Plan

Time and Place: Mondays, 13:30-15:30 from 6.10.25-15.12.25. Bernoulli Center, GA335.

Outline

In this seminar we discuss the basic mathematical methods present in quantum error correction. We will have three tutorial sessions to introduce some basic concepts and the rest of the seminar will run in a reading-club style, where we all read a paper over the course of the week and 1-2 participants prepare a presentation and host a discussion on the paper together.
The goal of this presentation is not to showcase the most polished performance but for everybody to learn something.
The Tutorial sessions will be as follows

- Tutorial 1 (6.10.25, JC): Quantum mechanics- and computing, Quantum Error Correction, Knill-Laflamme Conditions, the Shor Code.
- Tutorial 2 (13.10.25, JC): Stabilizer codes, Homological Codes : Toric Code and hyperbolic codes.
- Tutorial 3 (27.10.25, TV): LDPC codes, expanders, connections to complexity.

Papers

Here is a list of papers we propose. Contact me `jonathan.conrad@epfl.ch` with your preferred paper. We can also incorporate alternative suggestions outside this list.

Coding Theory: Duality, SDPs and bounds from locality

The following papers discuss existence results for codes based on a duality argument / linear programming bounds for their weight distributions. Refs. [1, 2] are elementary in this regard (it would be nice to have a presentation on 20.10.25 on this already). These techniques are extended to a hierarchy of SDPs in [3].

[1] P. Shor and R. Laflamme. *Quantum MacWilliams Identities*. 1996.
[2] E. M. Rains. *Quantum Weight Enumerators*. 1996.
[3] G. A. Munné, A. Nemec, and F. Huber. *SDP bounds on quantum codes*. 2025.

LDPC Codes

Low-density-Parity-Check (LDPC) codes are stabilizer quantum error correcting codes with bounded locality for their generating set (they are abelian groups which possess a generating set bounded in a Hamming-metric). Their study is motivated by physical implementation of quantum error correction but are also relevant to questions in complexity theory. The existence of *good* LDPC codes has been one of the bigger recent events in quantum error correction and their construction proceeds via products of fiber bundles. Here the core paper is ref. [8], which builds on the "older" ideas presented in ref. [6, 7].
Refs. [4, 5] discuss general bounds on code parameters for stabilizer codes with restricted locality and ref. [9] provides a fun way to construct such codes.

[4] S. Bravyi, D. Poulin, and B. Terhal. "Tradeoffs for Reliable Quantum Information Storage in 2D Systems". In: *Physical Review Letters* 104.5 (Feb. 2010).

[5] N. Baspin et al. *Improved rate-distance trade-offs for quantum codes with restricted connectivity*. 2023.

[6] J.-P. Tillich and G. Zemor. "Quantum LDPC Codes With Positive Rate and Minimum Distance Proportional to the Square Root of the Blocklength". In: *IEEE Transactions on Information Theory* 60.2 (Feb. 2014), pp. 1193–1202.

[7] M. B. Hastings, J. Haah, and R. O'Donnell. "Fiber bundle codes: breaking the n 1/2 polylog(n) barrier for Quantum LDPC codes". In: *Proceedings of the 53rd Annual ACM SIGACT Symposium on Theory of Computing*. STOC '21. ACM, June 2021, pp. 1276–1288.

[8] P. Panteleev and G. Kalachev. *Asymptotically Good Quantum and Locally Testable Classical LDPC Codes*. 2022.

[9] D. J. Williamson and N. Baspin. *Layer Codes*. 2024.

Codes and Manifolds

We will see how to construct quantum codes from chain complexes built from tessellating manifolds. In ref. [10] Freedman and Hastings show how to "invert" this procedure and build manifolds from codes that possess $\mathbb{Z}_2$-*systolic freedom*. Portnoy then used these ideas in [12] to show how one can construct LDPC codes. A nice pedagogical discussion is found in ref. [11].

[10] M. Freedman and M. B. Hastings. *Building manifolds from quantum codes*. 2021.

[11] V. Guemard. *Lifting a CSS code via its handlebody realization*. 2025.

[12] E. Portnoy. *Local Quantum Codes from Subdivided Manifolds*. 2023.

GKP codes

GKP codes are quantum error correcting codes that encode quantum information hosted in a finite Hilbert space into an infinite Hilbert space, such as associated to the position- and momenta state of a quantum particle in $\mathbb{R}^n$. This is done by choosing stabilizer groups that are given by symplectic lattices (or their sublattices), such that lattice theoretic properties translate into the coding theory of these codes. Here Ref. [13] introduces these codes, motivates them as a limit of a commuting operator algebra on finite dimensional Hilbert spaces and discusses some physical intuition. Ref. [14] then uses a symplectic version of Siegel's mean value formula for lattices by Buser and Sarnak to show the existence of "good" GKP codes.

[13] D. Gottesman, A. Kitaev, and J. Preskill. "Encoding a qubit in an oscillator". In: *Phys. Rev. A* 64 (2001), p. 012310.

[14] J. Harrington and J. Preskill. "Achievable rates for the Gaussian quantum channel". In: *Physical Review A* 64.6 (Nov. 2001).

Mathematical pysics: Topological order

This is a whole other perspective on quantum error correction. The idea is to host quantum computation in the low energy sector of a Hamiltonian. In the first paper [15] Kitaev constructs such a model of quantum computation based on Drinfield's quantum double, which describes the algebra of excitations of this Hamiltonian and computation is discussed in terms of its representation theory. Some of the inherent ideas – like building a Hamiltonian based on terms from a geometric length-3 chain complex, are further generalized in ref. [17].

[15] A. Kitaev. "Fault-tolerant quantum computation by anyons". In: *Annals of Physics* 303.1 (Jan. 2003), pp. 2–30.

[17] J. Haah. "Commuting Pauli Hamiltonians as Maps between Free Modules". In: *Communications in Mathematical Physics* 324.2 (Oct. 2013), pp. 351–399.

Grading

This course awards 2 credits to IC and MA students. Passing will be decided based on the presentation and active participation (if there are more attendees than papers).