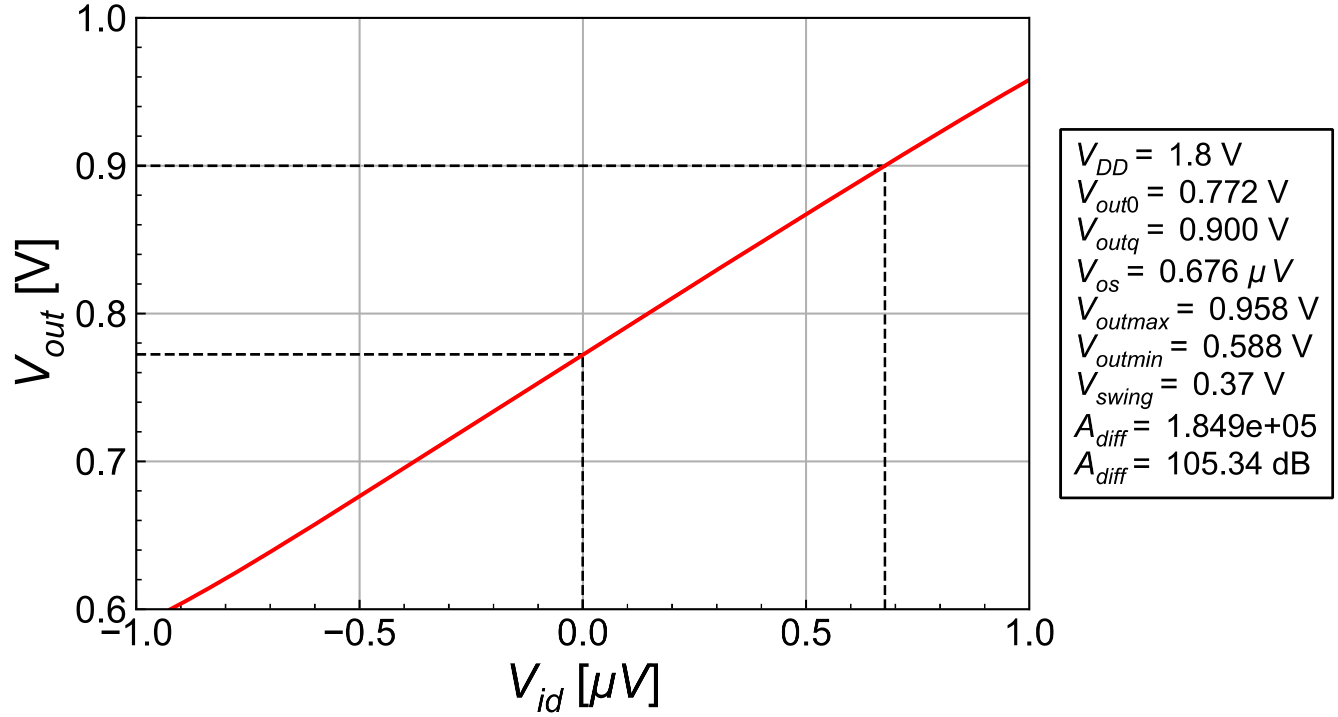
Transistor sizes

	Type	Function	W	L	ID	W/L	Ispec	IC	VP-VS	n	VG-VT0	VDSsat	Gspec	Gms	Gm	Gds	gamman
M1a	n	DP	3.3E-6	940.0E-9	250.0E-9	3.500	2.5E-6	0.10	-0.057	1.27	-0.045	0.105	96.7E-6	8.9E-6	7.0E-6	13.3E-9	0.65
M1b	n	DP	3.3E-6	940.0E-9	250.0E-9	3.500	2.5E-6	0.10	-0.057	1.27	-0.045	0.105	96.7E-6	8.9E-6	7.0E-6	13.3E-9	0.65
M2a	p	CM	1.3E-6	5.9E-6	1.1E-6	0.215	37.2E-9	29.55	0.298	1.31	0.228	0.300	1.4E-6	7.1E-6	5.5E-6	9.4E-9	0.83
M2b	p	CM	630.0E-9	5.9E-6	550.0E-9	0.108	18.6E-9	29.55	0.298	1.31	0.228	0.300	719.4E-9	3.6E-6	2.7E-6	4.7E-9	0.83
M2c	p	CM	630.0E-9	5.9E-6	550.0E-9	0.108	18.6E-9	29.55	0.298	1.31	0.228	0.300	719.4E-9	3.6E-6	2.7E-6	4.7E-9	0.83
M3a	n	CM	200.0E-9	10.9E-6	300.0E-9	0.018	13.1E-9	22.93	0.261	1.27	0.205	0.269	505.6E-9	2.2E-6	1.7E-6	1.4E-9	0.81
M3b	n	CM	200.0E-9	10.9E-6	300.0E-9	0.018	13.1E-9	22.93	0.261	1.27	0.205	0.269	505.6E-9	2.2E-6	1.7E-6	1.4E-9	0.81
M4a	p	CA	9.9E-6	570.0E-9	300.0E-9	17.333	3.0E-6	0.10	-0.057	1.31	-0.044	0.105	116.0E-6	10.6E-6	8.1E-6	26.3E-9	0.67
M4b	p	CA	9.9E-6	570.0E-9	300.0E-9	17.333	3.0E-6	0.10	-0.057	1.31	-0.044	0.105	116.0E-6	10.6E-6	8.1E-6	26.3E-9	0.67
M5a	n	CA	2.3E-6	540.0E-9	300.0E-9	4.204	3.0E-6	0.10	-0.057	1.27	-0.045	0.105	116.2E-6	10.6E-6	8.4E-6	27.8E-9	0.65
M5b	n	CA	2.3E-6	540.0E-9	300.0E-9	4.204	3.0E-6	0.10	-0.057	1.27	-0.045	0.105	116.2E-6	10.6E-6	8.4E-6	27.8E-9	0.65
M7a	n	CM	200.0E-9	8.2E-6	500.0E-9	0.025	17.5E-9	28.50	0.293	1.27	0.230	0.295	678.1E-9	3.3E-6	2.6E-6	3.1E-9	0.81
M7b	n	CM	200.0E-9	8.2E-6	500.0E-9	0.025	17.5E-9	28.50	0.293	1.27	0.230	0.295	678.1E-9	3.3E-6	2.6E-6	3.1E-9	0.81

