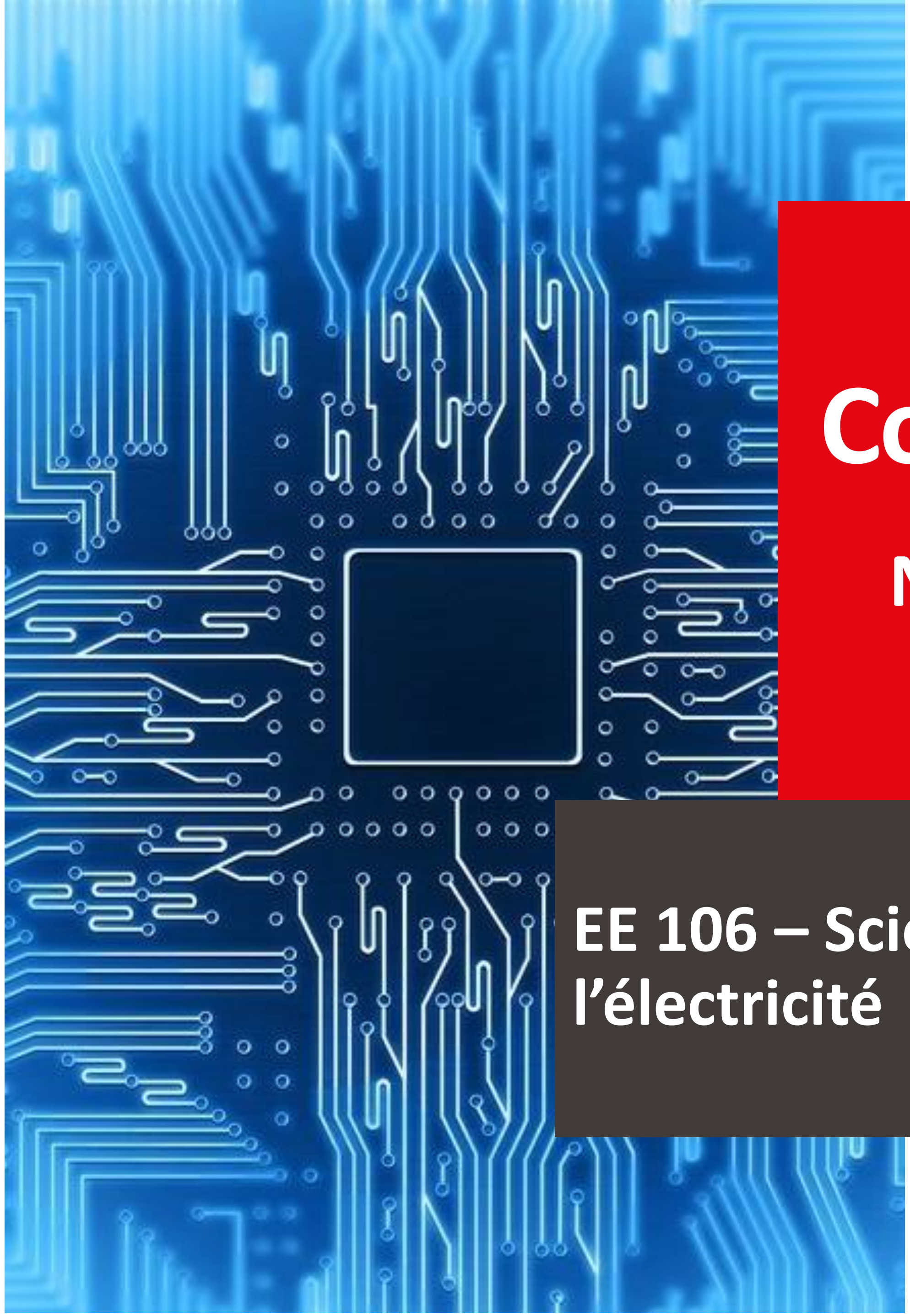
EPFL



Complément de cours

Nombres complexes

EE 106 – Sciences et Technologies de l’électricité

Ecole Polytechnique Fédérale de Lausanne

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Définitions

- Forme algébrique

$$\underline{z} = x + jy$$

- Forme trigonométrique

$$\underline{z} = \rho\left(\cos(\theta) + j\sin(\theta)\right)$$

- Forme exponentielle

$$\underline{z} = \rho e^{j\theta}$$

- $\mathrm{Re}\left(\underline{z}\right) = x = \rho \cdot \cos(\theta) \quad ; \quad \mathrm{Im}\left(\underline{z}\right) = y = \rho \cdot \sin(\theta)$

