

# Fundamentals of Electrical Circuits and Systems

## Chapter 1: Basic Concepts

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27.10 @ 1:15-3:00	lecture	Introduction, Basic concepts - circuit elements
27.10 @ 3:15-4:00	lecture	Kirchhoff's laws
27.10 @ 4:15-5:00	LTSpice	Video 1: Introduction to LTSpice + self exercise
03.11 @ 1:15-3:00	lecture	Fundamental Theorems
03.11 @ 3:15-5:00	problem set 1	Kirchhoff's law and fundamental theorems / LTSpice Exercise
10.11 @ 1:15-3:00	lecture	Nodal analysis
10.11 @ 3:15-5:00	lecture	Mesh analysis
17.11 @ 1:15-3:00	lecture	Sinusoidal regime 1: phasors, impedance, admittance
17.11 @ 3:15-5:00	problem set 2	Nodal and mesh analysis / LTSpice Exercise
24.11 @ 1:15-3:00	lecture	Sinusoidal regime 2: Thévenin/Norton equivalents
24.11 @ 3:15-5:00	lecture	Sinusoidal regime 3: Active and Reactive Power
01.12 @ 1:15-3:00	lecture	Three-phase circuits 1
01.12 @ 3:15-5:00	problem set 3	sinusoidal regime / LTSpice Exercise

08.12 @ 1:15-3:00	lecture	Three-phase circuits 2
08.12 @ 3:15-5:00	problem set 4	three-phase circuits / LTSpice Exercise
15.12 @ 1:15-3:00	lecture	Transient analysis
15.12 @ 3:15-5:00	problem set 5	transient analysis / LTSpice Exercise
22.12 @ 1:15-3:00	Q&A	

# Electrical circuits: Fundamentals

- What is an electrical circuit ?
- Unit systems
- Usual quantities
  - Charge, current, voltage
  - Power and energy
- Basic circuit elements
  - Resistance
  - Capacitance
  - Inductance
  - Voltage and current sources
- Kirchhoff's law
- Combination of elements
- Voltage / current dividers

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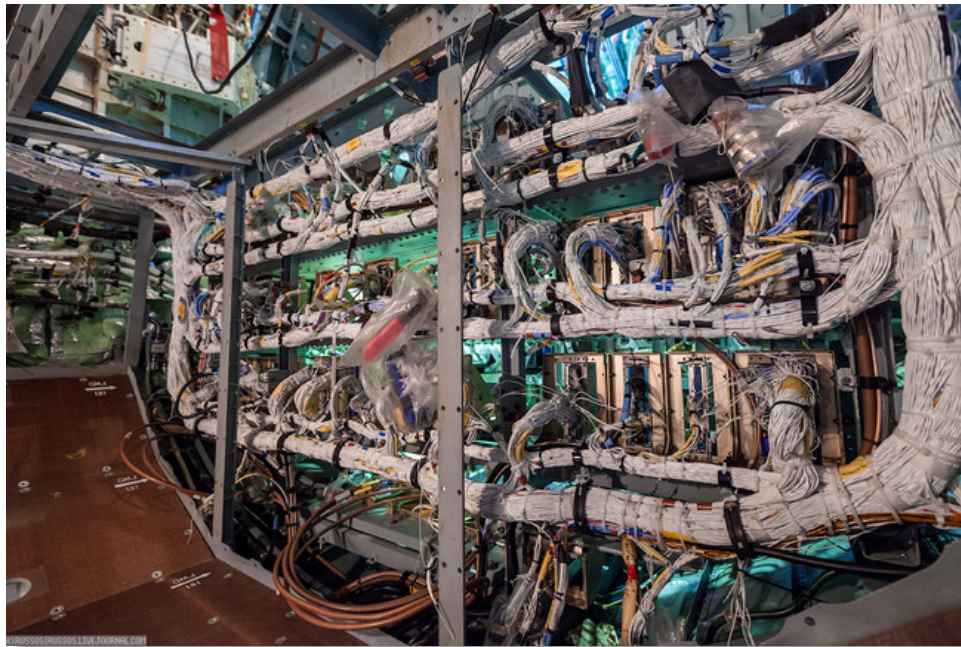
# What is an electrical circuit ?

- An electrical circuit allows
  - to transport energy from one point to another



# What is an electrical circuit ?

- An electrical circuit allows
  - to transport energy from one point to another
  - to transmit information



Wiring in an airplane

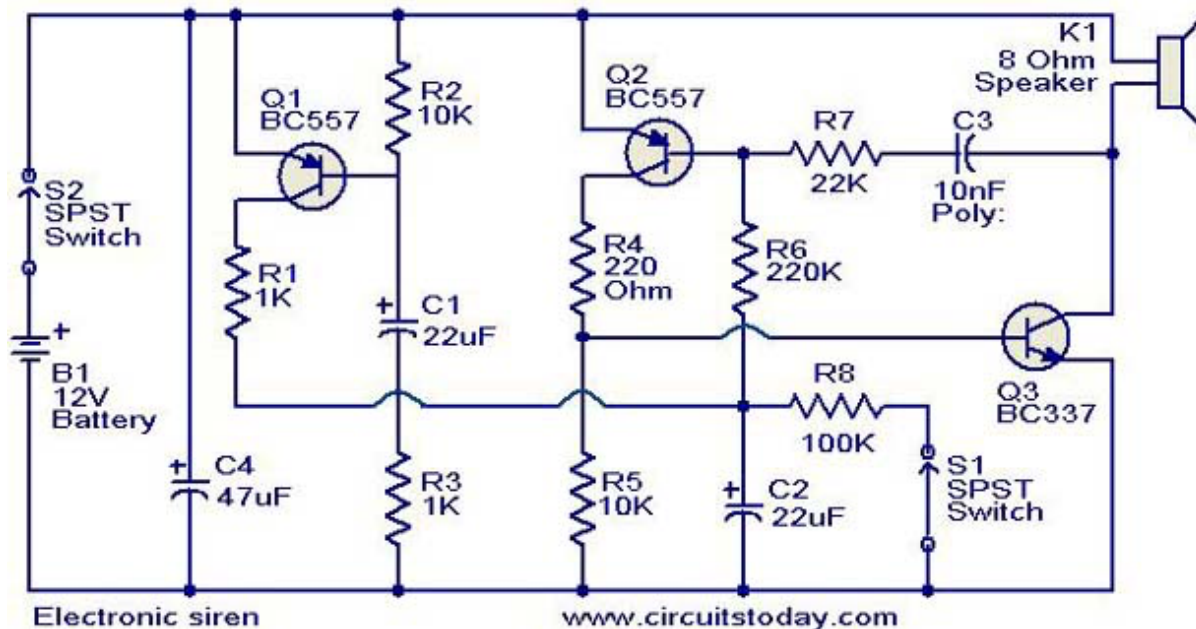
# What is an electrical circuit ?

- An electrical circuit allows
  - to transport energy from one point to another
  - to transmit information
  - to process information



# What is an electrical circuit ?

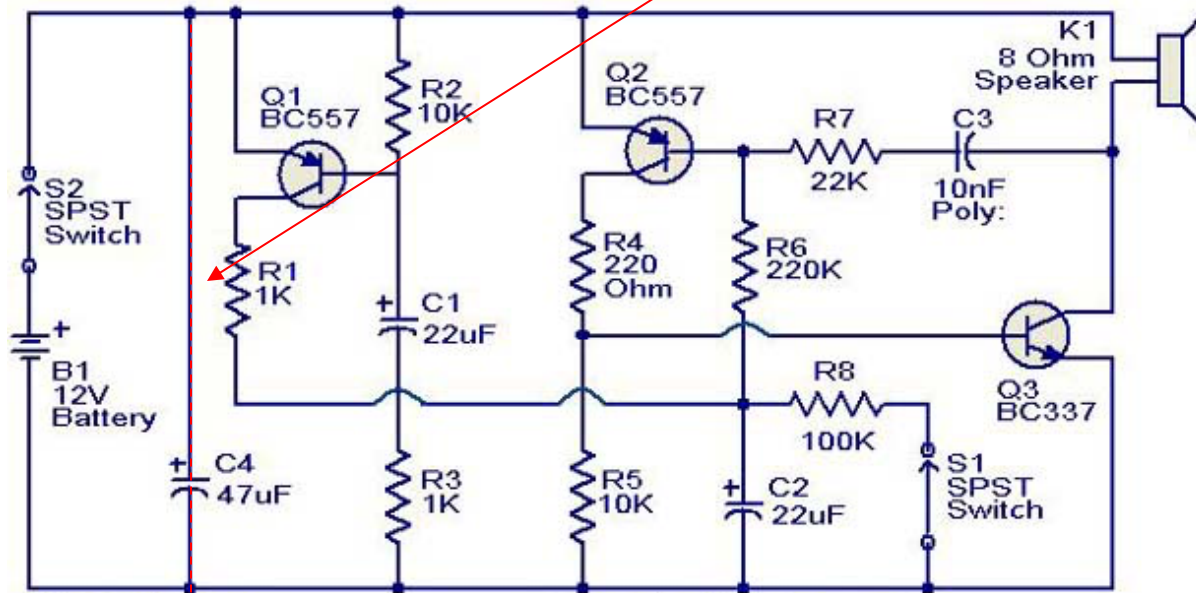
- An electrical circuit is an interconnection between electrical elements (components).



# What is an electrical circuit ?

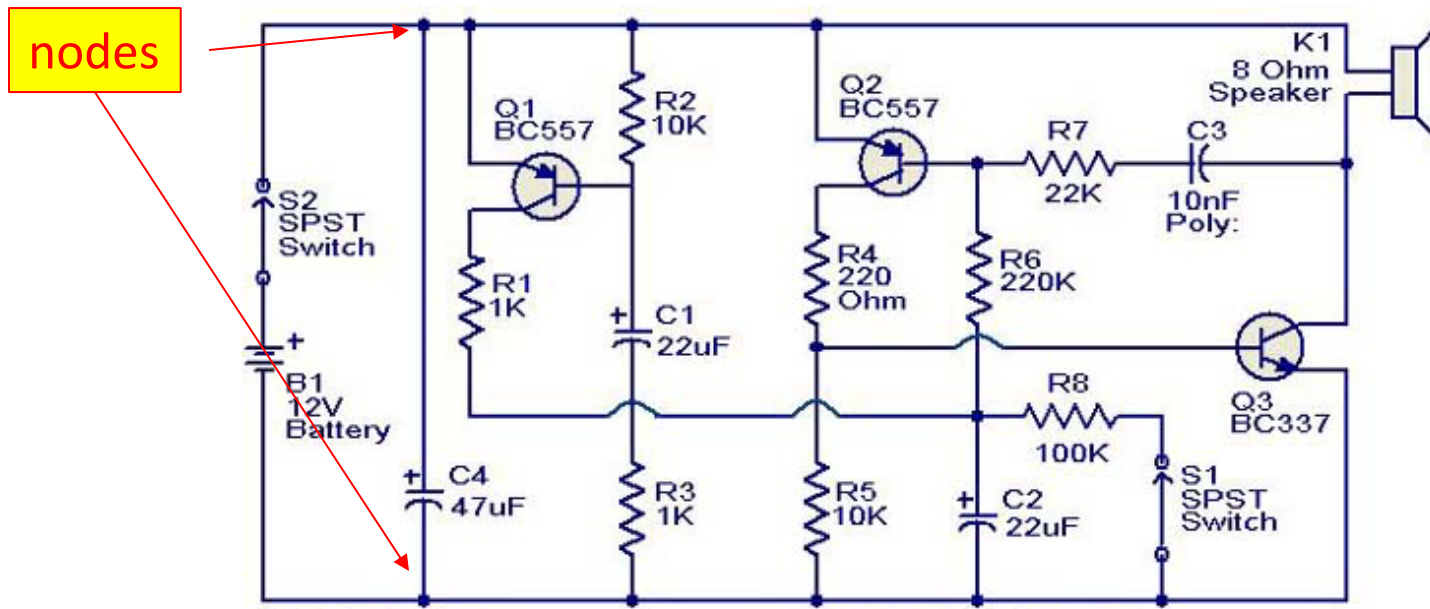
- An electrical circuit is an interconnection between electrical elements (components).
- It is formed of

branches



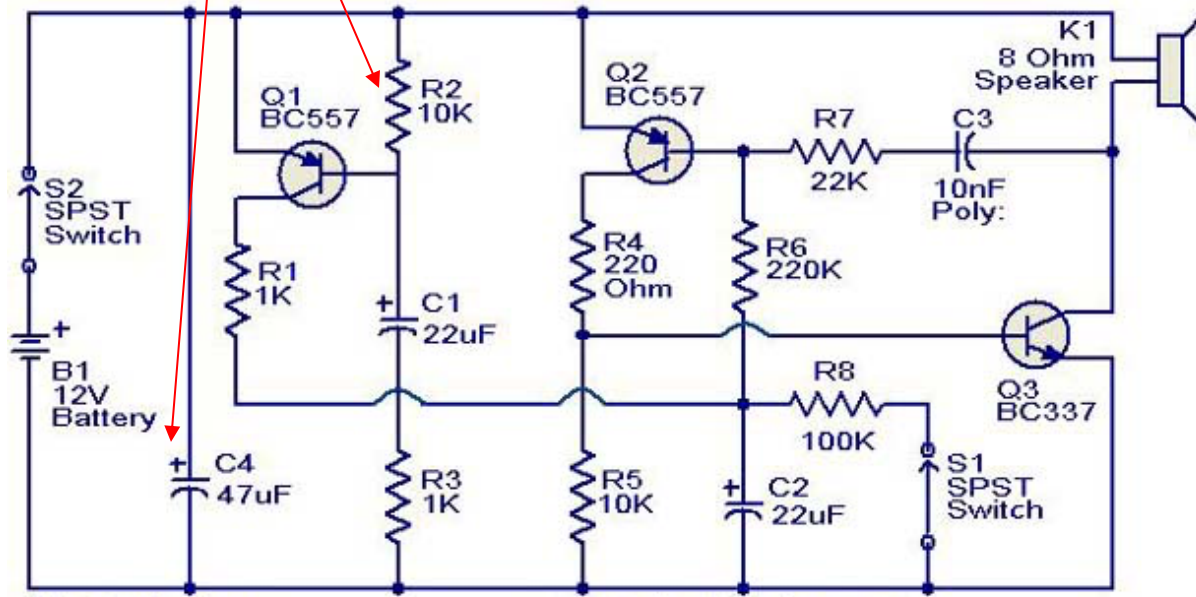
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- It is formed of



