1 Automatic Differentiation

1.1 Multiple choice (5 pts)

Which of the following statements is true? In all these questions, you can assume the functions are differentiable.

True	False	
☐	☐	Reverse-mode automatic differentiation computes the derivative of the input of a function with regards to its output.
☐	☐	To differentiate a function that has one input and 10 outputs, we should use reverse-mode automatic differentiation.
☐	☐	The Jacobian of a function is a symmetric matrix.
☐	☐	Reverse-mode differentiation is generally more expensive to use than forward-mode automatic differentiation because it requires recording all the operations.
☐	☐	Central finite differences always gives the same result as forward-mode automatic differentiation.

1.2 Jacobian (5 pts)

Consider a function $f : \mathbb{R}^n \to \mathbb{R}^m$. Find a way to use automatic differentiation in order to compute the Jacobian matrix $\mathbf{J}_f(\mathbf{x})$ of that function in the least number of differentiation passes possible. Describe the steps you would take and which AD mode you would use.

1.3 Differentiation of a function (5 pts)

Consider the following computation:

```
a = exp(-2*x)
b = 3 * a + 1
y = sqrt(b)
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We now want to apply automatic-differentiation to this computation. Write the sequence of steps that forward-mode automatic differentiation would take to compute the derivative of y with regards to x. Do the same for reverse-mode automatic differentiation.

1.4 Solving a linear system (5 pts)

We provide you with a black-box function $linear_solve(b)$, that takes as input a vector b and returns the solution x of the linear system $Ax = b$ for a constant symmetric matrix A.

(1) Is solving a linear system a differentiable operation ? Why ? (2 pts)

(2) similar to Homework 4, write the backward pass of the function: (3pts)