ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

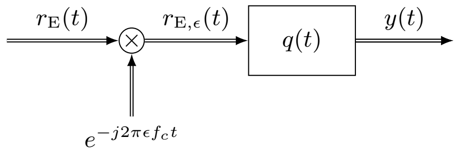
Handout 16 Modern Digital Communications
Solutions to Assignment 10 December 3, 2025

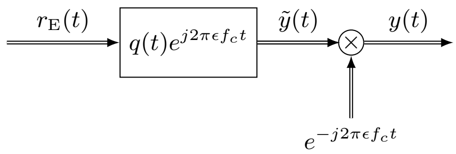
For the first 2 exercises, please see the provided MATLAB/Python code which is self-explanatory. Comparing the estimators, we observe (as expected) that the MMSE outperforms the LS.

SOLUTION 3.

1. $$r_{\mathrm{E},\epsilon}(t) = \sqrt{2}r(t)e^{-j2\pi f_c t - j2\pi f_c \epsilon t} = \sqrt{2}r(t)e^{-j2\pi f_c t}e^{-j2\pi f_c \epsilon t} = r_{\mathrm{E}}(t)e^{-j2\pi\epsilon f_c t}. \tag{1}$$

2. One can work out the calculations in the time domain and show that the systems in the two figures below are indeed equivalent.





Instead of that, we give here a more intuitive solution which uses the frequency domain. The output of the filter in the first figure is, in the frequency domain,

$$y_{\mathcal{F}}(f) = r_{\mathrm{E},\mathcal{F}}(f + \epsilon f_c) \cdot q_{\mathcal{F}}(f). \tag{2}$$

The output of the filter in the second figure is, in the frequency domain,

$$\tilde{y}_{\mathcal{F}}(f) = r_{\mathrm{E},\mathcal{F}}(f) \cdot q_{\mathcal{F}}(f - \epsilon f_c). \tag{3}$$

Clearly

$$y_{\mathcal{F}}(f) = \tilde{y}_{\mathcal{F}}(f + \epsilon f_c). \tag{4}$$