

Solutions to Exercise 5

Problem 1. Fetch-and-increment has a consensus number of 2, while compare-and-swap (CAS) has an infinite consensus number. Therefore we can use the universal construction to implement a fetch-and-increment object from consensus objects. Then we can replace consensus objects with their implementation¹ from CAS objects. The resulting algorithm is a wait-free implementation of fetch-and-increment from CAS.

Universal construction algorithm for fetch-and-increment: *Shared objects:*

- Array of n atomic registers $R[1, \dots, n]$, where n is the number of processes.
- Infinite list C of consensus objects.

Local objects:

- register seq the value of which is the number of executed operations by process $p[i]$, initially $seq = 0$.
- register k the value of which is the number of decided batches of requests, initially $k = 0$.
- list $Perf$ of performed requests.
- list Inv of requests which need to be performed.
- local copy f of fetch-and-increment.

Pseudocode for process $p[i]$:

```

fetch&inc()
  seq ++
  R[i] := (fetch&inc(), i, seq) // inform other processes about the request
  repeat
    Inv := Inv + R[1, .., n].read // add new requests of other processes to the list
    Inv := Inv - Perf // remove performed requests from the list
    if Inv  $\neq$   $\emptyset$  then // if there are requests that were not performed
      k++
      Dec := C[k].propose(Inv) // decide on requests to be performed
      Res := f.Dec // perform all requests from Dec on local copy f
                // and record the responses to list Res
      Perf := Perf + Dec // add the performed responses to list Perf
      if (fetch&inc(), i, seq)  $\in$  Dec then // if the request by  $p[i]$  is in
                // the list of decided responses
        return the result of (fetch&inc(), i, seq) from Res
        // return the corresponding response

```

¹For the implementation of consensus from CAS see the lecture on the limitations of registers