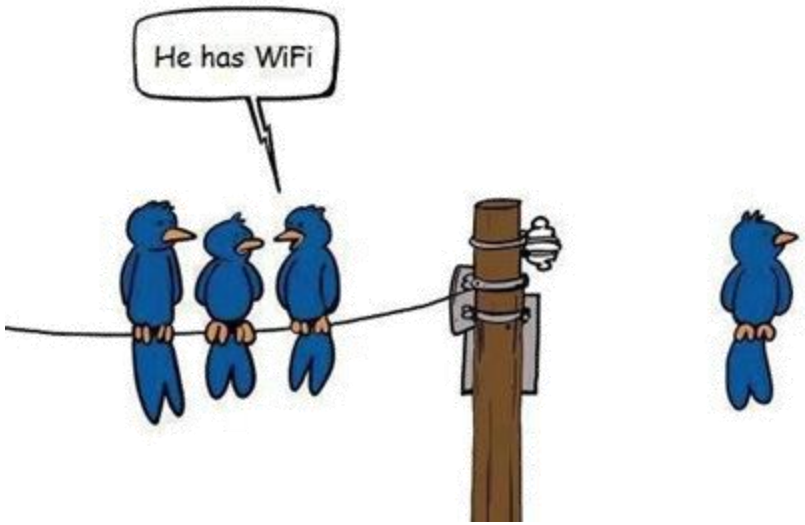
COM-405: Mobile Networks

Lecture 8.0: Radar

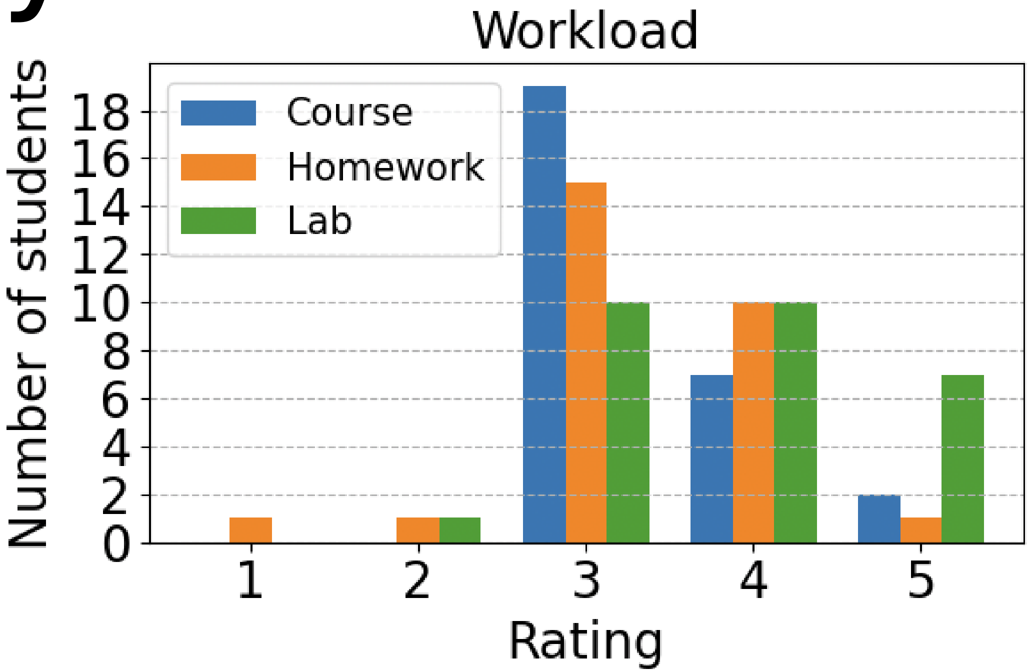
Haitham Hassanieh

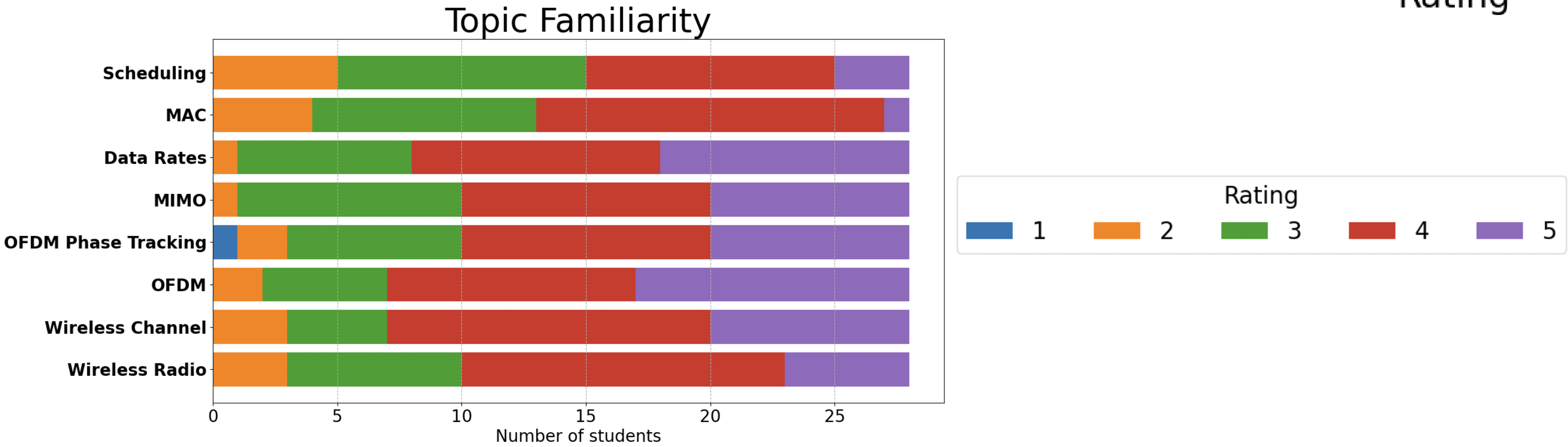


EPFL

Laboratory of SEnsing
& Networking Systems

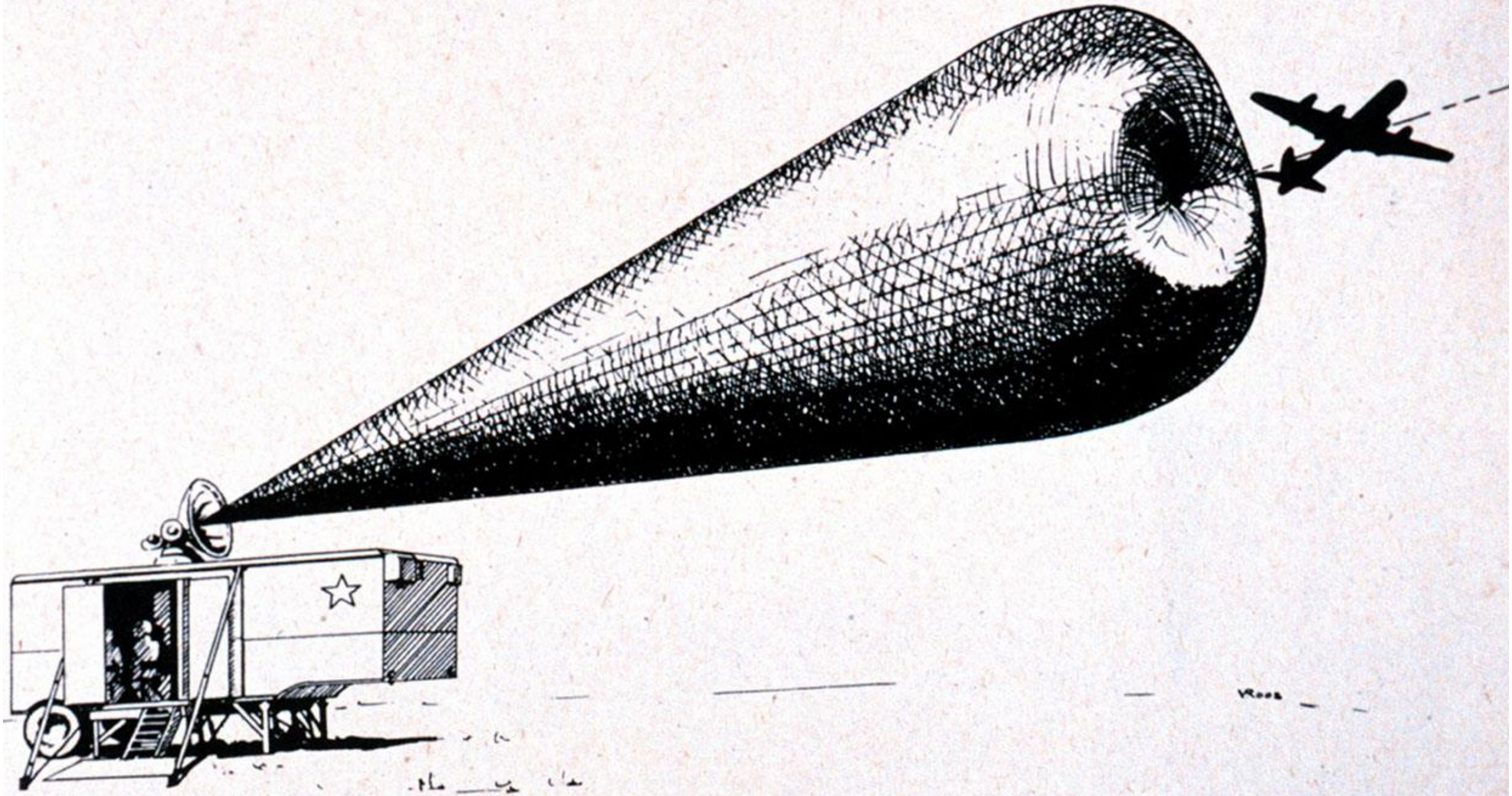
Feedback Survey Results





