Problem 1. 6 p. (Paper and Pencil)

Let $p_{\mathcal{F}}(f)$ be a frequency-domain rectangular pulse defined by

$$p_{\mathcal{F}}(f) = \begin{cases} 1, & |f| \leq B \\ 0, & \text{otherwise.} \end{cases}$$

Answer the following questions. (You can answer the questions in any order.)

(a) Sketch the time-domain pulse $p(t)$ and label its maximum value and its zero-crossings.

(b) Let $\mathcal{E} = \int p^2(t)dt$. Specify the value of $\mathcal{E}$ as a function of B.

(c) Let $p_i = p(iT_s)$ for some sampling time T_s and $i \in \mathbb{Z}$, where $T_s < \frac{1}{2B}$. Express

$$\sum_{i \in \mathbb{Z}} p_i^2$$

as a function of $\mathcal{E}$ and T_s.

Solution

(a) $p(t)$ is a `sinc` function. Its maximum value is at $t = 0$ and equals the area under the frequency-domain response, so $p(0) = 2B$. The zero crossings are at non-zero integers of c, where $2Bc = 1$ (the area of the triangle within the main lobe of the `sinc` equals $p_{\mathcal{F}}(0)$). So $p(t) = 2B\mathtt{sinc}(2Bt)$.

(b) The energy is preserved between the time and frequency domain representations (Parseval), so

$$\mathcal{E} = \int p^2(t)dt = \int p_{\mathcal{F}}^2(f)df = 2B.$$

(c) Since T_s respects the condition of the sampling theorem,

$$\mathcal{E} = \int p^2(t)dt = \sum_{i \in \mathbb{Z}} p^2(iT_s)T_s.$$

So

$$\sum_{i \in \mathbb{Z}} p_i^2 = \frac{\mathcal{E}}{T_s}.$$

Problem 2. 6 p. (Paper and Pencil)

If you type `help rcosdesign` in `MATLAB` you obtain the following (abridged) response:

```
rcosdesign Raised cosine FIR filter design
    B = rcosdesign(BETA, SPAN, SPS) returns square root raised cosine FIR
    filter coefficients, B, with a rolloff factor of BETA. The filter is
    truncated to SPAN symbols and each symbol is represented by SPS
    samples. rcosdesign designs a symmetric filter. Therefore, the filter
    order, which is SPS*SPAN, must be even. The filter energy is one.
```

Suppose that you issue the commands

```
h=rcosdesign(0.5,4,3)
xc=xcorr(h,h)
[y,i]=max(xc) % where y and i are the value and the index of the max
```

Answer the following questions:

(a) Evaluate `y`.

(b) Evaluate `i`.
Hint: the command `length(h)` returns `13`.

(c) Assuming that you know `i`, give at least two indices where `xc` is essentially zero (exactly zero if `SPAN` is sufficiently large).

Solution

(a) `y` equals the energy of the filter, so `y = 1`.

(b) `i` is the lag for which the two vectors of `xcorr` fully overlap, so `i = length(h) = 13`.

(c) `xc` is essentially zero at `13+k*SPS`, where `k` is a non-zero integer. This is so because `h` is essentially orthogonal with a shifted version of itself, whenever the shift is a non-zero integer multiple of `SPS`. So two indices are `i1 = 13-3=10, i2 =13+3= 16`.