ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE
School of Computer and Communication Sciences

Software-Defined Radio: A Hands-On Course

Midterm Exam
October 28, 2015

Name:

Note:

- You have 1 h 45 min to work at the exam.
- The exam is closed book, but your are allowed one page of handwritten notes. You are only allowed to use the workstations in the laboratory (not your own laptops). Resources from the internet as well as code written outside this exam are not allowed.
- There is a `MATLAB` part and a paper-and-pencil part.
- The code will be evaluated according to the usual criteria, namely correctness, speed, form, and readability. Short comments that allow us to follow what you are doing will improve readability.
- Start by downloading from Moodle and unzipping the file `sdr_midterm_2015.zip`. The files for Problem n, $n = 1, 2, 3$, are found in subfolder pn.
- You will upload to Moodle your solution to the problems that require writing `MATLAB` code. Do so in a single archive.
- The problems can be solved in any order.

Problem 1. 10 p. (MATLAB)

In this problem, you determine the matched filter outputs via a matrix multiplication.

Fill in the missing code in the function `n_tuple_output.m`.

To check your code you can make the following tests:

```
 y = n_tuple_output(7, 1, -30, 12889)
 y = 9.2429e+04 +1.1234e+05i

 y = n_tuple_output(29, 2, -3290, 3219)
 y = 1.0e+05 *
-1.7353 + 0.8145i  -1.7321 + 0.9275i
```

Problem 2. 11 p. (`MATLAB`)

Consider a communication chain using a 4-QAM constellation (with Gray mapping). Write a `MATLAB` routine according to the following instructions:

(a) Generate 1e6 random bits (0/1 values).

(b) Map the bit sequence into the corresponding sequence of 4-QAM symbols using the following (Gray) mapping: the bit-pairs [00, 01, 10, 11] are mapped into the symbols [1+1i, 1-1i, -1+1i, -1-1i] (in that order).

(c) Map the symbol sequence into the baseband-equivalent of the transmitted signal, according to what we call "symbol-by-symbol on a pulse train", where the pulse is a truncated and normalized sinc function. (The ith symbol is the scaling factor of the pulse delayed by iT for some appropriate T, where T is the smallest amount that makes the translated sinc pulses orthogonal to each other.) Use the `sinc` function to create the pulse. The truncated pulse should have length $40T$ ($20T$ on each side of the peak) and it should be sampled with 4 samples per symbol.

(d) Using either the `conv` or the `xcorr` command, filter the transmitted signal with the corresponding matched filter and sample the matched filter output appropriately. (No noise added.)

(e) Plot the sampled matched filter output sequence as a constellation of points in the complex plane. You should see 4 "clouds". Can you explain why, and how you can reduce the "diameter" of the clouds? Write your answer as a comment at the end of your code.

Problem 3. 11 p. (`MATLAB`)

In this problem, you estimate the Doppler frequency of a GPS satellite by means of the Fourier transform.

Write a function `dopplerEstimate = fftDoppler(sat,startOfBit)` that takes a satellite number `sat` and a sample-index `startOfBit` pointing to the first sample of a new bit of satellite `sat`. The function returns the Doppler estimate for that satellite. The resolution of your estimate should be 10 Hz or better.

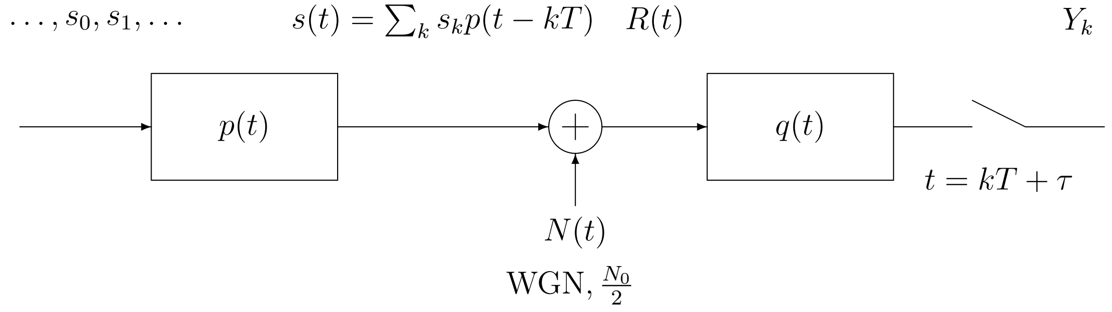
The step by step procedure is commented in the file `fftDoppler.m`.

To test your procedure, you can load the vectors `visibleSats` and `startOfBits`, both of length 6. At position i, the first has the satellite number of the ith visible satellite, and the second has the position of the first sample of the first bit of that satellite. (Visible are the satellites numbered 7, 9, 17, 18, 26 and 29.)

For comparison, the values of the Doppler found in class by `fineEstimate.m` were -30, 1030, -120, -1330, -3130, -3290 Hz.

Problem 4. 8 p. (Paper and Pencil)

Consider the following communication problem, where $p(t)$ is a pulse of support $[0, T_p]$, norm P, orthogonal to $p(t-kT)$ for all integers k.



(a) Describe, as a function of $p(t)$, the impulse response $q(t)$ of the matched filter. The matched filter must be causal and must create the smallest possible delay.

(b) Specify τ, so that the sample taken at $t = kT + \tau$ is a sufficient statistic for s_k.

(c) Suppose that $s_k = 1$. Describe the statistical property of Y_k, knowing that $N(t)$ is white Gaussian noise of power spectral density $\frac{N_0}{2}$.

(d) Now suppose that you are simulating the above system. Specifically, instead of $s(t)$ you send its sampled version, instead of $N(t)$ you add a noise sequence $\ldots, z_0, z_1, \ldots$, and the matched filter convolves the received sequence $\ldots, r_0, r_1, \ldots$ with a properly chosen impulse response $q = (q_1, q_2, \ldots, q_L)$. Suppose that the norm of q is Q. You want the noise variance at the matched filter output to be $\sigma^2 = 1$. Describe the statistical property of the stochastic process $\ldots, Z_0, Z_1, \ldots$ of which $\ldots, z_0, z_1, \ldots$ is a sample path.