# What is an electrical circuit ?

Passive linear elements



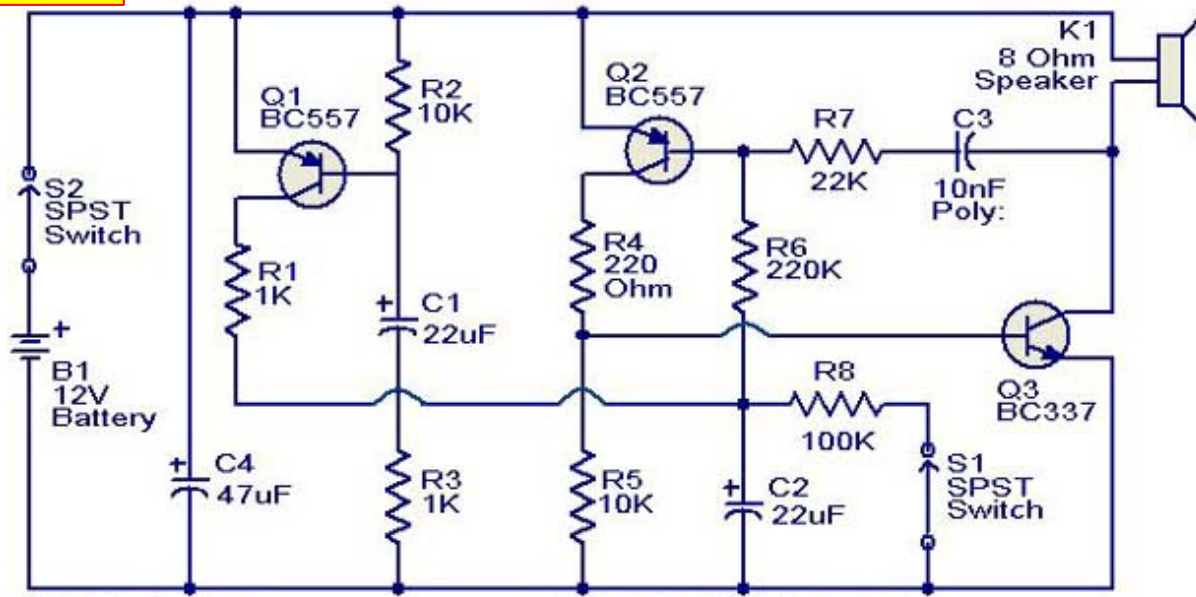


# What is an electrical circuit ?

- An electrical circuit is an interconnection between electrical elements (components).
- It is formed of branches and nodes
- Passive linear elements are resistors, inductors and capacitors.
- **Active elements (sources) are sources of voltage and current.**

# What is an electrical circuit ?

Active elements  
(sources)



# What is an electrical circuit ?

- An electrical circuit is an interconnection between electrical elements (components).
- It is formed of branches and nodes
- Passive linear elements are resistors, inductors and capacitors.
- Active elements are sources of voltage and current.
- The circuits are expressed in terms of two basic quantities: current and voltage.

# Electrical circuits: Fundamentals

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- Kirchoff's law
- Combination of elements
- Voltage / current dividers

# Electric charge

- The electric charge is an electrical property of the atomic particles that make up the matter, it is measured in Coulomb (C).
- $1 \text{ C} = 1/1.602 \times 10^{-19} = 6.24 \times 10^{18}$  electrons
- The only charges that exist in nature at our scale are integer multiples of the elementary charge  $e = -1.602 \times 10^{-19} \text{ C}$ .

Charles de **Coulomb** (1736-1806),  
French physicist.



# Charge and electric current

- The electric current is the time rate of change of the charge, measured in Ampere (A).

André-Marie **Ampère** (1775-1836),  
French physicist, founder of  
electrodynamics. We owe him the  
terms of voltage and current.



# Charge and electric current

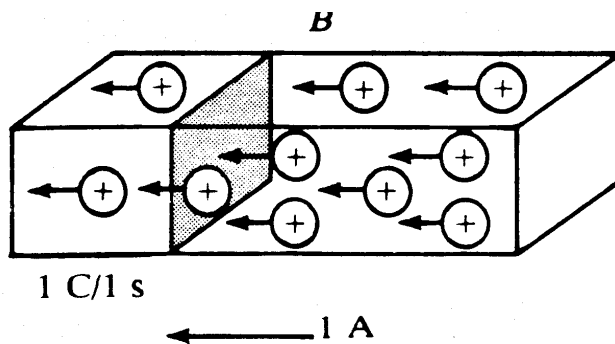
- The electric current is the time rate of change of the charge, measured in Ampere (A).
- The relationship between the charge  $q$  and the current  $i$  is given by

$$i(t) = \frac{dq}{dt}$$

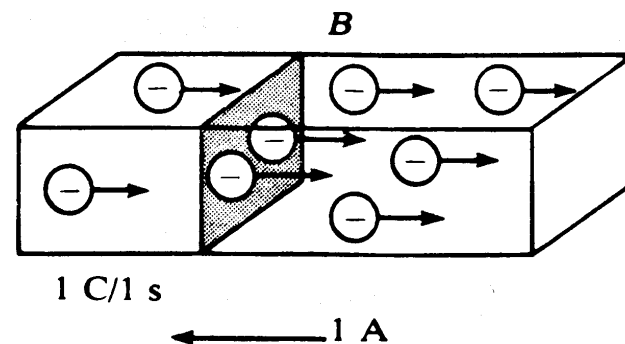
$$q(t) = \int_0^t i(\tau) d\tau$$

# The direction of the electric current

- By convention, the direction of the current is given by the movement of positive charges



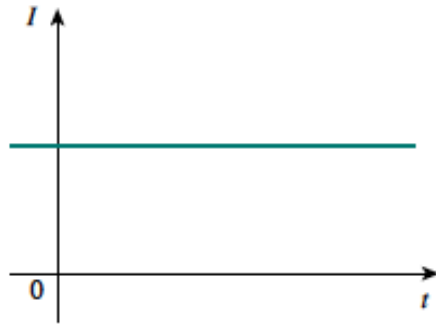
(a)



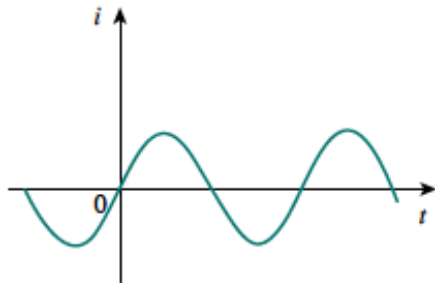
(b)

# Charge and electric current

- A direct current  $I$  is a current independent of time.



- An alternating current is a current that periodically changes direction.



Sinusoidal  
alternating current

# Voltage, potential difference, electromotive force

- Moving electrons in a conductor in a particular direction requires some work or energy transfer.
- This work is performed by an external force to the charge provided by a power source and called electromotive force, voltage or potential difference.
- The voltage or potential difference  $v_{ab}$  between two points of a circuit is the energy required to move a unit of electric charge from point a to point b.

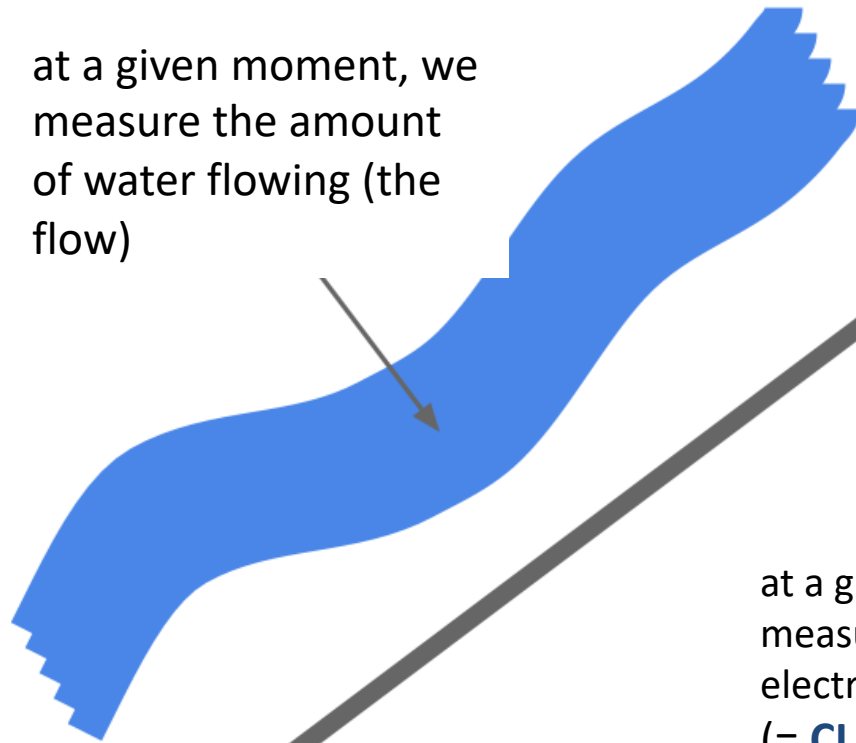
Comte Alessandro **Volta** (1745-1827), italian physicist, inventor of the electric battery (1800).



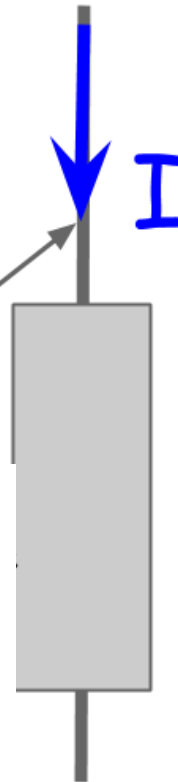
# Analogy

## Water - Flow

at a given moment, we measure the amount of water flowing (the flow)



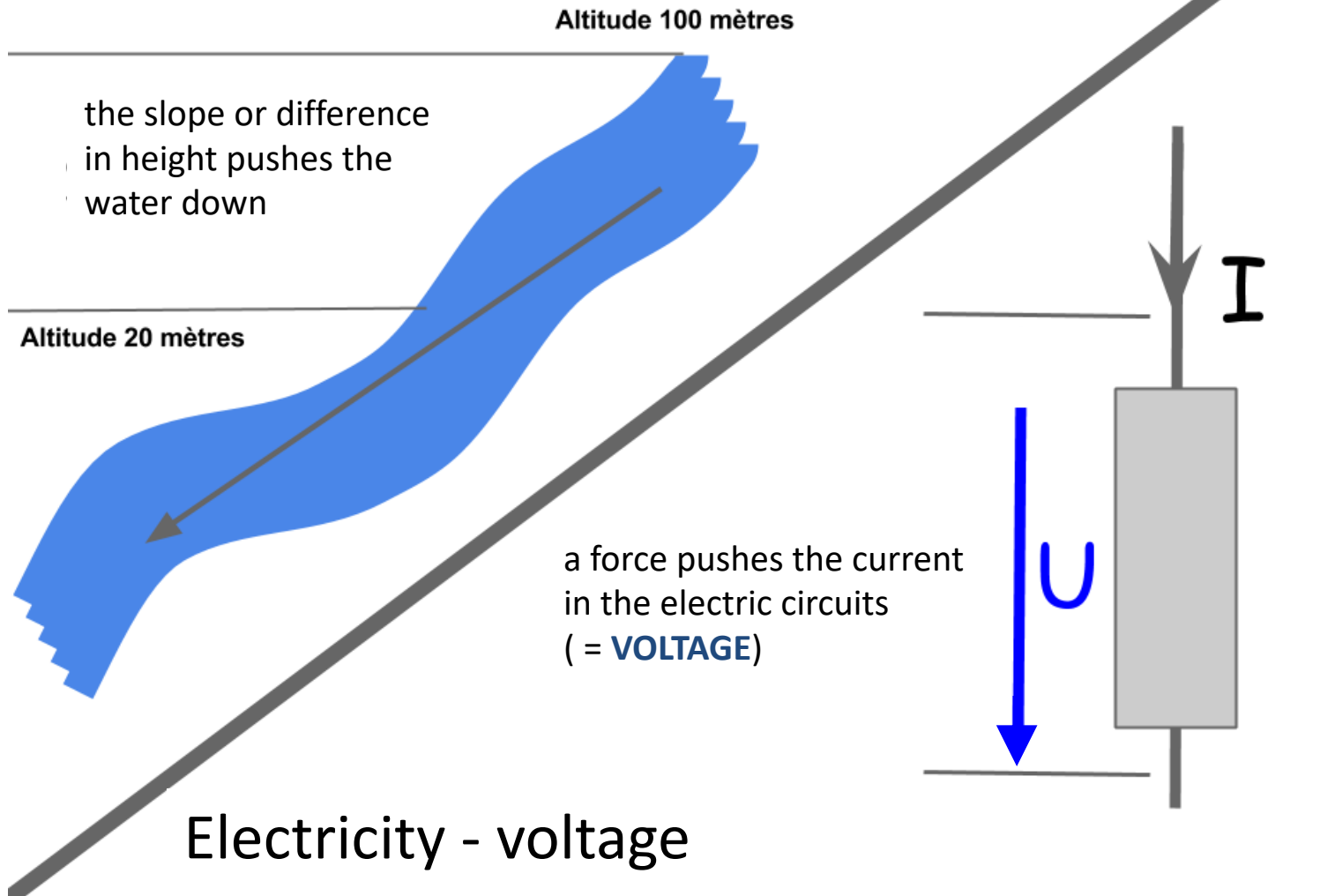
at a given moment, we measure the number of electrons circulating (= **CURRENT**)



## Electricity - current

# Analogy

## Water – Slope

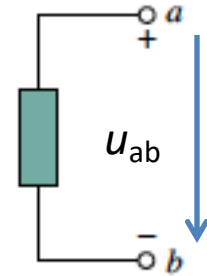


# Voltage, potential difference, electromotive force

- Mathematically :

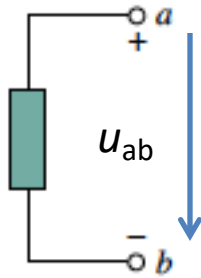
$$u_{ab} = dw / dq$$

- $w$  is the energy (J) and  $q$  is the charge(C).

