ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 15 Modern Digital Communications
Assignment 10 November 26, 2025 (due Dec 2, 2025)

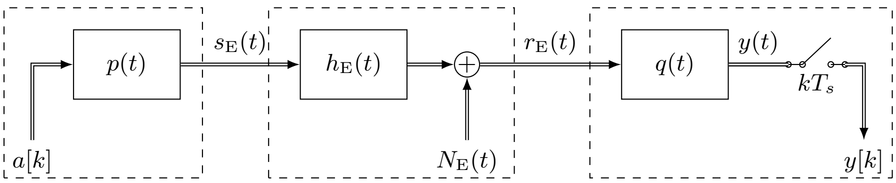
Channel Estimation

In the previous assignment, you implemented an OFDM transmitter and receiver assuming that the channel was somehow estimated and given to you. In the first part of this assignment we focus on the estimation of the channel coefficients. In fact, we implement two channel estimation methods. For each channel estimation method, run the script `test_ofdmChEst` to test your channel estimation.
As in the previous assignments, we provide the solutions for every function that you should write. For MATLAB, the script `function_mapper_mmse` is used to select for each function whether to use the provided solution or your own implementation. For Python, the functions that you need to write are grouped under `my_utilOFDM.py`.
In the second part of the assignment, we derive a baseband-equivalent channel model which incorporates the effect of the carrier frequency offset (CFO). The CFO accounts for the offset between the local oscillator at the transmitter and that at the receiver. Once we know the CFO, we can correct for it.

Exercise 1. Implement the function `channel_estMMSE()`.

Exercise 2. The MMSE channel estimation method of Exercise 1 is optimal, but it needs a relatively large amount of information (channel delays, covariance matrix of channel amplitudes, noise variance) and computational resources (for all the matrix multiplications and inversions). A simpler method for channel estimation is the Least Squares (LS) method which consists of simply dividing the received symbols corresponding to the preamble by the preamble symbols. Implement the function `channel_estLS()` that implements this simple estimation method.

Exercise 3. So far, we have been considering a *baseband* communication system described by the following block-diagram:



The transmitter converts complex-valued data symbols $a[k]$ to a complex-valued signal via the pulse-shaping (interpolation) filter $p(t)$:

$$s_{\mathrm{E}}(t) = \sum_k a[k]p(t - kT_s).$$

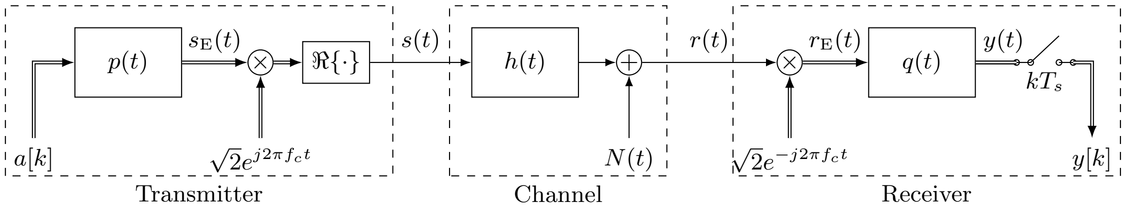
The receiver filters the noisy received signal $r_{\mathrm{E}}(t)$ with a matched filter (lowpass filter in the sampling theorem terminology) of impulse response $q(t)$ and samples its output at multiples of T_s to form decision statistics $y[k]$.

We have seen that, in general, decision statistics are in the form of

$$y[n] = \sum_k h[k]a[n - k] + z[n],$$

where $z[n]$ is a Gaussian noise process and $h[n]$ is the effective symbol-level channel. We have learned that OFDM is a powerful technique that sends the symbols over carriers that are channel eigenfunctions. This results in a number of parallel AWGN channels, one per carrier, that are ISI-free. Each parallel channel has an attenuation determined by the Fourier transform of the channel impulse-response evaluated at the carrier frequency.

In practice, most communication media are not capable of carrying baseband signals. The baseband-equivalent signal $s_{\mathrm{E}}(t)$ is up-converted to the passband signal $s(t) = \sqrt{2}\Re\left\{s_{\mathrm{E}}(t)e^{j2\pi f_c t}\right\}$ to be transmitted over the physical channel. The receiver then converts the received signal back to a baseband-equivalent signal via a down-converter. (See Principles of Digital Communications for details.)

