Problem 1. 14 p. (Paper and Pencil)

Consider a toy (2D) GPS system that has 2 satellites on a circular orbit of radius R, see figure. The receiver is on the surface of a circle of radius r, (a toy Earth), with $r < R$.

Since the orbit is circular, the easiest way to describe the position of a satellite is to use polar coordinates $(\rho(t), \phi(t))$, where t refers to the GPS time. Suppose that the position of satellite 1 is $\left(R, \frac{2\pi}{T}t\right)$ and that of satellite 2 is $\left(R, \frac{2\pi}{T}t + \frac{\pi}{2}\right)$. We suppose that a receiver has this information. The figure shows the positions at GPS time $t = 0$. It takes T seconds for a satellite to make a revolution.

For the purpose of making distance measurements (range/pseudorange), each satellite sends a pseudonoise (PN) sequence (e.g. a C/A code) with chip of duration T_c. For both satellites, a new cycle of the PN sequence starts at GPS time $t = 0$. We assume that each satellite's clock is perfectly synchronized with the GPS time.

The receiver clock t_r has an unknown offset, i.e., $t_r = t + b$. Suppose that we want a pseudorange measurement at $t_r = \tau$. The receiver makes the following observations:

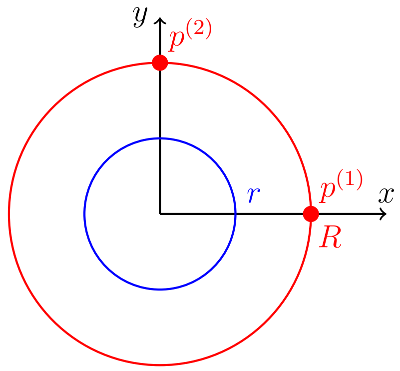
- The start of a new PN sequence, transmitted at $t = 0$, is received at $t_r = \tau^{(1)} < \tau$ and $t_r = \tau^{(2)} < \tau$, respectively.
- The number of chips between τ and $\tau^{(i)}$ is $n^{(i)}$, $i = 1, 2$.

(a) Using the parameters defined so far, express the time of flight $T_f^{(1)}(\tau^{(1)})$. This is the time of flight for a signal transmitted by satellite 1 and received at receiver time $t_r = \tau^{(1)}$.

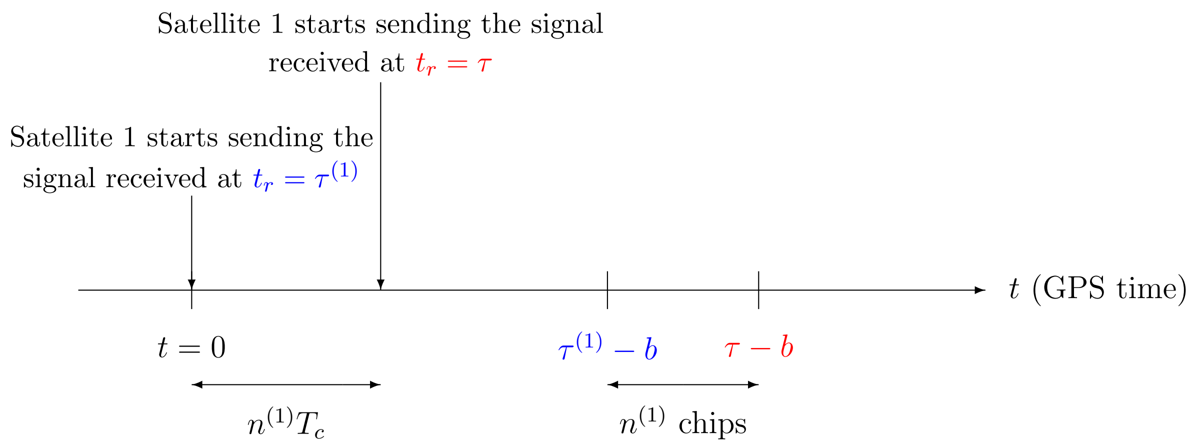
(b) More generally, express $T_f^{(1)}(\tau)$. (Hint: sketch the various events on the GPS time line.)

(c) Let p be the receiver position. Describe, in polar coordinates, the position $p^{(1)}$ of satellite 1 for which $\|p^{(1)} - p\| = c\,T_f^{(1)}(\tau)$, where c is the speed of light.

(d) Write down the equations, the solution of which allows us to determine the receiver position p and the offset b at $t_r = \tau$. Note: To avoid dependencies from previous questions, it is sufficient that you write those equations using notation introduced thus far. For instance, you may assume that the functions $T_f^{(i)}$, $i = 1, 2$ are known.



Solution



(a) $T_f^{(1)}(\tau^{(1)}) = \tau^{(1)} - b$.

(b) $T_f^{(1)}(\tau) = \tau - b - n^{(1)} T_c$.

(c) $p^{(1)} = \left(R, \frac{2\pi}{T} n^{(1)} T_c\right)$.

(d) $$\begin{cases} \|p^{(1)} - p\| = c\, T_f^{(1)}(\tau) = c(\tau - b - n^{(1)} T_c) \\ \|p^{(2)} - p\| = c\, T_f^{(2)}(\tau) = c(\tau - b - n^{(2)} T_c) \\ \|p\| = r \end{cases}$$

where $\|p^{(i)} - p\|$ denotes the distance between $p^{(i)}$ and p, $(i = 1, 2)$ and

$$\begin{cases} p^{(1)} = \left(R, \frac{2\pi}{T} n^{(1)} T_c\right) \\ p^{(2)} = \left(R, \frac{2\pi}{T} n^{(2)} T_c + \frac{\pi}{2}\right) \end{cases}$$