RADAR: **RA**dio **D**etection **A**nd **R**anging

- Invented during world war 2 to detect planes
- Changed the course of the war
- Transmit wireless signals, detect the reflections

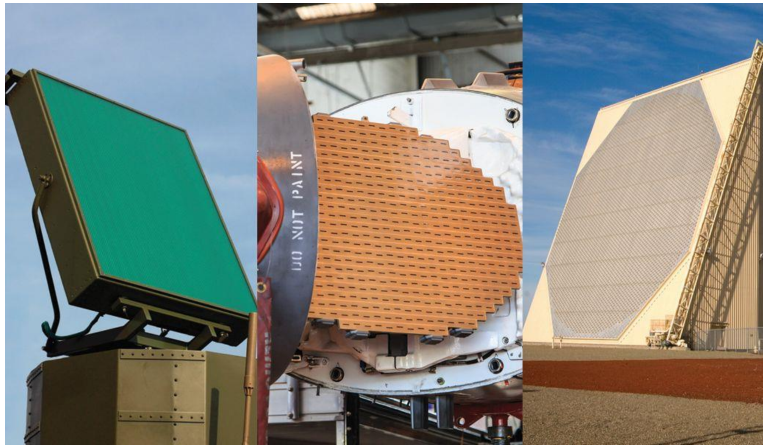


Radar: Interested in Detecting

- Range: Distance from the radar