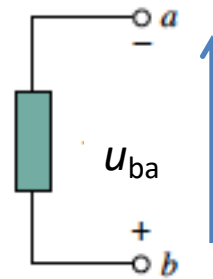


- The voltage  $u_{ab}$  is defined between two points of the circuit a and b.
- if  $u_{ab} > 0$ : the potential of point a is *greater* than the potential of point b
- if  $u_{ab} < 0$ : the potential of point a is *smaller* than the potential of point b

# Voltage, potential difference, electromotive force



$$U_{ab} = 9 \text{ V}$$



$$U_{ba} = -9 \text{ V}$$

# Voltage and electric field

- The voltage between two points a and b can also be defined as the integral of the electric field between these two points

$$u_{ab} = \int_a^b \vec{E} \cdot \vec{dl}$$

# Power and Energy

- Power is the rate at which you consume energy.
- The watt (W), unit of power, corresponding to the transfer of 1 joule of energy over one second.
- Mathematically :

$$p = \frac{dw}{dt}$$

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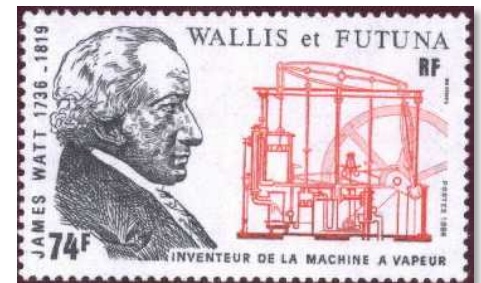
$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = u \cdot i$$

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James **Watt** (1736-1819), mechanical engineer and British inventor. We owe him the steam engine.



# Power and Energy

- In an electrical circuit, the instantaneous power  $p(t)$  is equal to the product of the instantaneous voltage and the instantaneous current, and the average power  $P$  is equal to the mean value of the instantaneous power

$$p(t) = u(t) \cdot i(t)$$

$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} u(t)i(t)dt \quad \text{ou} \quad P = \frac{1}{T} \int_0^T u(t)i(t)dt$$

# Power and Energy

- The energy absorbed or supplied by an element during the time interval from 0 to  $t$  is :

$$w(t) = \int_0^t p(\tau) d\tau$$

- The law of energy conservation implies that the algebraic sum of the power in a circuit, at any instant, must be zero :

$$\sum P = 0$$

# Joule

- The Joule (J) is equivalent to the work produced by a force of 1 N whose point of application moves one meter in the direction of the force.

$$w(t) = \int_0^t p(\tau) d\tau$$

James prescott **Joule** (1818-1890),  
British physicist. He studied the  
heat released by the electric  
currents (Joule heating).

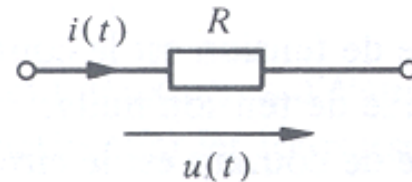


# Passive elements

# Resistance

- Circuit element that satisfies Ohm's law

$$u(t) = Ri(t)$$

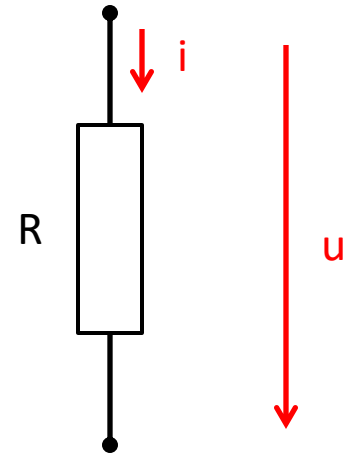
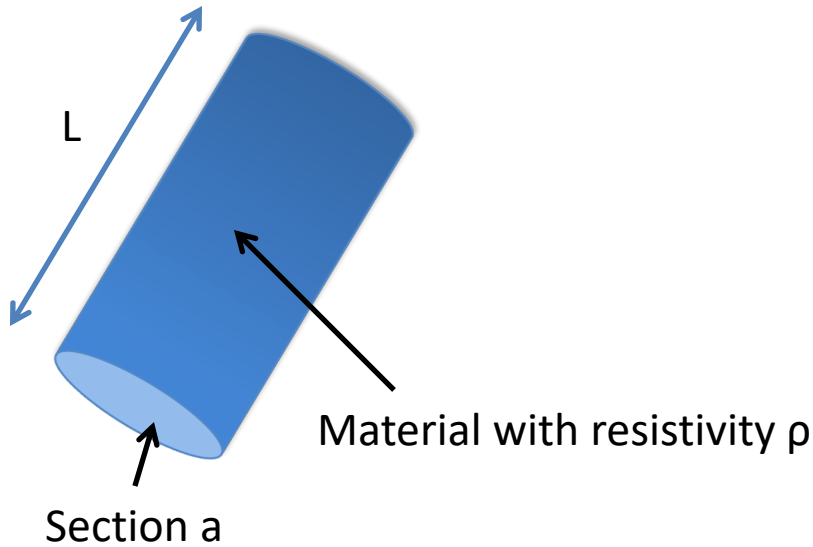


- The unit of resistance is the ohm ( $\Omega$ )

Georg Simon **Ohm** (1789-1854), German physicist. He discovered in 1827 the fundamental laws of electric currents.



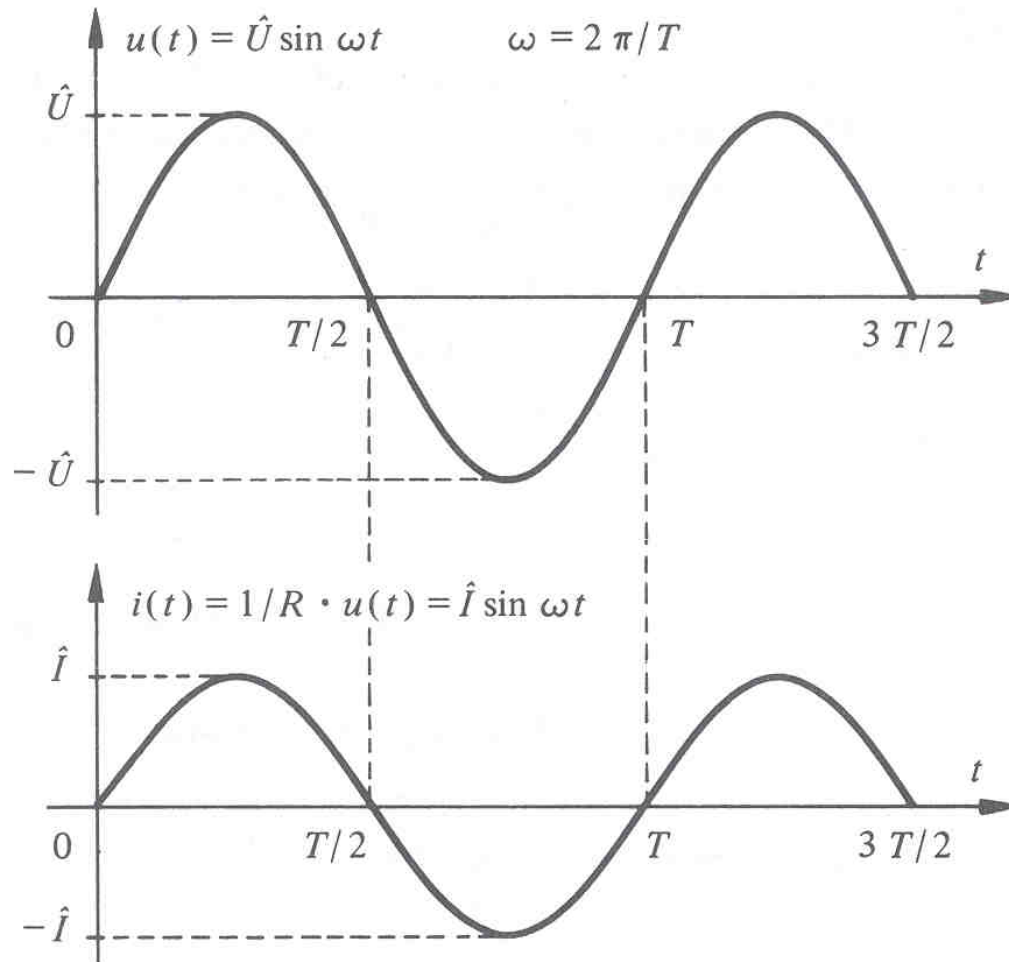
# Ohm's Law and Resistance



Ohm's law :  $E = \rho J$       J: Current density A/m<sup>2</sup>

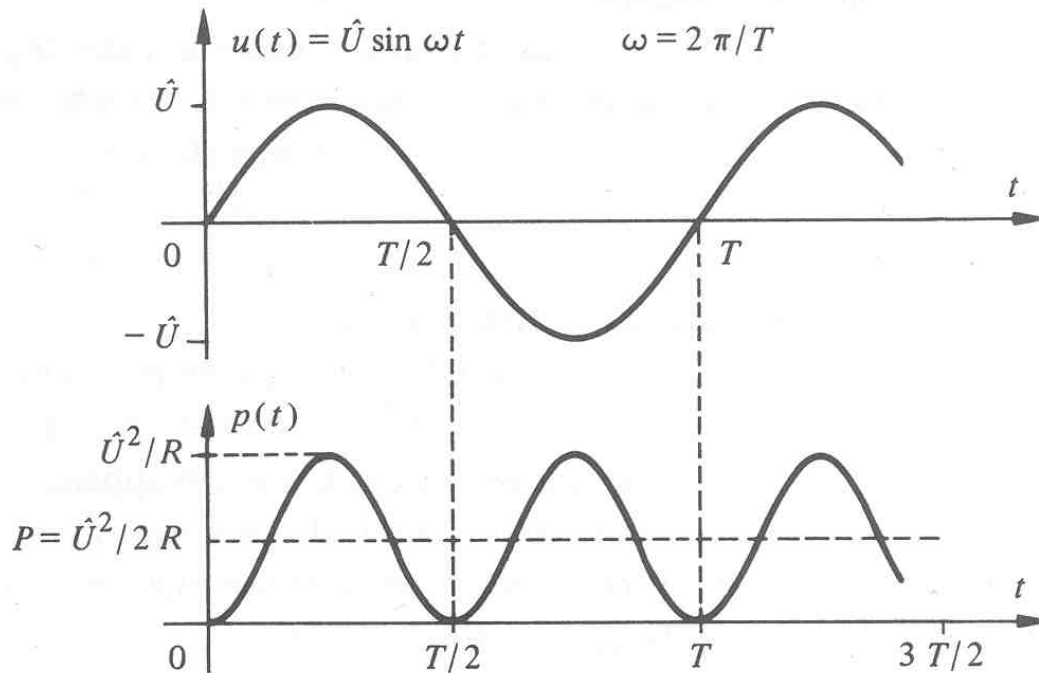
$\Rightarrow u = R \cdot i$        $R = \rho \frac{L}{S}$

# Resistance: current and voltage



# Resistance: dissipated power

$$p(t) = u(t)i(t) = Ri^2(t) = \frac{u^2(t)}{R}$$



# Resistance: dissipated energy

- The electrical energy supplied to a resistor for a duration  $t$  is dissipated in the form of heat