  $\left|\underline{z}\right| = \rho = \sqrt{x^2 + y^2} \quad ; \quad \arg\left(\underline{z}\right) = \theta$

- $\cos(\theta) = \frac{x}{\rho} \quad ; \quad \sin(\theta) = \frac{y}{\rho} \quad ; \quad \tan(\theta) = \frac{y}{x}$

Complexe conjugué

- Forme algébrique

$$\underline{z} = x + jy \;\Rightarrow \underline{z}^* = x - jy$$

- Forme trigonométrique

$$\underline{z} = \rho\big(\cos(\theta) + j\sin(\theta)\big) \Rightarrow \underline{z}^* = \rho(\cos(\theta) - j\sin(\theta))$$

- Forme exponentielle

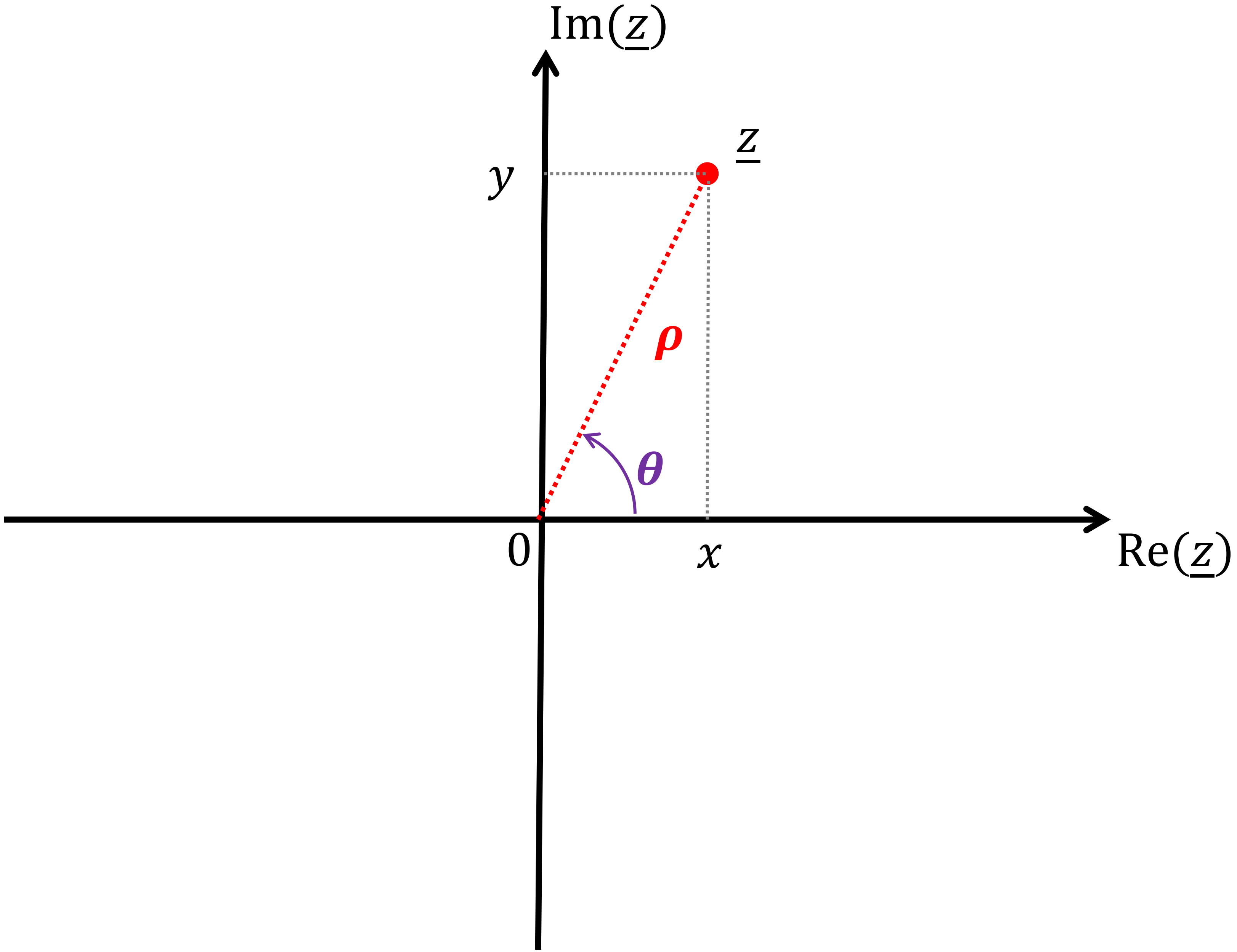
$$\underline{z} = \rho e^{j\theta} \;\Rightarrow \underline{z}^* = \rho e^{-j\theta}$$

