

Analog IC design (EE-320), Lecture 6

Prof. Mahsa Shoaran

Institute of Electrical and Micro Engineering, School of Engineering, EPFL

Homework 1, TP Schedule

Week	Subject by week – EE-320: Analog IC design – Fall 2025	Suggested Chapters
Week 1: 08/09 – 14/09	Introduction, organization, review of MOS transistors + Exercise1	Ch 1, Ch 2.1-2.4, Slides on Moodle
Week 2: 15/09 – 21/09	MOS large and small-signal models, regimes of operations + Exercise2	Ch 2.1-2.4
Week 3: 22/09 – 28/09	Holiday – No class	
Week 4: 29/09 – 05/10	MOS parasitic effects, layout basic, single-stage amplifiers + Exercise3	Ch 2.1-2.4, Ch 3.1
Week 5: 06/10 – 12/10	Single-stage amplifiers + Exercise4	Ch 3.1-3.7
Week 6: 13/10 – 19/10	Single-stage amplifiers + Exercise5	Ch 3.1-3.7
Week 7: 20/10 – 26/10	Break – No class	
Week 8: 27/10 – 02/11	Single-stage amplifiers + Cascode + Exercise6 + Homework1	Ch 3.1-3.7
Week 9: 03/11 – 09/11	Differential amplifiers + Exercise7	Ch 4.1-4.4
Week 10: 10/11 – 16/11	TP1 Practical exercise session on Cadence	Tutorial on Moodle
Week 11: 17/11 – 23/11	TP2 Practical exercise session on Cadence	Tutorial on Moodle
Week 12: 24/11 – 30/12	TP3 Practical exercise session on Cadence + Homework2	Tutorial on Moodle
Week 13: 01/12 – 07/12	TP4 Practical exercise session on Cadence	Tutorial on Moodle
Week 14: 08/12 – 14/12	Differential amplifiers, current mirrors + Exercise8	Ch 4.1-4.4, Ch 5.1-5.3
Week 15: 15/12 – 22/12	Current mirrors + Exercise9	Ch 5.1-5.3

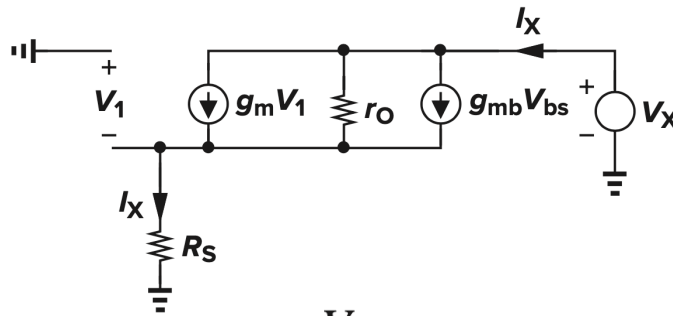
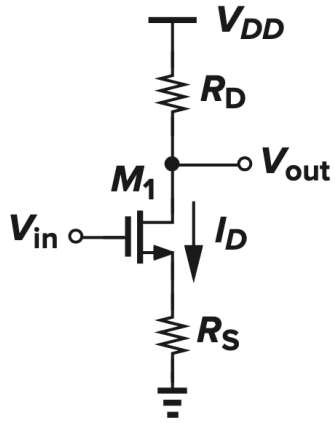
Assessment:

- **Written final exam:** 70% of the final grade
- **Homework (2 in total):** 30% of the final grade

EDA Statement

- **Assignment for EDA document**, to be signed and uploaded via moodle, please complete the assignment **by Nov 3**
- **First TP session on Nov 10 in **BC07-08** from **3:15pm to 6pm****
- **Homework 1 assignment** will be posted later this week
- **Homework 2 assignment** will be posted after TP3

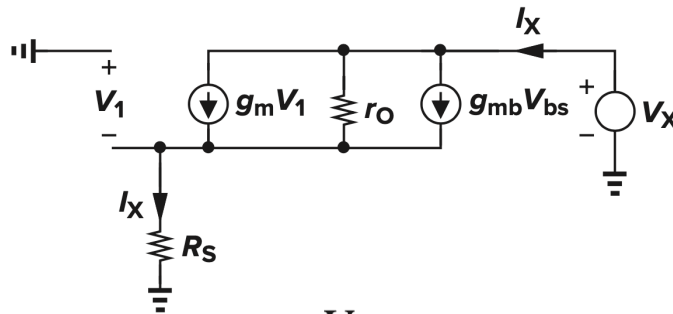
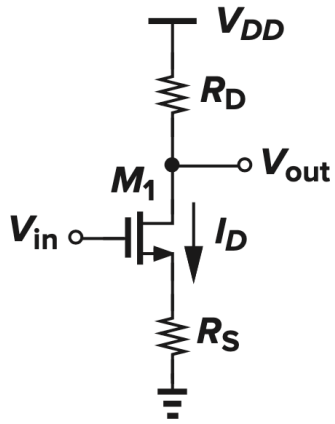
Review: Common-Source



$$R_{out} = [1 + (g_m + g_{mb})R_S]r_O + R_S$$
$$= [1 + (g_m + g_{mb})r_O]R_S + r_O$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_m r_O R_D}{R_D + R_S + r_O + (g_m + g_{mb})R_S r_O}$$

Review: Common-Source, Source-Follower

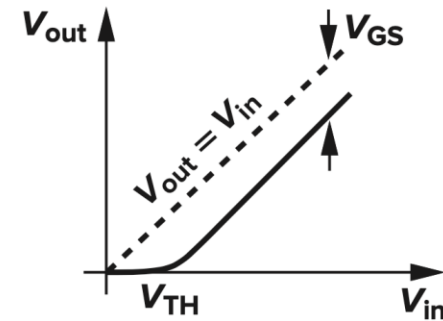
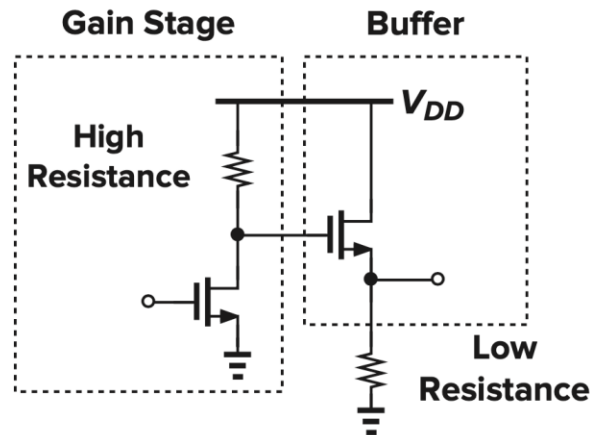
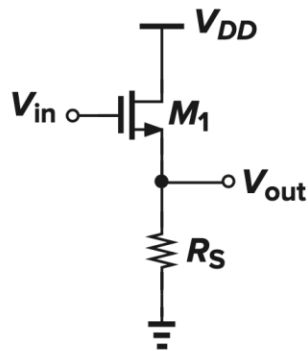


