PROBLEM 3. 14 points (Paper and Pencil)

1. The signal at the output of the matched filter is

$$\begin{aligned} Y(t) &= R(t) \star h_{\mathrm{MF}}(t) \\ &= R(t) \star \psi^*(-t) \\ &= \int R(\alpha)\psi^*(\alpha - t)d\alpha \\ &= \langle R(\alpha), \psi(\alpha - t)\rangle. \end{aligned}$$

2.

$$\begin{aligned} \rho(\upsilon) &= \langle R(t), p(t-\upsilon)\rangle \\ &= \left\langle R(t), \sum_k b_k \psi(t - \upsilon - kT) \right\rangle \\ &= \sum_k b_k^* \langle R(t), \psi(t - \upsilon - kT)\rangle \\ &= \sum_k b_k^* Y(\upsilon + kT). \end{aligned}$$

Hence, $\rho(mT_s) = \sum_k b_k^* Y(mT_s + kT)$.

3.

$$\begin{aligned} \rho[m] \coloneqq \sqrt{T_s}\rho(mT_s) &= \sum_k b_k^* \sqrt{T_s} Y(mT_s + kT) \\ &= \sum_k b_k^* Y[m + kN] \\ &= \sum_k \hat{b}_k^* Y[m + k], \end{aligned} \tag{1}$$

where $\hat{b}_k$ is the upsampled preamble sequence with upsampling factor N, i.e.,

$$\hat{b}_k \coloneqq \begin{cases} b_{k/N} & \text{if } k/N \text{ is an integer,} \\ 0 & \text{otherwise.} \end{cases}$$