Fundamentals of Electrical Circuits and Systems

Chapter 7: Three-Phase Circuits

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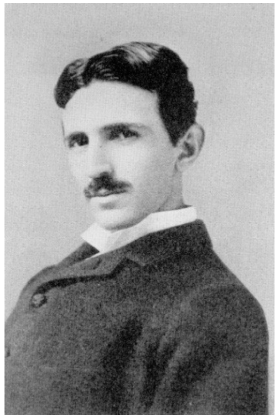
Three-Phase circuits

- Three-phase circuits allow an optimal use of the electrical networks both at the source and the charge.
- They were proposed by Tesla in 1888 and commercially implemented for the first time at Niagara Falls on November 16, 1896.

Nicola **Tesla** (1856-1943),
Serbian engineer and physicist.



Three-Phase circuits

- Circuits powered by three sinusoidal sources out of phase with each other by 120º.
- Symmetrical (balanced) source: the three source voltages have the same amplitude (rms value)
- Balanced load: the three loads connected to the three-phase source consume the same active and reactive powers.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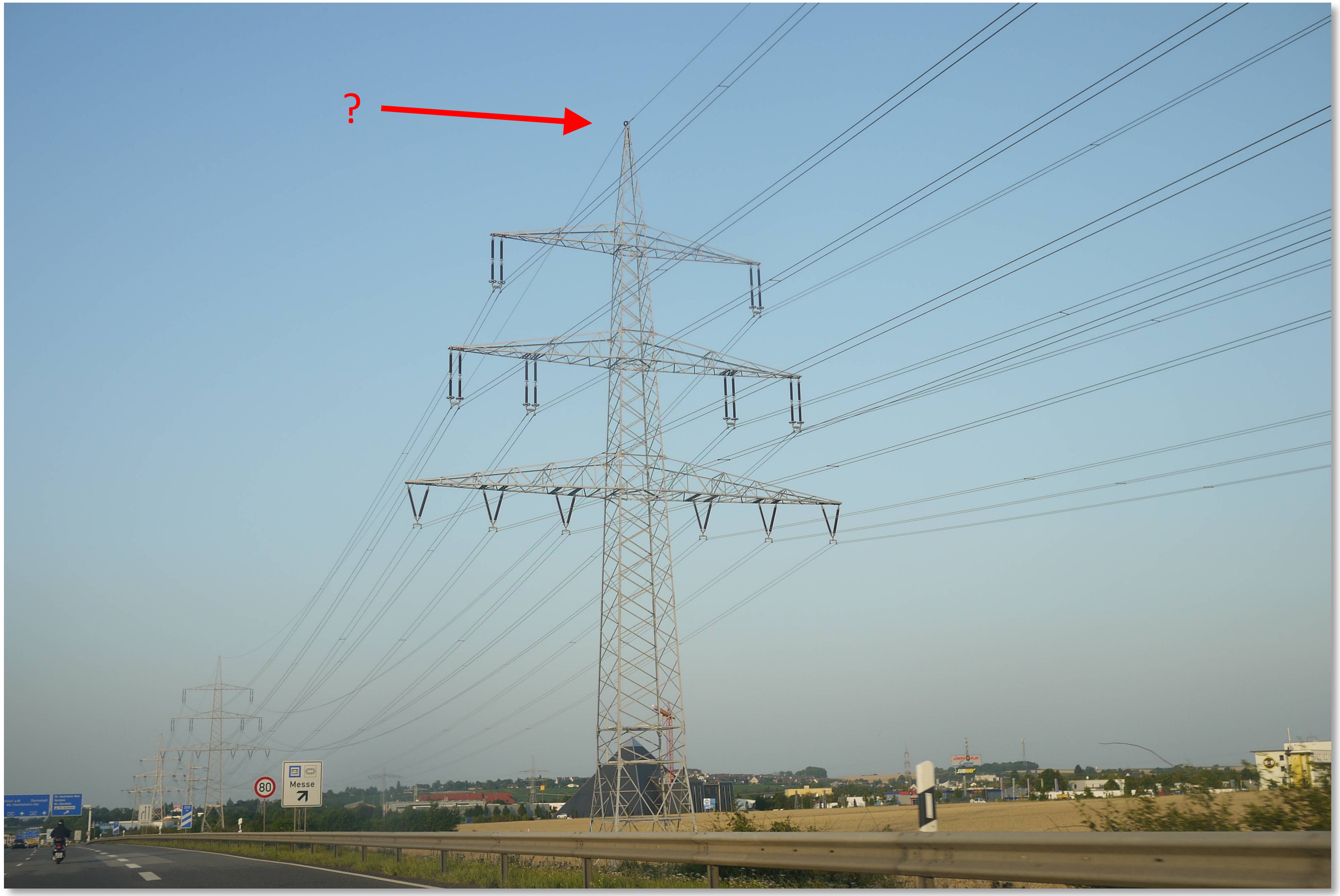
Messe


80
Messe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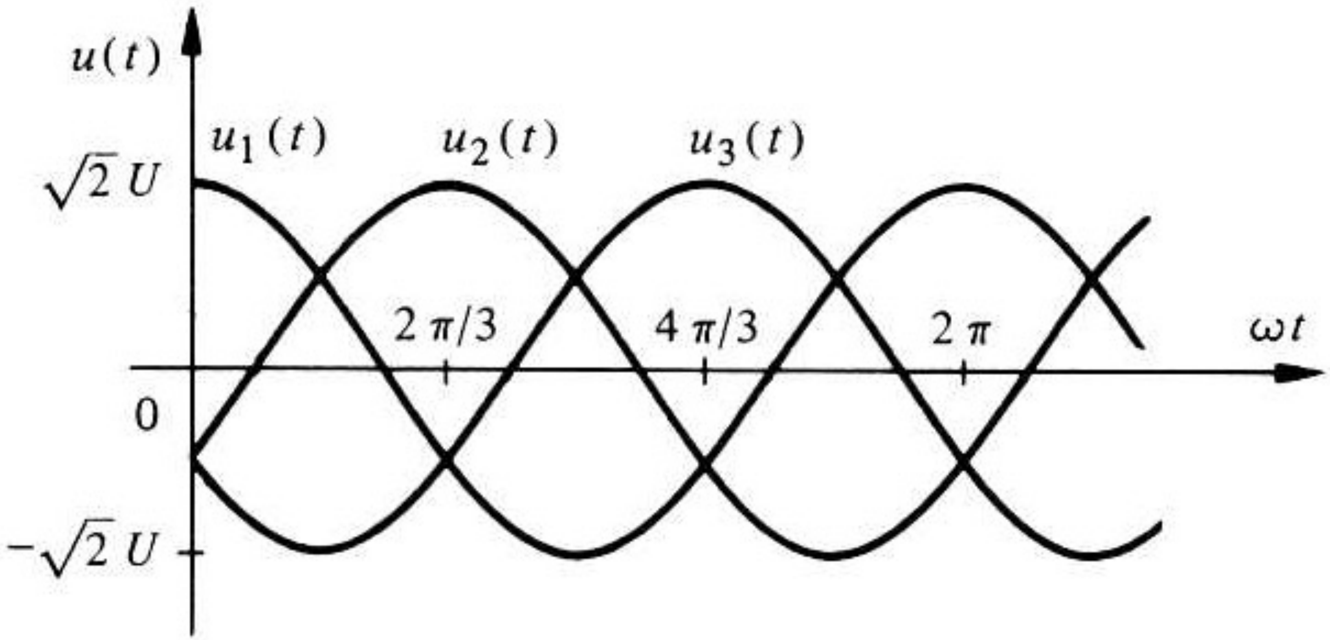
Balanced three-phase systems

Direct system:

$$u_1(t) = U\sqrt{2}\cos(\omega t) \quad u_2(t) = U\sqrt{2}\cos(\omega t - 2\pi/3) \quad u_3(t) = U\sqrt{2}\cos(\omega t + 2\pi/3)$$

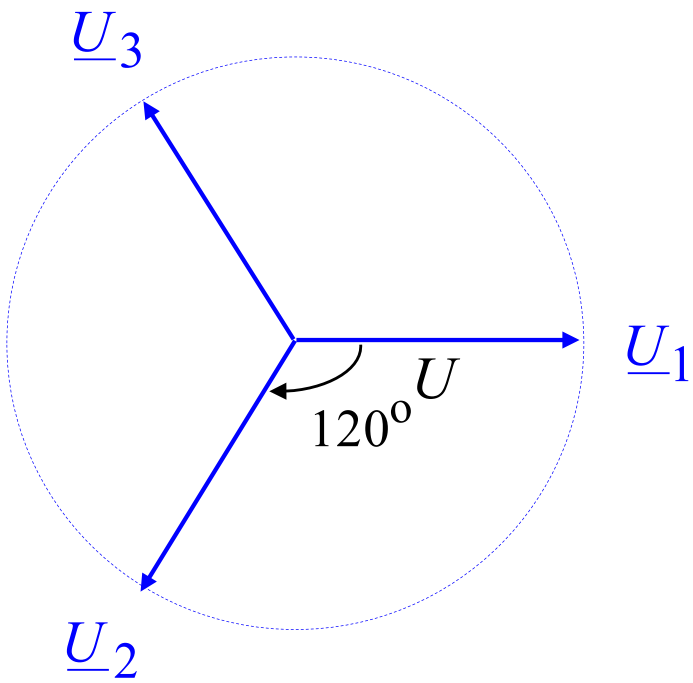


$$u_1(t) + u_2(t) + u_3(t) = 0$$

Balanced three-phase systems

Direct system:

$$\underline{U}_1 = Ue^{j\alpha} \quad \underline{U}_2 = Ue^{j(\alpha - 2\pi/3)} \quad \underline{U}_3 = Ue^{j(\alpha + 2\pi/3)}$$

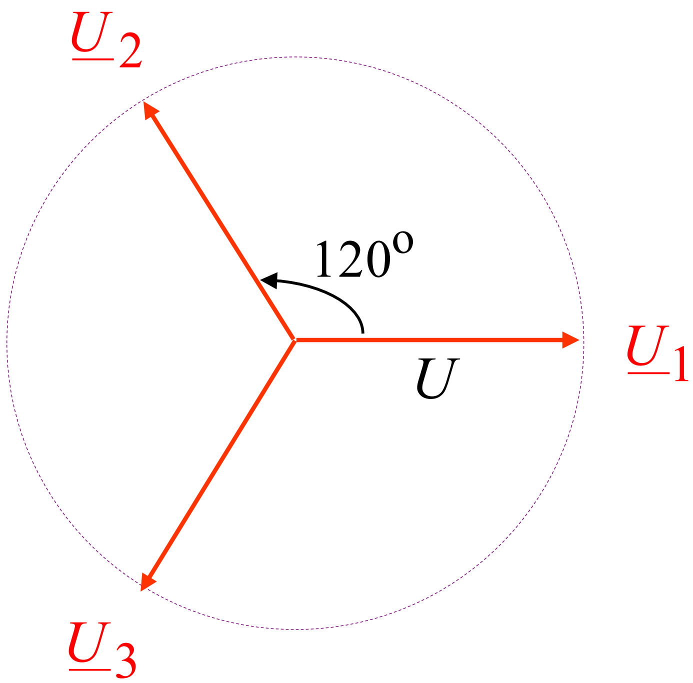


$$\underline{U}_1 + \underline{U}_2 + \underline{U}_3 = 0$$

Balanced three-phase systems

Reverse system:

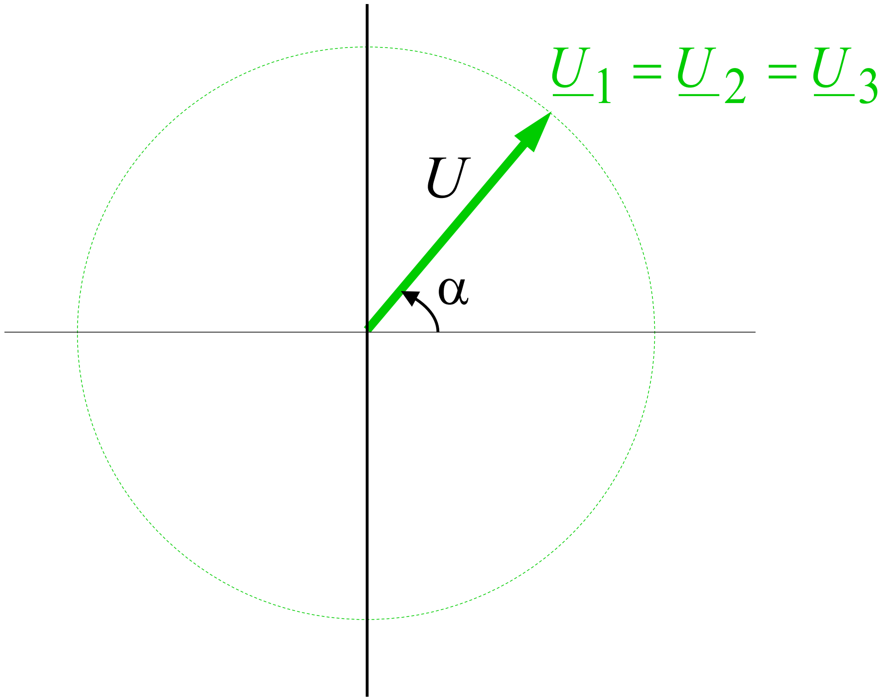
$$\underline{U}_1 = Ue^{j\alpha} \quad \underline{U}_2 = Ue^{j(\alpha+2\pi/3)} \quad \underline{U}_3 = Ue^{j(\alpha-2\pi/3)}$$