- $\mathrm{Re}(\underline{z}^*) = x = \rho \cdot \cos(\theta) \quad ; \quad \mathrm{Im}(\underline{z}^*) = -y = -\rho \cdot \sin(\theta)$

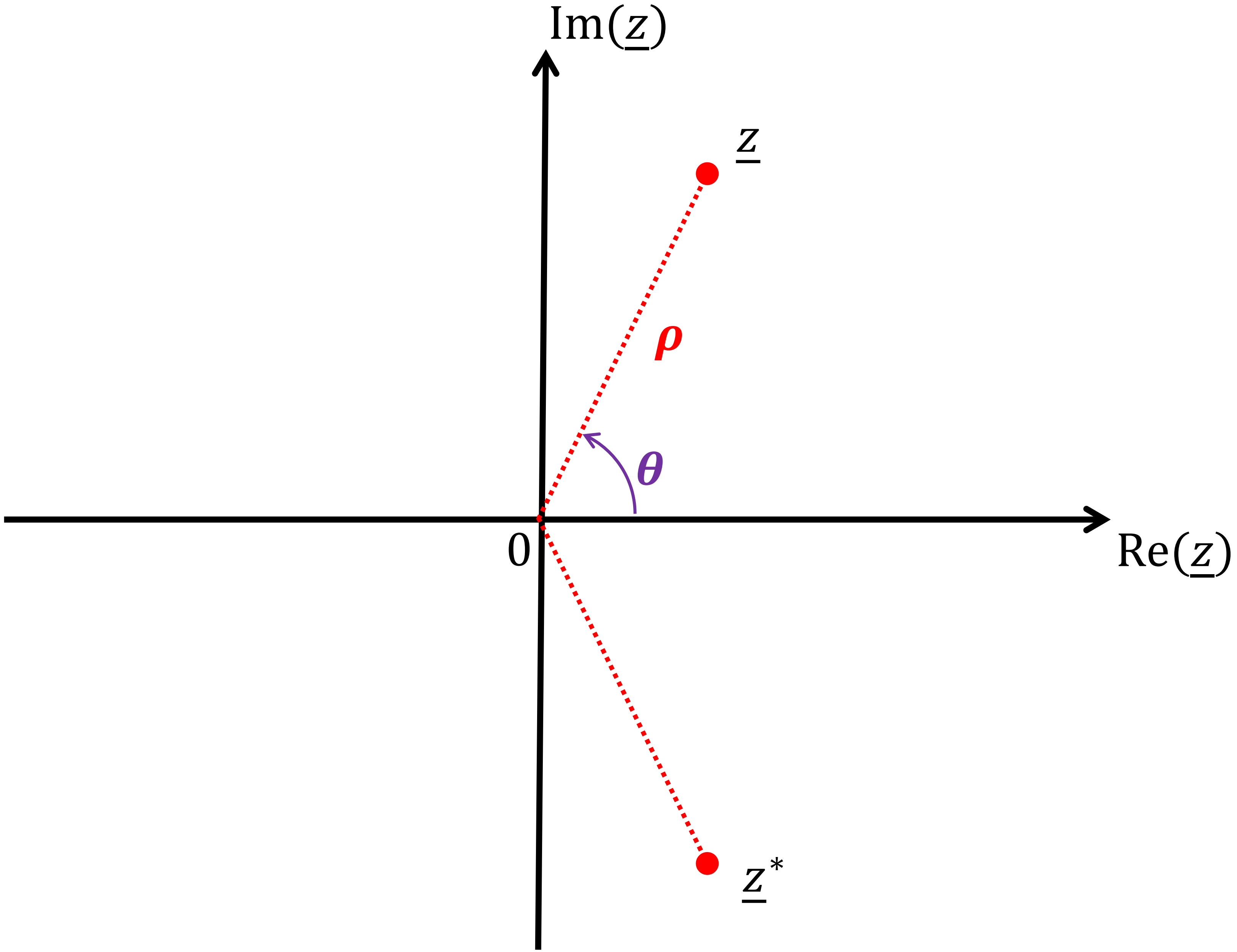
  $|\underline{z}^*| = \rho = \sqrt{x^2 + y^2} \quad ; \quad \arg(\underline{z}^*) = -\theta$

