CS-472: Design Technologies for Integrated Systems

Exercise Problem Set 4 Solution — Date: 30/10/2025

Topic: Scheduling (cf. slide set 5)

For all problems, consider the following sequencing graph.

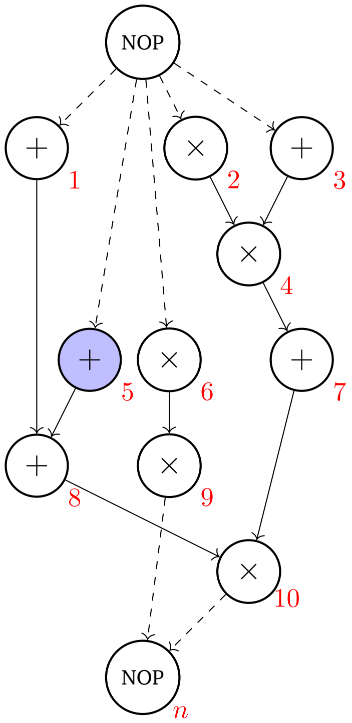
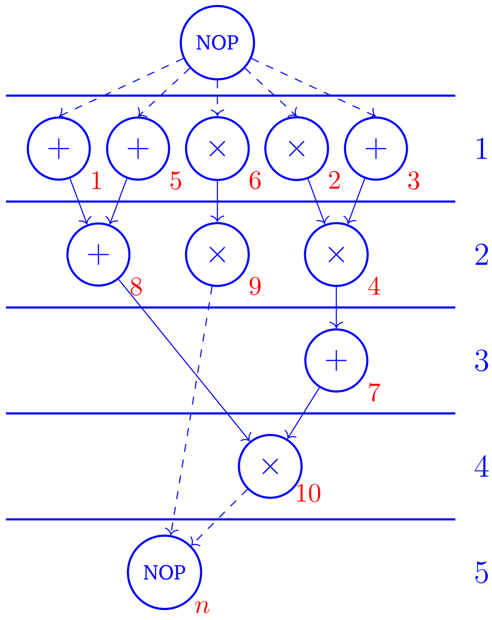


Figure 1: Sequencing graph

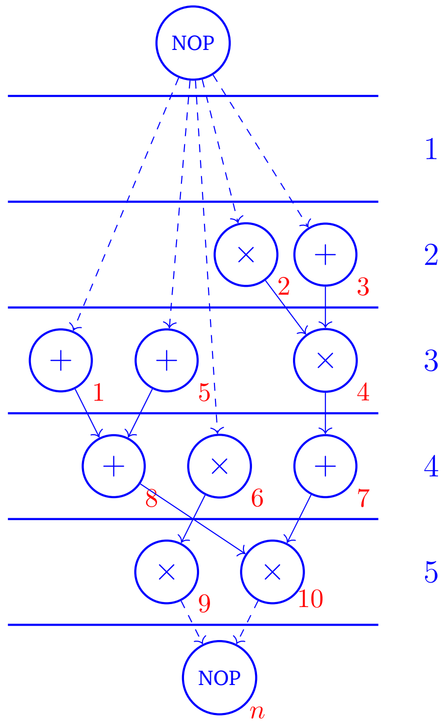
Problem 1

Assume all operations have unit execution delay.

(a) Schedule the graph with ASAP.



(b) Schedule the graph with ALAP and a latency bound $\bar{\lambda}$ of 5 cycles.



(c) Compute the mobility of each operation.