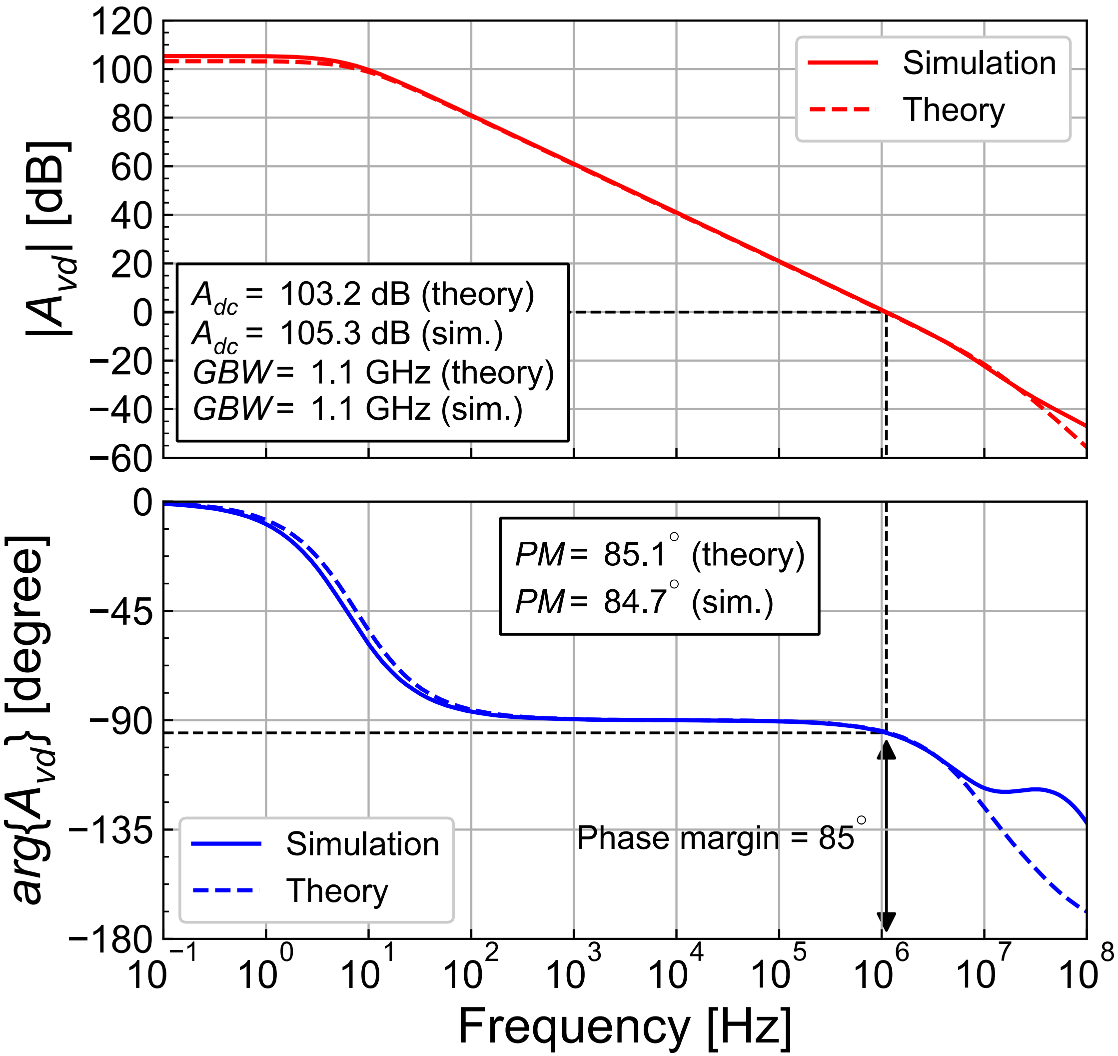
Large-signal Transfer Characteristic – Simulations



