

CS-472: Design Technologies for Integrated Systems

Exercise Problem Set 6

Date: 27/11/2025

Topics: Boolean operators, recursive paradigm (cf. slide set 8), symbolic and encoding optimization (cf. slide set 9)

Problem 1

Given the Boolean function $F = \bar{a}\bar{c}d + \bar{a}cd + a\bar{b}\bar{c} + abc + ac$

- (a) Check if F is negative or positive unate in the variables a , b , c and d .
- (b) Is F negative or positive unate?

Problem 2

Given the Boolean function $G = \bar{a}\bar{b} + \bar{a}bc + \bar{a}b\bar{c}\bar{d} + ab\bar{c}d + a\bar{b} + abc$, compute:

- (a) The Boolean difference $\partial G/\partial a$.
- (b) The smoothing $S_a(G)$.
- (c) The consensus $C_a(G)$.

Problem 3

Given the Boolean function $H = \bar{a}d + ac + a\bar{b}\bar{c}$, use the positional cube notation and recursive paradigm to show if the following cubes are contained in H :

- (a) cd
- (b) ad

Problem 4

Given the constraint mapping A , find the minimum encoding matrix E that satisfies the constraints of A .

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

- (a) Write the dichotomies considering that the columns in A correspond to the operations *AND*, *OR*, *JMP* and *ADD*.

- (b) Write the seed dichotomies.
- (c) Find the compatible seed dichotomies and draw the compatibility graph.
- (d) Find the prime dichotomies.
- (e) Write the covering matrix and find a minimum cover.
- (f) Write the encoding matrix.