$$R_{out} = [1 + (g_m + g_{mb})R_S]r_O + R_S$$

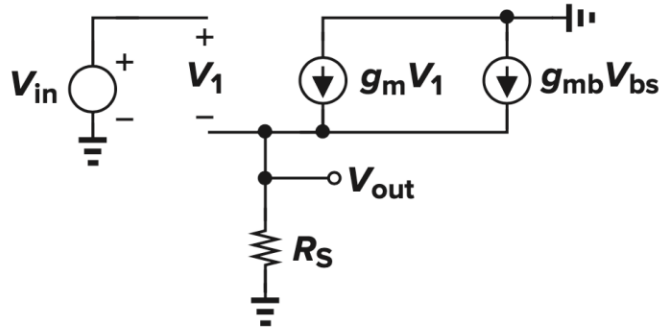
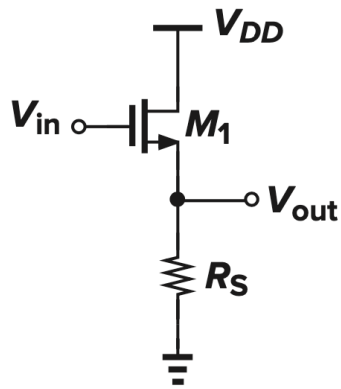
$$= [1 + (g_m + g_{mb})r_O]R_S + r_O$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_m r_O R_D}{R_D + R_S + r_O + (g_m + g_{mb})R_S r_O}$$

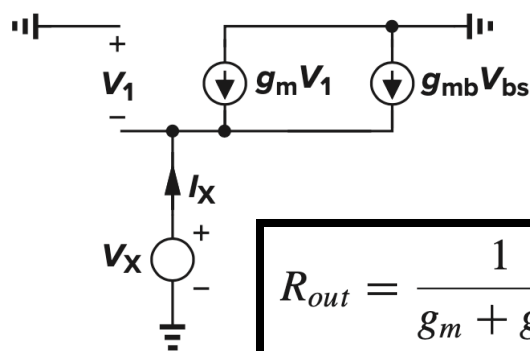
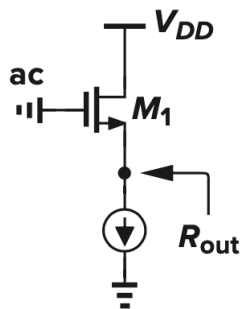
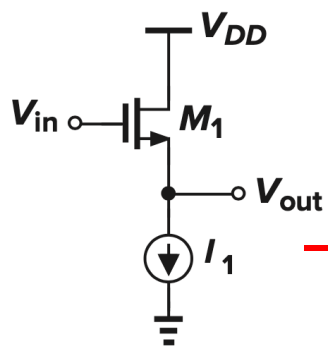
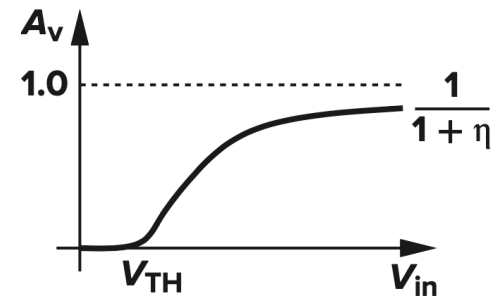
Source follower: common-drain or voltage buffer



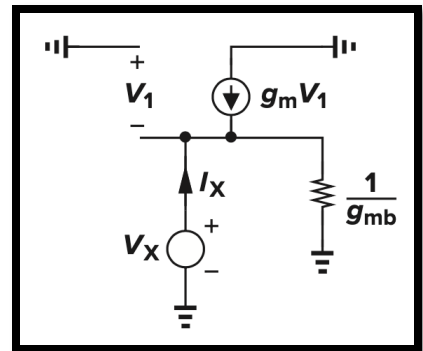
Review: Source-Follower



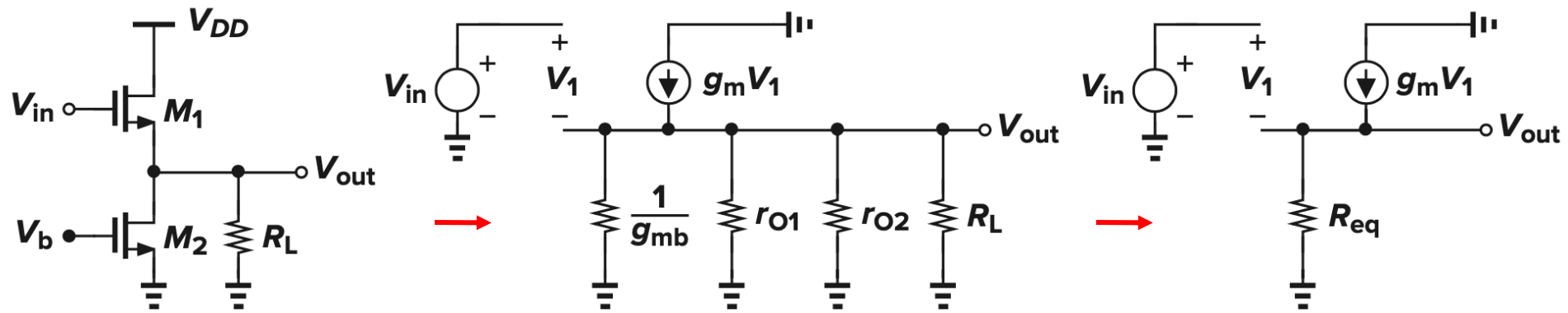
$$A_v = \frac{g_m R_S}{1 + (g_m + g_{mb}) R_S}$$



$$R_{out} = \frac{1}{g_m + g_{mb}}$$



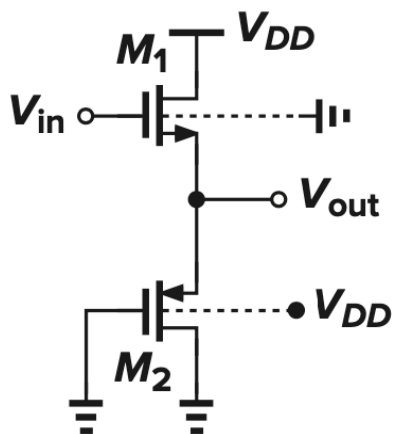
Review: Source Follower with a finite load resistance