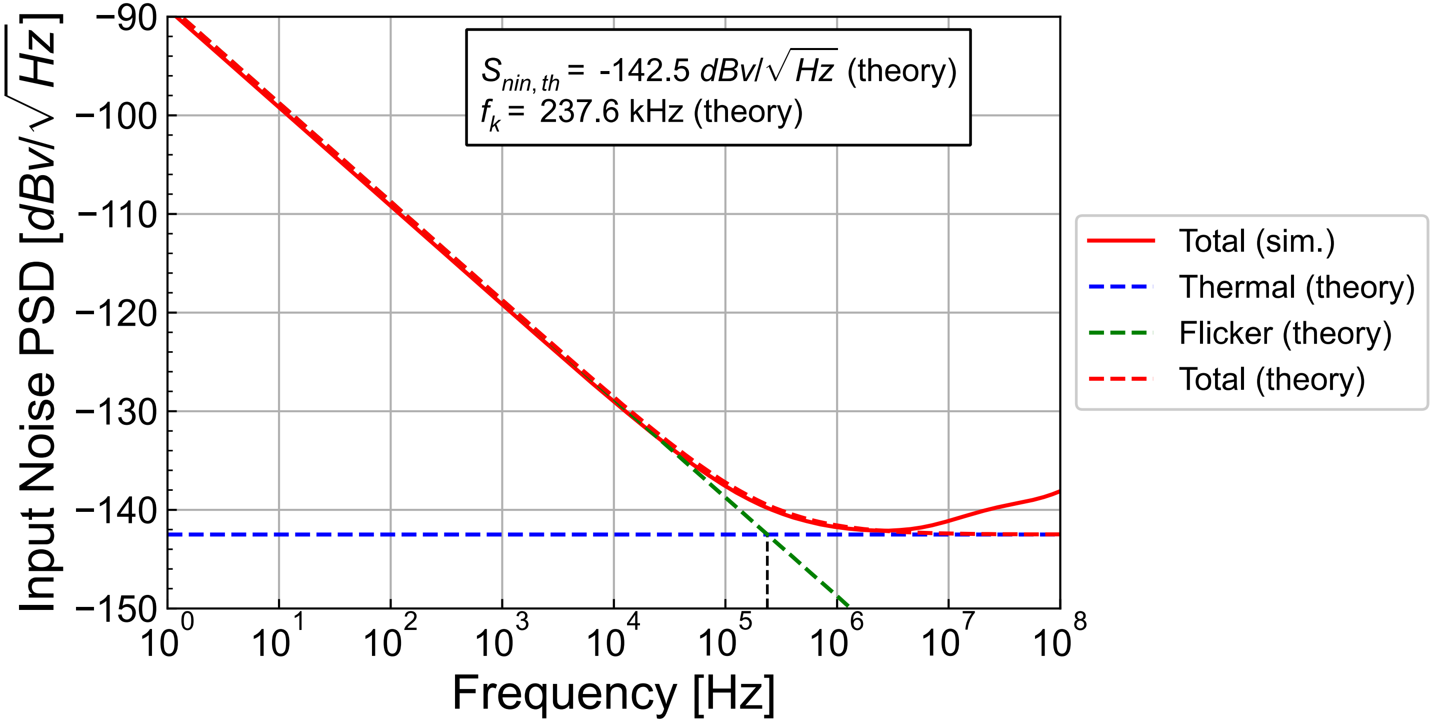
Large-signal Transfer Characteristic – Simulations