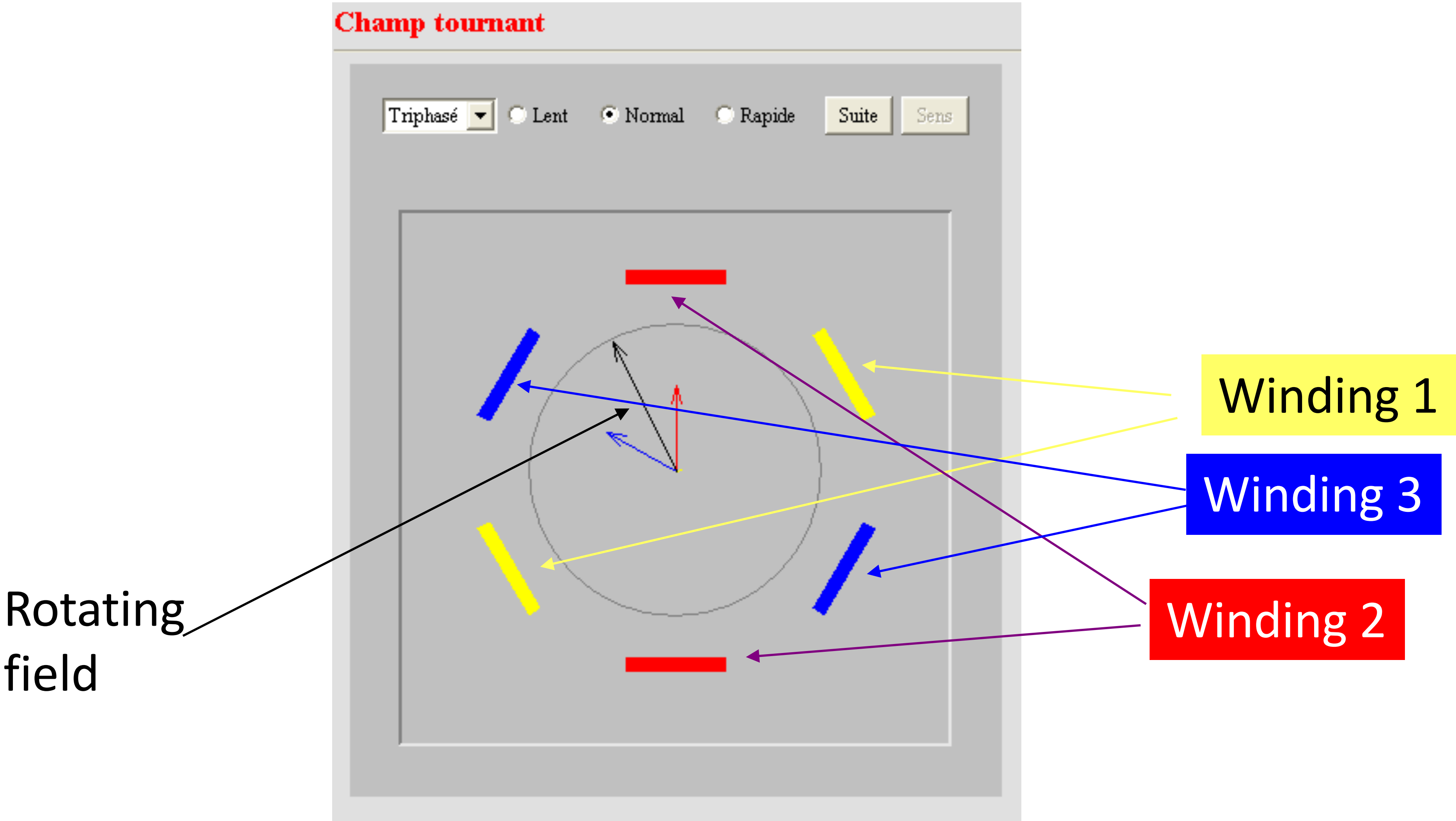
$$\underline{U}_1 + \underline{U}_2 + \underline{U}_3 = 0$$

Homopolar system

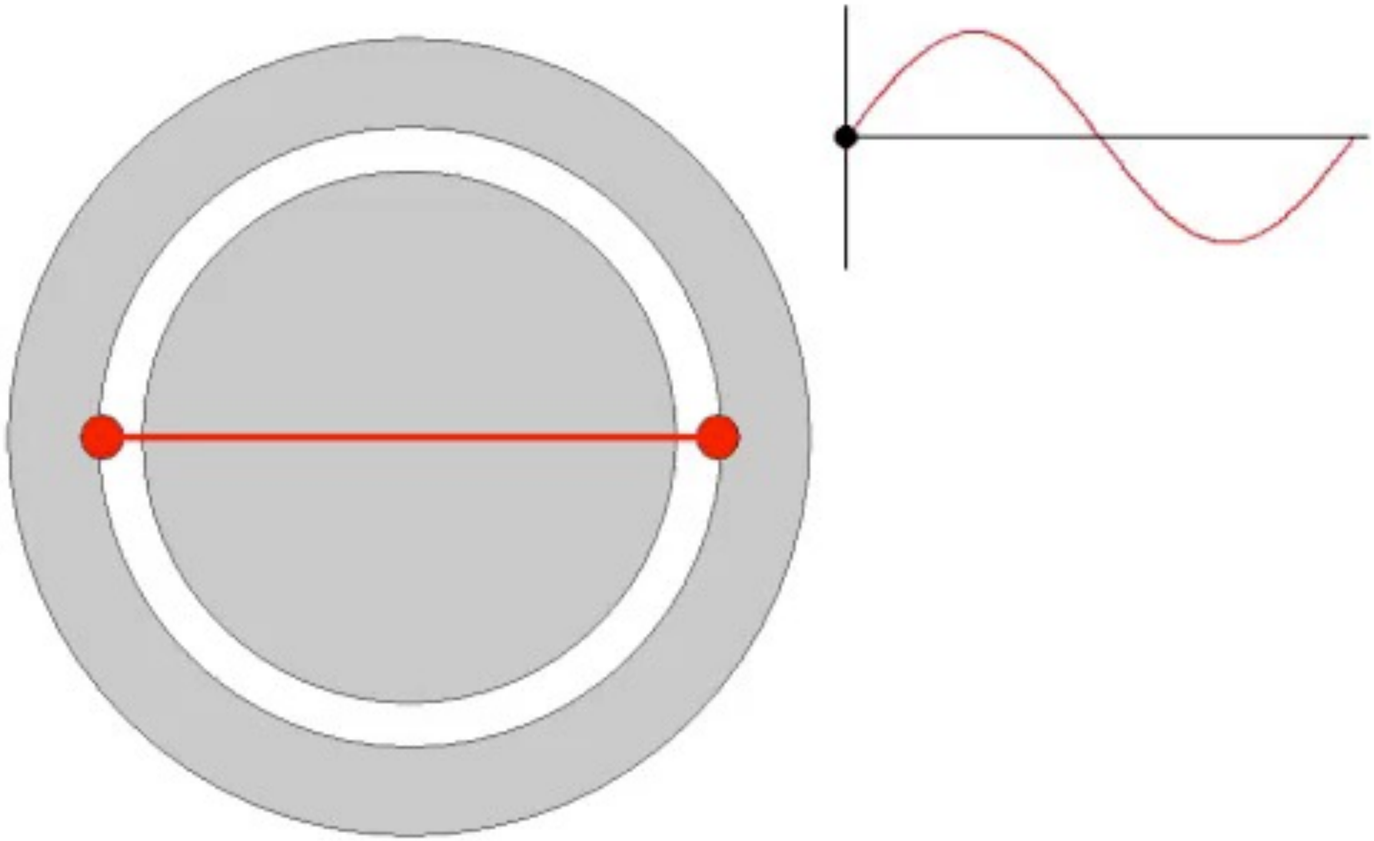
$$\underline{U}_1 = \underline{U}_2 = \underline{U}_3 = Ue^{j\alpha}$$



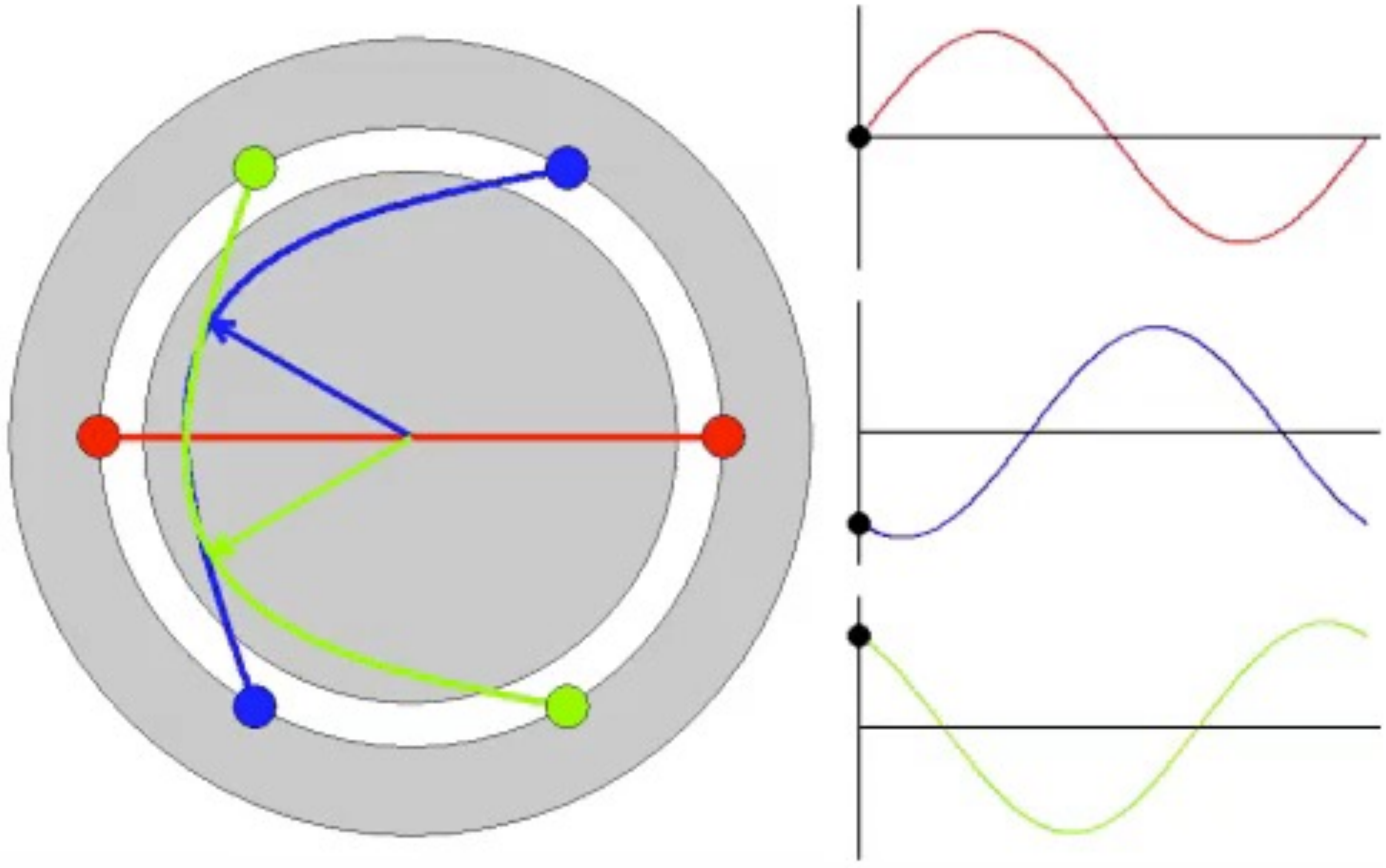
How to generate a balanced three-phase system?



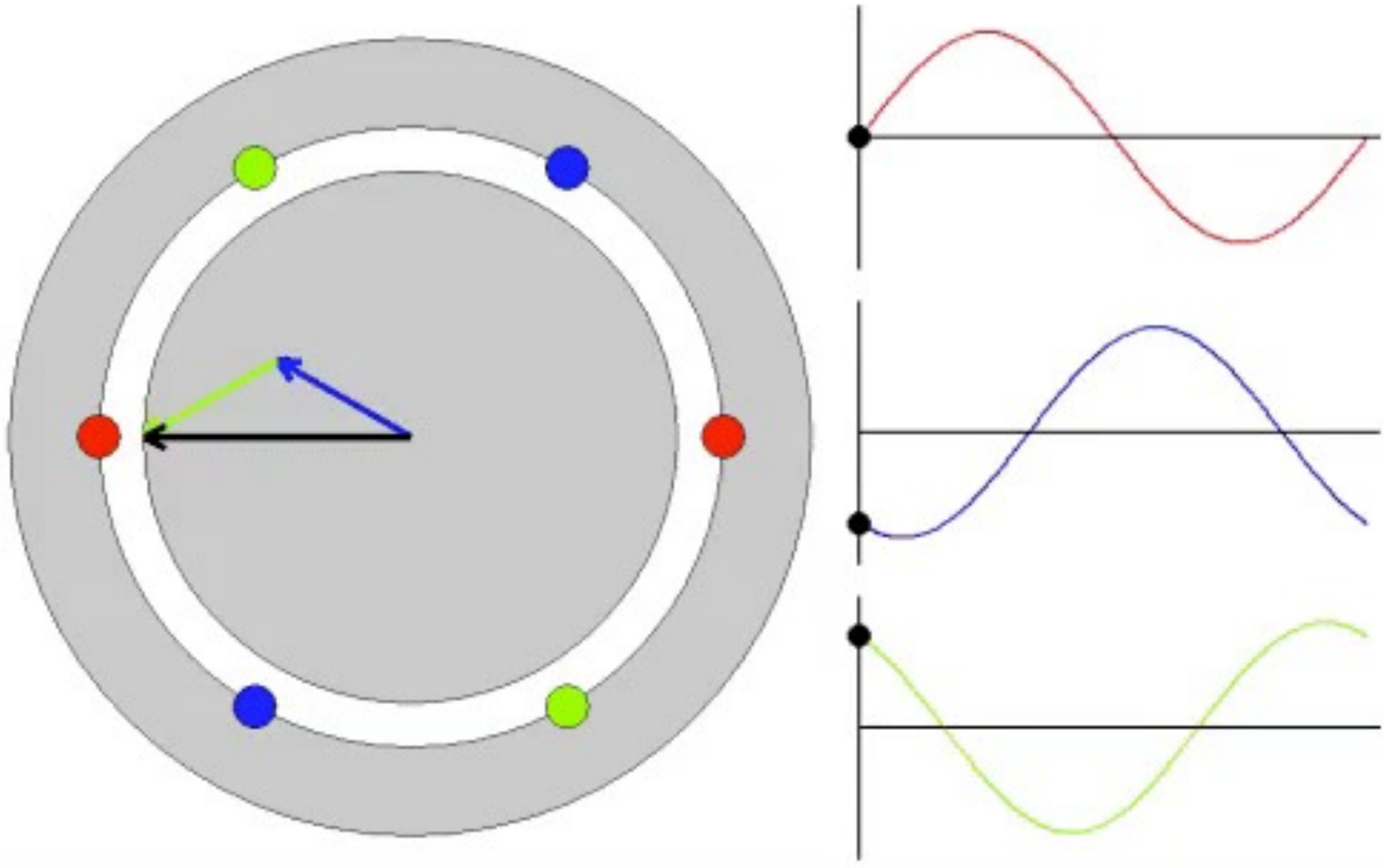
How to generate a balanced three-phase system?