- $|\underline{z}|^2 = \underline{z} \cdot \underline{z}^*$

Plan complexe

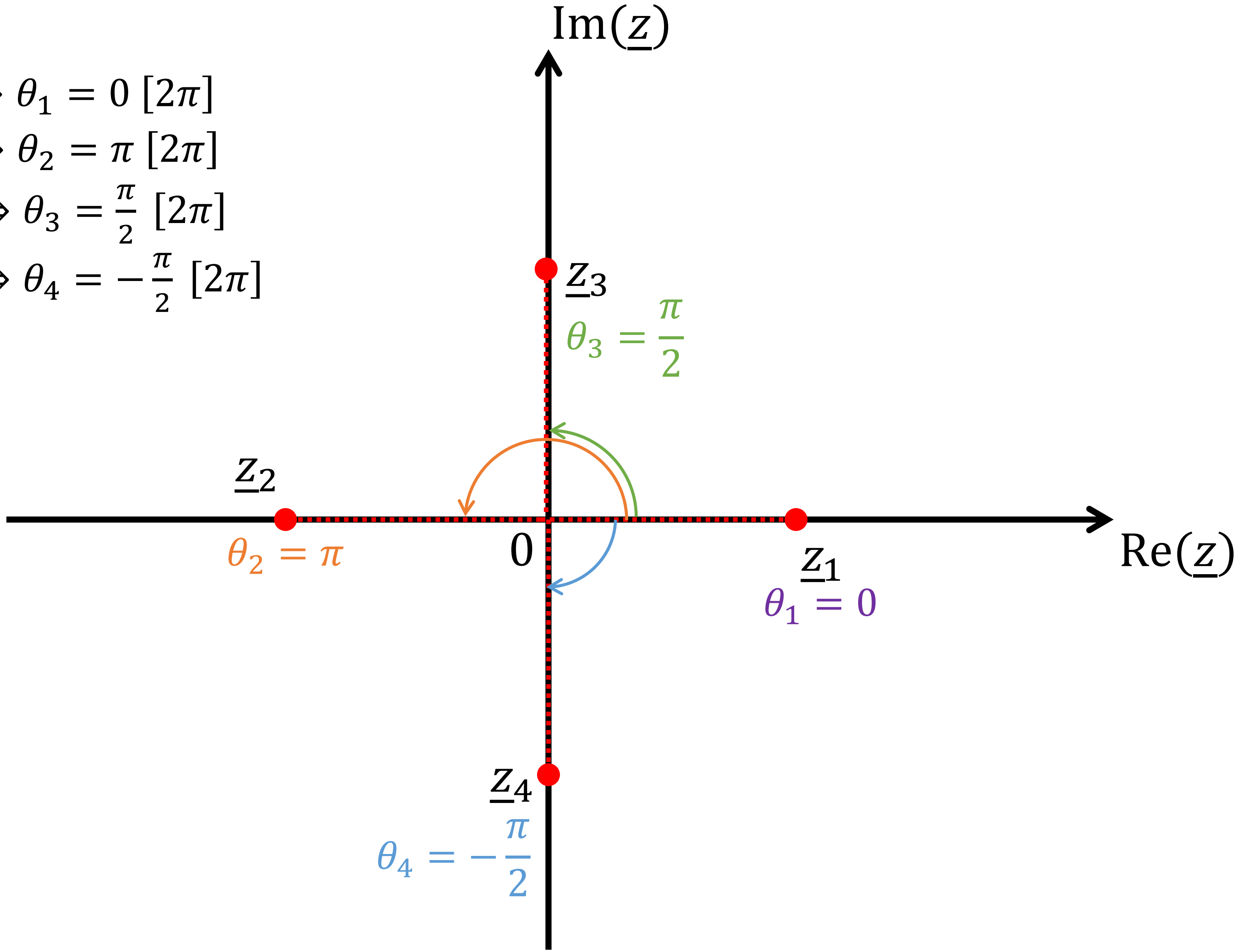


Plan complexe



Cas particuliers

- $\underline{z}_1 \in \mathbb{R}_+^* \Rightarrow \theta_1 = 0\ [2\pi]$
- $\underline{z}_2 \in \mathbb{R}_-^* \Rightarrow \theta_2 = \pi\ [2\pi]$
- $\underline{z}_3 \in j\mathbb{R}_+^* \Rightarrow \theta_3 = \frac{\pi}{2}\ [2\pi]$
- $\underline{z}_4 \in j\mathbb{R}_-^* \Rightarrow \theta_4 = -\frac{\pi}{2}\ [2\pi]$