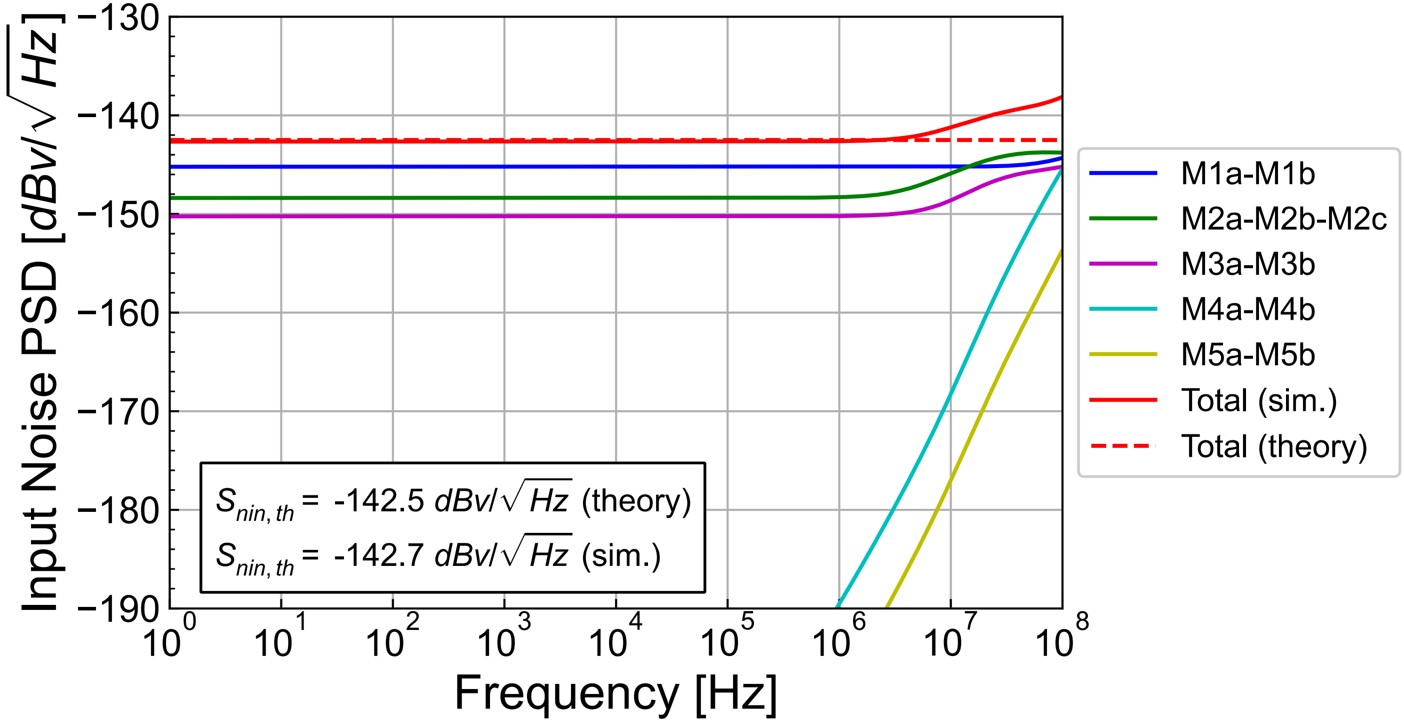
Open-loop Gain Response



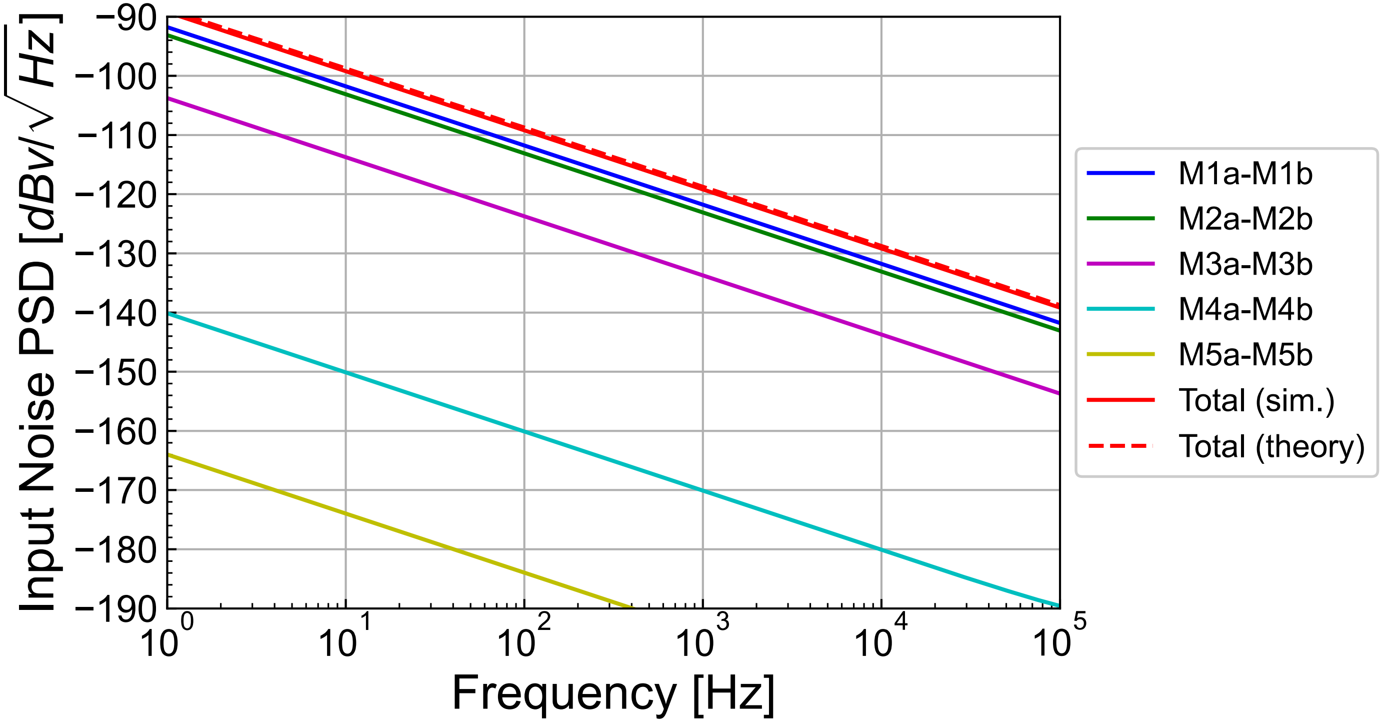
Input-referred Noise PSD