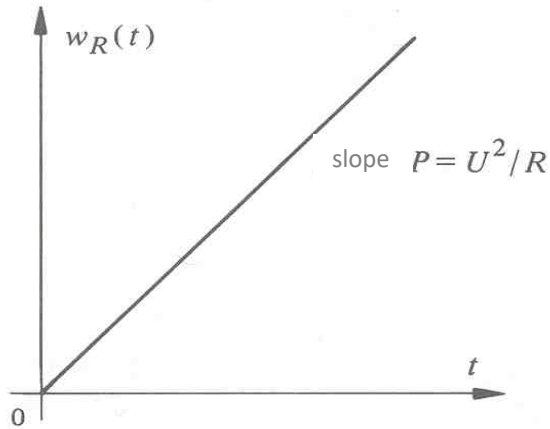
$$w_R(t) = R \int_0^t i^2(\tau) d\tau = \frac{1}{R} \int_0^t u^2(\tau) d\tau$$

- Continuous regime:

$$i(t) = I \quad u(t) = U \Rightarrow w_R(t) = RI^2t = \frac{U^2t}{R}$$

# Resistance: dissipated energy

- Continuous regime :

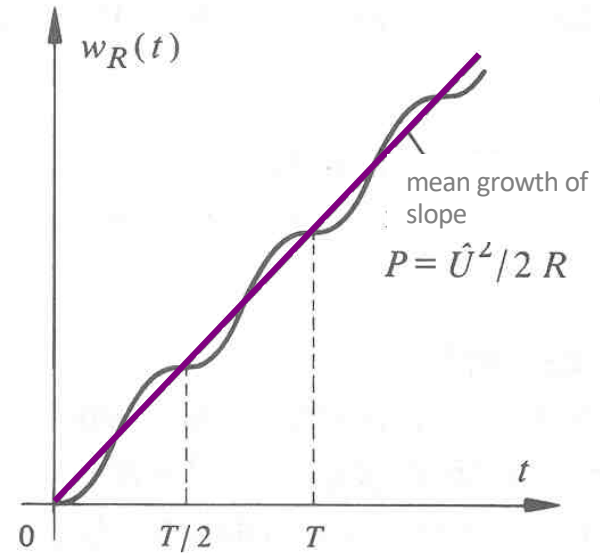


$$w_R(t) = RI^2t = \frac{U^2t}{R}$$

- Sinusoidal regime:

$$u(t) = \hat{U} \sin \omega t$$

$$w_R(t) = \frac{\hat{U}^2}{2R} \left[ t - \frac{1}{2\omega} \sin(2\omega t) \right]$$



# Short circuit

- Let a circuit whose two points are connected by a resistor. If the value of this resistance tends to zero, these two points become short circuited.
- In practice, a piece of wire is often (not always!) a short circuit.

# Open circuit

- Let a circuit whose two points are connected by a resistor. If one makes the value of this resistance tend towards infinity, these two points become in open circuit.
- In practice, an open circuit is realized by removing all the elements that connect the two points of the circuit.

# Conductance

- Inverse of resistance  $G=1/R$

$$i(t) = Gu(t)$$

- The unit of conductance is the Siemens (S)

Werner von **Siemens** (1816-1892),  
German engineer and industrialist.



# Capacitance

- Circuit element that obeys the following relationship :

$$q(t) = Cu(t)$$

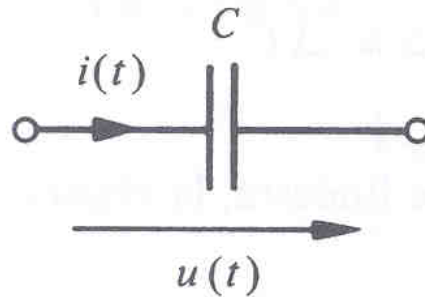
- Knowing that the current is the flow of charges :

$$i(t) = \frac{dq}{dt}$$

$$i(t) = C \frac{du}{dt} \quad u(t) = \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau = u(0) + \frac{1}{C} \int_0^t i(\tau) d\tau$$

# Capacitance

The value  $C$  of the capacitance is expressed in farad (F).

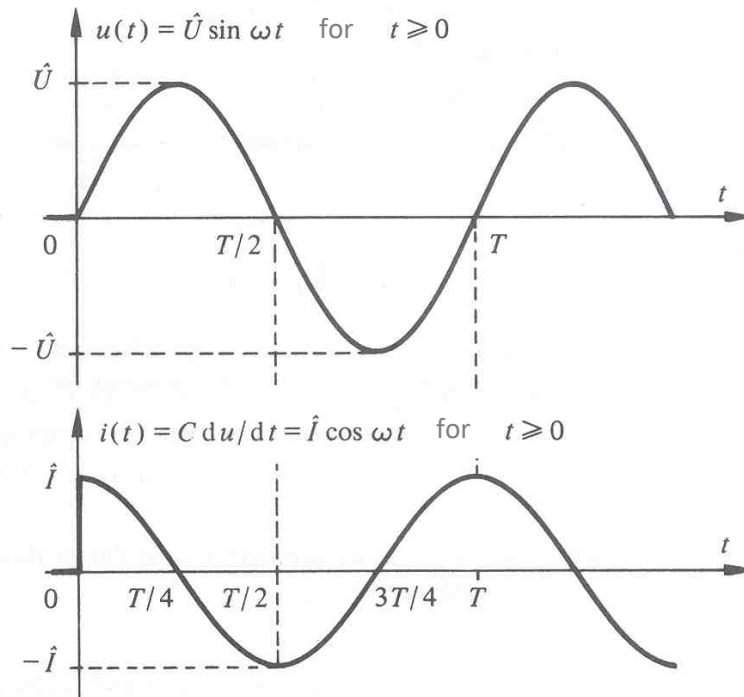


Michael **Faraday** (1791-1867),  
British physicist.



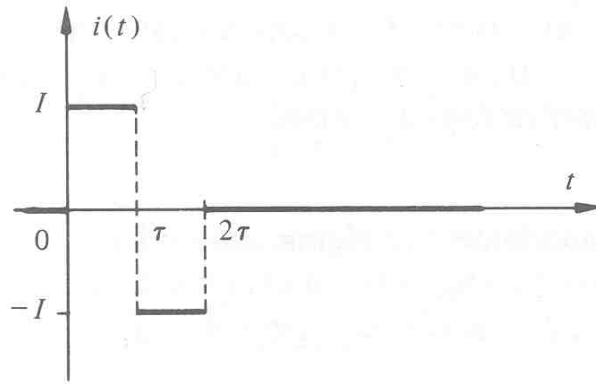
# Capacitance: current and voltage

- In continuous regime : open circuit
- In sinusoidal regime :



$$\hat{I} = \omega C \hat{U}$$

# Capacitance: no voltage step!



$$u(t) = u(0) + \frac{1}{C} \int_0^t i(\tau) d\tau$$

# Capacitance: electrostatic energy

$$w_c(t) = \frac{1}{2} C u^2(t)$$

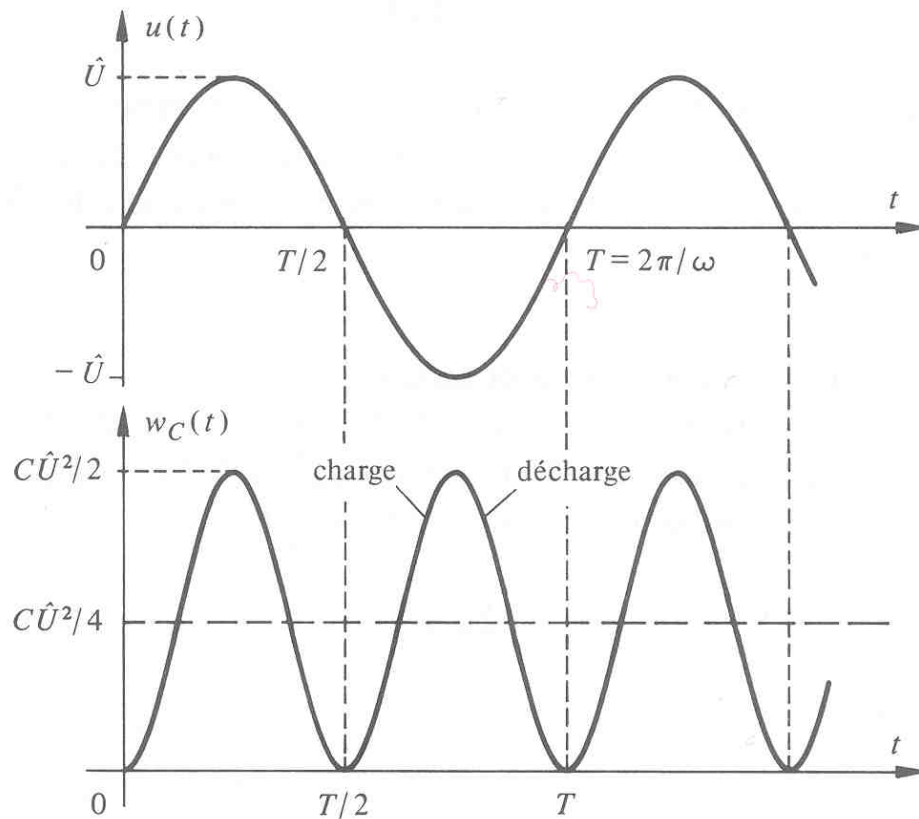
- Sinusoidal regime:  $u(t) = \hat{U} \sin \omega t$

$$w_c(t) = \frac{C \hat{U}^2}{2} \sin^2 \omega t$$

$$w_c(t) = \frac{C \hat{U}^2}{4} [1 - \cos 2\omega t]$$

# Capacitance: electrostatic energy

- Sinusoidal regime:

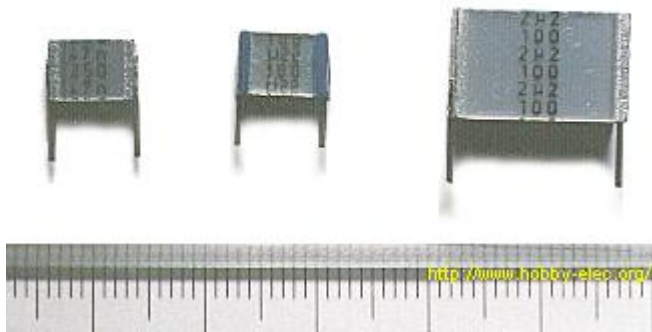


$$w_C(t) = \frac{C\hat{U}^2}{4} [1 - \cos 2\omega t]$$

# Capacitor: physical element



Ceramic (100 pF, 10 nF)

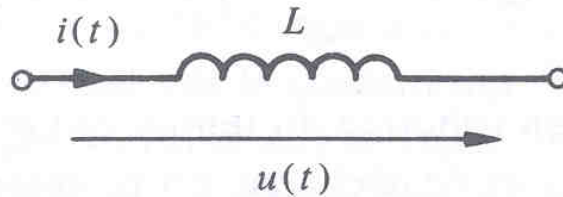


Metallic polyester film  
(1 nF, 220 nF, 2.2 mF)

# Inductance

- Circuit element that obeys the following relationship :

$$u(t) = L \frac{di}{dt} \quad i(t) = \frac{1}{L} \int_{-\infty}^t u(\tau) d\tau = i(0) + \frac{1}{L} \int_0^t u(\tau) d\tau$$



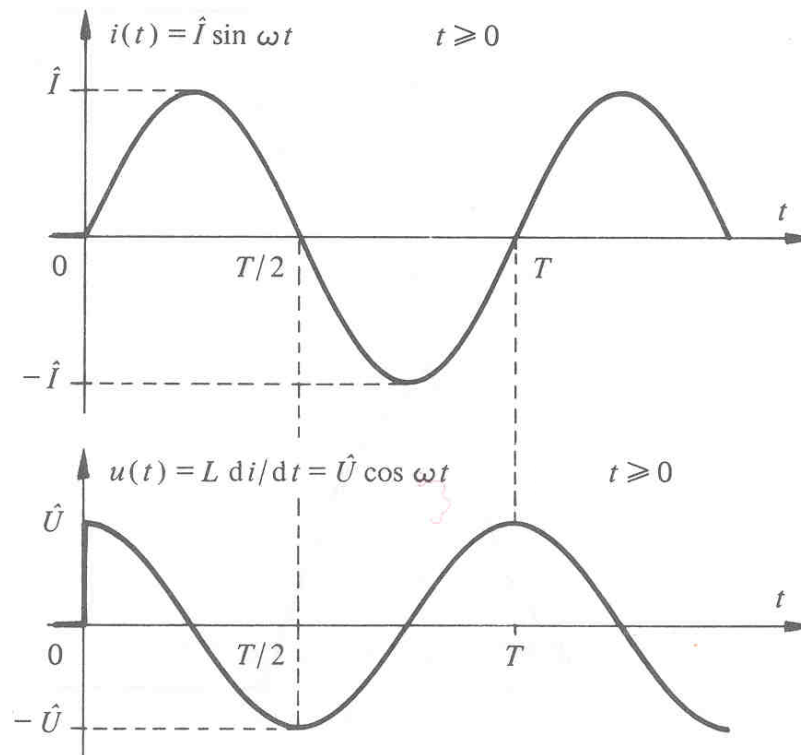
- The unit of inductance is henry (H).

Joseph **Henry** (1797-1878),  
american physicist.