    - UWB: Ultra Wide Band
    - FMCW: Frequency Modulated Carrier Wave
- Direction: Angle from which the object is coming

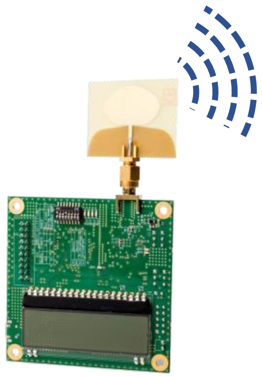
Directional antenna that is rotating.

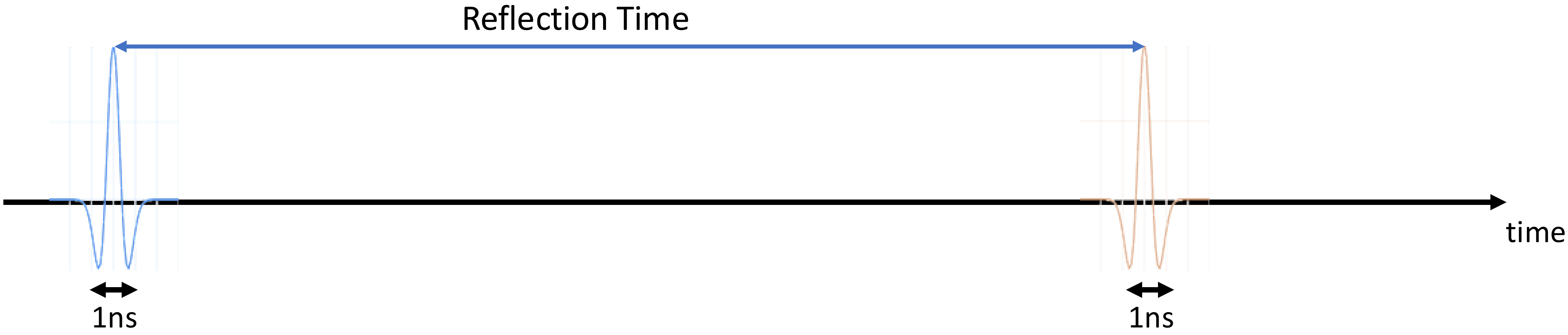
Phased Antenna Array that is always Scanning.