$$A_v = \frac{R_{eq}}{R_{eq} + \frac{1}{g_m}}$$

$$R_{eq} = (1/g_{mb}) \parallel r_{O1} \parallel r_{O2} \parallel R_L$$

Example: Calculate the voltage gain



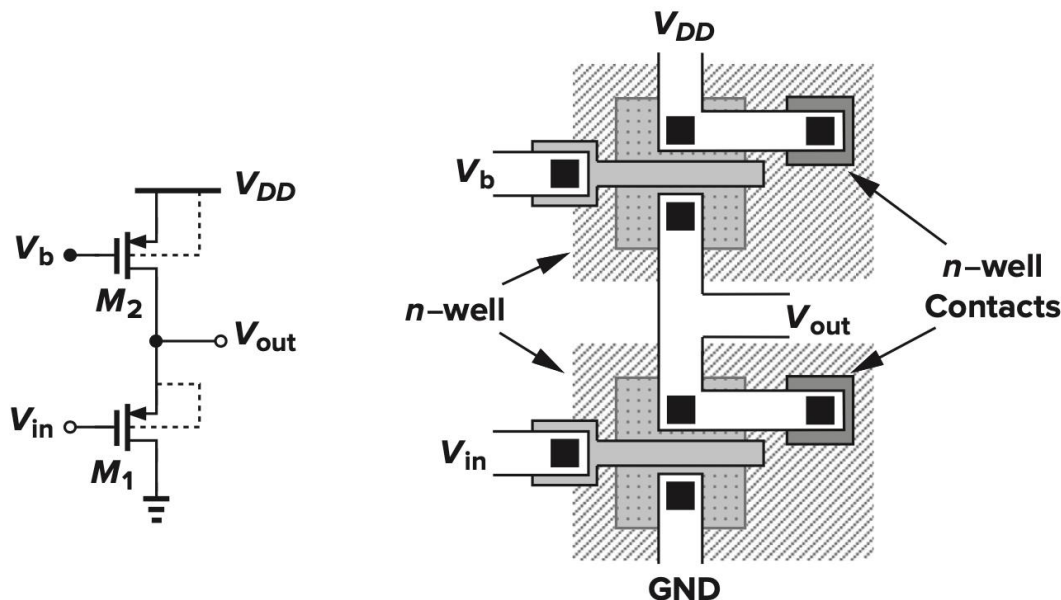
$$A_v = \frac{\frac{1}{g_{m2} + g_{mb2}} \parallel r_{O2} \parallel r_{O1} \parallel \frac{1}{g_{mb1}}}{\frac{1}{g_{m2} + g_{mb2}} \parallel r_{O2} \parallel r_{O1} \parallel \frac{1}{g_{mb1}} + \frac{1}{g_{m1}}}$$

PMOS Source Follower with no body effect

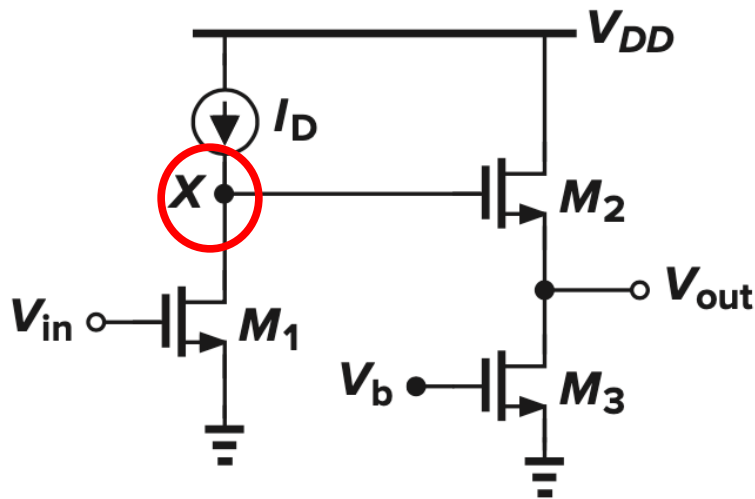
- Source follower drawbacks:
 - **nonlinearity**
 - **voltage headroom limitation**
- Even if biased by an ideal current source, some nonlinearity exists due to the **nonlinear** dependence of V_{TH} upon the **source** potential

PMOS Source Follower with no body effect

- Source follower drawbacks:
 - **nonlinearity**
 - **voltage headroom limitation**
- Even if biased by an ideal current source, some nonlinearity exists due to the **nonlinear** dependence of V_{TH} upon the **source** potential
- The nonlinearity due to body effect can be eliminated if the bulk is tied to source



Source Follower: a level shifter

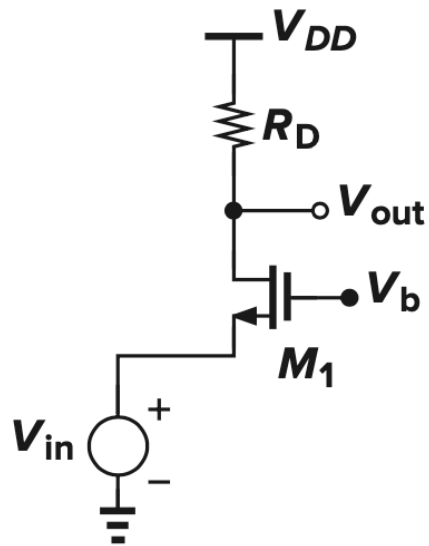


$$V_{GS1} - V_{TH1} \quad \rightarrow \quad V_{GS2} + (V_{GS3} - V_{TH3})$$

- Source followers shift the dc level by V_{GS} , thereby consuming voltage headroom and limiting the swings
- One application of source followers is in performing **voltage-level shift**

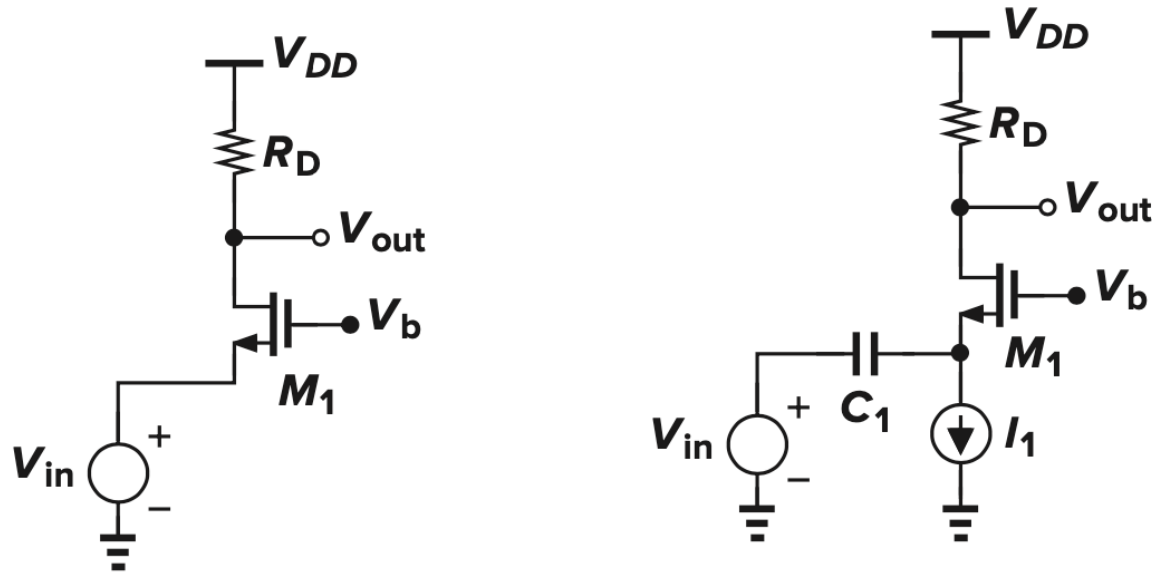
Common-Gate Stage

- A common-gate (CG) stage senses the input at the **source** and produces the output at the **drain**