How to generate a balanced three-phase system?

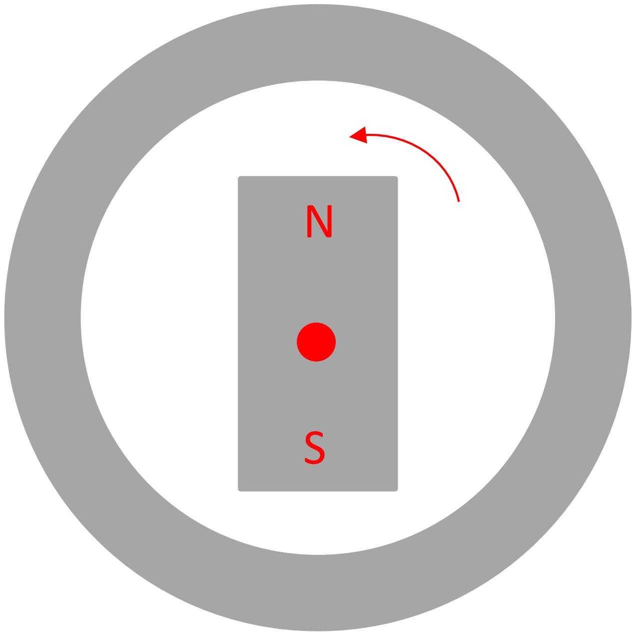


How to generate a balanced three-phase system?

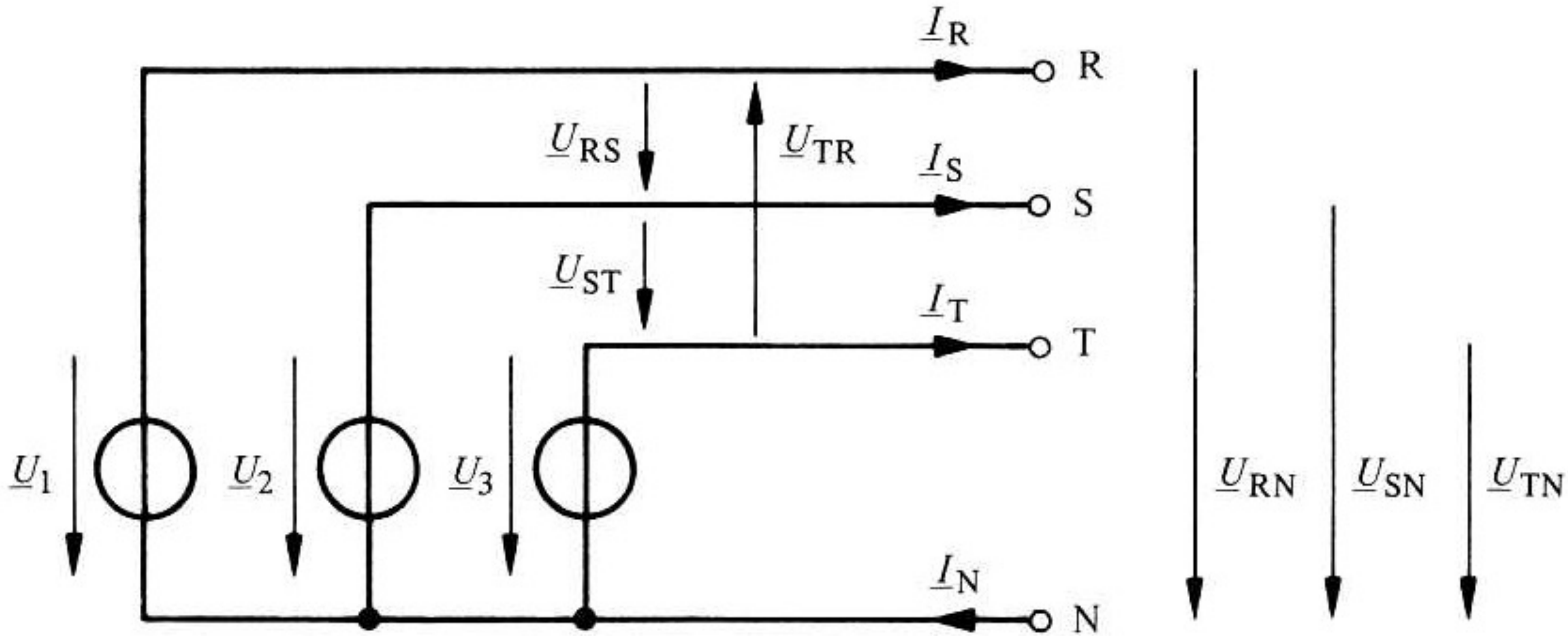


How to generate a balanced three-phase system?

- The total magnetic field generated by the 3 windings is a rotating field
- A rotating field (produced by a magnet fixed on the rotor) will induce three-phase currents in the coils arranged symmetrically.



Three-phase source
(Wye connection)

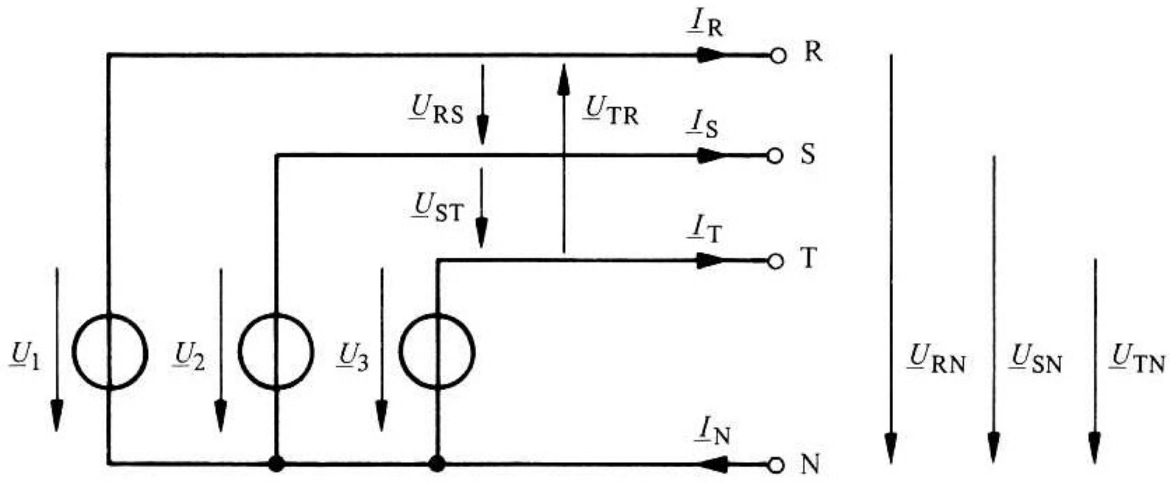


R, S, T: phase conductors
N: neutral conductor

Three-phase source
Phase voltages and line voltages

Phase voltages
or line-to-neutral voltages:

$$\underline{U}_{RN}, \underline{U}_{SN}, \underline{U}_{TN}$$



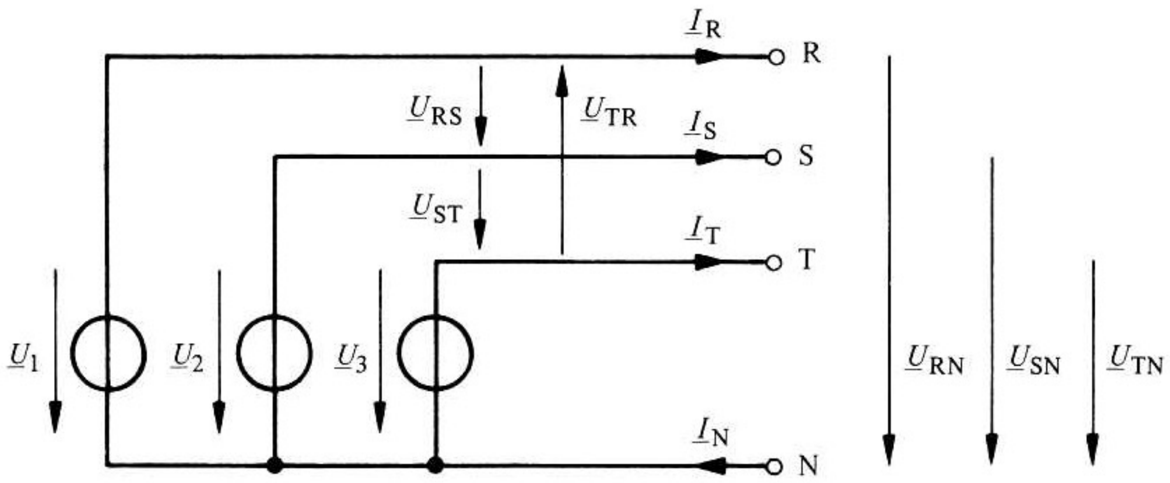
with:

$$\underline{U}_{RN} = \underline{U}_1 = U$$

$$\underline{U}_{SN} = \underline{U}_2 = Ue^{-j2\pi/3}$$

$$\underline{U}_{TN} = \underline{U}_3 = Ue^{+j2\pi/3}$$

Three-phase source
Phase voltages and line voltages



Line-to-line voltages or line voltages:

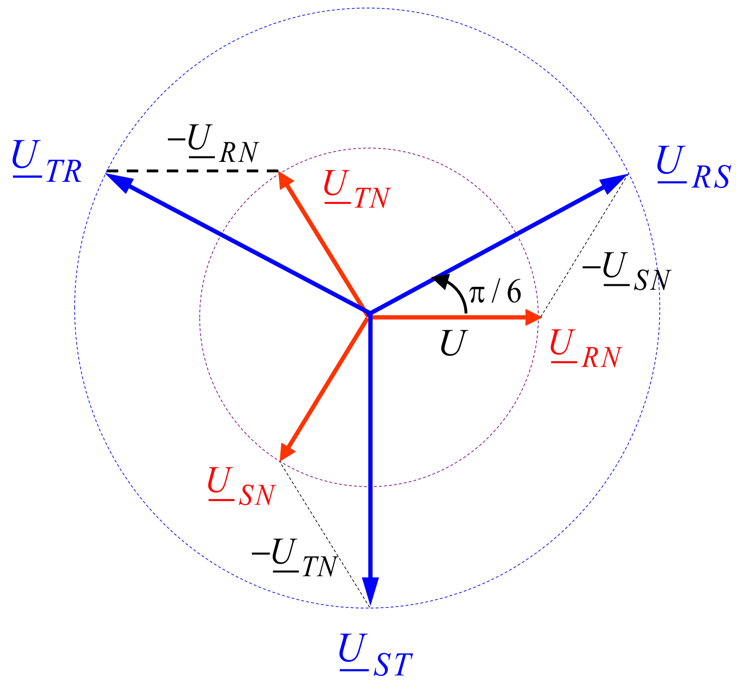
$$\underline{U}_{RS}, \underline{U}_{ST}, \underline{U}_{TR}$$

Relationships between phase voltages and line voltages

$$\underline{U}_{RS} = \underline{U}_{RN} - \underline{U}_{SN}$$
$$\underline{U}_{ST} = \underline{U}_{SN} - \underline{U}_{TN}$$
$$\underline{U}_{TR} = \underline{U}_{TN} - \underline{U}_{RN}$$

Three-phase source
Phase voltages and line voltages

$$\underline{U}_{RS} = \underline{U}_{RN} - \underline{U}_{SN}$$
$$\underline{U}_{ST} = \underline{U}_{SN} - \underline{U}_{TN}$$
$$\underline{U}_{TR} = \underline{U}_{TN} - \underline{U}_{RN}$$



$$\underline{U}_{RS} = \sqrt{3}Ue^{j\pi/6}$$
$$\underline{U}_{ST} = \sqrt{3}Ue^{-j\pi/2}$$
$$\underline{U}_{TR} = \sqrt{3}Ue^{+j5\pi/6}$$

Balanced three-phase system

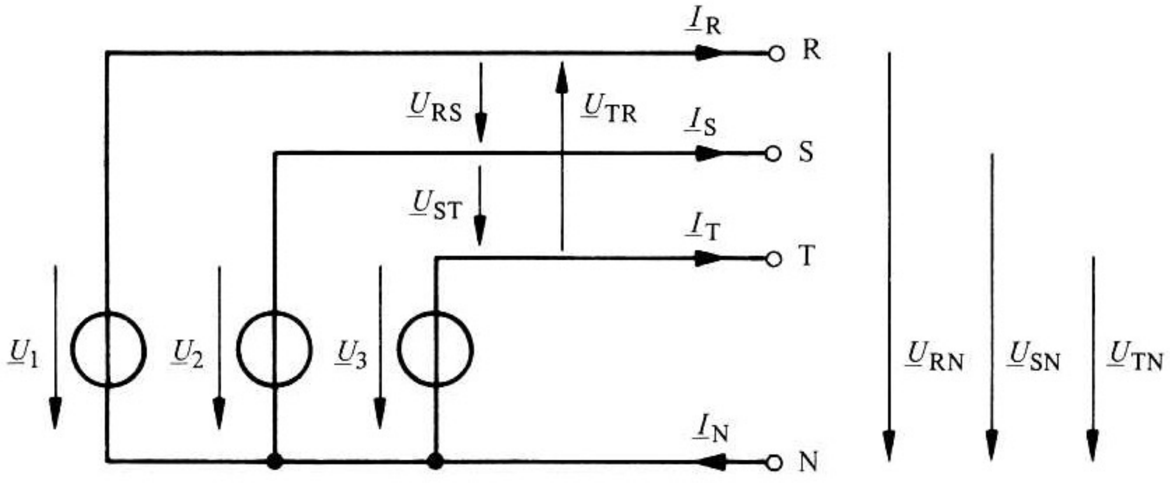
Compared to phase voltages :

