Problem 4.

(a) The matched filter which is causal and creates the smallest possible delay is $q(t) = p^*(T_p - t)$.

Alternatively, we could use its normalized version $q(t) = \frac{p^*(T_p - t)}{P}$.

(b) Since the matched filter is delayed, we have to sample at $kT + T_p$, so $\tau = T_p$.

(c) $R(t) = s(t) + N(t)$. Sampling at $kT + T_p$ we obtain $Y_k = PQ + N_k$, where N_k is Gaussian, zero-mean and of variance $\sigma^2 = Q^2 \frac{N_0}{2}$, and Q is the norm of $q(t)$.

(d) $1 = \sigma^2 = \sigma_Z^2 Q^2$. So the process Z should have variance $\sigma_Z^2 = \frac{1}{Q^2}$.