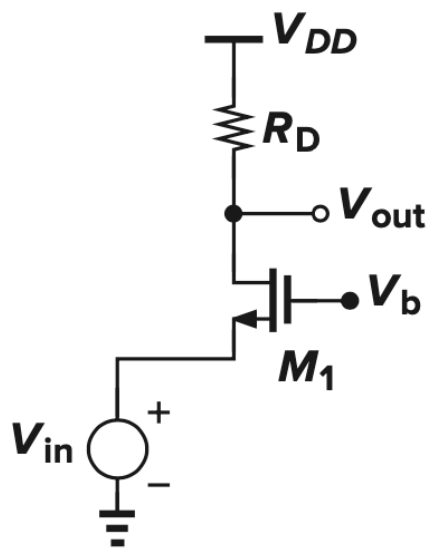


Common-Gate Stage

- A common-gate (CG) stage senses the input at the **source** and produces the output at the **drain**



Common-Gate Stage: large signal analysis



in saturation

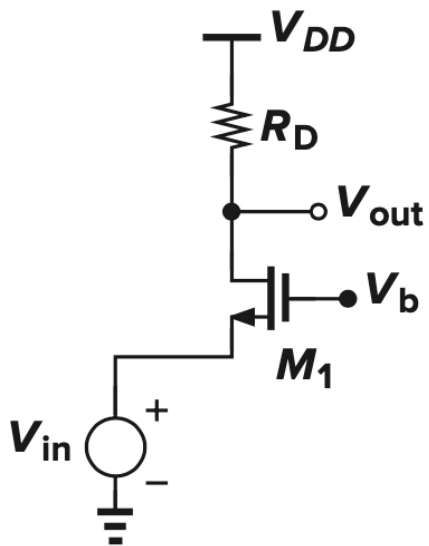
$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2$$

$$V_{out} = V_{DD} - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2 R_D$$

driving M_1 into the triode region if:

$$V_{DD} - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2 R_D = V_b - V_{TH}$$

Common-Gate Stage: large signal analysis



in saturation

$$V_{out} = V_{DD} - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2 R_D$$

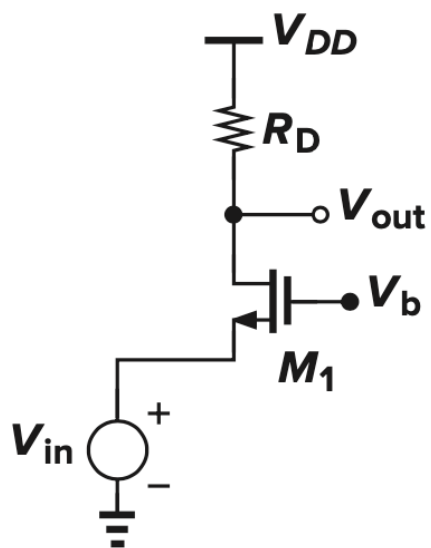
$$\frac{\partial V_{out}}{\partial V_{in}} = -\mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH}) \left(-1 - \frac{\partial V_{TH}}{\partial V_{in}} \right) R_D$$

$$\boxed{\frac{\partial V_{TH}}{\partial V_{in}} = \frac{\partial V_{TH}}{\partial V_{SB}} = \eta}$$

$$= \mu_n C_{ox} \frac{W}{L} R_D (V_b - V_{in} - V_{TH}) (1 + \eta)$$

$$\boxed{= g_m (1 + \eta) R_D}$$

Common-Gate Stage: large signal analysis



in saturation

$$V_{out} = V_{DD} - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2 R_D$$

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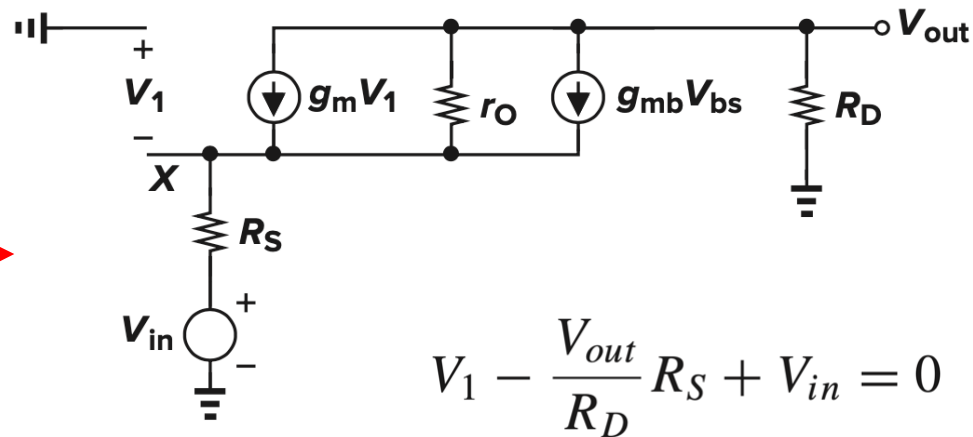
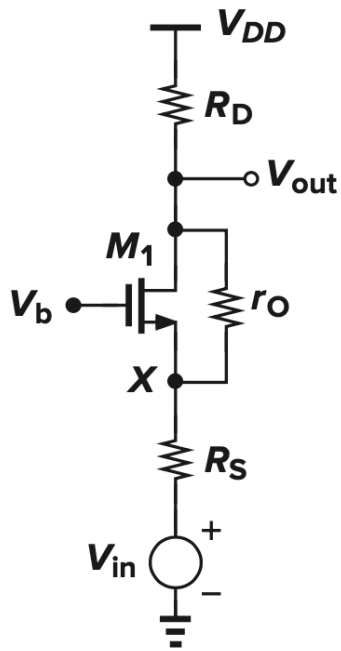
$$= \mu_n C_{ox} \frac{W}{L} R_D (V_b - V_{in} - V_{TH}) (1 + \eta)$$

$$\boxed{= g_m (1 + \eta) R_D}$$

- for $\lambda = 0$, the **input impedance** (seen at the source of M_1): $\boxed{1/(g_m + g_{mb})}$

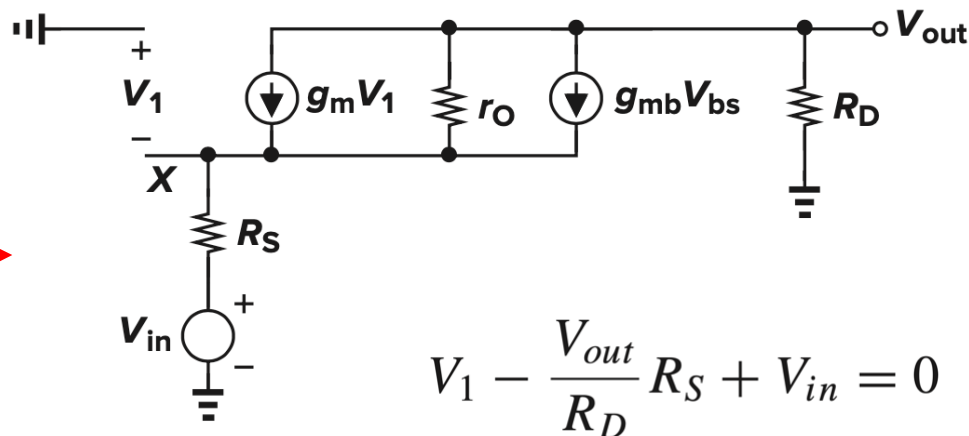
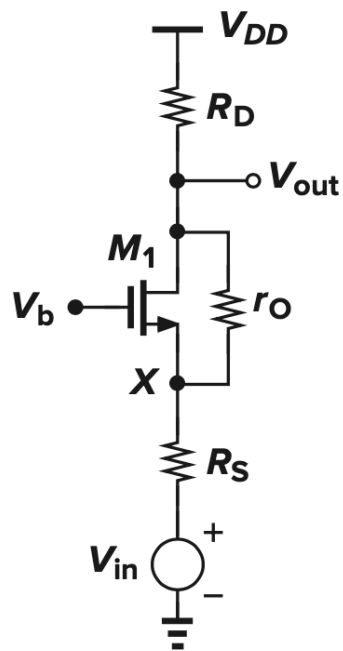
$$\boxed{1/[g_m (1 + \eta)]}$$