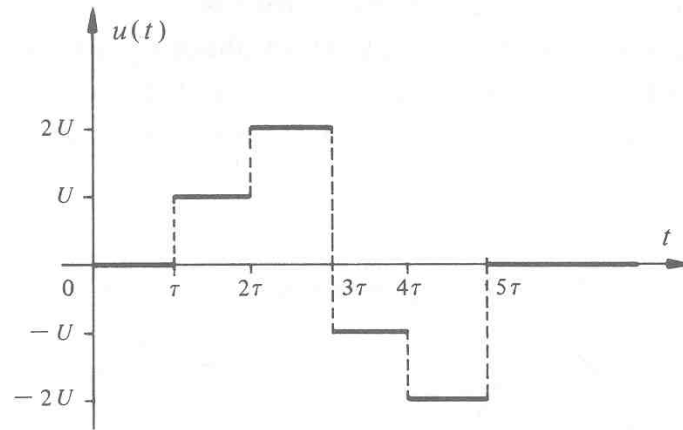
# Inductance: current and voltage

- In continuous regime: short circuit
- In sinusoidal regime:



$$\hat{U} = \omega L \hat{I}$$

# Inductance: no current step!



## Inductance: magnetic energy

$$w_L(t) = \frac{1}{2} L i^2(t)$$

- Sinusoidal regime:  $i(t) = \hat{I} \sin \omega t$

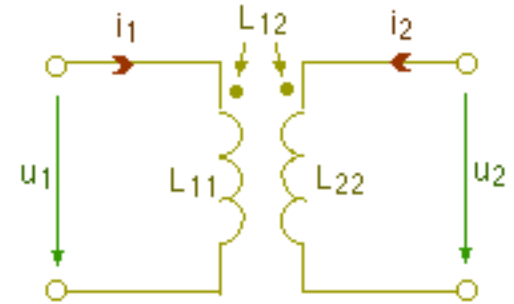
$$w_L(t) = \frac{L \hat{I}^2}{2} \sin^2 \omega t = \frac{L \hat{I}^2}{4} [1 - \cos 2\omega t]$$

# Coupled (Mutual) inductors

- Coupled inductors constitute a biporte defined by the constitutive relations :

$$u_1 = L_{11} \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

$$u_2 = L_{12} \frac{di_1}{dt} + L_{22} \frac{di_2}{dt}$$

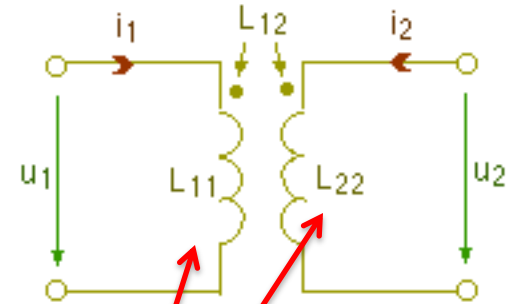


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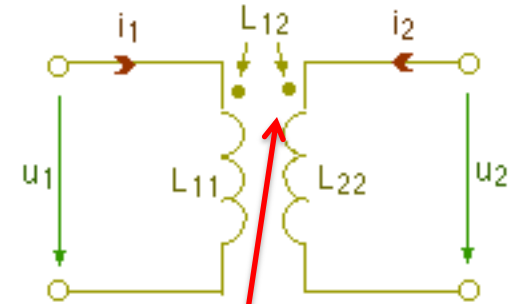
Inductors

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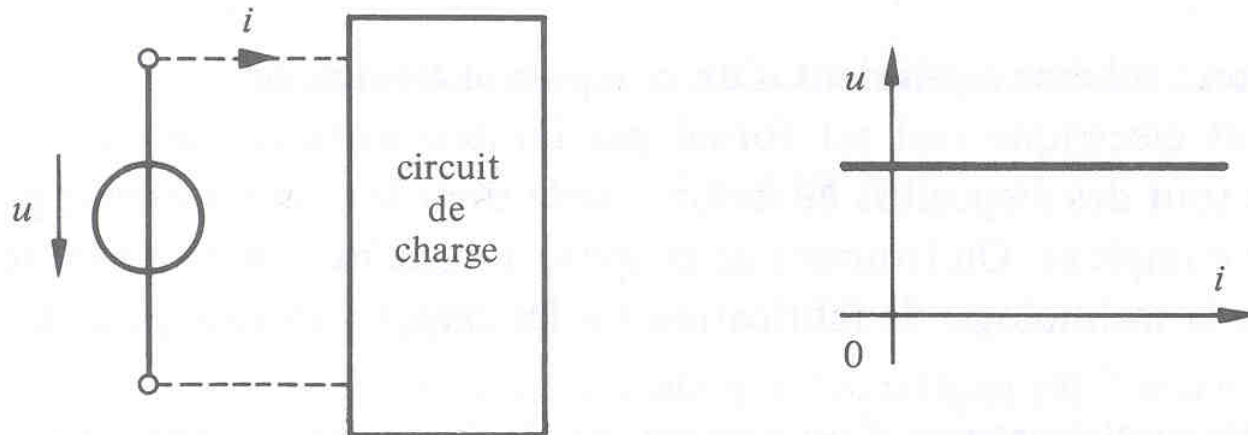


Mutual inductances

# Active circuit elements

# Ideal voltage source

- Circuit element capable of supplying or absorbing electrical energy and whose voltage across its terminals is constant and independent of the charge connected to it.
- A voltage source provides no energy when it is in open circuit.
- Symbol :

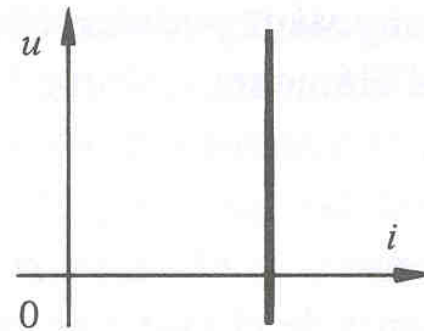
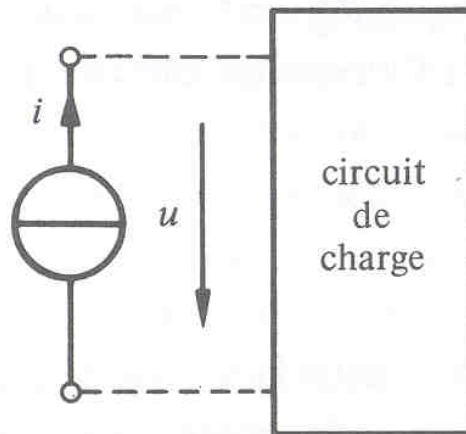


# Examples of sources



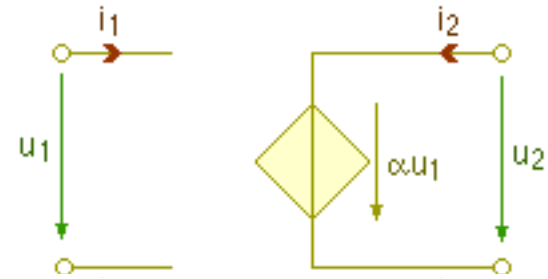
# Ideal current source

- Circuit element capable of supplying or absorbing electrical energy and whose current flowing through it is constant and independent of the charge connected to it.
- A current source provides no energy when it is short-circuited.
- Symbol :



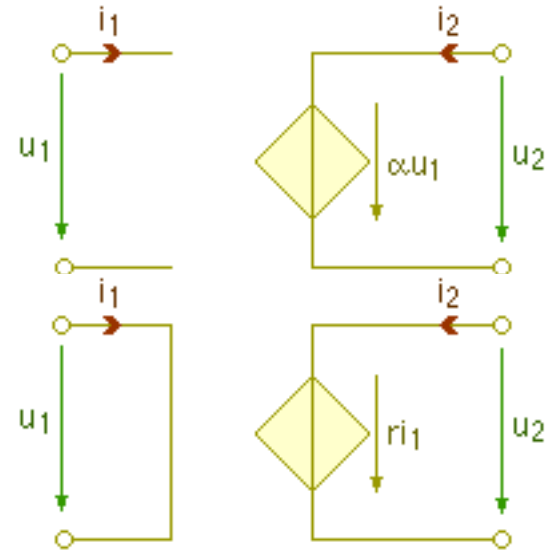
# Dependent sources

- Voltage source controlled by a voltage



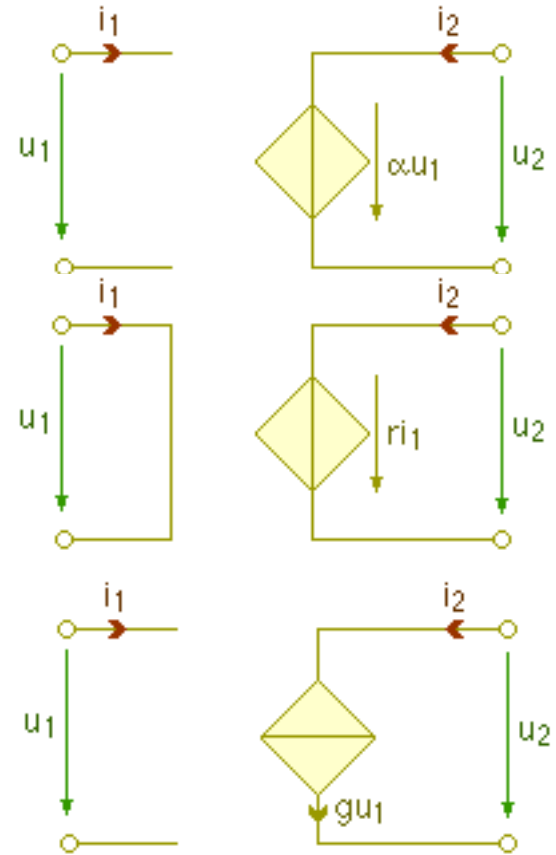
# Dependent sources

- Voltage source controlled by a voltage
- Voltage source controlled by a current



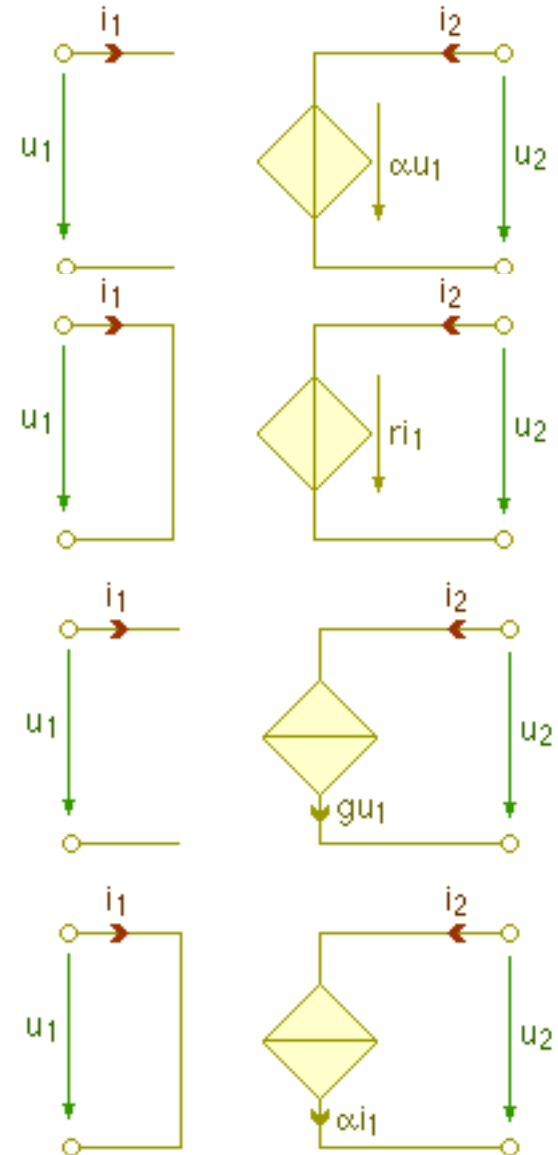
# Dependent sources

- Voltage source controlled by a voltage
- Voltage source controlled by a current
- Current source controlled by a voltage



# Dependent sources

- Voltage source controlled by a voltage
- Voltage source controlled by a current
- Current source controlled by a voltage
- Current source controlled by a current



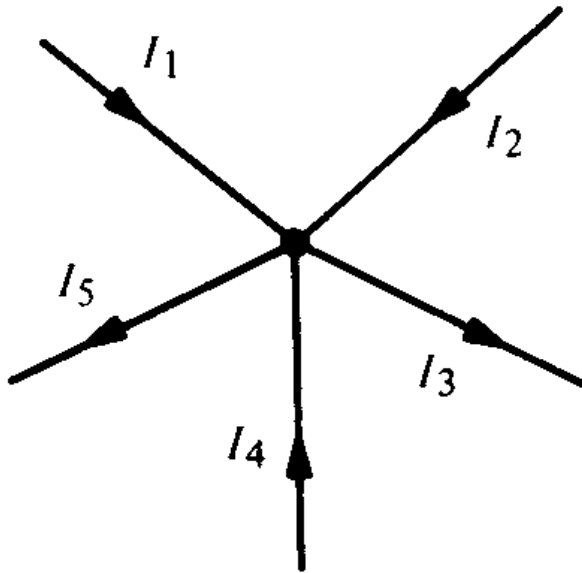
# Kirchhoff's Laws

- Fundamental laws of electrical circuits

Gustav Robert **Kirchhoff** (1824-1897),  
German physicist.