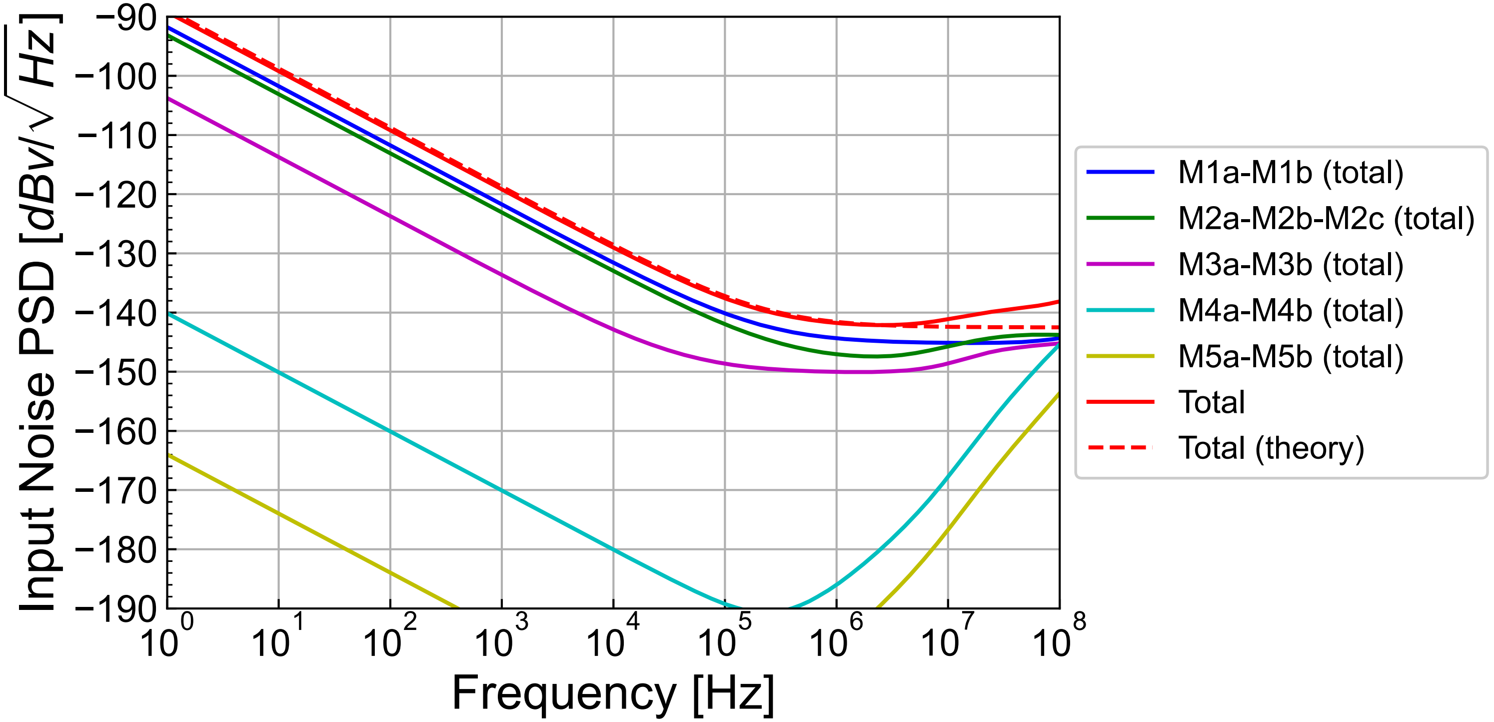
Input-referred Noise PSD – White Noise



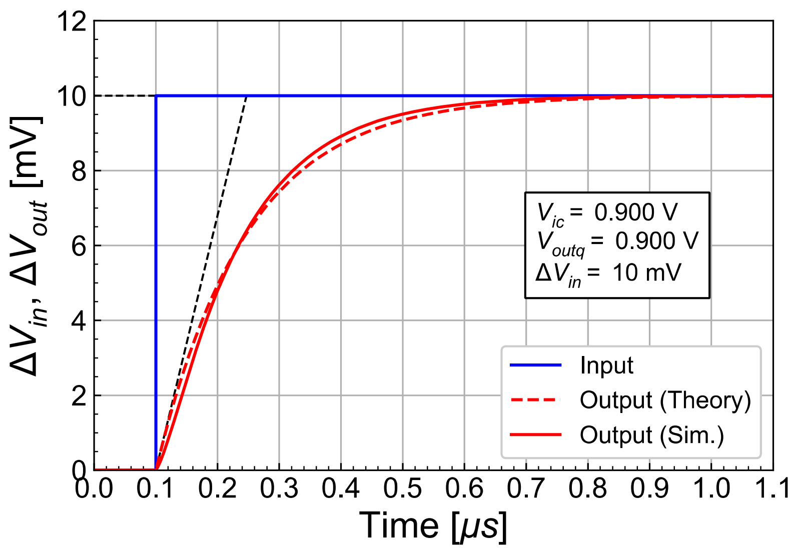
Input-referred Noise PSD – Flicker Noise