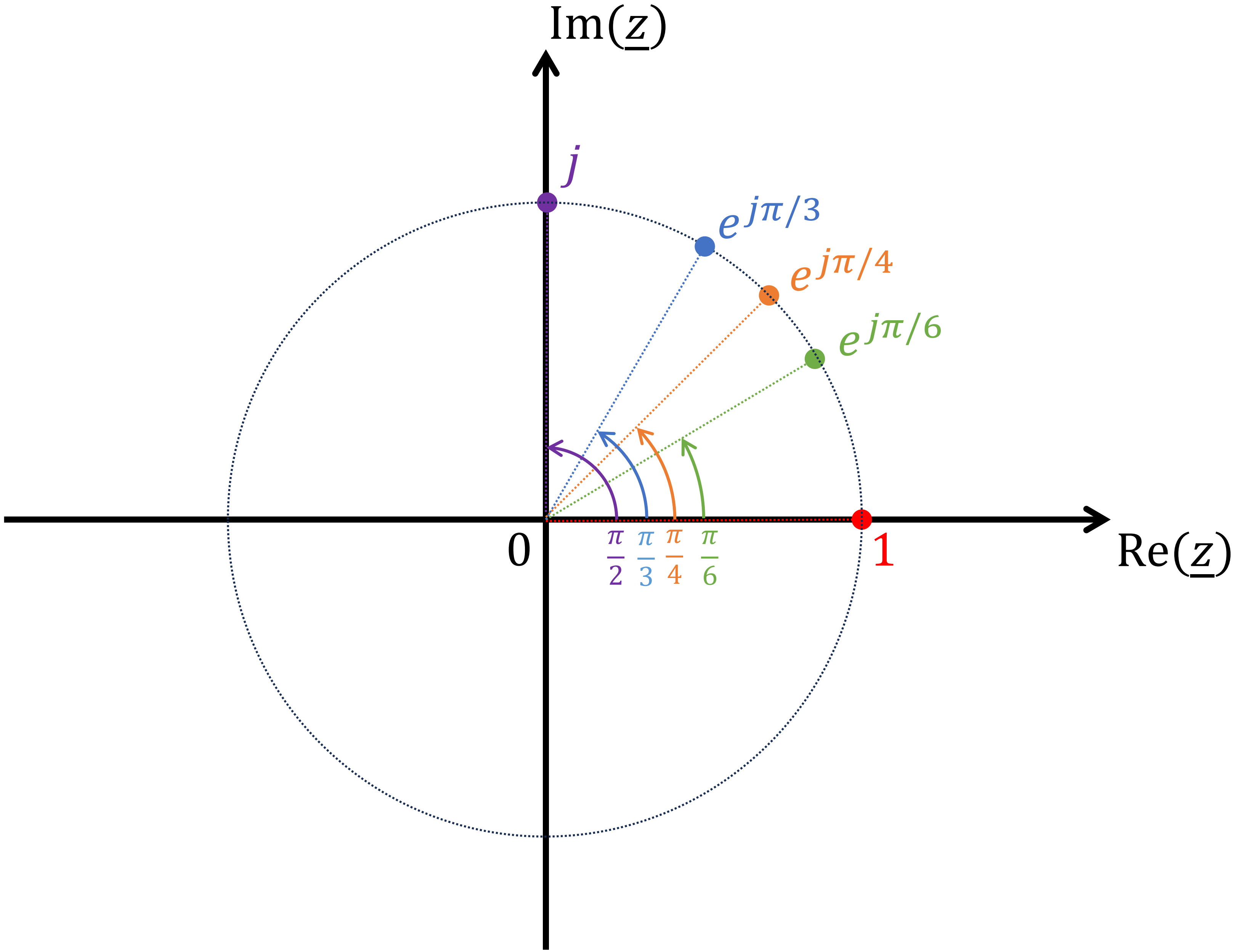


Cas particuliers

Points du cercle trigonométrique:

Angle (rad)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$+\infty$

Cas particuliers



Formules

- $\mathrm{Re}(\underline{z}_1 + \underline{z}_2) = \mathrm{Re}(\underline{z}_1) + \mathrm{Re}(\underline{z}_2)$
- $\mathrm{Im}(\underline{z}_1 + \underline{z}_2) = \mathrm{Im}(\underline{z}_1) + \mathrm{Im}(\underline{z}_2)$
- $|\underline{z}_1 \cdot \underline{z}_2| = |\underline{z}_1| \cdot |\underline{z}_2|$
- $(\underline{z}_1 \cdot \underline{z}_2)^* = \underline{z}_1^* \cdot \underline{z}_2^*$
- $\arg(\underline{z}_1 \cdot \underline{z}_2) = \arg(\underline{z}_1) + \arg(\underline{z}_2)$
- $\left|\frac{\underline{z}_1}{\underline{z}_2}\right| = \frac{|\underline{z}_1|}{|\underline{z}_2|}$
- $\arg\left(\frac{\underline{z}_1}{\underline{z}_2}\right) = \arg(\underline{z}_1) - \arg(\underline{z}_2)$
- $|\underline{z}^n| = |\underline{z}|^n$
- $\arg(\underline{z}^n) = n \cdot \arg(\underline{z})$

Cas particuliers

$\underline{z} = x + j \cdot y = \rho \cdot e^{j\theta}$

- ❑ $x \in \mathbb{R}_+^* \Rightarrow \theta = \arctan\left(\frac{y}{x}\right)$
- ❑ $x \in \mathbb{R}_-^*, y \in \mathbb{R}_+^* \Rightarrow \theta = \arctan\left(\frac{y}{x}\right) + \pi$
- ❑ $x \in \mathbb{R}_-^*, y \in \mathbb{R}_-^* \Rightarrow \theta = \arctan\left(\frac{y}{x}\right) - \pi$