# Kirchhoff's current Law

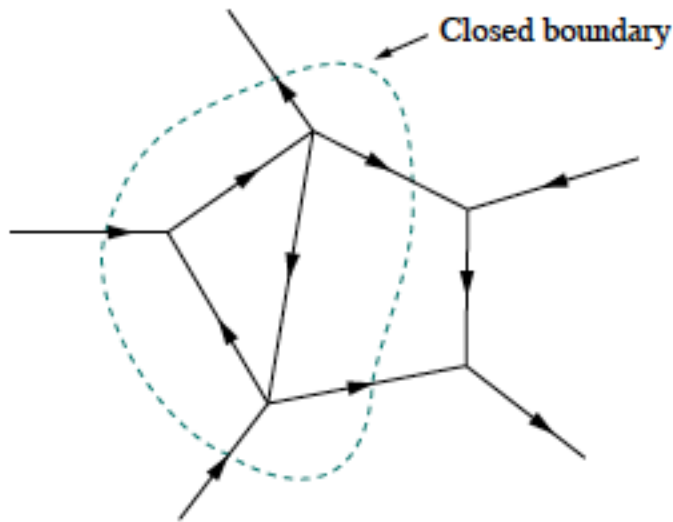


$$\sum_{j=1}^N i_j = 0$$

$$i_1 + i_2 - i_3 + i_4 - i_5 = 0$$

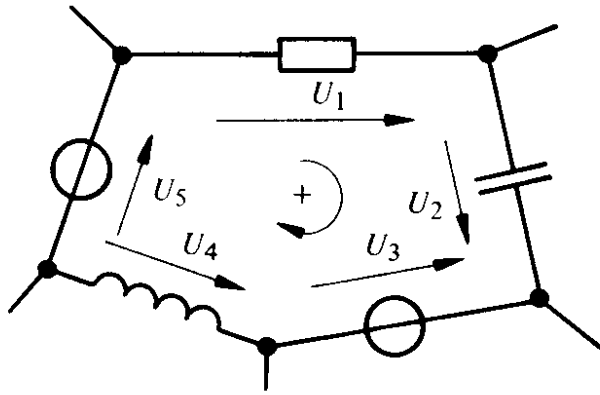
# Kirchhoff's current Law

- The sum of currents in a **closed boundary** is zero



$$\sum_{j=1}^N i_j = 0$$

# Kirchhoff's voltage Law

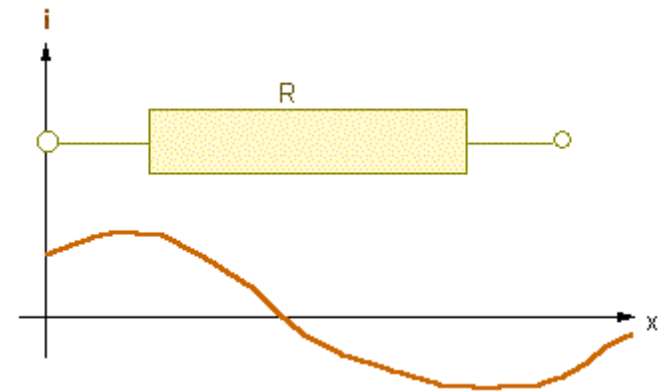


$$\sum_{j=1}^N u_j = 0$$

$$U_1 + U_2 - U_3 - U_4 + U_5 = 0$$

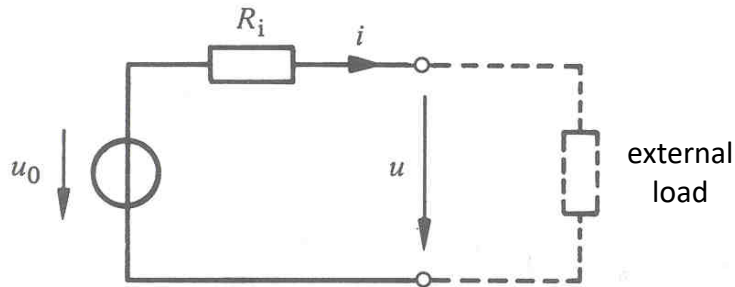
# Limitations of the Kirchhoff model

- Kirchhoff's model is a special case of a more general model, the Maxwell model.
- Maxwell's equations describe the phenomenon of wave propagation. A basic assumption of Kirchhoff's model is the equality of the current entering and leaving a node.
- For example, the current at both ends of a resistor is the same at all times. This is no longer the case if a wave propagates along the resistance.
- However, if the wavelength is much greater than the length of the resistance, the Kirchhoff model remains valid.



# 'Real' voltage source

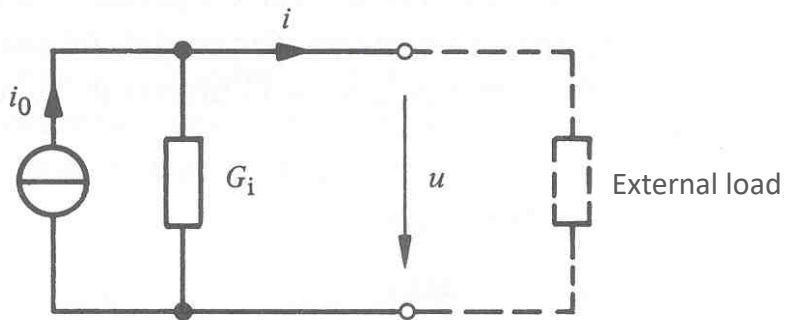
- The voltage of a real voltage source such as a battery decreases with the current it provides. We can model such a source by an ideal source in series with a resistance :



$$u = u_0 - R_i i$$

# 'Real' current source

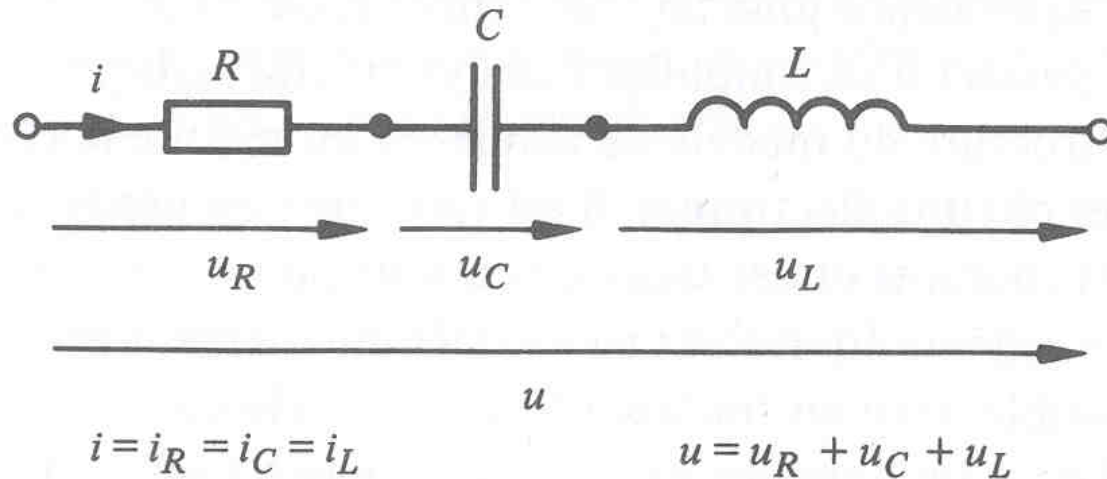
- Current-voltage characteristic and equivalent circuit



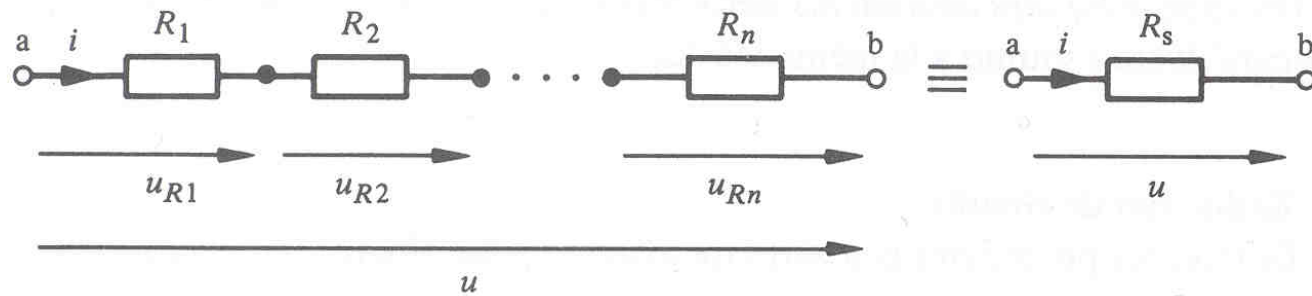
$$i = i_0 - G_i u$$

# Elements in series

- Elements connected in series are traversed by the same current. The voltage across the circuit is equal to the sum of the voltages relative to each element.

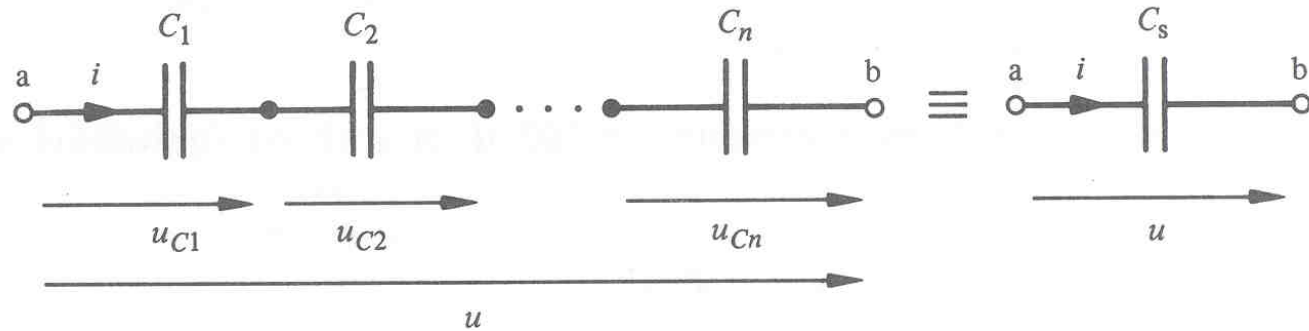


# Resistors in series



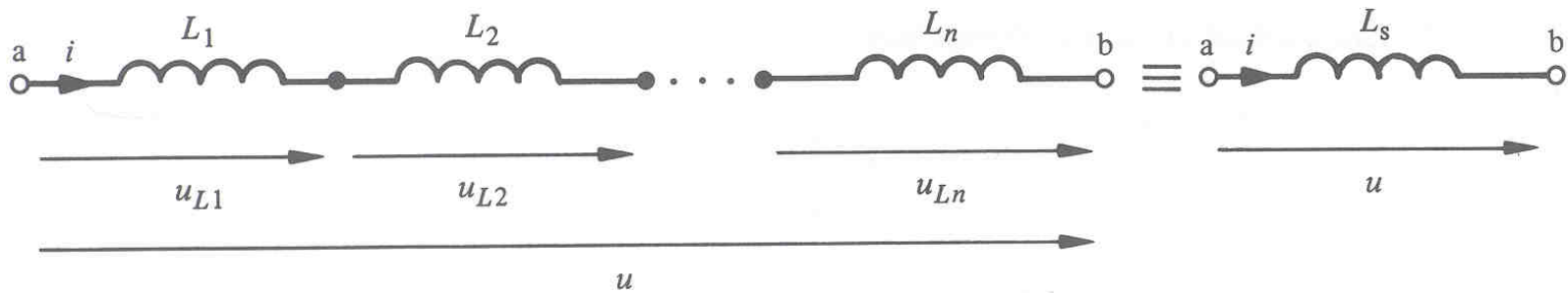
$$R_s = \sum_{k=1}^n R_k$$

# Capacitors in series



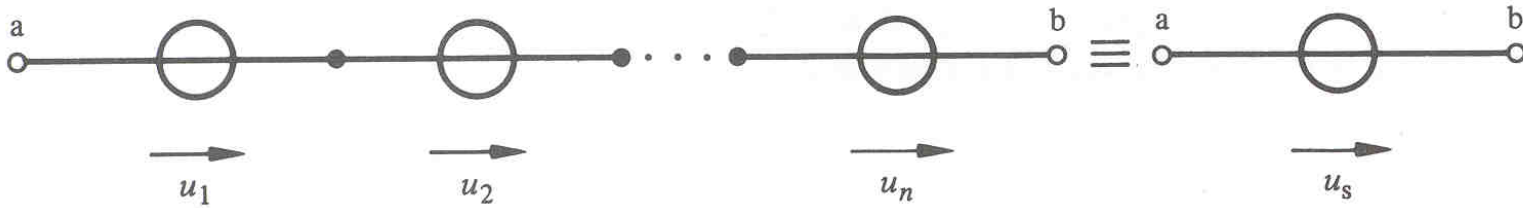
$$\frac{1}{C_s} = \sum_{k=1}^n \frac{1}{C_k} \quad \text{and} \quad u(0) = \sum_{k=1}^n u_{C_k}(0)$$