Common-Gate Stage: general case



$$V_1 - \frac{V_{out}}{R_D} R_S + V_{in} = 0$$

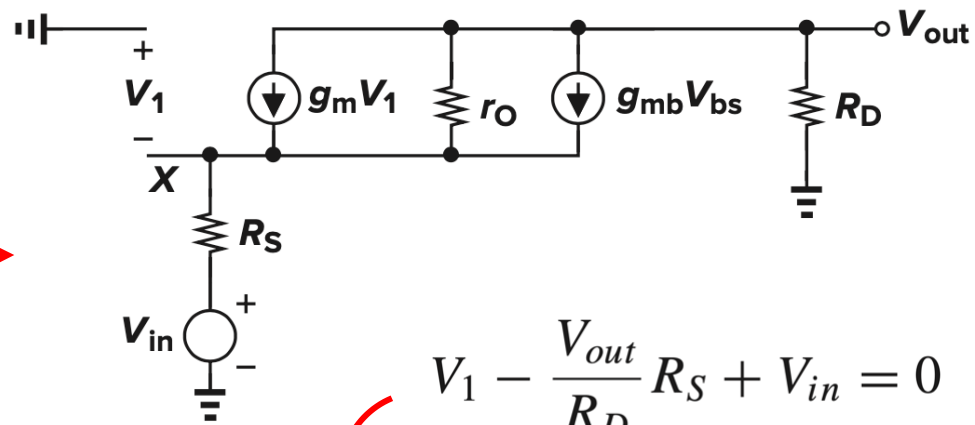
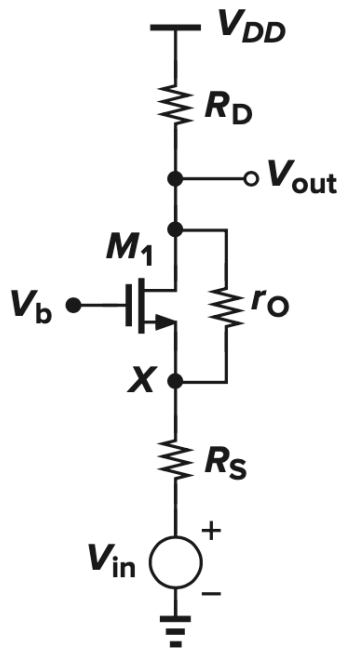
Common-Gate Stage: general case



$$V_1 - \frac{V_{out}}{R_D} R_S + V_{in} = 0$$

$$r_O \left(\frac{-V_{out}}{R_D} - g_m V_1 - g_{mb} V_1 \right) - \frac{V_{out}}{R_D} R_S + V_{in} = V_{out}$$

Common-Gate Stage: general case

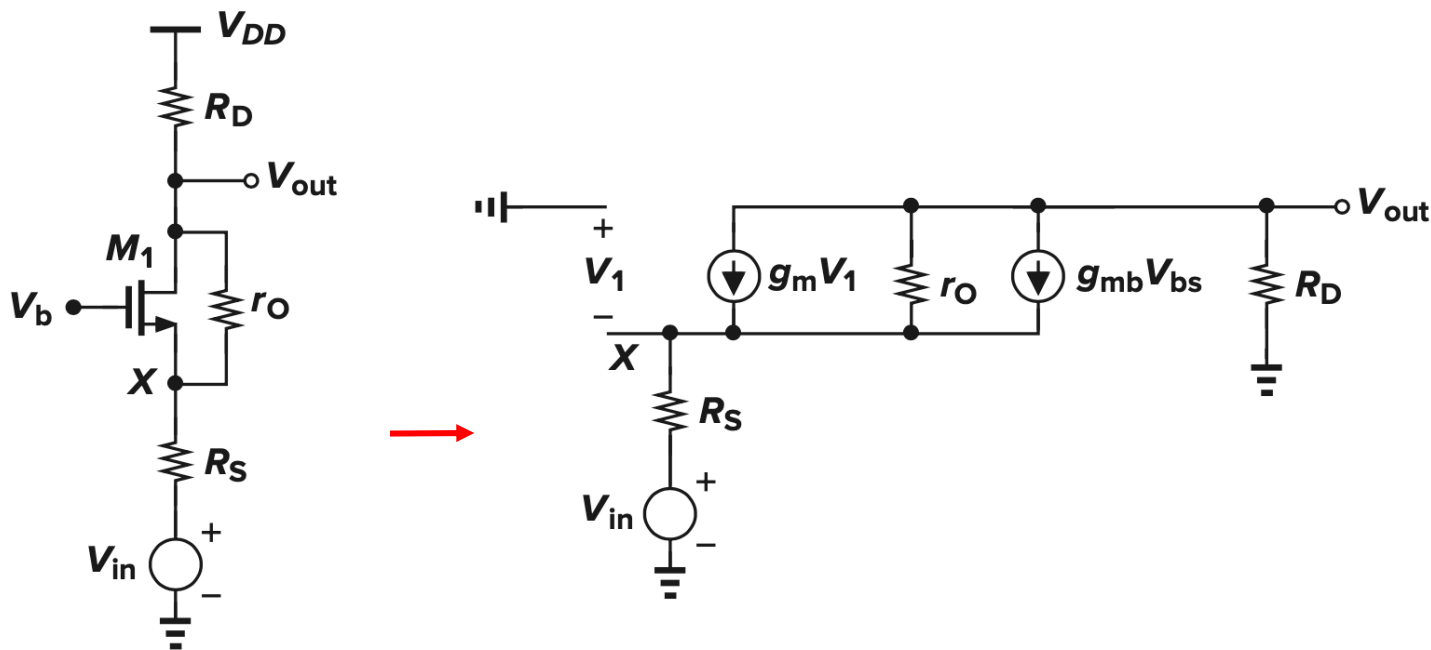


$$V_1 - \frac{V_{out}}{R_D} R_S + V_{in} = 0$$

$$r_o \left(\frac{-V_{out}}{R_D} - g_m V_1 - g_{mb} V_1 \right) - \frac{V_{out}}{R_D} R_S + V_{in} = V_{out}$$

$$r_o \left[\frac{-V_{out}}{R_D} - (g_m + g_{mb}) \left(V_{out} \frac{R_S}{R_D} - V_{in} \right) \right] - \frac{V_{out} R_S}{R_D} + V_{in} = V_{out}$$