- Ahead of 30º,
- Module is $\sqrt{3}$ times the one of phase voltages

Three-phase source
Line currents

Line currents:

$$\underline{I}_R, \underline{I}_S, \underline{I}_T$$



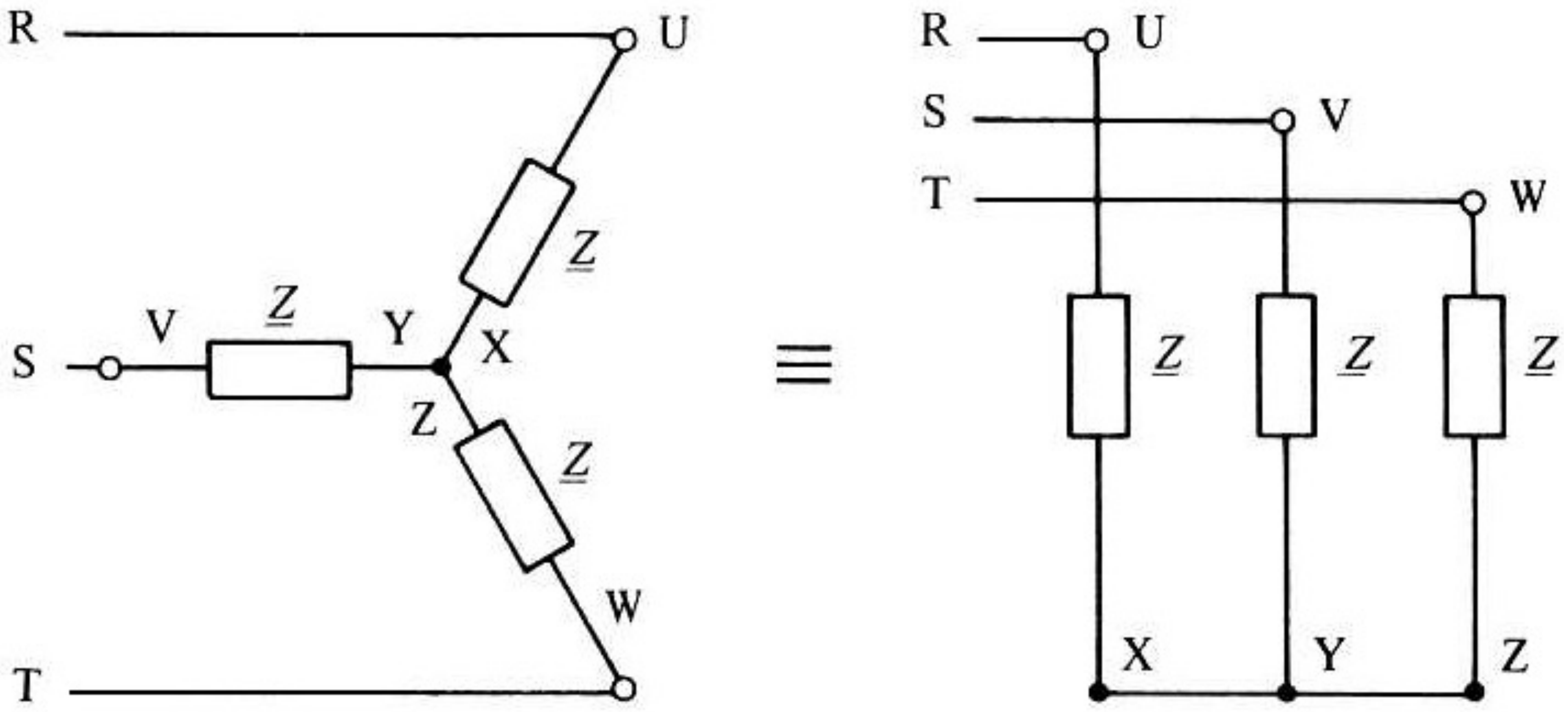
Return current:

$$\underline{I}_N = \underline{I}_R + \underline{I}_S + \underline{I}_T$$

Balanced three-phase load

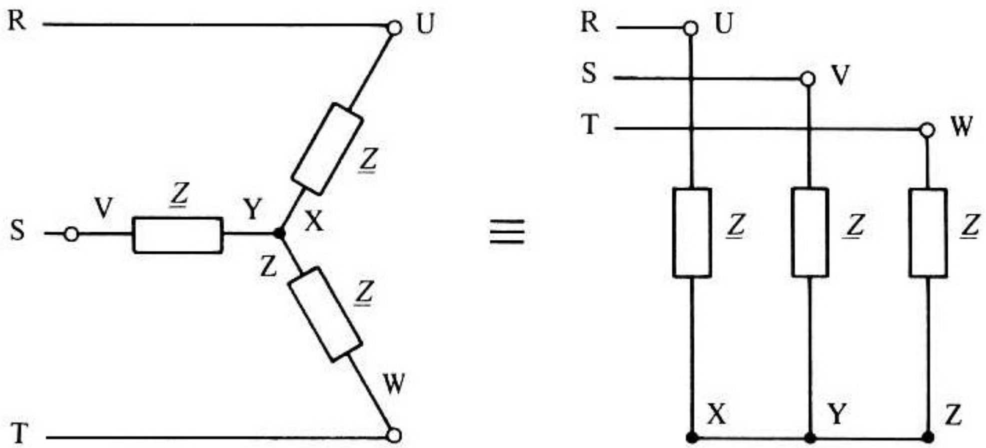
- A balanced three-phase load is characterized by three identical impedances (same module and argument) $\underline{Z} = Ze^{j\varphi}$ that we call the three phases of the user.
- The three impedances can be connected in ‘wye (star)’ or ‘delta (triangle)’.

Balanced three-phase load
Wye connection



Balanced three-phase load
Wye connection

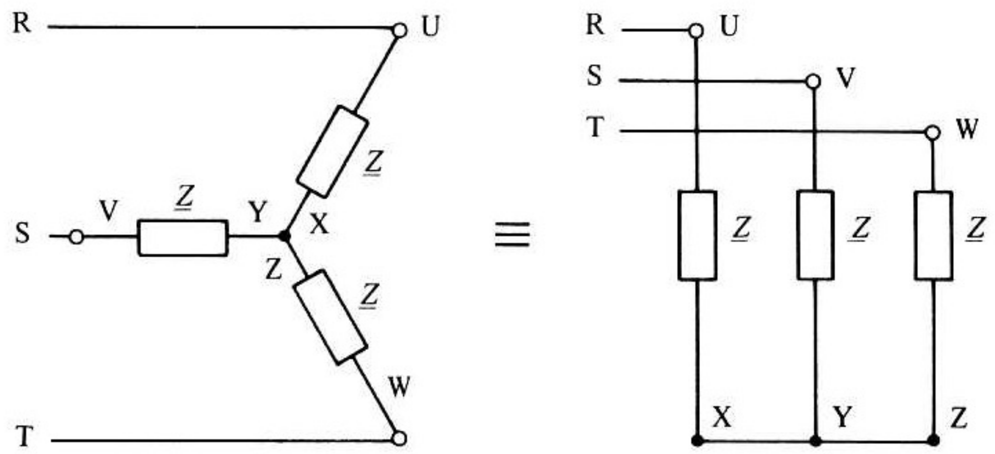
$$\underline{U}_{UX} = \underline{U}_{RN}$$
$$\underline{U}_{VY} = \underline{U}_{SN}$$
$$\underline{U}_{WZ} = \underline{U}_{TN}$$



Load voltages are the same as the line-to-neutral voltages

Balanced three-phase load
Wye connection

Load currents:



$$\underline{I}_{UX} = \frac{\underline{U}_{UX}}{\underline{Z}} = \frac{\underline{U}_{RN}}{\underline{Z}} = \frac{U}{Z} e^{j(\alpha-\varphi)}$$

$$\underline{I}_{VY} = \frac{\underline{U}_{VY}}{\underline{Z}} = \frac{\underline{U}_{SN}}{\underline{Z}} = \frac{U}{Z} e^{j(\alpha-\varphi-2\pi/3)}$$

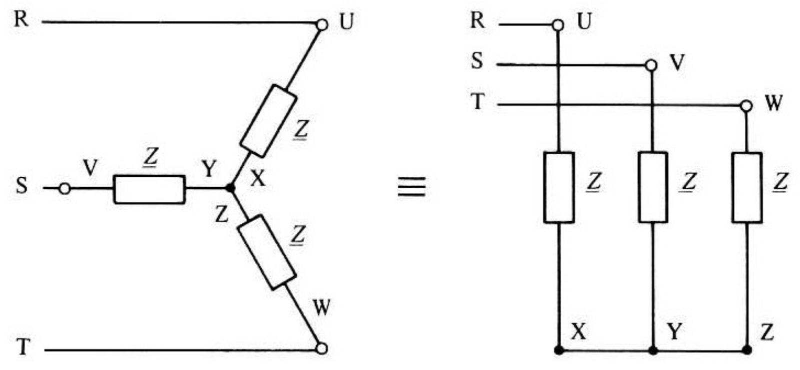
$$\underline{I}_{WZ} = \frac{\underline{U}_{WZ}}{\underline{Z}} = \frac{\underline{U}_{TN}}{\underline{Z}} = \frac{U}{Z} e^{j(\alpha-\varphi+2\pi/3)}$$

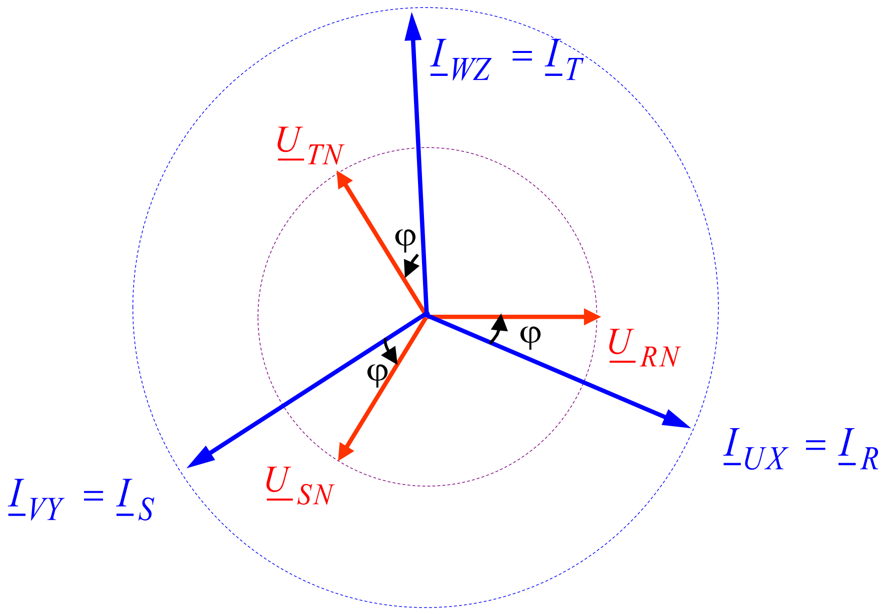
Balanced three-phase load
Wye connection

$$\underline{I}_{UX} = \frac{\underline{U}_{UX}}{\underline{Z}} = \frac{\underline{U}_{RN}}{\underline{Z}} = \frac{U}{Z} e^{j(\alpha-\varphi)}$$

$$\underline{I}_{VY} = \frac{\underline{U}_{VY}}{\underline{Z}} = \frac{\underline{U}_{SN}}{\underline{Z}} = \frac{U}{Z} e^{j(\alpha-\varphi-2\pi/3)}$$

$$\underline{I}_{WZ} = \frac{\underline{U}_{WZ}}{\underline{Z}} = \frac{\underline{U}_{TN}}{\underline{Z}} = \frac{U}{Z} e^{j(\alpha-\varphi+2\pi/3)}$$



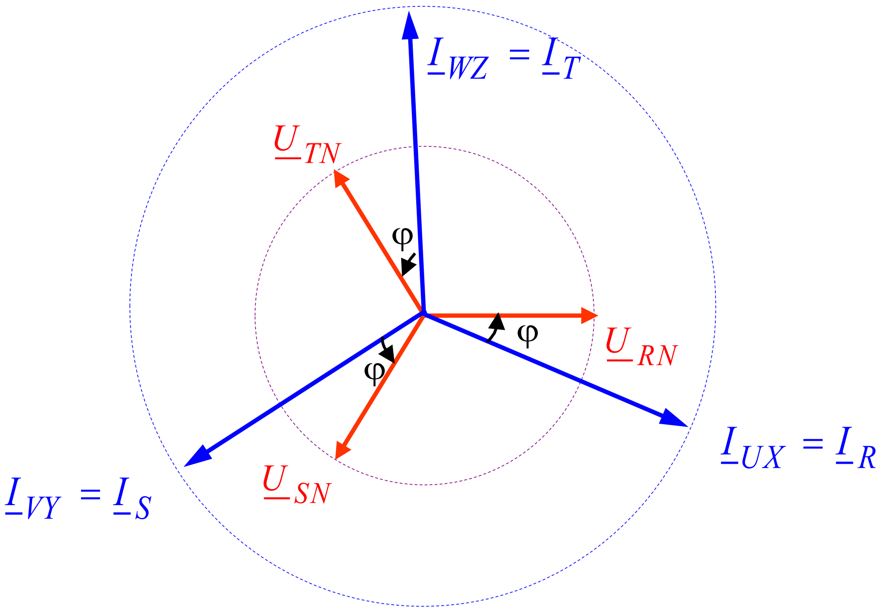


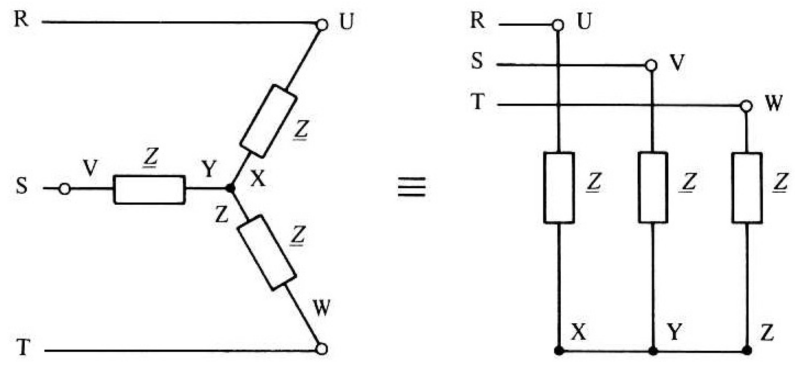
$$\underline{I}_R = \underline{I}_{UX} \qquad \underline{I}_S = \underline{I}_{VY} \qquad \underline{I}_T = \underline{I}_{WZ}$$