UWB: Ultra Wide Band

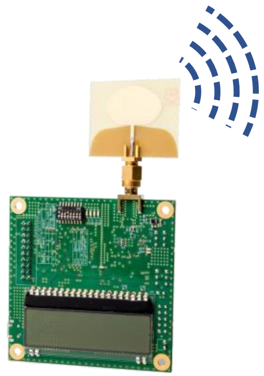
- Transmit a Narrow Pulse in Time (1 ns)



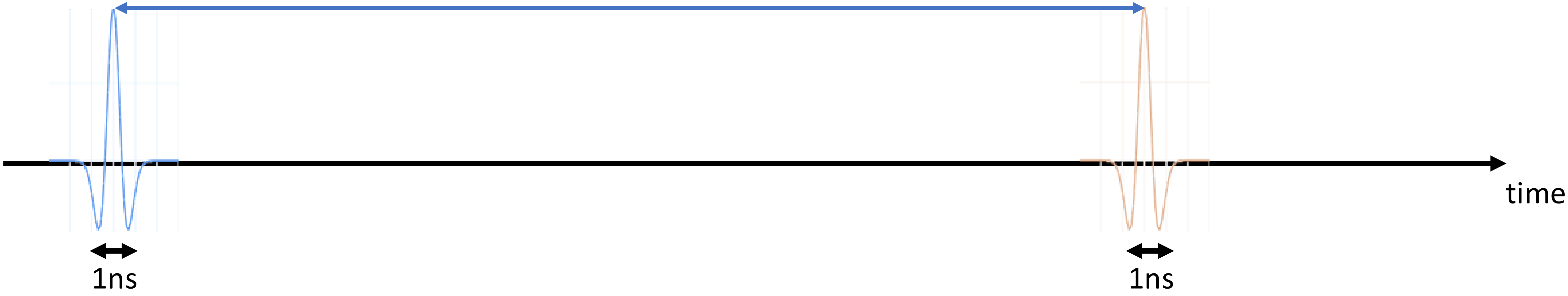


UWB: Ultra Wide Band

- Transmit a Narrow Pulse in Time (1 ns)



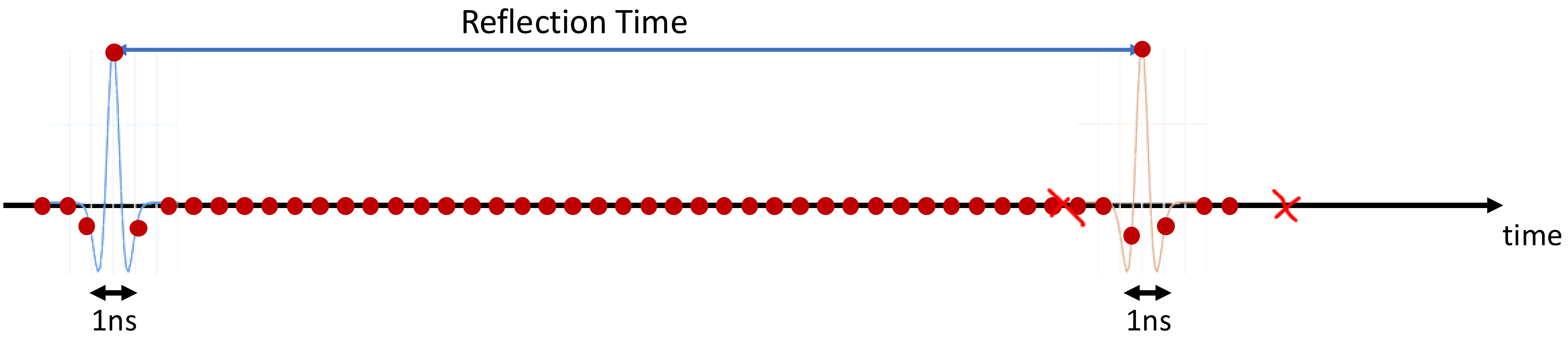
Distance = $\left(\text{Reflection Time}\ /2\right) \times$ Speed of Light



