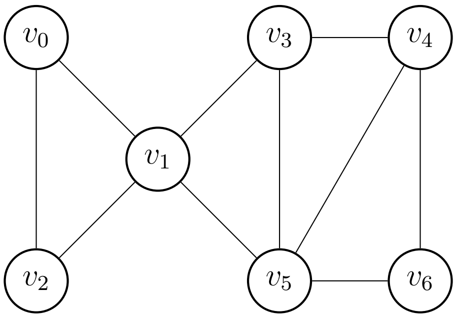
CS-472: Design Technologies for Integrated Systems

Exercise Problem Set 1 Solution Date: 18/09/2025

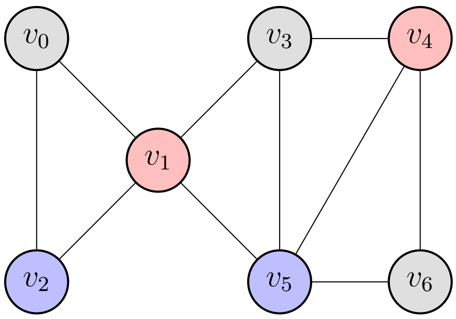
Problem 1

Given the graph $G(V, E)$ below:



cf: Textbook pp.40-42.

(a) Color the graph with the smallest number of colors.
Ans:



(b) Show a minimum clique cover.
Ans: (For example; not unique) $\{v_0, v_1, v_2\}, \{v_1, v_3, v_5\}, \{v_4, v_5, v_6\}$

(c) Show a minimum clique partition.
Ans: (For example; not unique) $\{v_0, v_1, v_2\}, \{v_3, v_4, v_5\}, \{v_6\}$ (This could also be an answer for (b).)

(d) Is G a *perfect graph*? Why or why not?
Ans:

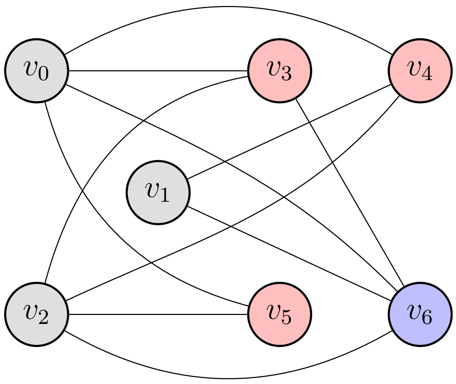
1. The *clique number* $\omega(G) = 3$ (i.e. size of the largest clique, e.g. $\{v_0, v_1, v_2\}$).
2. The *chromatic number* $\chi(G) = 3$ (i.e. minimum number of colors needed, as in (a)).
3. The *stability number* $\alpha(G) = 3$ (i.e. size of the largest stable set, e.g. $\{v_0, v_3, v_6\}$).

4. The *clique cover number* $\kappa(G) = 3$ (i.e. size of the minimum clique cover, as in (b)).

$\omega(G) = \chi(G)$ and $\alpha(G) = \kappa(G)$, so G is a perfect graph.

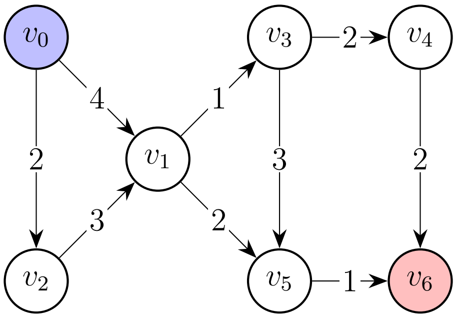
(e) Draw the complement graph.

(f) Color the complement graph with the smallest number of colors.
Ans: (e)+(f). Note that this coloring corresponds to the solution of (c).



Problem 2

Given the directed acyclic graph $G(V, E, W)$ below:



Find the shortest path from the *source* v_0 to the *sink* v_6 by applying the following algorithms:
cf: Textbook pp.55.

(a) Dijkstra algorithm.

$v_q =$	v_0	v_2	v_1	v_3	v_5	v_4	v_6
s_0	0	0	0	0	0	0	0
s_1	4	4	4	4	4	4	4
s_2	2	2	2	2	2	2	2
s_3	∞	∞	$(4+1)$	5	5	5	5
s_4	∞	∞	∞	$(5+2)$	7	7	7
s_5	∞	∞	$(4+2)$	6	6	6	6
s_6	∞	∞	∞	∞	$(6+1)$	7	7

Ans: Shortest path: $v_0 \to v_1 \to v_5 \to v_6$ [cost = 7]

(b) Bellman-Ford algorithm.

j	1	2	3	4
s_0	0	0	0	0
s_1	4	4	4	4
s_2	2	2	2	2
s_3	∞	$(4+1)$	5	5
s_4	∞	∞	$(5+2)$	7
s_5	∞	$(4+2)$	6	6
s_6	∞	∞	$(6+1)$	7

Ans: Shortest path: $v_0 \to v_1 \to v_5 \to v_6$ [cost = 7]