The line current is equal to
load current and they have as module:

$$I_l = I_{load} = \frac{U}{Z}$$

Balanced three-phase load
Wye connection

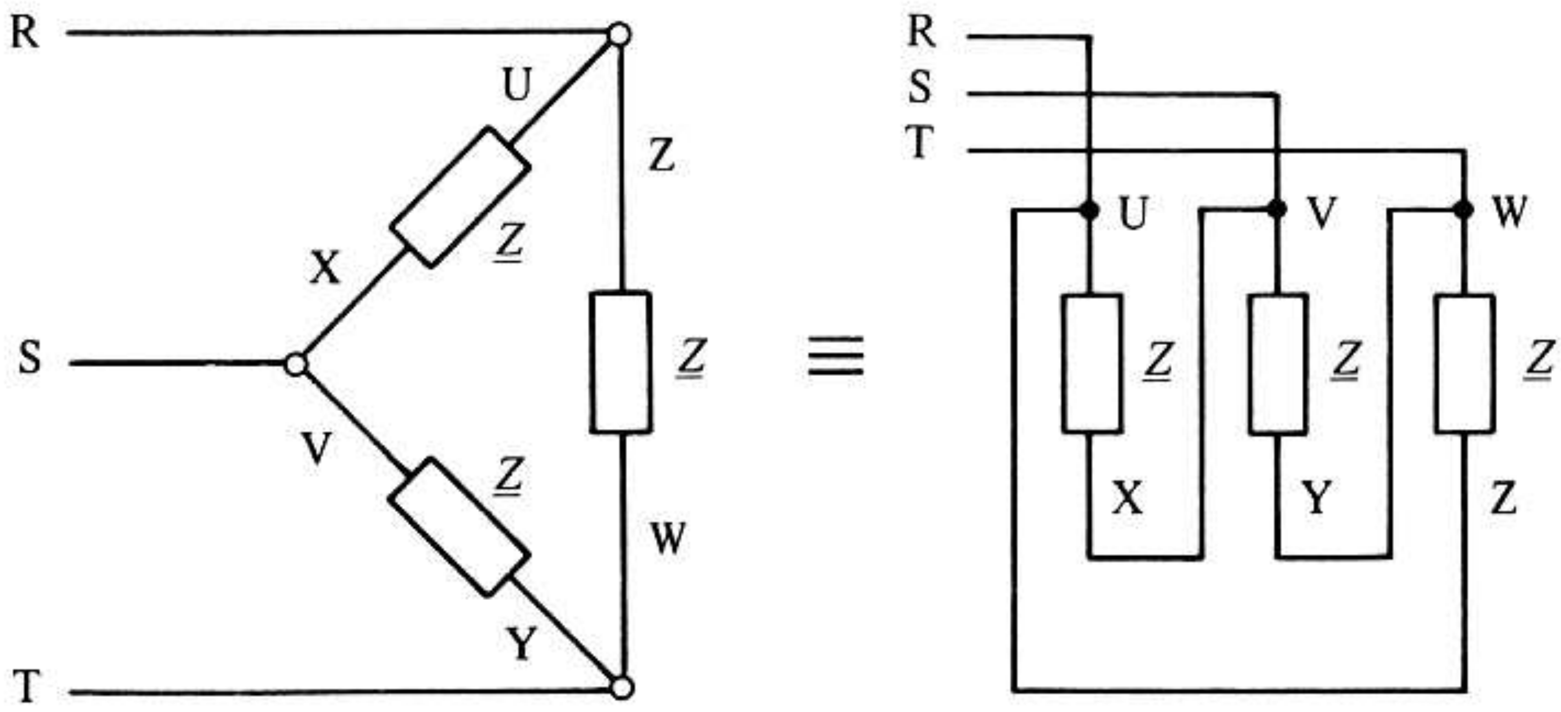




$$\underline{I}_R + \underline{I}_S + \underline{I}_T = 0$$

➡ Thus in the case of a symmetrical source with balanced load, it is not necessary to connect the neutral point of the load to the one of the source.

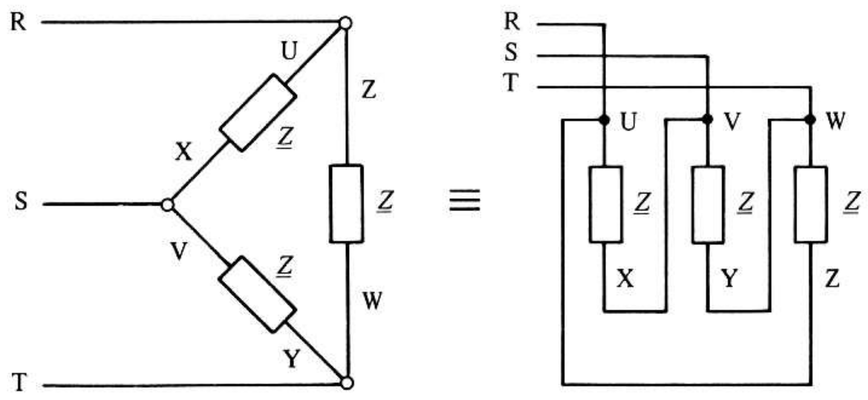
Balanced three-phase load
Delta connection



Balanced three-phase load
Delta connection

Phase voltages:

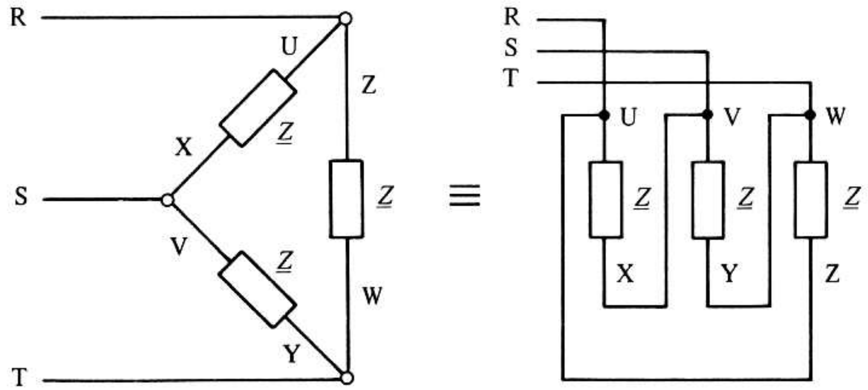
$$\underline{U}_{UX} = \underline{U}_{RS}$$
$$\underline{U}_{VY} = \underline{U}_{ST}$$
$$\underline{U}_{WZ} = \underline{U}_{TR}$$



Load voltages are the same as line voltages

Balanced three-phase load
Delta connection

Load currents:



$$\underline{I}_{UX} = \frac{\underline{U}_{UX}}{\underline{Z}} = \frac{\underline{U}_{RS}}{\underline{Z}} = \frac{\sqrt{3}U}{Z} e^{j(\alpha - \varphi + \pi/6)}$$

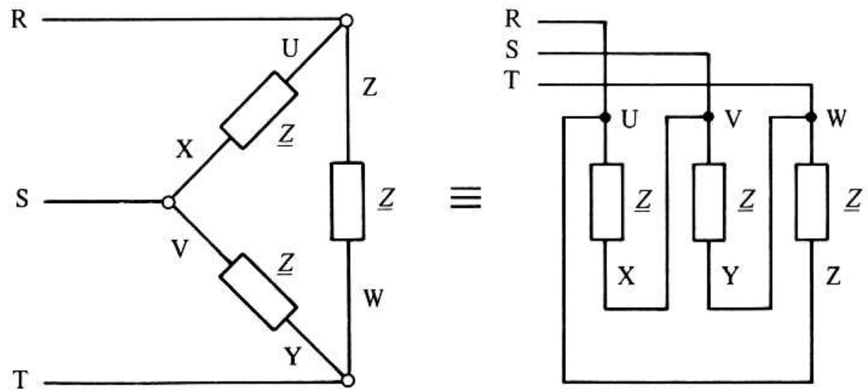
$$\underline{I}_{VY} = \frac{\underline{U}_{VY}}{\underline{Z}} = \frac{\underline{U}_{ST}}{\underline{Z}} = \frac{\sqrt{3}U}{Z} e^{j(\alpha - \varphi - \pi/2)}$$

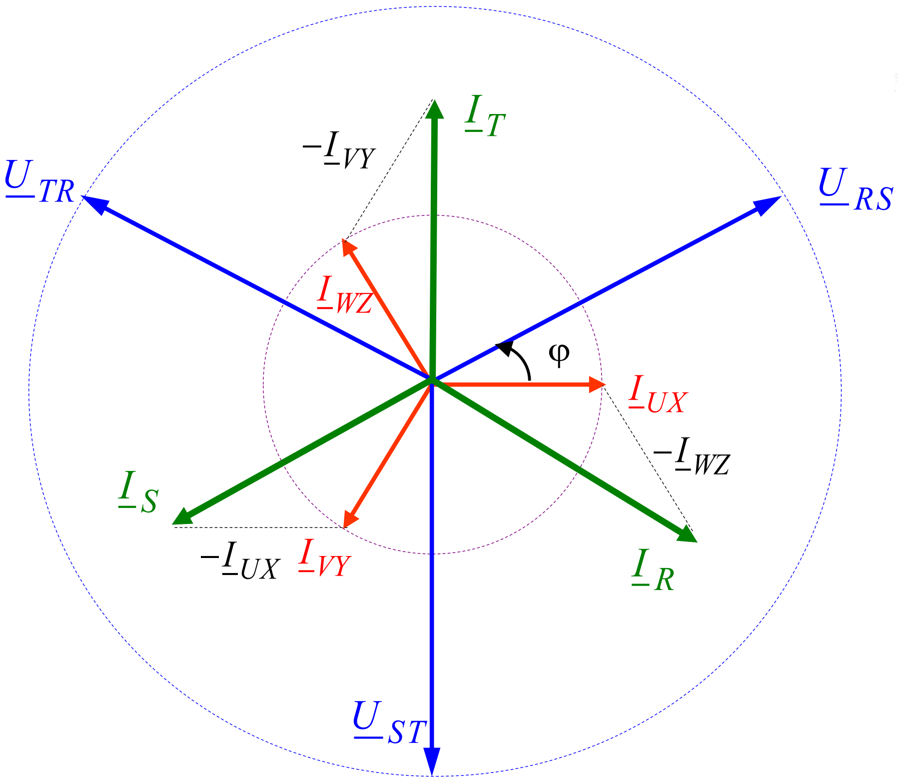
$$\underline{I}_{WZ} = \frac{\underline{U}_{WZ}}{\underline{Z}} = \frac{\underline{U}_{TR}}{\underline{Z}} = \frac{\sqrt{3}U}{Z} e^{j(\alpha - \varphi + 5\pi/6)}$$

Balanced three-phase load
Delta connection

Line currents:

$$\underline{I}_R = \underline{I}_{UX} - \underline{I}_{WZ}$$
$$\underline{I}_S = \underline{I}_{VY} - \underline{I}_{UX}$$
$$\underline{I}_T = \underline{I}_{WZ} - \underline{I}_{VY}$$



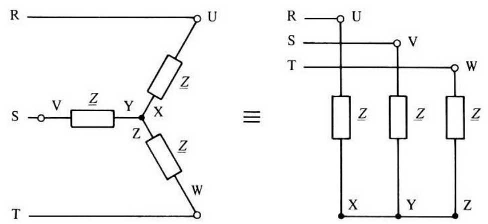


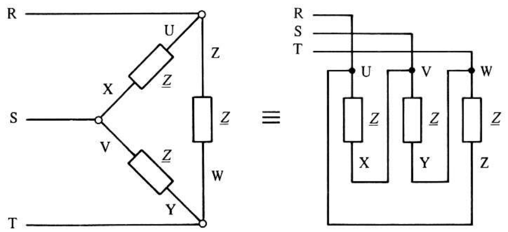
$$\underline{I}_R = \sqrt{3} I_{load} e^{j(\alpha-\phi)}$$
$$\underline{I}_S = \sqrt{3} I_{load} e^{j(\alpha-\phi-2\pi/3)}$$
$$\underline{I}_T = \sqrt{3} I_{load} e^{j(\alpha-\phi+2\pi/3)}$$

with $I_{load} = \sqrt{3} U / Z$

$$I_l = \sqrt{3} I_{load} = 3U / Z$$

Summary
Load quantities expressed in terms of line quantities





$$U_{load-Y} = \frac{U_l}{\sqrt{3}}$$

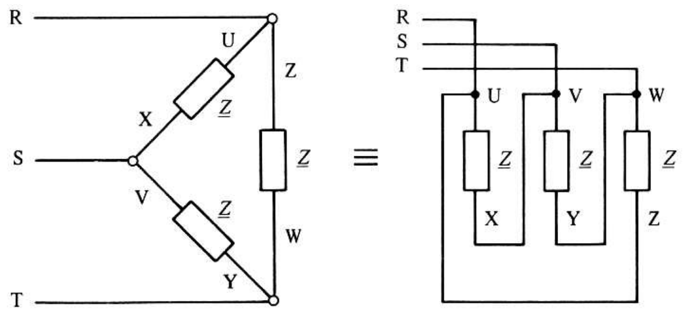
$$I_{load-Y} = I_l$$

$$U_{load-\Delta} = U_l$$

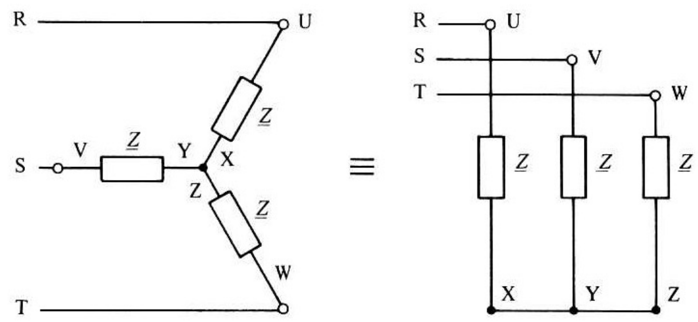
$$I_{load-\Delta} = \frac{I_l}{\sqrt{3}}$$