Common-Gate Stage: general case

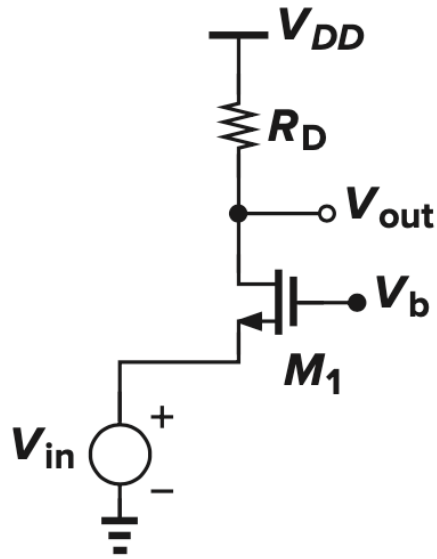
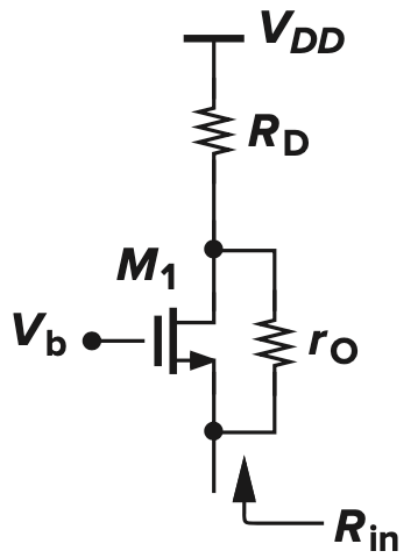


$$\frac{V_{out}}{V_{in}} = \frac{(g_m + g_{mb})r_O + 1}{r_O + (g_m + g_{mb})r_O R_S + R_S + R_D} R_D$$

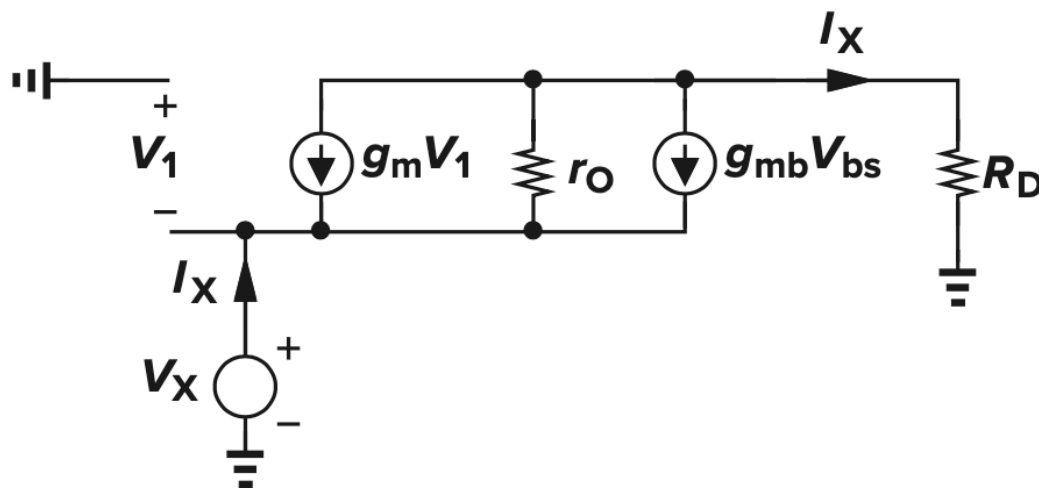
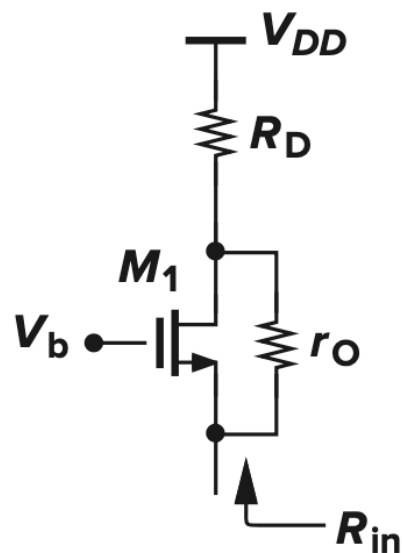
Gain of a degenerated CS stage

$$\frac{V_{out}}{V_{in}} = \frac{-g_m r_O R_D}{R_D + R_S + r_O + (g_m + g_{mb}) R_S r_O}$$

Input resistance of a CG stage



Input resistance of a CG stage



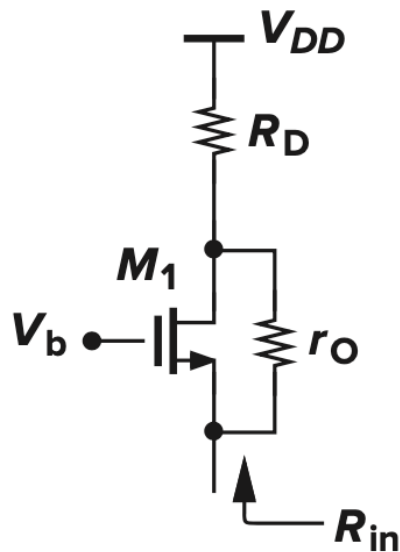
$$R_D I_X + r_O [I_X - (g_m + g_{mb}) V_X] = V_X$$



$$\frac{V_X}{I_X} = \frac{R_D + r_O}{1 + (g_m + g_{mb}) r_O}$$

$$\approx \frac{R_D}{(g_m + g_{mb}) r_O} + \frac{1}{g_m + g_{mb}}$$

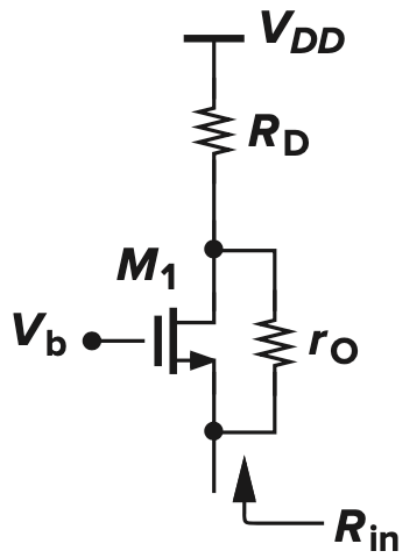
Input resistance of a CG stage



$$\frac{V_X}{I_X} = \frac{R_D + r_o}{1 + (g_m + g_{mb})r_o}$$
$$\approx \frac{R_D}{(g_m + g_{mb})r_o} + \frac{1}{g_m + g_{mb}}$$

- ✓ the **drain impedance** is **divided** by $(g_m + g_{mb})r_o$ when seen at the **source**