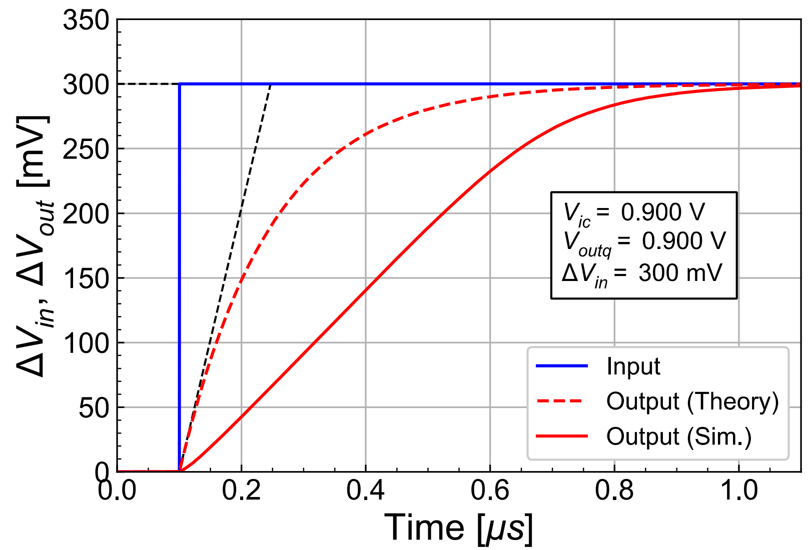


Input-referred Noise PSD – Total Noise

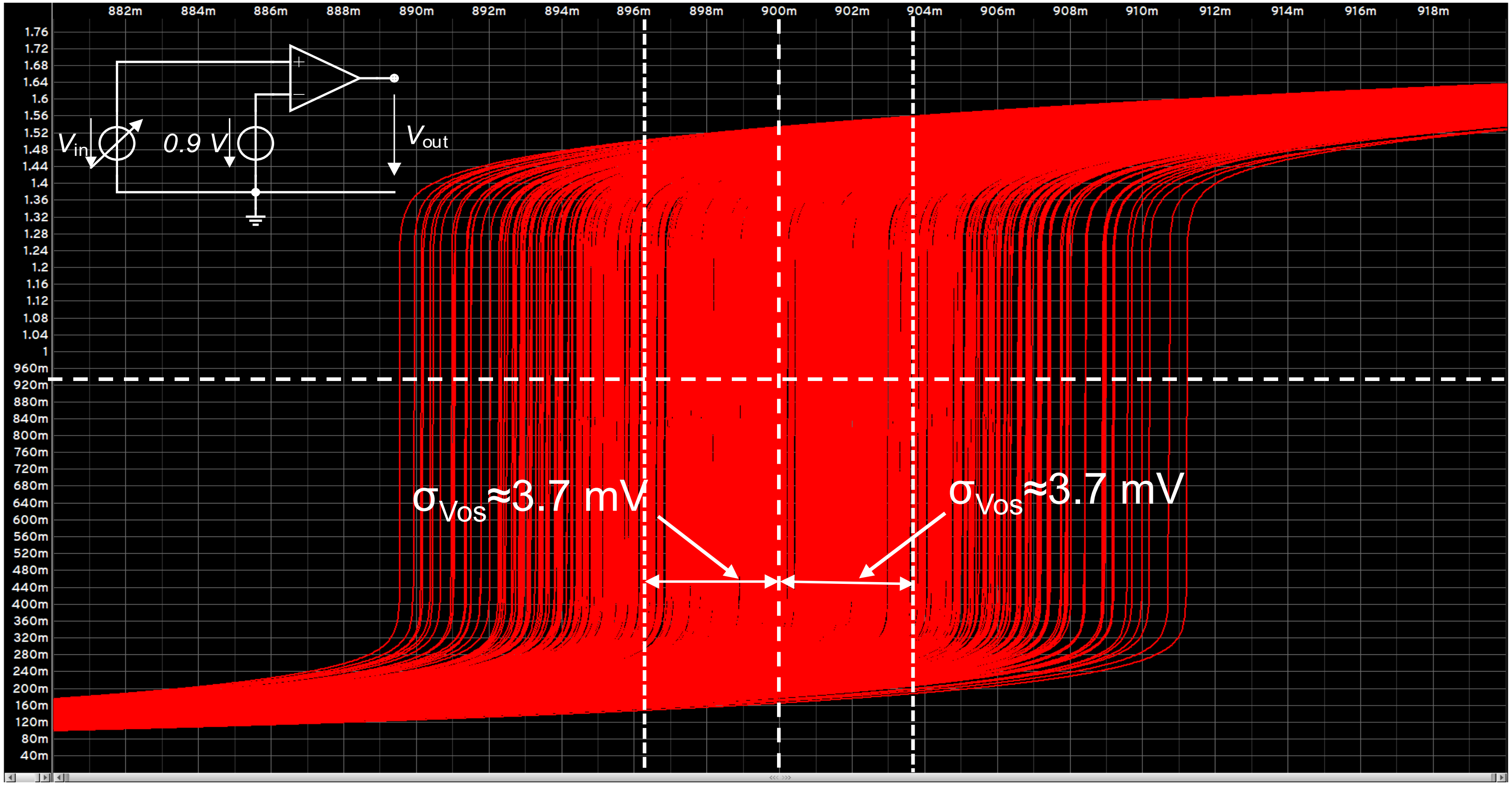