Power in a three-phase systems

Delta load



Wye load



Instantaneous power:

$$p(t) = u_{UX}(t)i_{UX}(t) + u_{VY}(t)i_{VY}(t) + u_{WZ}(t)i_{WZ}(t)$$

Active and reactive powers:

$$P = U_{UX} I_{UX} \cos\varphi_{UX} + U_{VY} I_{VY} \cos\varphi_{VY} + U_{WZ} I_{XZ} \cos\varphi_{WZ}$$

$$Q = U_{UX} I_{UX} \sin\varphi_{UX} + U_{VY} I_{VY} \sin\varphi_{VY} + U_{WZ} I_{XZ} \sin\varphi_{WZ}$$

Power in balanced three-phase systems

Phase UX:

$$u_{UX}(t) = U_{load}\sqrt{2}\cos(\omega t + \alpha)$$

$$i_{UX}(t) = I_{load}\sqrt{2}\cos(\omega t + \alpha - \phi)$$

Phase VY:

$$u_{VY}(t) = U_{load}\sqrt{2}\cos(\omega t + \alpha - 2\pi/3)$$

$$i_{VY}(t) = I_{load}\sqrt{2}\cos(\omega t + \alpha - \phi - 2\pi/3)$$

Phase WZ:

$$u_{WZ}(t) = U_{load}\sqrt{2}\cos(\omega t + \alpha + 2\pi/3)$$

$$i_{WZ}(t) = I_{load}\sqrt{2}\cos(\omega t + \alpha - \phi + 2\pi/3)$$

$$p(t) = u_{UX}(t)i_{UX}(t) + u_{VY}(t)i_{VY}(t) + u_{WZ}(t)i_{WZ}(t)$$

Power in balanced three-phase systems

$$p(t) = u_{UX}(t)i_{UX}(t) + u_{VY}(t)i_{VY}(t) + u_{WZ}(t)i_{WZ}(t)$$

$$= 3U_{load}I_{load}\cos\phi + U_{load}I_{load}\underbrace{\Big[\cos(2\omega t + 2\alpha - \phi) + \cos(2\omega t + 2\alpha - \phi + 2\pi/3) + \cos(2\omega t + 2\alpha - \phi - 2\pi/3)\Big]}_{=0}$$

$$p(t) = P = 3U_{load}I_{load}\cos\phi$$

Instantaneous power has no pulsed component and it is equal to the active power.

Power in balanced three-phase systems

Active, reactive and apparent power

$$P = 3U_{load} I_{load} \cos\phi$$

In the same way:

$$Q = 3U_{load} I_{load} \sin\phi$$

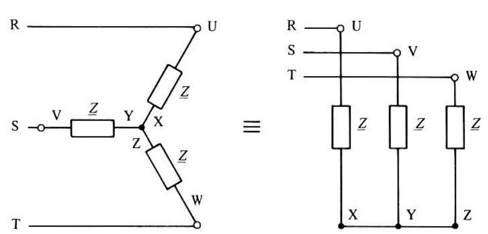
$$S = 3U_{load} I_{load}$$

U_{load} and I_{load} being the rms values of the phase voltages and phase currents

Complex apparent power: $\underline{S} = 3U_{load} I_{load} e^{j\phi}$

Active, reactive and apparent power

The powers expressed according to the line quantities:

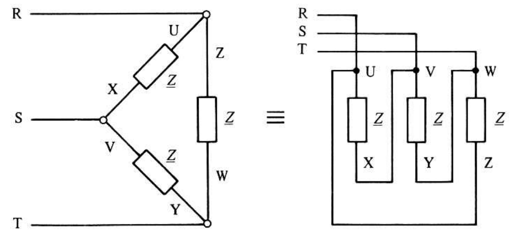


$$I_{l\mathrm{Y}} = I_{load\,\mathrm{Y}} \qquad U_l = \sqrt{3}U_{load\,\mathrm{Y}}$$

$$P_{\mathrm{Y}} = \sqrt{3}U_l I_{l\mathrm{Y}} \cos\varphi$$

$$Q_{\mathrm{Y}} = \sqrt{3}U_l I_{l\mathrm{Y}} \sin\varphi$$

$$S_{\mathrm{Y}} = \sqrt{3}U_l I_{l\mathrm{Y}}$$



$$U_l = U_{load\,\Delta} \qquad I_{l\Delta} = \sqrt{3}I_{load\,\Delta}$$

$$P_{\Delta} = \sqrt{3}U_l I_{l\Delta} \cos\varphi$$

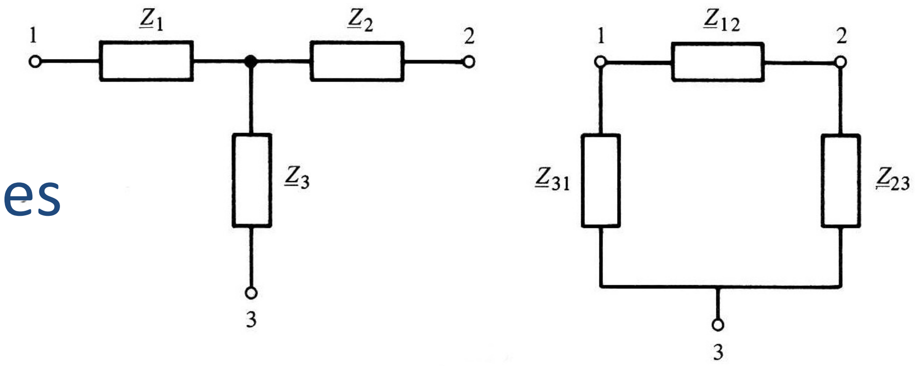
$$Q_{\Delta} = \sqrt{3}U_l I_{l\Delta} \sin\varphi$$

$$S_{\Delta} = \sqrt{3}U_l I_{l\Delta}$$

$$P_{\mathrm{Y}} = \frac{1}{3}P_{\Delta}$$

Delta-Wye conversion

Reminder: Equivalent tripoles



$$\underline{Z}_{12} = \frac{\underline{Z}_1\underline{Z}_2 + \underline{Z}_2\underline{Z}_3 + \underline{Z}_3\underline{Z}_1}{\underline{Z}_3}$$

$$\underline{Z}_{23} = \frac{\underline{Z}_1\underline{Z}_2 + \underline{Z}_2\underline{Z}_3 + \underline{Z}_3\underline{Z}_1}{\underline{Z}_1}$$

$$\underline{Z}_{31} = \frac{\underline{Z}_1\underline{Z}_2 + \underline{Z}_2\underline{Z}_3 + \underline{Z}_3\underline{Z}_1}{\underline{Z}_2}$$

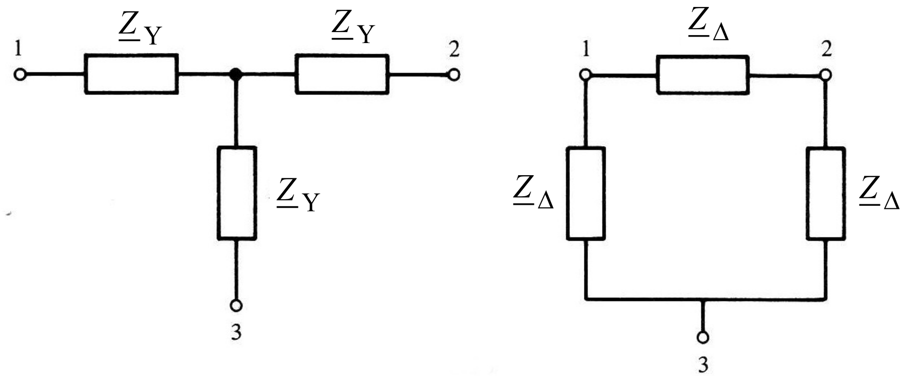
$$\underline{Z}_1 = \frac{\underline{Z}_{12}\underline{Z}_{31}}{\underline{Z}_{12} + \underline{Z}_{23} + \underline{Z}_{31}}$$

$$\underline{Z}_2 = \frac{\underline{Z}_{12}\underline{Z}_{23}}{\underline{Z}_{12} + \underline{Z}_{23} + \underline{Z}_{31}}$$

$$\underline{Z}_3 = \frac{\underline{Z}_{23}\underline{Z}_{31}}{\underline{Z}_{12} + \underline{Z}_{23} + \underline{Z}_{31}}$$

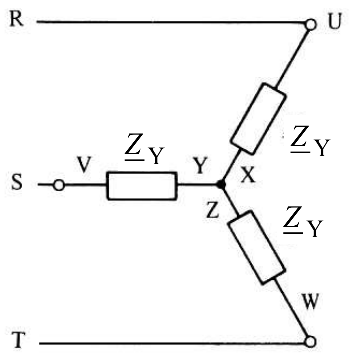
Delta-Wye conversion

Equivalent tripoles
(equal impedances)



$$\underline{Z}_Y = \frac{\underline{Z}_\Delta}{3} \quad \Leftrightarrow \quad \underline{Z}_\Delta = 3\underline{Z}_Y$$

Delta-Wye conversion



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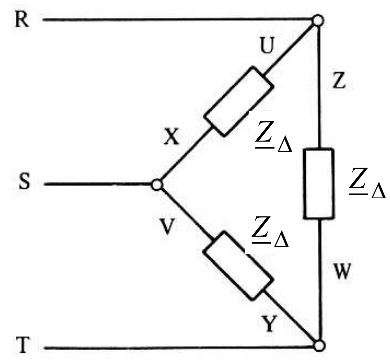


Both charges are equivalent provided that:

$$\underline{Z}_Y = \frac{\underline{Z}_\Delta}{3} \quad \text{ou} \quad \underline{Z}_\Delta = 3\underline{Z}_Y$$

Case of a capacitor bank:

$$\frac{1}{\omega C_Y} = \frac{1}{3}\frac{1}{\omega C_\Delta} \Rightarrow C_Y = 3C_\Delta$$

Example 1

A balanced three-phase load is connected in wye on a direct three-phase network at 6kV. The active power consumed is 48 kW, with a power factor of 0.94. Calculate the rms value of the line current. Repeat the calculation for the case where the load is connected in delta (with the same active power and the same power factor) and determine the rms value of the current flowing in the user's phases.

Example 2

The phase impedance of a balanced three-phase load is formed of a resistor R = 10 Ω in series with a capacitor C = 185 µF. This load is connected in star to a 50 Hz network whose line voltage is 380 V. Determine the load voltage, the load current and the phase shift. Also calculate the active and reactive powers absorbed by each load and by the total load.

Example 3

The active power and the apparent power consumed by a user having a balanced three-phase load are known:
$P = 20$ kW and $S = 30$ kVA. The supply voltage (line) is 500 V. It is asked to calculate the impedance Z for the two connection modes (wye and delta).

Example 4

A balanced three-phase load in delta is formed by putting in series a resistor R = 15 Ω and an inductor L = 32 mH. The network supply is 380 V (line voltage). Determine the active power P supplied by the network.

Unbalanced three-phase systems

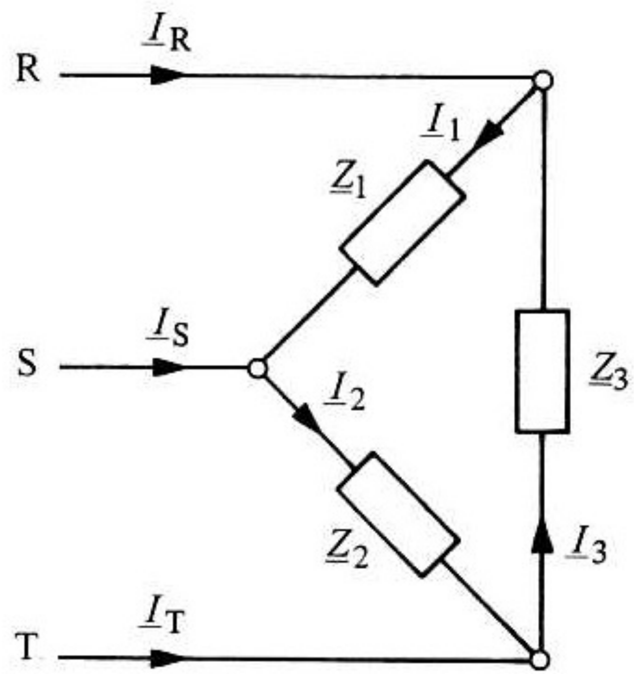
- An unbalanced state occurs when the impedances of the load are not identical.
- Such a situation is caused by the connection of single-phase loads connected either between a phase conductor and the neutral conductor, or between two phase conductors.
- It can also intervene in case of disturbances such as short-circuit, lightning strike on a phase conductor, etc.