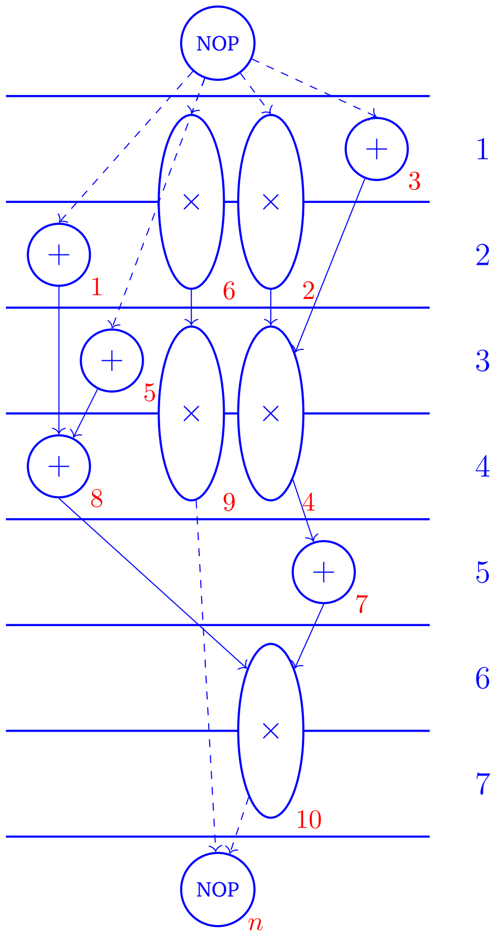
Ans: $\mu_1 = 2; \mu_2 = 1; \mu_3 = 1; \mu_4 = 1; \mu_5 = 2; \mu_6 = 3; \mu_7 = 1; \mu_8 = 2; \mu_9 = 3; \mu_{10} = 1;$

Problem 2

Schedule the sequencing graph using the list algorithm with at most two multipliers and one adder (at the same time per level). Assume that the multiplier takes two units of time and the adder one. Try to obtain the minimum latency subject to the resource bounds.

cf: Textbook pp. 208; slide set 5 pp. 40.

Ans: The minimum latency is 7. Following is one possible scheduling (the solution is not unique).



Problem 3

Assume again that the multiplier takes two units of time and the adder one, and that at most two multipliers and one adder are available at the same time.

(a) Write down the integer linear programming (ILP) inequalities describing the sequencing graph and subject to the resource constraints. Use an upper bound on the latency $\bar{\lambda} = 9$.
cf: Textbook pp. 198–202; slide set 5 pp. 23–25.
Ans:
Operations start once:

$$\sum_{l=1}^{10} x_{i,l} = 1, i = 1, \ldots, 10, n$$

Sequencing relations:

$$\sum_{l=1}^{10} (l \cdot x_{1,l}) - 1 \geq 0 \qquad (\text{NOP} \to v_1)$$

$$\sum_{l=1}^{10} (l \cdot x_{8,l}) - \sum_{l=1}^{10} (l \cdot x_{1,l}) - 1 \geq 0 \qquad (v_1 \to v_8)$$

$$\sum_{l=1}^{10} (l \cdot x_{10,l}) - \sum_{l=1}^{10} (l \cdot x_{8,l}) - 1 \geq 0 \qquad (v_8 \to v_{10})$$

$$\sum_{l=1}^{10} (l \cdot x_{n,l}) - \sum_{l=1}^{10} (l \cdot x_{10,l}) - 2 \geq 0 \qquad (v_{10} \to v_n)$$

$$\sum_{l=1}^{10} (l \cdot x_{5,l}) - 1 \geq 0 \qquad (\text{NOP} \to v_5)$$

$$\sum_{l=1}^{10} (l \cdot x_{8,l}) - \sum_{l=1}^{10} (l \cdot x_{5,l}) - 1 \geq 0 \qquad (v_5 \to v_8)$$

$$\sum_{l=1}^{10} (l \cdot x_{6,l}) - 1 \geq 0 \qquad (\text{NOP} \to v_6)$$

$$\sum_{l=1}^{10} (l \cdot x_{9,l}) - \sum_{l=1}^{10} (l \cdot x_{6,l}) - 2 \geq 0 \qquad (v_6 \to v_9)$$

$$\sum_{l=1}^{10} (l \cdot x_{n,l}) - \sum_{l=1}^{10} (l \cdot x_{9,l}) - 2 \geq 0 \qquad (v_9 \to v_n)$$

$$\sum_{l=1}^{10}(l \cdot x_{2,l}) - 1 \geq 0 \qquad (\text{NOP} \rightarrow v_2)$$