UWB: Ultra Wide Band

- Transmit a Narrow Pulse in Time (1 ns)
- Bandwidth = 1/pulse time = 1 GHz → UWB
- To detect pulse need to sample at Multi-GigaSample/sec

Distance = (Reflection Time /2) × Speed of Light



UWB: Ultra Wide Band

Pros:

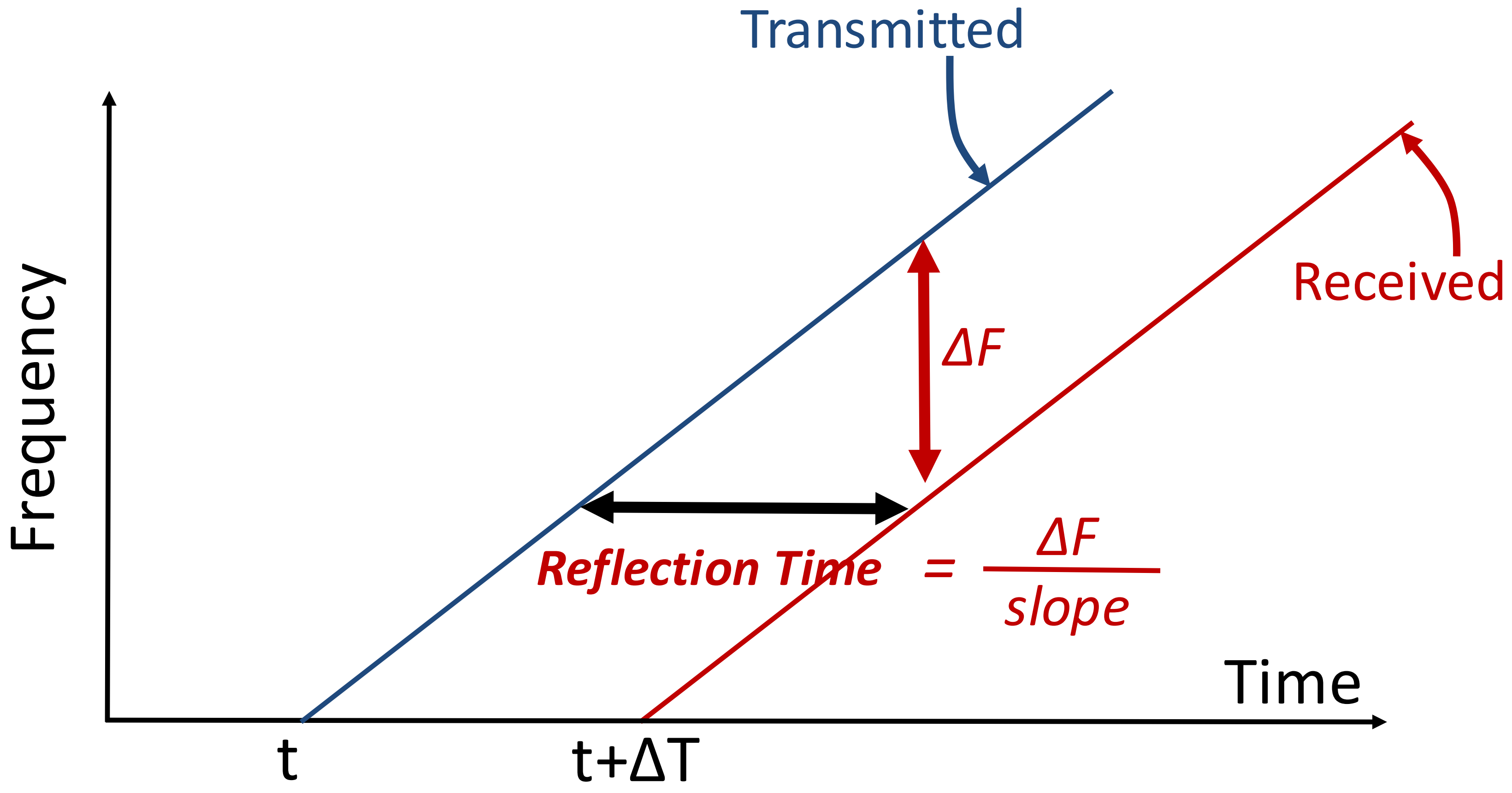
- High resolution
- Simple idea
- Used also for communication (UWB 802.15.3a, 3 - 10 GHz)