# Inductors in series

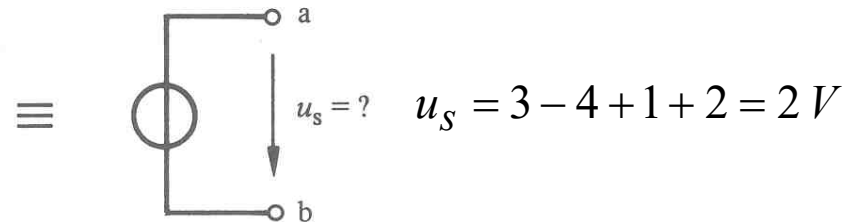
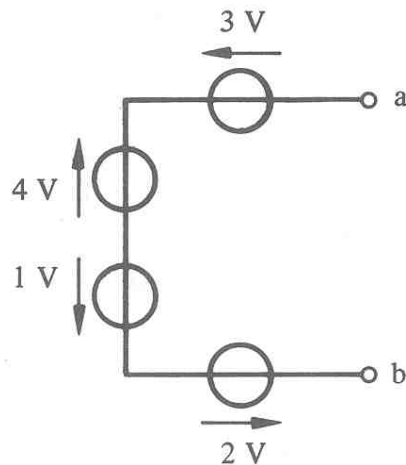


$$L_s = \sum_{k=1}^n L_k \quad \text{and} \quad i(0) = i_{Lk}(0)$$

# Voltage sources in series



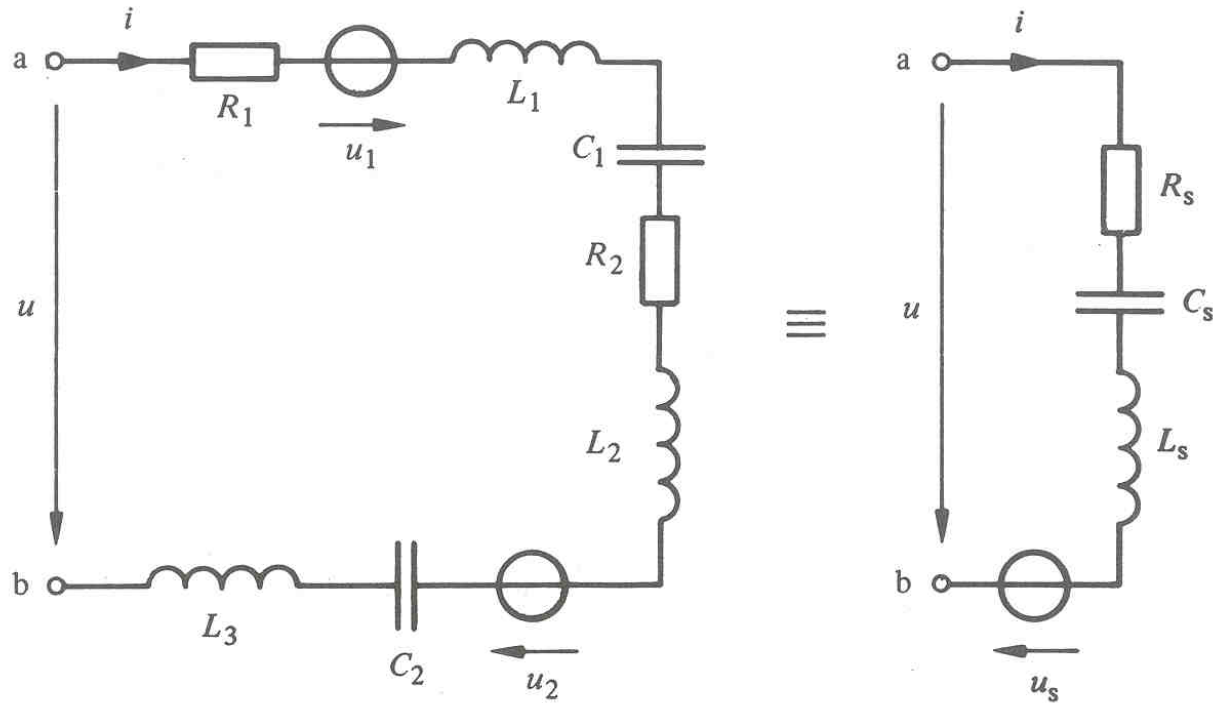
$$u_S = \sum_{k=1}^n u_k$$



## Current sources in series

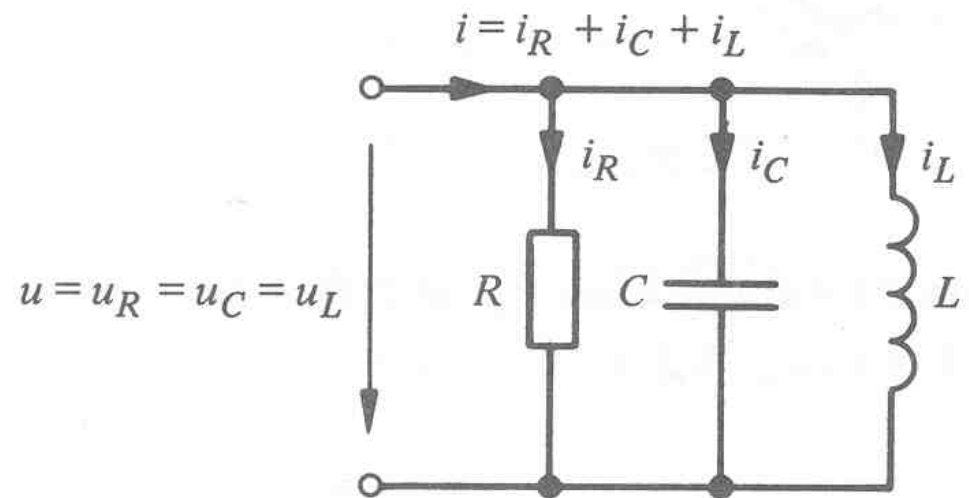
- Impossible *unless all individual sources produce the same current*

# Series circuit: example

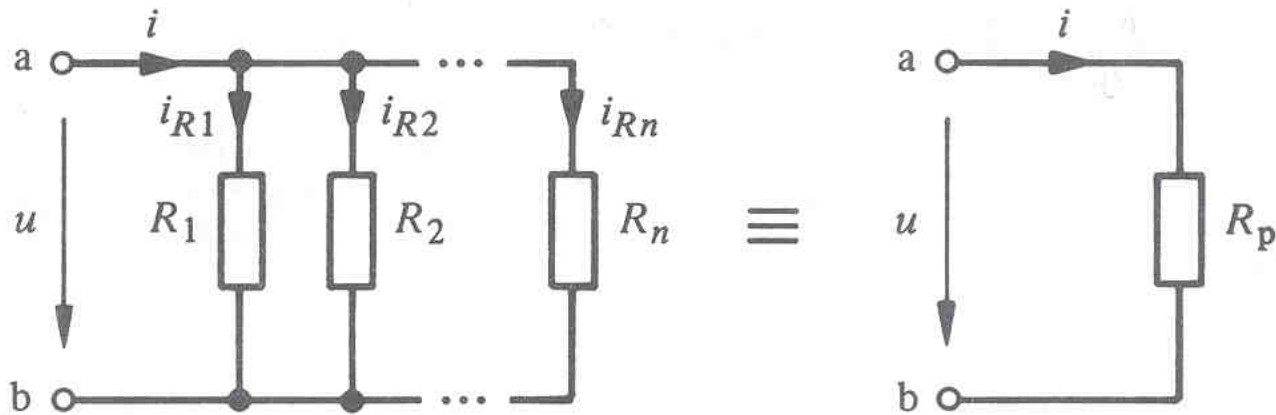


# Elements in parallel

- Elements connected in parallel are subjected to the same voltage. The total current is the sum of the individual currents.

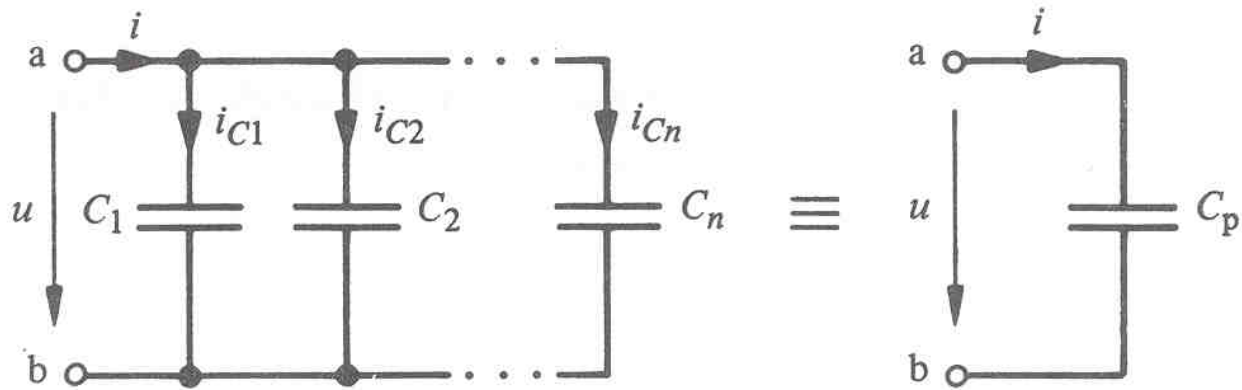


# Resistors in parallel



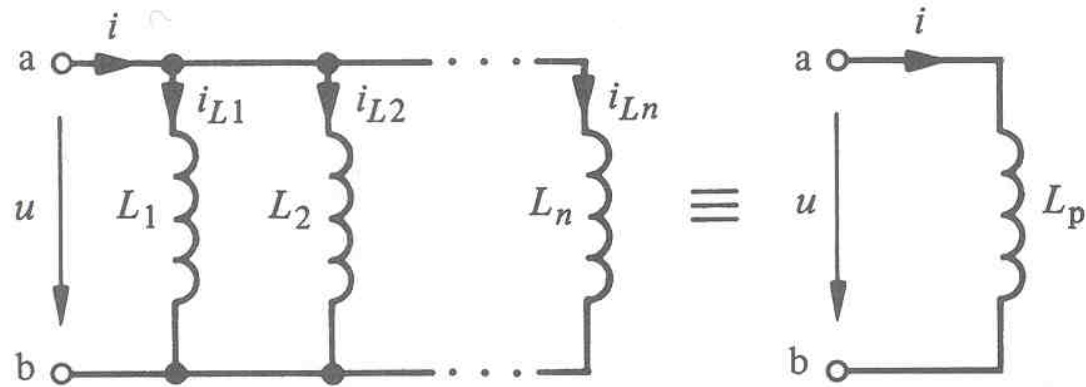
$$\frac{1}{R_p} = \sum_{k=1}^n \frac{1}{R_k}$$