Step Response



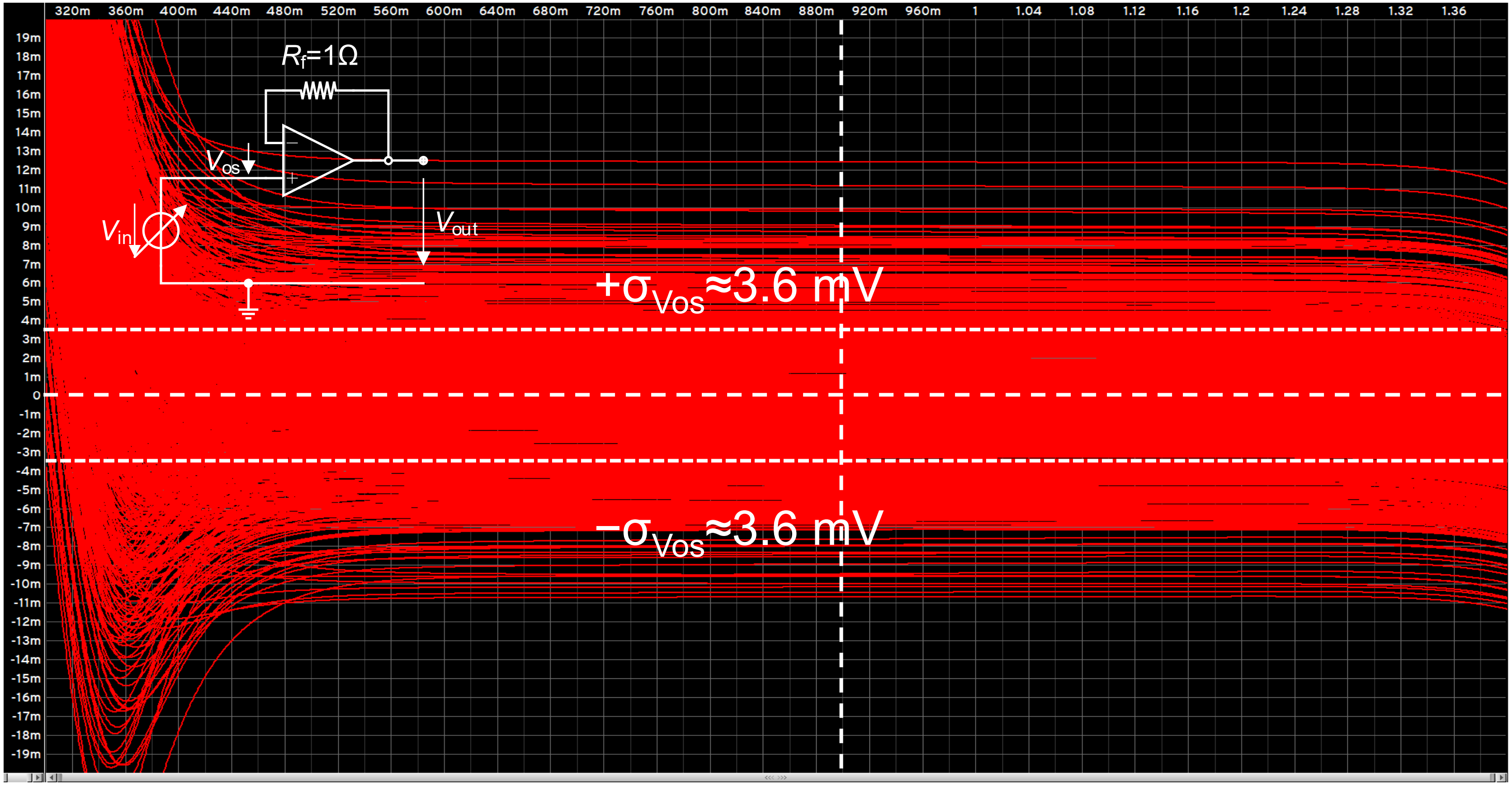


Monte Carlo Simulation of Offset Voltage (open-loop)