$$\sum_{l=1}^{10}(l \cdot x_{4,l}) - \sum_{l=1}^{10}(l \cdot x_{2,l}) - 2 \geq 0 \qquad (v_2 \rightarrow v_4)$$

$$\sum_{l=1}^{10}(l \cdot x_{7,l}) - \sum_{l=1}^{10}(l \cdot x_{4,l}) - 2 \geq 0 \qquad (v_4 \rightarrow v_7)$$

$$\sum_{l=1}^{10}(l \cdot x_{10,l}) - \sum_{l=1}^{10}(l \cdot x_{7,l}) - 1 \geq 0 \qquad (v_7 \rightarrow v_{10})$$

$$\sum_{l=1}^{10}(l \cdot x_{3,l}) - 1 \geq 0 \qquad (\text{NOP} \rightarrow v_3)$$

$$\sum_{l=1}^{10}(l \cdot x_{4,l}) - \sum_{l=1}^{10}(l \cdot x_{3,l}) - 1 \geq 0 \qquad (v_3 \rightarrow v_4)$$

Resource bounds:

At most one adder: $x_{1,l} + x_{3,l} + x_{5,l} + x_{7,l} + x_{8,l} \leq 1, l = 1, \ldots, 10$
At most two multipliers: $x_{2,1} + x_{4,1} + x_{6,1} + x_{9,1} + x_{10,1} \leq 2$
$x_{2,l-1} + x_{2,l} + x_{4,l-1} + x_{4,l} + x_{6,l-1} + x_{6,l} + x_{9,l-1} + x_{9,l} + x_{10,l-1} + x_{10,l} \leq 2, l = 2, \ldots, 10$

(b) What variable assignment correspond to the solution you obtained in Problem 2? Plug these values into the inequalities. Are they all satisfied?
Ans: $x_{1,2} = x_{2,1} = x_{3,1} = x_{4,3} = x_{5,3} = x_{6,2} = x_{7,5} = x_{8,4} = x_{9,4} = x_{10,6} = x_{n,8} = 1$; all other variables are 0. All inequalities are satisfied as this is a valid scheduling.

Problem 4

Assume now all operations have unit delays. Consider an upper bound on the latency $\bar{\lambda} = 5$. Use a force calculation schedule the colored operation (node 5) to reduce concurrency.

cf: Textbook pp. 211–215; slide set 5 pp. 45–53.

Step 0: Obtain ASAP and ALAP schedules (done in Problem 1).

Step 1: Compute the time frames, mobility, probabilities, and type distributions.

Table 1: Time frames obtained from ASAP and ALAP ($\bar{\lambda} = 5$), mobility μ and probabilities $p_i(l)$

	t_S	t_L	L_1	L_2	L_3	L_4	L_5	μ	$p_i(1)$	$p_i(2)$	$p_i(3)$	$p_i(4)$	$p_i(5)$	type
v_1	1	3	✓	✓	✓	×	×	2	1/3	1/3	1/3	0	0	adder
v_2	1	2	✓	✓	×	×	×	1	1/2	1/2	0	0	0	multiplier
v_3	1	2	✓	✓	×	×	×	1	1/2	1/2	0	0	0	adder
v_4	2	3	×	✓	✓	×	×	1	0	1/2	1/2	0	0	multiplier
v_5	1	3	✓	✓	✓	×	×	2	1/3	1/3	1/3	0	0	adder
v_6	1	4	✓	✓	✓	✓	×	3	1/4	1/4	1/4	1/4	0	multiplier
v_7	3	4	×	×	✓	✓	×	1	0	0	1/2	1/2	0	adder
v_8	2	4	×	✓	✓	✓	×	2	0	1/3	1/3	1/3	0	adder
v_9	2	5	×	✓	✓	✓	✓	3	0	1/4	1/4	1/4	1/4	multiplier
v_{10}	4	5	×	×	×	✓	✓	1	0	0	0	1/2	1/2	multiplier

Table 2: Type distributions $q_k(l)$

adder						multiplier				
$q_a(1)$	$q_a(2)$	$q_a(3)$	$q_a(4)$	$q_a(5)$		$q_m(1)$	$q_m(2)$	$q_m(3)$	$q_m(4)$	$q_m(5)$
1.2	1.5	1.5	0.8	0		0.75	1.5	1	1	0.75