# Capacitors in parallel



$$C_p = \sum_{k=1}^n C_k \quad \text{and} \quad u(0) = u_k(0)$$

# Inductors in parallel



$$\frac{1}{L_p} = \sum_{k=1}^n \frac{1}{L_k}$$

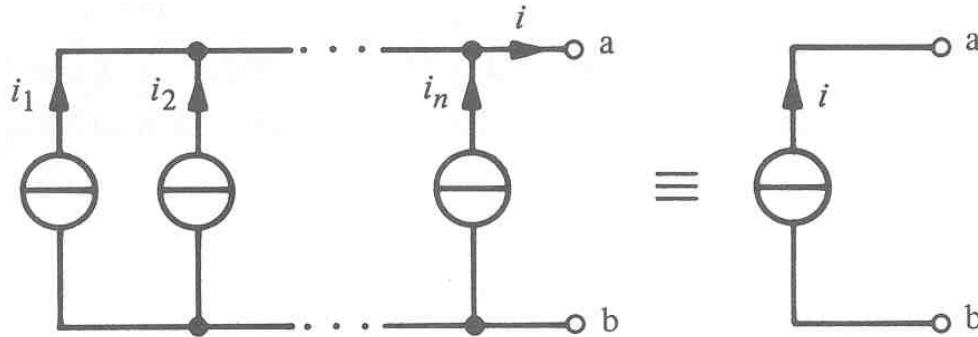
and

$$i(0) = \sum_{k=1}^n i_{Lk}(0)$$

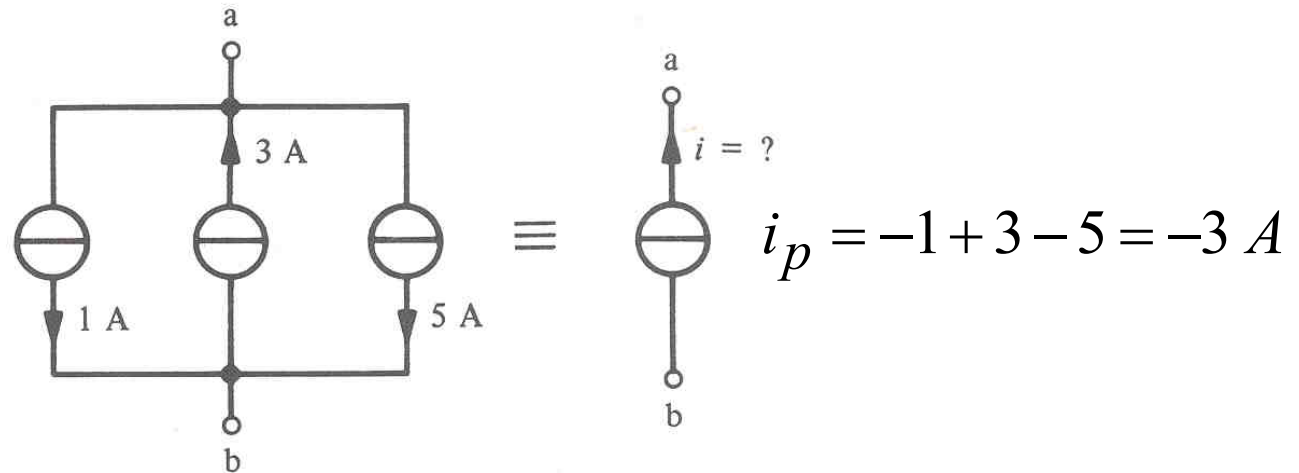
# Voltage sources in parallel

- Impossible *unless all individual sources produce the same voltage*

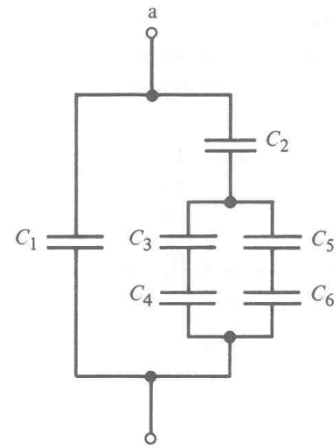
# Current sources in parallel



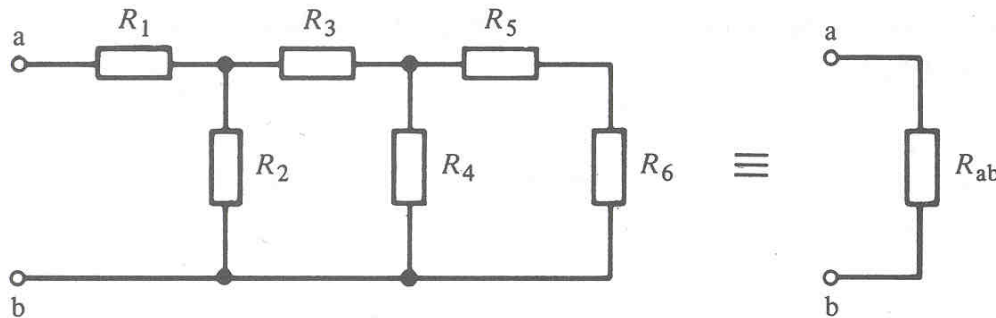
$$i_p = \sum_{k=1}^n i_k$$



# Series-parallel circuits

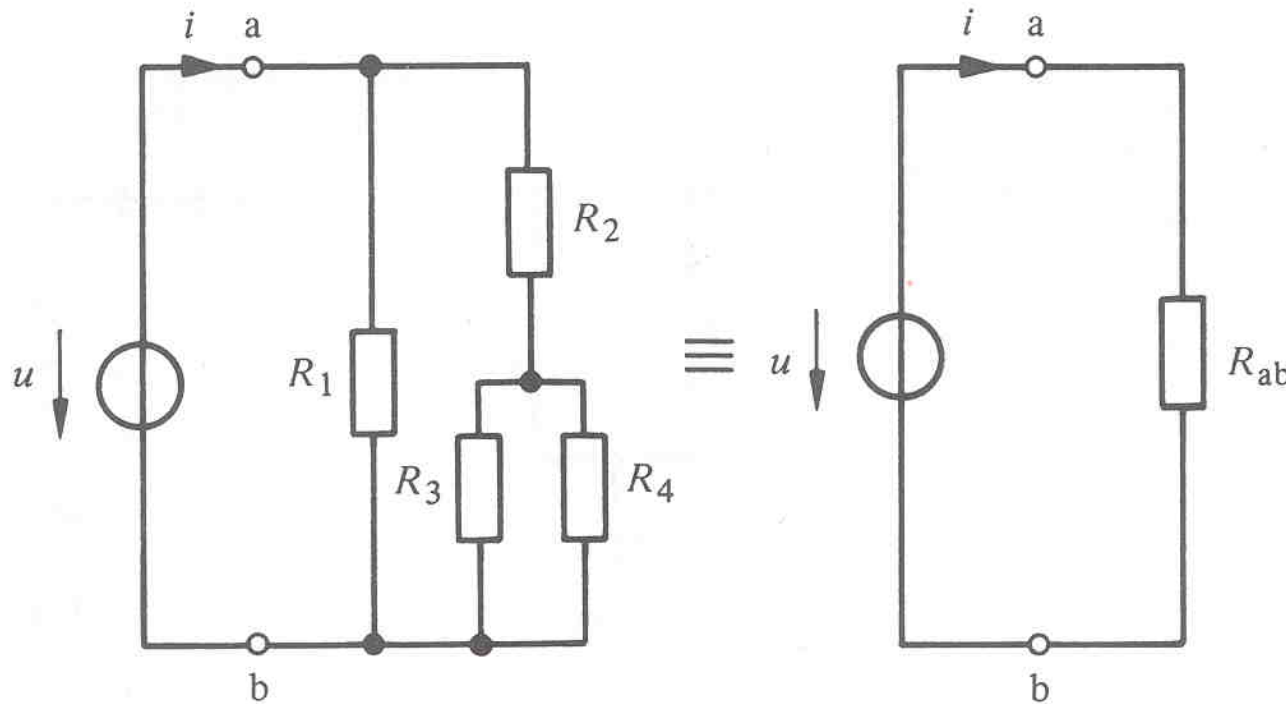


# Series-parallel circuits



$$R_{ab} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3 + \frac{1}{\frac{1}{R_4} + \frac{1}{R_5 + R_6}}}}$$

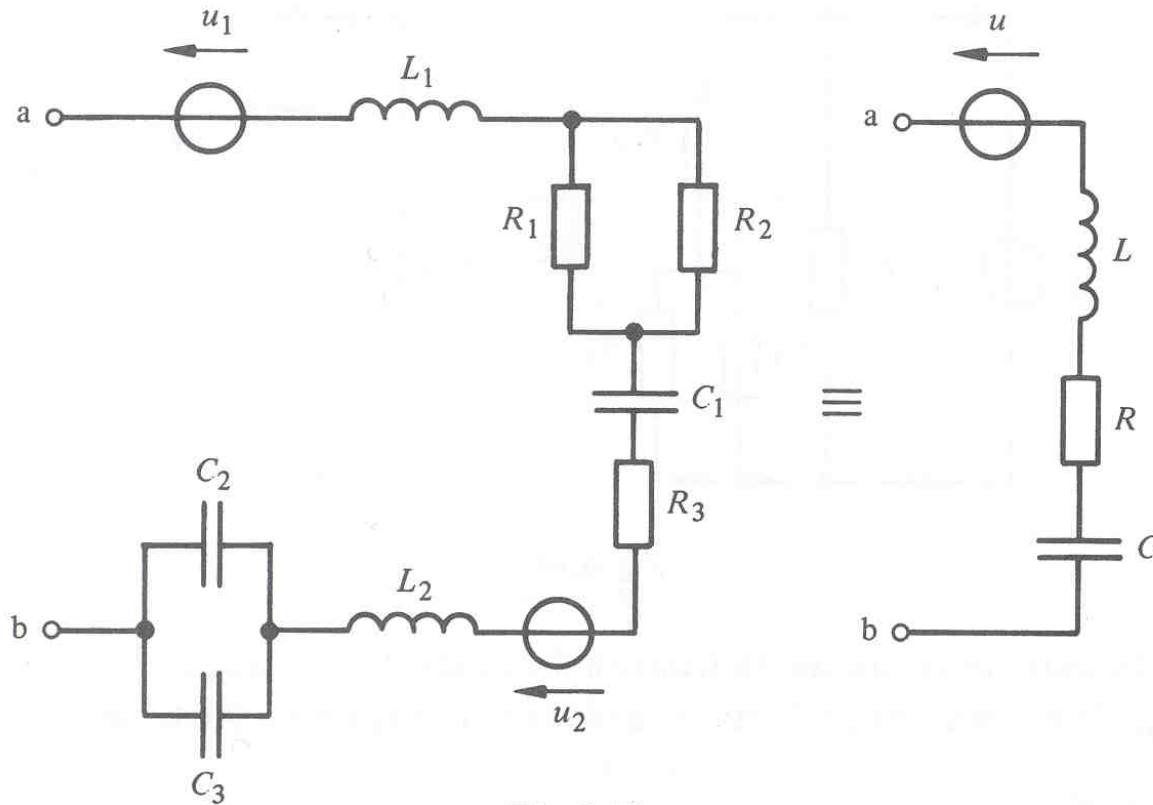
# Series-parallel circuits



$$\begin{aligned}u &= 73,45 \text{ V} \\R_1 &= 22 \Omega \\R_2 &= 36 \Omega \\R_3 &= 18 \Omega \\R_4 &= 15 \Omega\end{aligned}$$

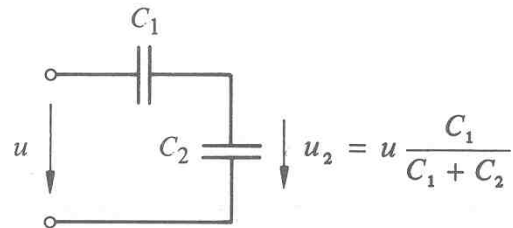
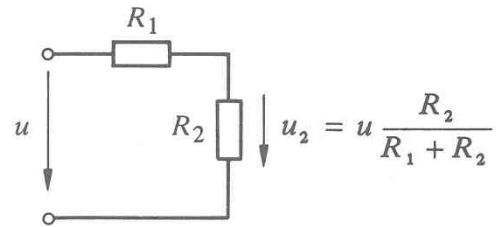
$$R_{ab} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + \frac{R_3 R_4}{R_3 + R_4}}}$$

# Series-parallel circuits

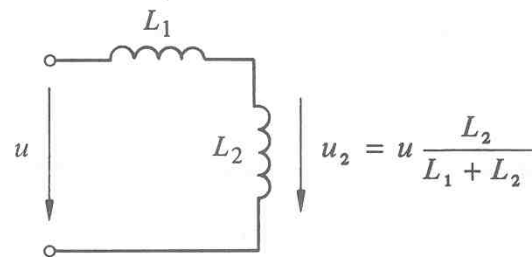


$$R = R_3 + \frac{R_1 R_2}{R_1 + R_2} \quad u = u_1 - u_2 \quad L = L_1 + L_2 \quad C = \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$

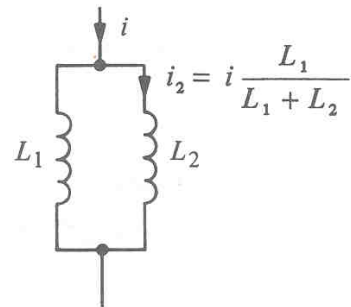
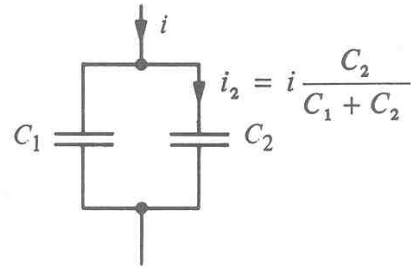
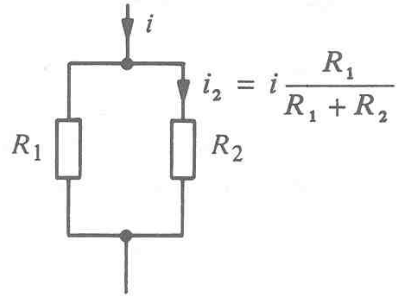
# Voltage divider



(No initial voltages)

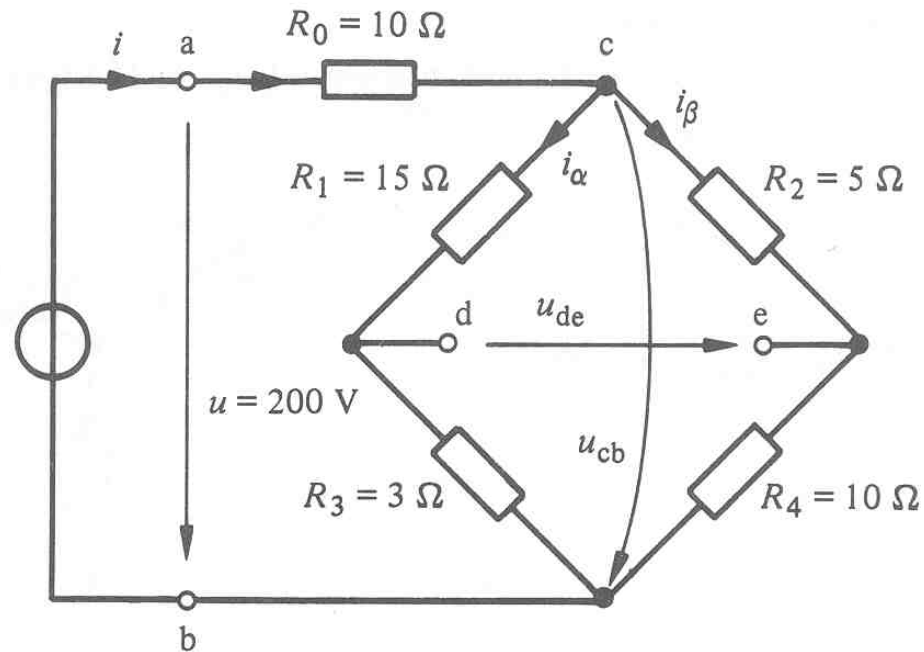


# Current divider



(No initial currents)

# Voltage / current divider : example



Solution on the board