Unbalanced three-phase systems
Delta load

$$\underline{I}_1 = \frac{\underline{U}_{RS}}{\underline{Z}_1}$$

$$\underline{I}_2 = \frac{\underline{U}_{ST}}{\underline{Z}_2}$$

$$\underline{I}_3 = \frac{\underline{U}_{TR}}{\underline{Z}_3}$$



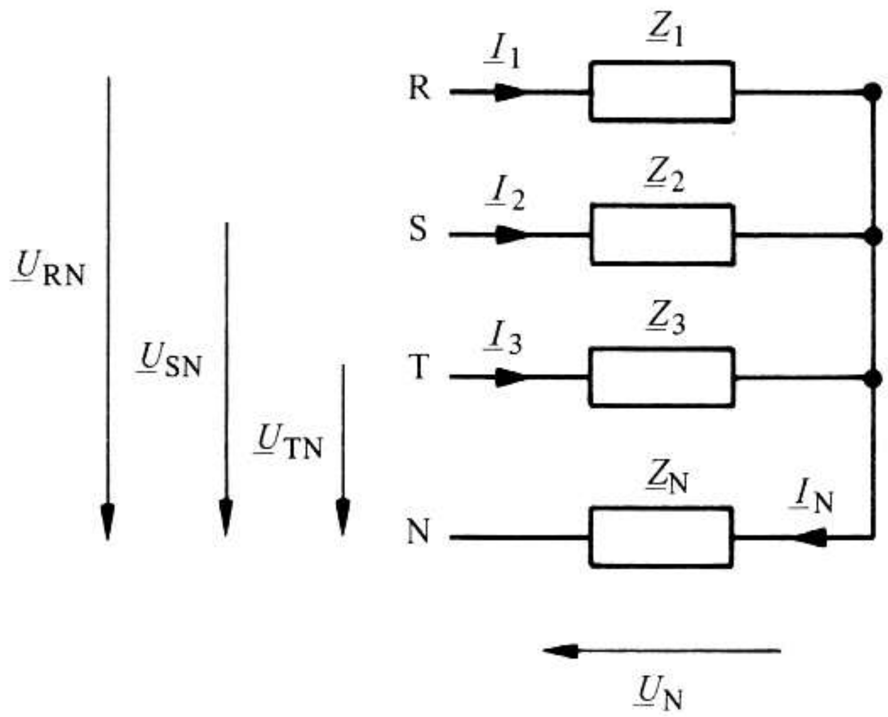
$$\underline{I}_R = \underline{I}_1 - \underline{I}_3 \qquad \underline{I}_S = \underline{I}_2 - \underline{I}_1 \qquad \underline{I}_T = \underline{I}_3 - \underline{I}_2$$

Unbalanced three-phase systems
Wye load

$$\underline{I}_R = \underline{I}_1 = \frac{\underline{U}_{RN} - \underline{U}_N}{\underline{Z}_1}$$

$$\underline{I}_S = \underline{I}_2 = \frac{\underline{U}_{SN} - \underline{U}_N}{\underline{Z}_2}$$

$$\underline{I}_T = \underline{I}_3 = \frac{\underline{U}_{TN} - \underline{U}_N}{\underline{Z}_3}$$

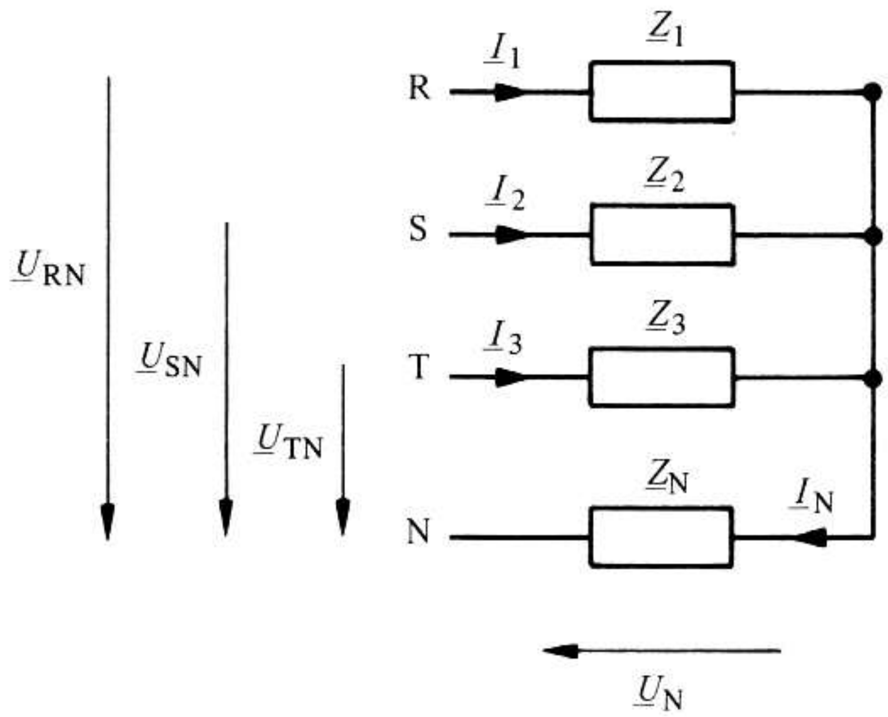


And the return current: $$\underline{I}_N = \frac{\underline{U}_N}{\underline{Z}_N}$$

Unbalanced three-phase systems
Wye load

$$\underline{I}_N = \frac{\underline{U}_N}{\underline{Z}_N}$$

On the other hand, from Kirchhoff's law on currents :



$$\underline{I}_N = \underline{I}_R + \underline{I}_S + \underline{I}_T$$

$$= \frac{\underline{U}_{RN}}{\underline{Z}_1} + \frac{\underline{U}_{SN}}{\underline{Z}_2} + \frac{\underline{U}_{TN}}{\underline{Z}_3} - \underline{U}_N \left(\frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} + \frac{1}{\underline{Z}_3} \right)$$

Unbalanced three-phase systems
Wye load

By eliminating $\underline{I}_N$ in the last two equations, we obtain the expression of the voltage $\underline{U}_N$:

$$\underline{U}_N = \underline{Z}_p \underline{I}_{N0}$$

with:

$$\frac{1}{\underline{Z}_p} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} + \frac{1}{\underline{Z}_3} + \frac{1}{\underline{Z}_N} \quad \text{et} \quad \underline{I}_{N0} = \frac{\underline{U}_{RN}}{\underline{Z}_1} + \frac{\underline{U}_{SN}}{\underline{Z}_2} + \frac{\underline{U}_{TN}}{\underline{Z}_3}$$

The impedance $\underline{Z}_p$ is equivalent to putting in parallel all impedances of the charge, including the neutral conductor. The current $\underline{I}_{N0}$ represents the return current in the neutral that we would observe if the impedance $\underline{Z}_N$ of the neutral was zero.

Unbalanced three-phase systems
Wye load

In summary, we first calculate:

$$\frac{1}{\underline{Z}_p} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} + \frac{1}{\underline{Z}_3} + \frac{1}{\underline{Z}_N} \quad \text{et} \quad \underline{I}_{N0} = \frac{\underline{U}_{RN}}{\underline{Z}_1} + \frac{\underline{U}_{SN}}{\underline{Z}_2} + \frac{\underline{U}_{TN}}{\underline{Z}_3}$$

then:

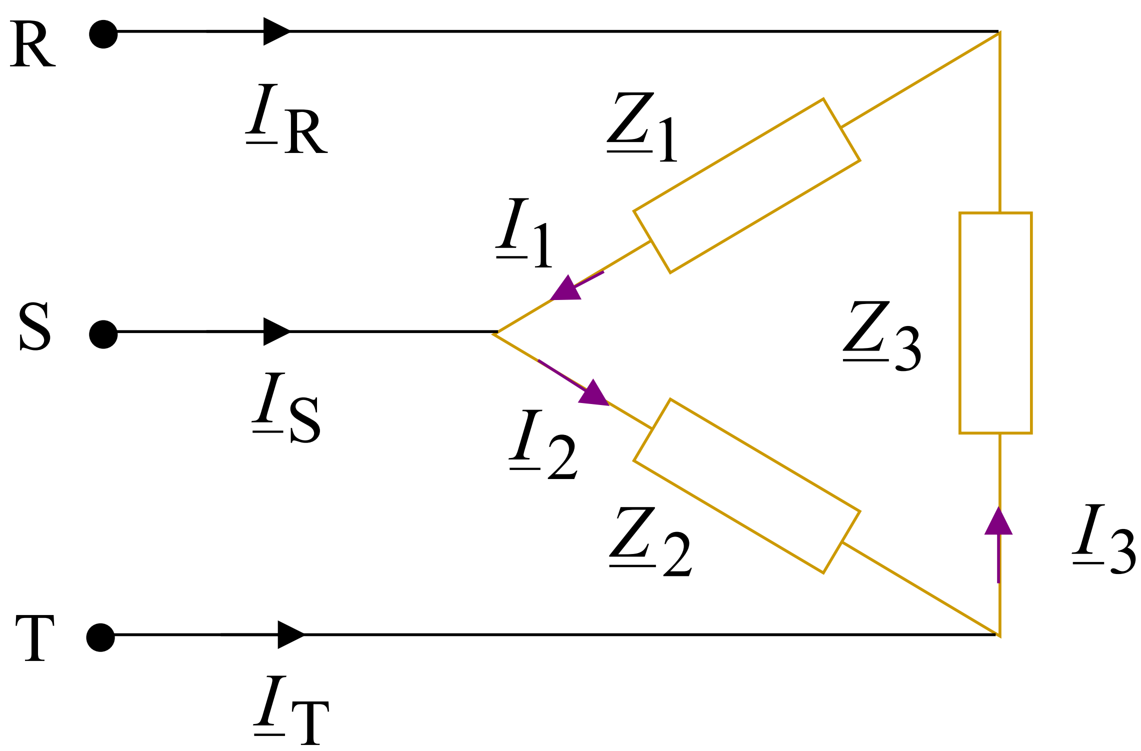
$$\underline{U}_N = \underline{Z}_p \underline{I}_{N0} \quad \text{et} \quad \underline{I}_N = \frac{\underline{U}_N}{\underline{Z}_N}$$

and finally:

$$\underline{I}_R = \frac{\underline{U}_{RN} - \underline{U}_N}{\underline{Z}_1} \qquad \underline{I}_S = \frac{\underline{U}_{SN} - \underline{U}_N}{\underline{Z}_2} \qquad \underline{I}_T = \frac{\underline{U}_{TN} - \underline{U}_N}{\underline{Z}_3}$$

Example: Unbalanced three-phase circuit

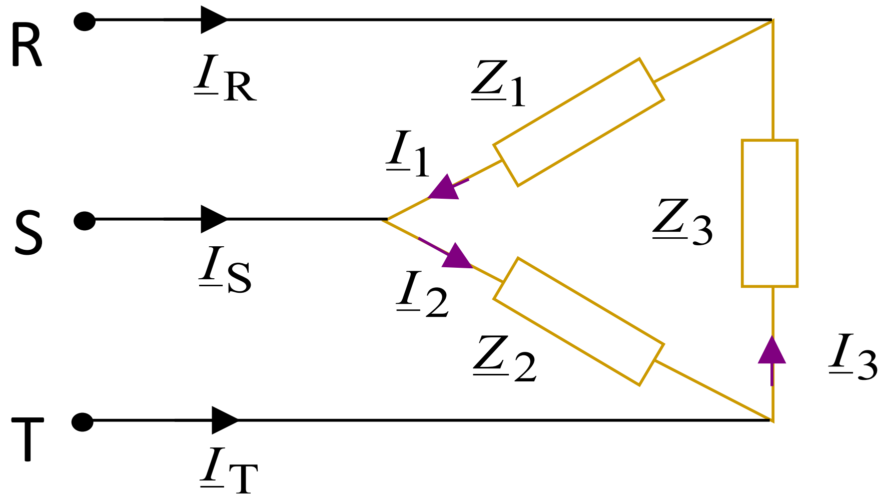
Between the line conductors R, S and T are connected two identical receivers whose active powers are P_1=P_2= 70 kW for a power factor of 0.92 (inductive). The third receiver connected between the T and R conductors has a power factor cos ϕ = 1 and an active power P_3 = 30.4 kW. Determine the active power of the circuit, as well as the currents $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$ and $\underline{I}_R$, if the line voltages are equal to 380 V.



Example: Unbalanced three-phase circuit

Total active power:

$$P = P_1 + P_2 + P_3$$
$$= 70 + 70 + 30.4 = 170.4 \text{ kW}$$



$$\underline{Z}_1 = \underline{Z}_2$$

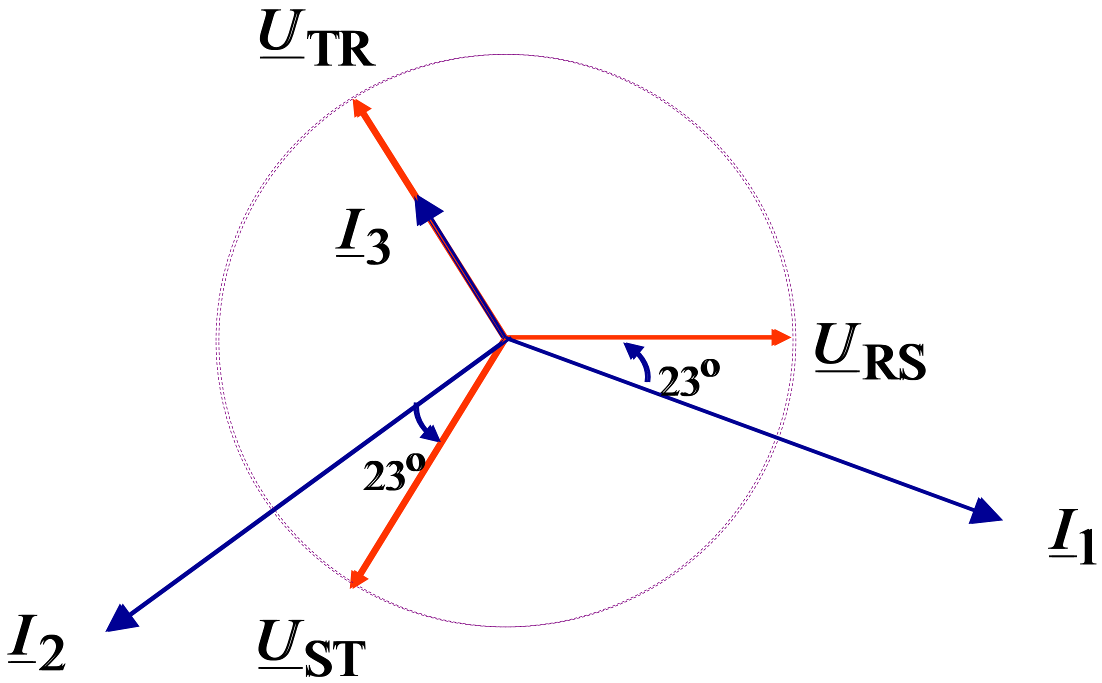
$$I_1 = I_2 = \frac{P_1}{U_{RS} \cos\varphi_1} = \frac{70 \cdot 10^3}{380 \cdot 0.92} = 200 \text{ A}$$

$$I_3 = \frac{P_3}{U_{TR} \cos\varphi_3} = \frac{30.4 \cdot 10^3}{380 \cdot 1} = 80 \text{ A}$$