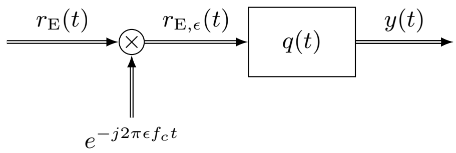


The baseband-equivalent channel model considered so far is accurate if the transmitter and the receiver have the exact same frequency f_c. Since the carrier signals are generated locally at the transmitter and at the receiver, in practice, their frequencies are not identical. This phenomena, called *carrier frequency offset (CFO)*, can be modeled by replacing the carrier signal at the receiver by $\exp(-j2\pi[1+\epsilon]f_c t)$, where ϵ is the relative frequency offset between the transmitter and the receiver ($\epsilon = 0$ means the carrier signals are perfectly synchronized). In this exercise, we will see the effects of CFO on the output of the parallel channels of an OFDM system.

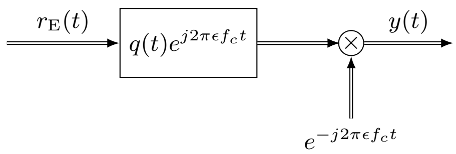
1. Let $r_{\mathrm{E},\epsilon}(t)$ denote the output of the baseband-equivalent channel when the relative CFO is ϵ and let $r_{\mathrm{E}}(t) = r_{\mathrm{E},0}(t)$. Show that

$$r_{\mathrm{E},\epsilon}(t) = r_{\mathrm{E}}(t)e^{-j2\pi\epsilon f_c t}.$$

2. At this point, the picture at the receiver around the filter $q(t)$ looks as follows:



Show that the system in the above picture is equivalent to the following one.



Discussion:

We have seen that the effect of the CFO is to modify the filter $q(t)$ into $q(t)e^{j2\pi\epsilon f_c t}$ and to multiply the output of this filter by a complex exponential $e^{-j2\pi\epsilon f_c t}$. We can ignore the fact that the filter $q(t)$ changes, as it has the same impact as a different channel impulse response. The estimated vector of channel coefficients will take into account all the channel impulse responses experienced by the communication signal, including the modified $q(t)$. Thus it suffices to model the effect of the multiplication by $e^{-j2\pi\epsilon f_c t}$ after the filter. The effect is to multiply the samples $y[n] = y(nT_s)$ by $e^{-j2\pi f_0 n}$ where $f_0 = \epsilon f_c T_s$. In principle, to undo the CFO, it is sufficient to estimate f_0 and remove the multiplicative factor $e^{-j2\pi f_0 n}$ from the sampled output of the matched filter.

In practice, the estimate of f_0 will not be perfect. Due to the estimation errors, there will be some residual CFO, leading to

$$\hat{y}[n] = e^{j2\pi\hat{f}_0 n}(e^{-j2\pi f_0 n}y[n]) = e^{-j2\pi\delta n}y[n]$$

where $\hat{f}_0$ is the estimated value of f_0 and $\delta \coloneqq f_0 - \hat{f}_0$ is the estimation error.

Define $\gamma = e^{-j2\pi\delta}$ and let L and N be the length of the cyclic prefix and of the OFDM blocks, respectively. The situation before we take the FFT is as follows:

$$\underbrace{y[0], \gamma y[1], \dots, \gamma^{N+L-1}y[N+L-1]}_{\text{0-th block}}, \quad \underbrace{\gamma^{N+L}y[N+L], \dots, \gamma^{2N+2L-1}y[2N+2L-1]}_{\text{1-st block}}, \quad \dots$$

If $\gamma^{N+L-1} = e^{-j2\pi\delta(N+L-1)}$ is a small rotation, say $2\pi\delta(N+L-1) \ll$ angle between different constellation points, then the relative rotation between elements of the same OFDM block can be neglected. In this case we can approximate by

$$\underbrace{(y[0], y[1], \ldots, y[N+L-1])}_{\text{0-th block}}, \quad \underbrace{\gamma^{N+L}(y[N+L], \ldots, y[2N+2L-1])}_{\text{1-st block}}, \quad \ldots$$

After the FFT, the blocks are

$$Y^{(0)}, \quad \gamma^{N+L}Y^{(1)}, \quad (\gamma^{N+L})^2 Y^{(2)}, \quad \ldots$$

If γ^{N+L} is sufficiently small, which is often the case, one strategy is to decode the 0-th OFDM block, estimate the rotation, and correct for it in subsequent blocks. The procedure is repeated at every block.