Cons:

- Very High Speed Sampling Rate!

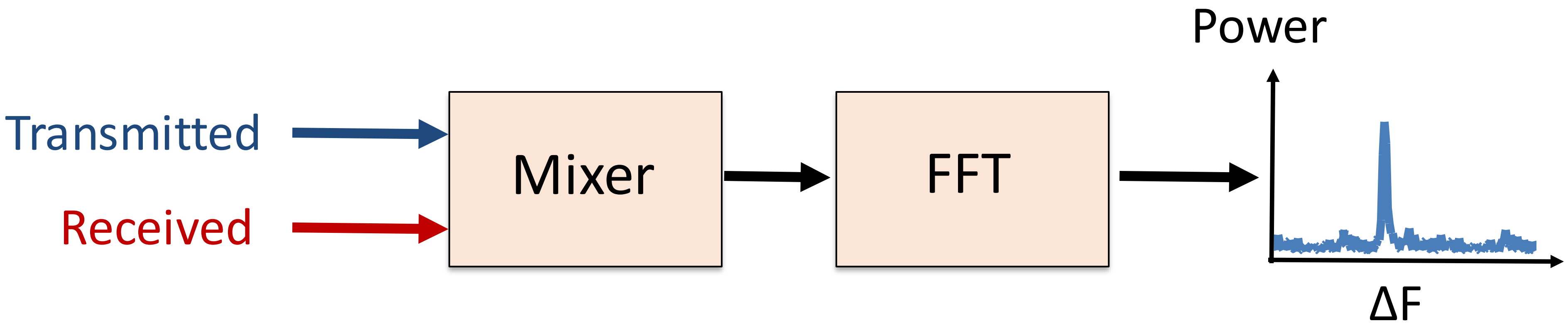
FMCW: Frequency Modulated Carrier Wave

- Use chirps: signals where the frequency changes with time
- Measure time using a frequency change



How do we measure ΔF ?

- Subtracting frequencies is easy (e.g., removing carrier in WiFi)
- Done using a mixer (low-power; cheap)



Signal whose frequency is ΔF

ΔF → Reflection Time → Distance

FMCW: Frequency Modulated Carrier Wave

Slope

- FMCW Transmitted Signal: $x(t) = e^{j2\pi\left(\frac{\alpha}{2}t^2 + f_0 t + \phi_0\right)}$

Reflection Time

- FMCW Received Signal: $y(t) = Ae^{j2\pi\left(\frac{\alpha}{2}(t-\tau)^2 + f_0(t-\tau) + \phi_0\right)}$
  (Single reflector)

$$= Ae^{j2\pi\left(\frac{\alpha}{2}t^2 - \alpha\tau t + \frac{\alpha}{2}\tau^2 + f_0 t - f_0\tau + \phi_0\right)}$$

Phase

- Mix TX and RX Signal: $y_b(t) = y(t) \times x^*(t) = Ae^{-j2\pi(\alpha\tau t + f_0\tau)}$

Frequency at $f = \alpha\tau$

FMCW: Frequency Modulated Carrier Wave

- FMCW Transmitted Signal: $x(t) = e^{j2\pi\left(\frac{\alpha}{2}t^2 + f_0 t + \phi_0\right)}$

Slope

- FMCW Received Signal: (Multiple reflectors) $y(t) = \sum_i A_i e^{j2\pi\left(\frac{\alpha}{2}(t-\tau_i)^2 + f_0(t-\tau_i) + \phi_0\right)}$

