Holiday quiz solutions

Quiz 1

The answer is: yes, it is possible to play a fair game with a single biased coin, whatever the bias is (as soon as $p = \mathbb{P}(\text{heads})$ satisfies $0 < p < 1$).

Throw the coin twice for each game:

- if HT comes out, player A wins;
- if TH comes out, player B wins;
- in the other two cases, the game is restarted.

I let you check formally that chances are equal for both players.

Quiz 2 (a.k.a. "The Monty Hall Problem")

You should definitely change your mind!

- If you stick to your first choice, your chances of winning are $1/3$ (as this was your initial winning probability).
- If you change your mind, then here is what happens:
  - If your initial choice was the right one (this happens wp $\frac{1}{3}$), then you lose.
  - If your initial choice was a bad one (which happens wp $\frac{2}{3}$), then you necessarily win!

In total, you win wp $\frac{2}{3}$ in this case! Try this at home!

Quiz 3

Yes, you can beat 50% ! Here is the strategy:
Once revealed the number in the first envelope (say x), draw an exponential r.v. (say Y). If $Y > x$, choose the other envelope, else stick to the first one.
Now what happens? Call z the amount in the other envelope:
- if $Y < \min(x,z)$ or $Y > \max(x,z)$, your chances of winning will be equal, whatever the envelope you choose.
- but if $x < z$ and $x < Y < z$ (which happens with positive probability), then you switch, and this is the right choice: and if $z < x$ and $z < Y < x$, then you stay, and this is the right choice also. Overall, your winning probability is $> 50\%$.