Input resistance of a CG stage

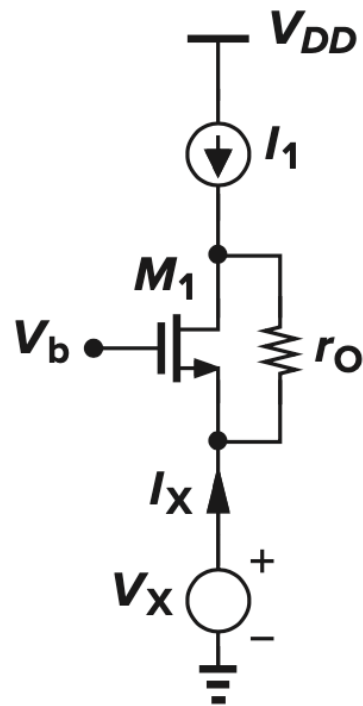


$$\frac{V_X}{I_X} = \frac{R_D + r_o}{1 + (g_m + g_{mb})r_o}$$
$$\approx \frac{R_D}{(g_m + g_{mb})r_o} + \frac{1}{g_m + g_{mb}}$$

✓ the **drain impedance** is **divided** by $(g_m + g_{mb})r_o$ when seen at the **source**

$$R_D = 0 \quad \rightarrow \quad \frac{V_X}{I_X} = \frac{r_o}{1 + (g_m + g_{mb})r_o}$$
$$= \frac{1}{\frac{1}{r_o} + g_m + g_{mb}}$$

Input resistance of a CG stage

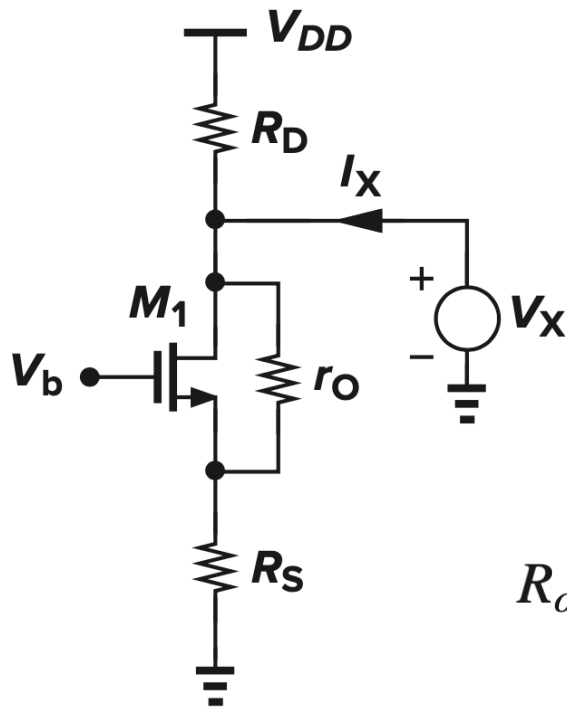


$$\frac{V_X}{I_X} = \frac{R_D + r_O}{1 + (g_m + g_{mb})r_O}$$
$$\approx \frac{R_D}{(g_m + g_{mb})r_O} + \frac{1}{g_m + g_{mb}}$$



the input impedance approaches infinity

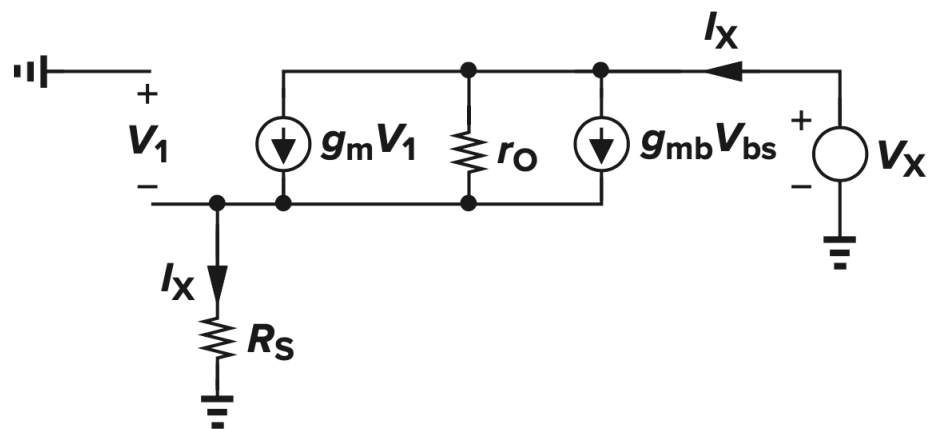
Output resistance of a CG stage



$$R_{out} = \{[1 + (g_m + g_{mb})r_O]R_S + r_O\} \parallel R_D$$

Recall: Output resistance of a degenerated CS

- Source degeneration **increases** the output resistance



$$r_O [I_X + (g_m + g_{mb}) R_S I_X] + I_X R_S = V_X$$

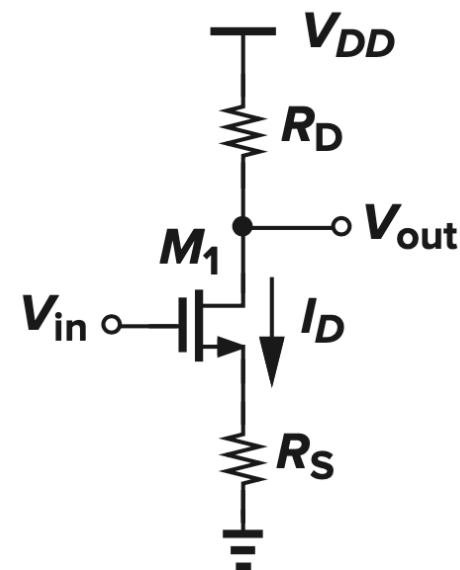
$$R_{out} = [1 + (g_m + g_{mb}) R_S] r_O + R_S$$

$$= [1 + (g_m + g_{mb}) r_O] R_S + r_O$$

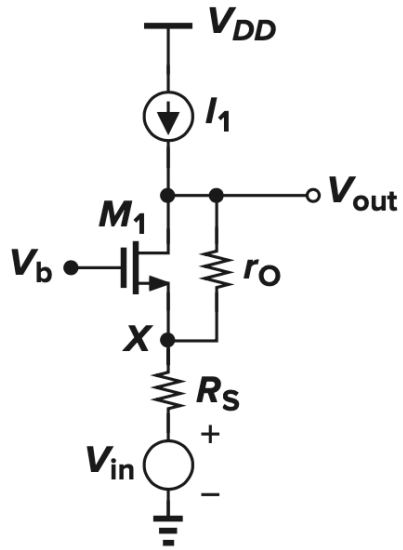
$$\approx (g_m + g_{mb}) r_O R_S + r_O$$