Visualization of type distributions (for your information only; not necessary to plot):

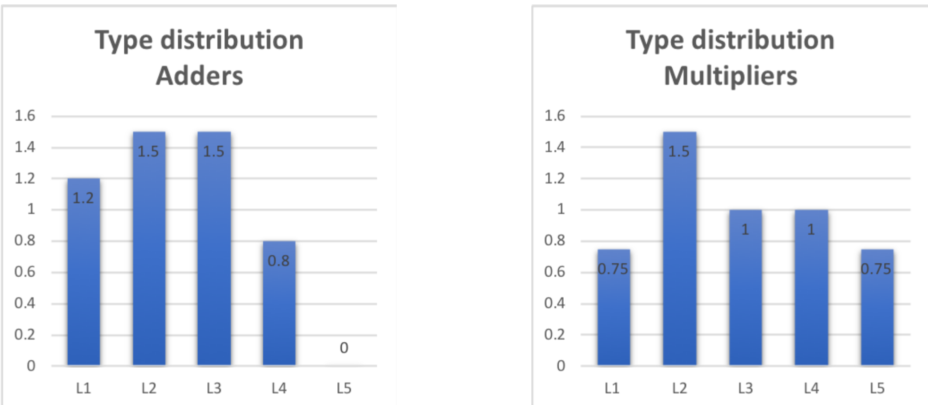


Figure 2: Type distribution for adders and multipliers.

Step 2: Compute self-force, PS-force and total force for each possible steps v_5 can be assigned to.

1. Self-force

- L_1: $\mathit{self\_force}(5,1) = q_{\text{adder}}(1) - \frac{1}{\mu_5+1}(q_{\text{adder}}(1) + q_{\text{adder}}(2) + q_{\text{adder}}(3))$
$= 1.2 - \frac{1}{3}(1.2 + 1.5 + 1.5) = -0.2$
- L_2: $\mathit{self\_force}(5,2) = 1.5 - \frac{1}{3}(1.2 + 1.5 + 1.5) = 0.1$
- L_3: $\mathit{self\_force}(5,3) = 1.5 - \frac{1}{3}(1.2 + 1.5 + 1.5) = 0.1$

2. Predecessor/successor-force (PS-force)

- L_1: When v_5 is in L_1, it is not influencing other nodes' time frame.
- L_2: When v_5 is in L_2, it is influencing v_8's time frame:
$\mathit{PS\_force}(8,2) = 1/2(q_{\text{adder}}(3) + q_{\text{adder}}(4)) - 1/3(q_{\text{adder}}(2) + q_{\text{adder}}(3) + q_{\text{adder}}(4))$
$= 1/2(1.5 + 0.8) - 1/3(1.5 + 1.5 + 0.8) = -0.12$
- L_3: When v_5 is in L_3, it is influencing both v_8 and v_{10}'s time frames:
$\mathit{PS\_force}(8,3) = 1(q_{\text{adder}}(4)) - 1/3(q_{\text{adder}}(2) + q_{\text{adder}}(3) + q_{\text{adder}}(4)) = -0.47$
$\mathit{PS\_force}(10,3) = 1(q_{\text{mult}}(5)) - 1/2(q_{\text{mult}}(4) + q_{\text{mult}}(5)) = -0.125$

3. Total force

- L_1: $\mathit{total\_force}(5,1) = \mathit{self\_force}(5,1) = -0.2$
- L_2: $\mathit{total\_force}(5,2) = \mathit{self\_force}(5,2) + \mathit{PS\_force}(8,2) = 0.1 - 0.12 = -0.02$
- L_3: $\mathit{total\_force}(5,3) = \mathit{self\_force}(5,3) + \mathit{PS\_force}(8,3) + \mathit{PS\_force}(10,3)$
$= 0.1 - 0.47 - 0.125 = -0.5$

Step 3: Find the smallest total force, thus conclude.

Ans: The smallest force is in $\underline{L_3}$, thus we should schedule v_5 at this level to reduce concurrency.