Principles of Online Decision-Making (CS-303)

Problem Set 4

Problem 1

(a) Let X be a Markov chain. Which of the following are also Markov chains?

1. X_{m+r} for $r \geq 0$.
2. X_{2m} for $m \geq 0$.
3. The sequence of pairs (X_n, X_{n+1}) for $n \geq 0$.

(b) Let X be a Markov chain. Show that for $1 < r < n$,

$$\mathbb{P}[X_r = k | X_i = x_i \text{ for } i = 1, 2, \ldots, r-1, r+1, \ldots, n] = \mathbb{P}[X_r = k | X_{r-1} = x_{r=1}, X_{r+1} = x_{r+1}] \quad (1)$$

Problem 2

(a) Show that a state i is recurrent iff the expected number of visits to state i, having started at i, is infinite.

(b) Let X be a Markov chain containing an absorbing state s with which all other states communicate (i.e., $p_{is}(n) > 0$ for some $n = n(i)$). Show that all states other than s are transient.

Problem 3

We have a Markov chain with the following transition matrix:

$$P = \begin{pmatrix} 1-2p & 2p & 0 \\ p & 1-2p & p \\ 0 & 2p & 1-2p \end{pmatrix} \quad (2)$$

Compute $p_{ij}(n)$ and the mean recurrence time of all states.