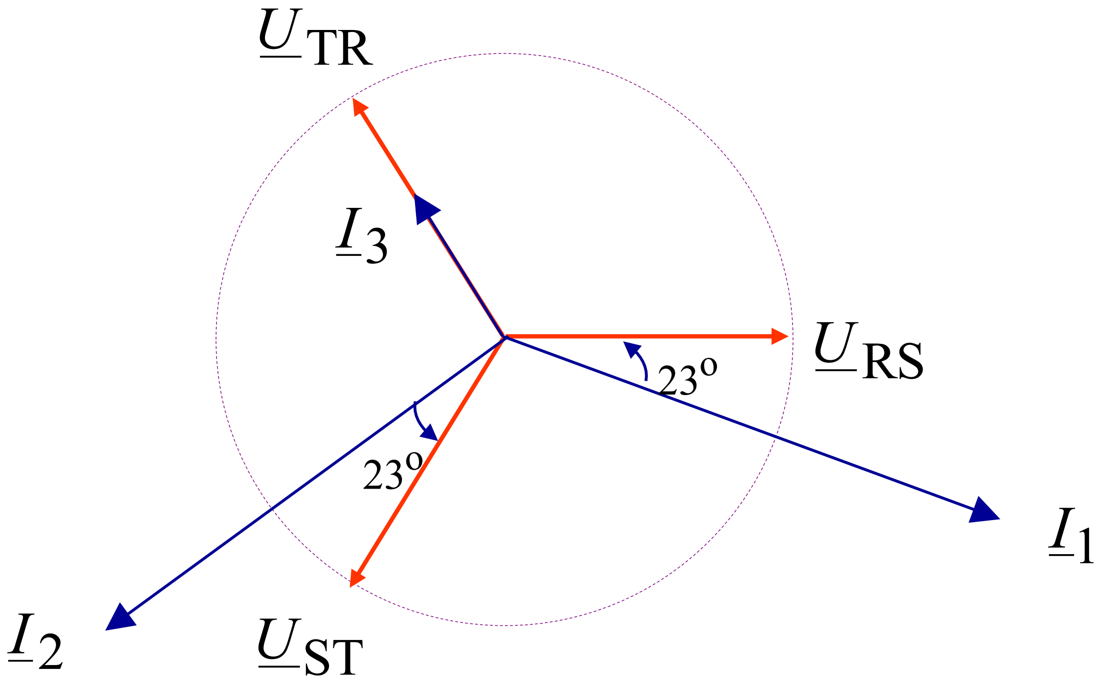
Example: Unbalanced three-phase circuit

$\underline{I}_1$ and $\underline{I}_2$ are in phase delay of an angle ϕ_1=arccos 0.92=23° relative to the corresponding voltages.

$\underline{I}_3$ is in phase with $\underline{U}_{TR}$



Example: Unbalanced three-phase circuit

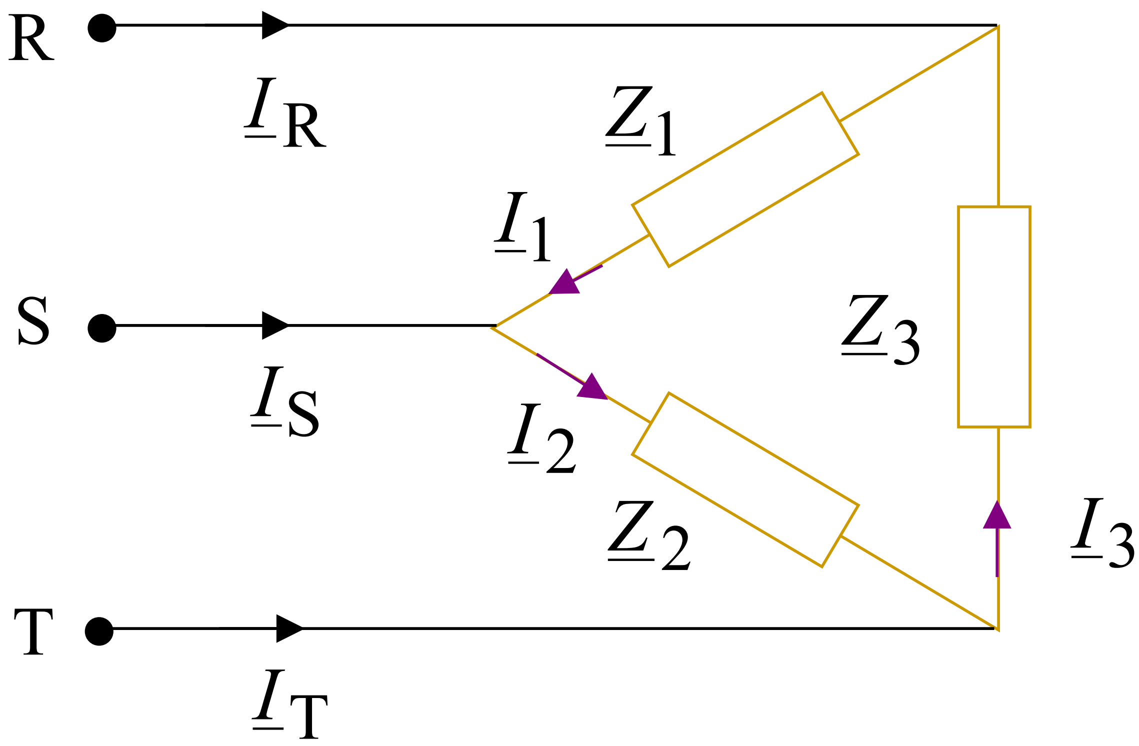


$$\underline{I}_1 = 200e^{-j23^\circ} = 184.1 - j78.1 \text{ A}$$

$$\underline{I}_2 = 200e^{-j(120+23)^\circ} = 200e^{-j143^\circ} = -159.7 - j120.4 \text{ A}$$

$$\underline{I}_3 = 80e^{j120^\circ} = -40 + j69.3 \text{ A}$$

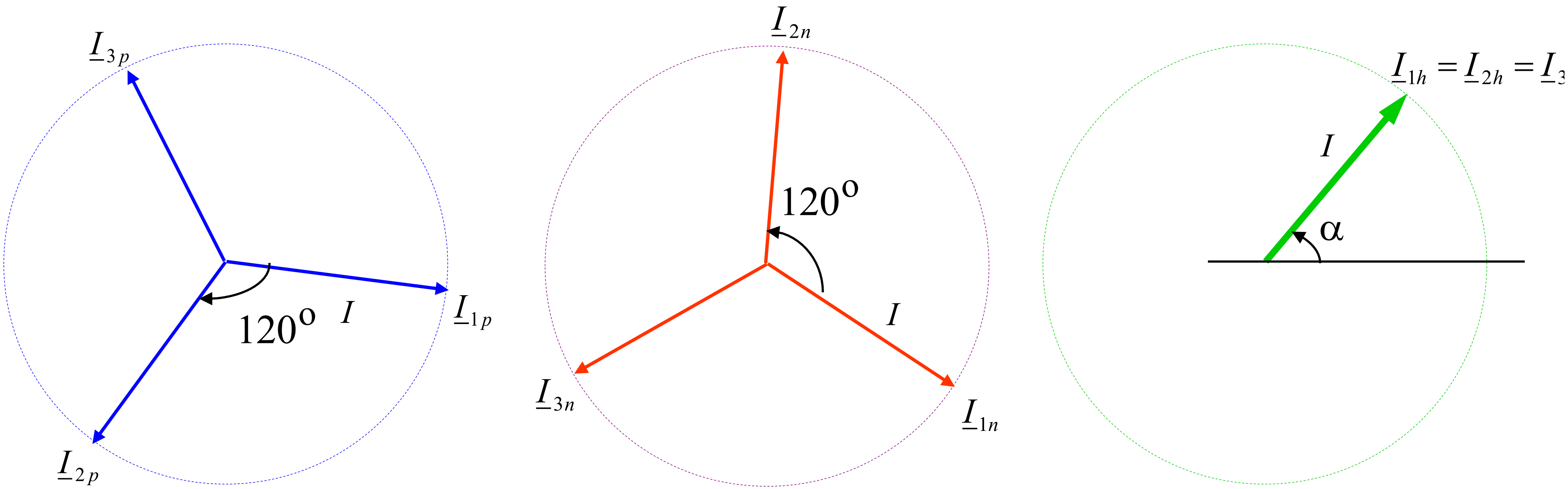
Example: Unbalanced three-phase circuit



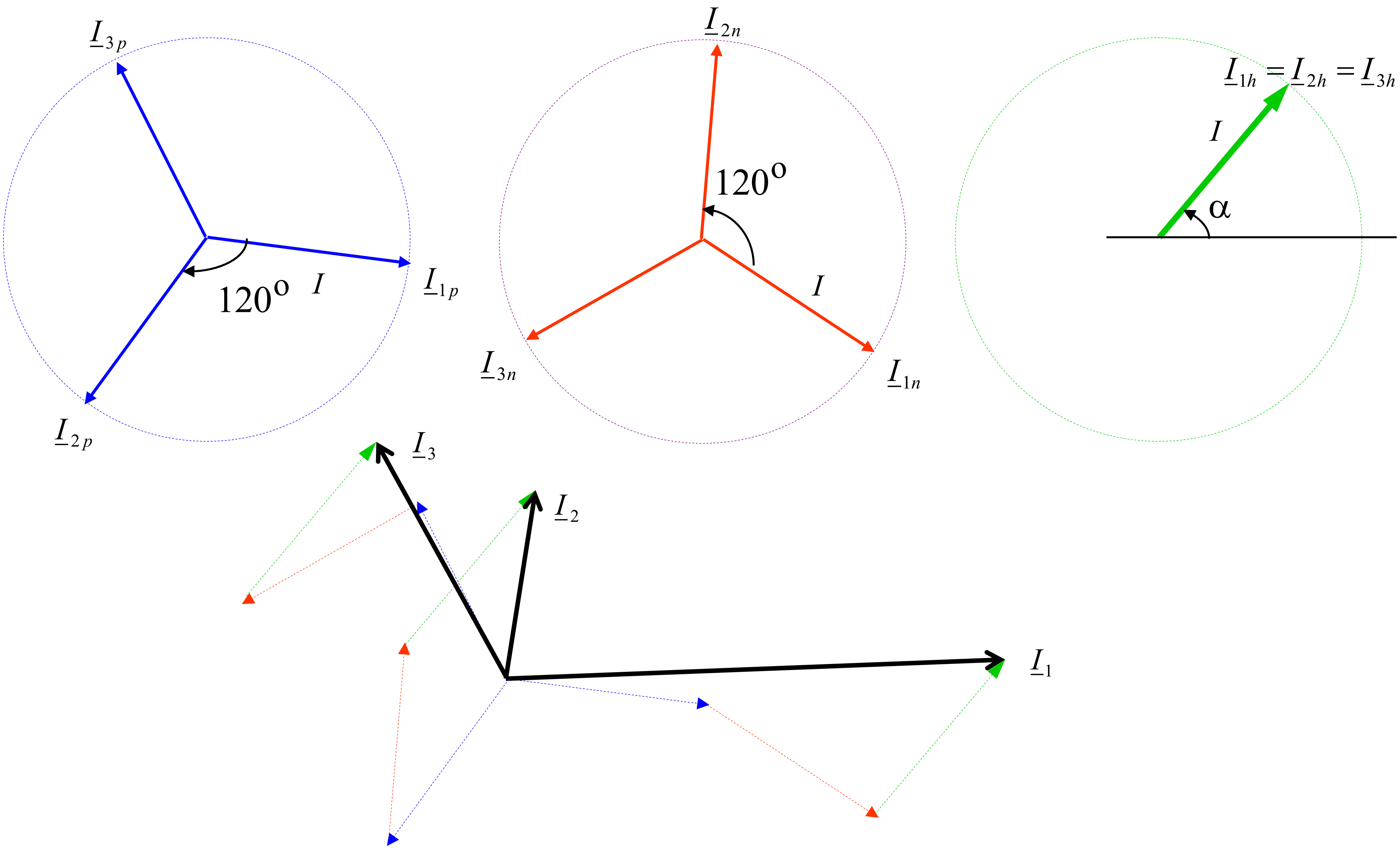
$$\underline{I}_R = \underline{I}_1 - \underline{I}_3 = 224.1 - j147.4 \text{ A} = 268.2 e^{-j33.3^o} \text{ A}$$

Symmetrical components method

- A non-symmetric system can be broken down into three symmetrical systems: a direct system, an inverse system and a homopolar system.



Symmetrical components method



Symmetrical components method

$$\underline{I}_1 = \underline{I}_{1p} + \underline{I}_{1n} + \underline{I}_{1h}$$

$$\underline{I}_2 = \underline{I}_{2p} + \underline{I}_{2n} + \underline{I}_{2h}$$

$$\underline{I}_3 = \underline{I}_{3p} + \underline{I}_{3n} + \underline{I}_{3h}$$

denoting $a = \exp(j2\pi/3)$

and noticing that $a^2 = \exp(j4\pi/3) = \exp(-j2\pi/3);\quad a^3 = 1$

the following relationships can be established

$$\underline{I}_{3p} = a\underline{I}_{1p};\quad \underline{I}_{2p} = a^2\underline{I}_{1p}$$

$$\underline{I}_{2n} = a\underline{I}_{1n};\quad \underline{I}_{3n} = a^2\underline{I}_{1n}$$

$$\underline{I}_{1h} = \underline{I}_{2h} = \underline{I}_{3h}$$

Symmetrical components method

Denoting for simplicity

$$\underline{I}_{1p} = \underline{I}_p, \underline{I}_{1n} = \underline{I}_n, \underline{I}_{1h} = \underline{I}_h$$

The system of equations is then written:

$$\underline{I}_1 = \underline{I}_p + \underline{I}_n + \underline{I}_h$$

$$\underline{I}_2 = a^2 \underline{I}_p + a\underline{I}_n + \underline{I}_h$$

$$\underline{I}_3 = a\underline{I}_p + a^2 \underline{I}_n + \underline{I}_h$$

In matrix form:

$$\begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} \underline{I}_p \\ \underline{I}_n \\ \underline{I}_h \end{bmatrix} \Longleftrightarrow \begin{bmatrix} \underline{I}_p \\ \underline{I}_n \\ \underline{I}_h \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_3 \end{bmatrix}$$

Symmetrical components method