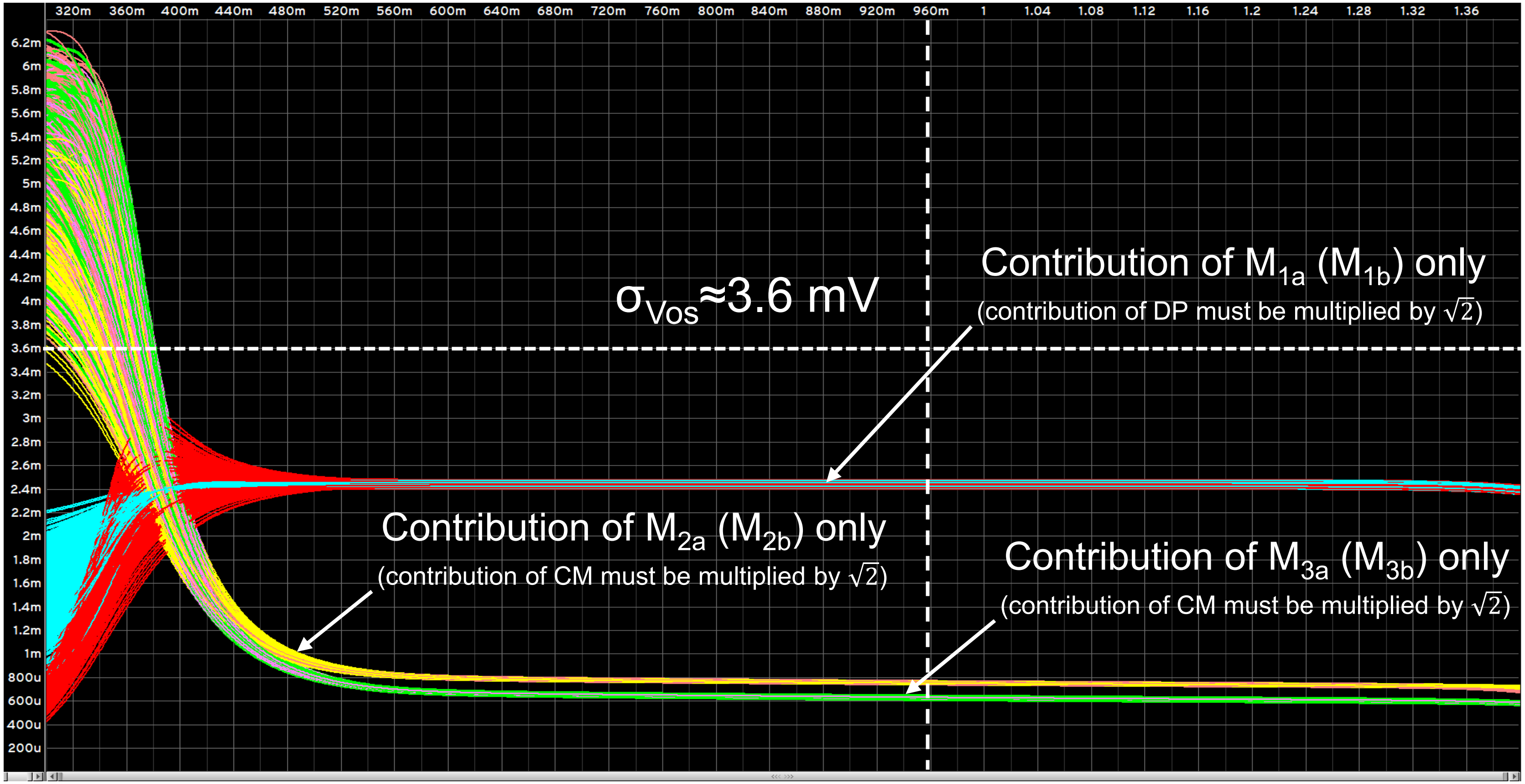
- Monte Carlo simulation of V_{os} versus V_{in} for 1000 runs in open-loop mode
- The standard deviation of V_{os} is about 3.56 mV which is slightly smaller than the dispersion simulation giving 3.72 mV and close to the 3.65 mV theoretical prediction

Monte Carlo Simulation of Offset Voltage (closed-loop)



- Monte Carlo simulation of V_{os} versus V_{in} for 1000 runs in voltage follower mode
- The standard deviation of V_{os} is about 3.62 mV which is consistent with the dispersion simulation giving 3.72 mV and close to the 3.65 mV theoretical prediction

Monte Carlo Simulation of Offset Voltage



- Offset simulation using Monte Carlo simulations with 1000 runs
- As expected the contribution of the differential pair (M_{1a}-M_{1b}) dominates within the linear range