$$= [1 + (g_m + g_{mb}) R_S] r_O$$

- The overall output resistance is the parallel combination of R_{out} and R_D

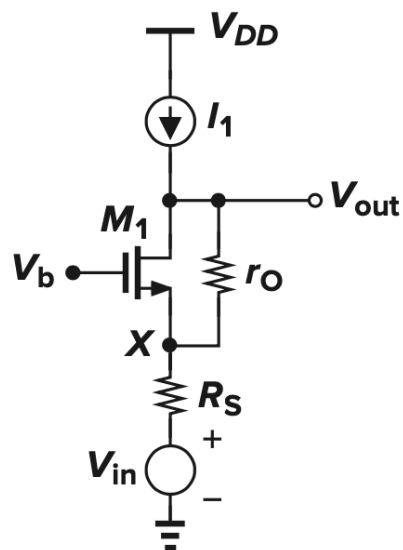


Example: Calculate the voltage gain




- A common-gate stage with a **current-source load**

Example: Calculate the voltage gain



- A common-gate stage with a current-source load

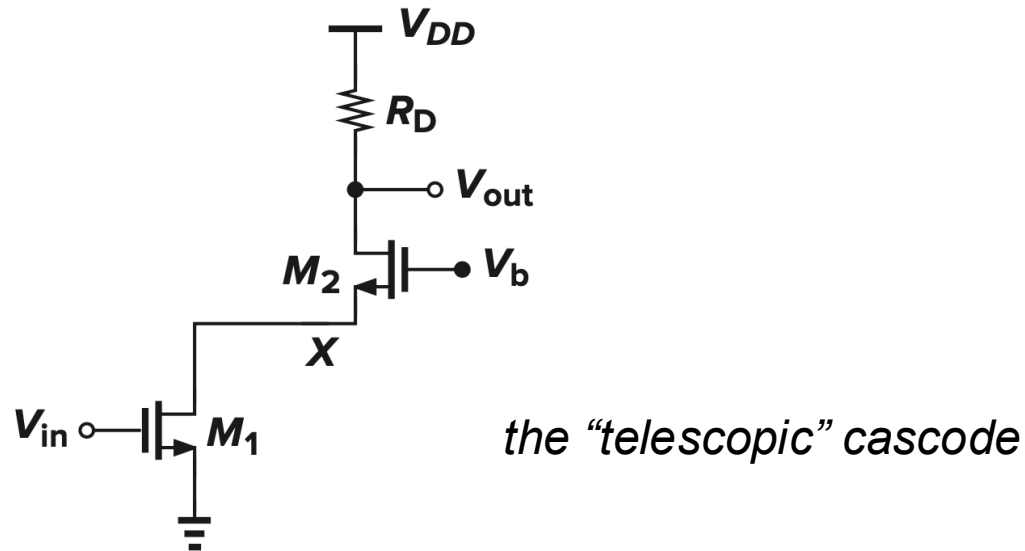
$$\frac{V_{out}}{V_{in}} = \frac{(g_m + g_{mb})r_O + 1}{r_O + (g_m + g_{mb})r_O R_S + R_S + R_D} R_D$$

$$R_D \rightarrow \infty$$


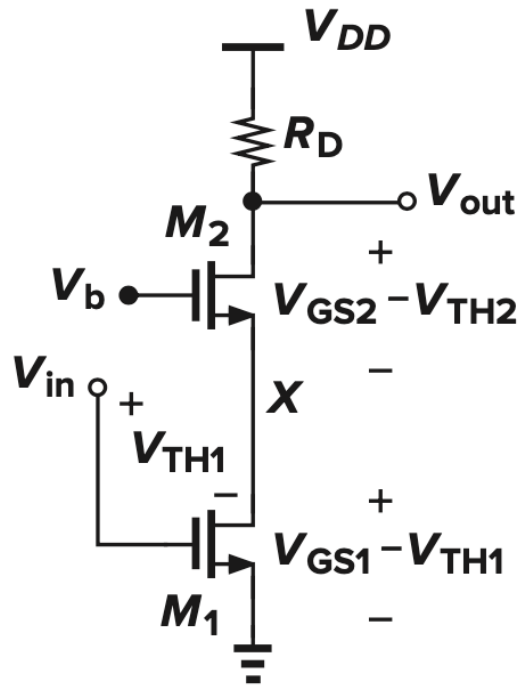
$$A_v = (g_m + g_{mb})r_O + 1$$

Cascode stage

- The cascade of a **CS stage** and a **CG stage** is called a “cascode” topology, providing many useful properties.



Cascode: bias condition



$$V_b - V_{GS2} \geq V_{in} - V_{TH1}$$

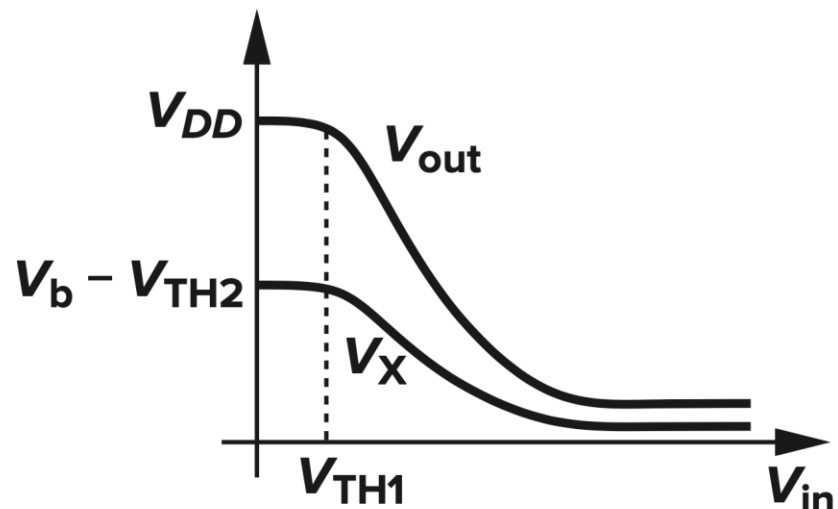
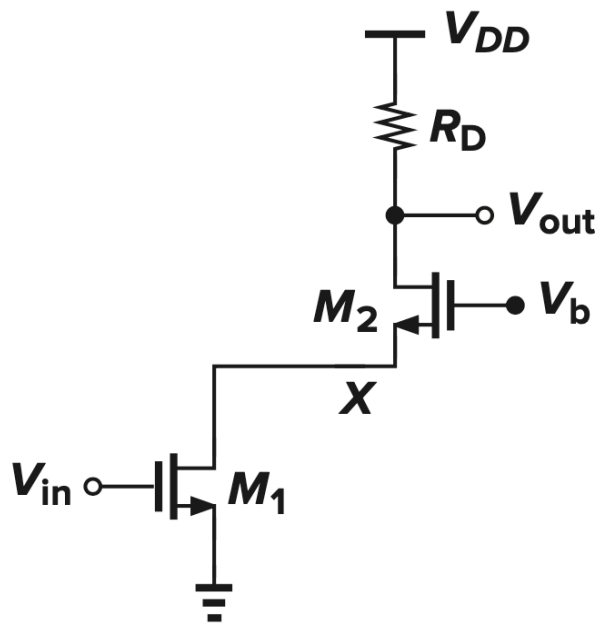
$$V_{out} \geq V_b - V_{TH2}$$



$$\begin{aligned} V_{out} &\geq V_{in} - V_{TH1} + V_{GS2} - V_{TH2} \\ &= (V_{GS1} - V_{TH1}) + (V_{GS2} - V_{TH2}) \end{aligned}$$

- ✓ the minimum output level for both transistors to operate in saturation is equal to the overdrive voltage of M_1 plus that of M_2

Cascode: large-signal behavior



Cascode stage: Small-signal model