Reflection Time

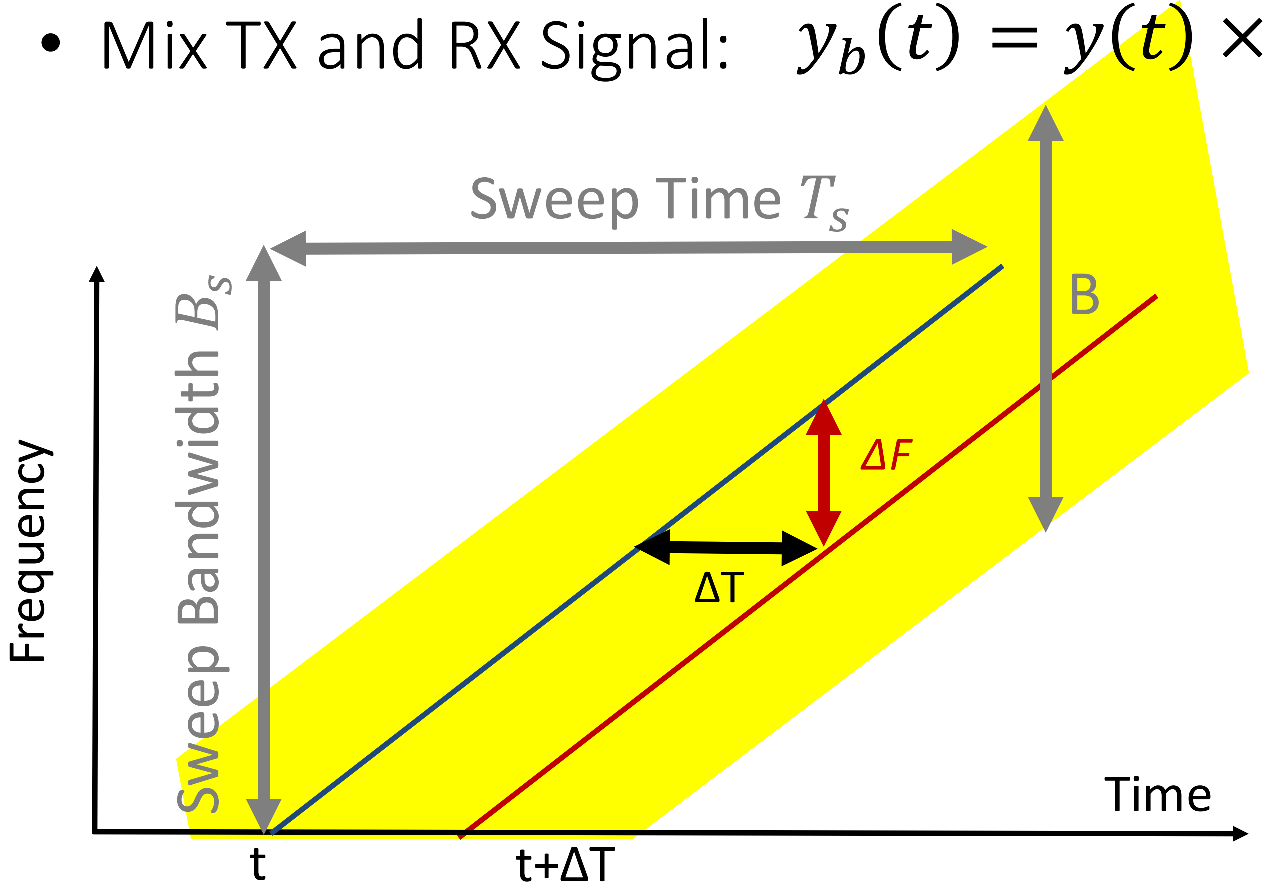
- Mix TX and RX Signal: $y_b(t) = y(t) \times x^*(t) = \sum_i A_i e^{-j2\pi(\alpha\tau_i t + f_0\tau_i)}$

Phase

Frequencies at $f = \alpha\tau_i$

FMCW: Frequency Modulated Carrier Wave

- FMCW Transmitted Signal: $x(t) = e^{j2\pi\left(\frac{\alpha}{2}t^2+f_0t+\phi_0\right)}$
- Mix TX and RX Signal: $y_b(t) = y(t) \times x^*(t) = \sum_i A_i e^{-j2\pi(\alpha\tau_i t + f_0\tau_i)}$



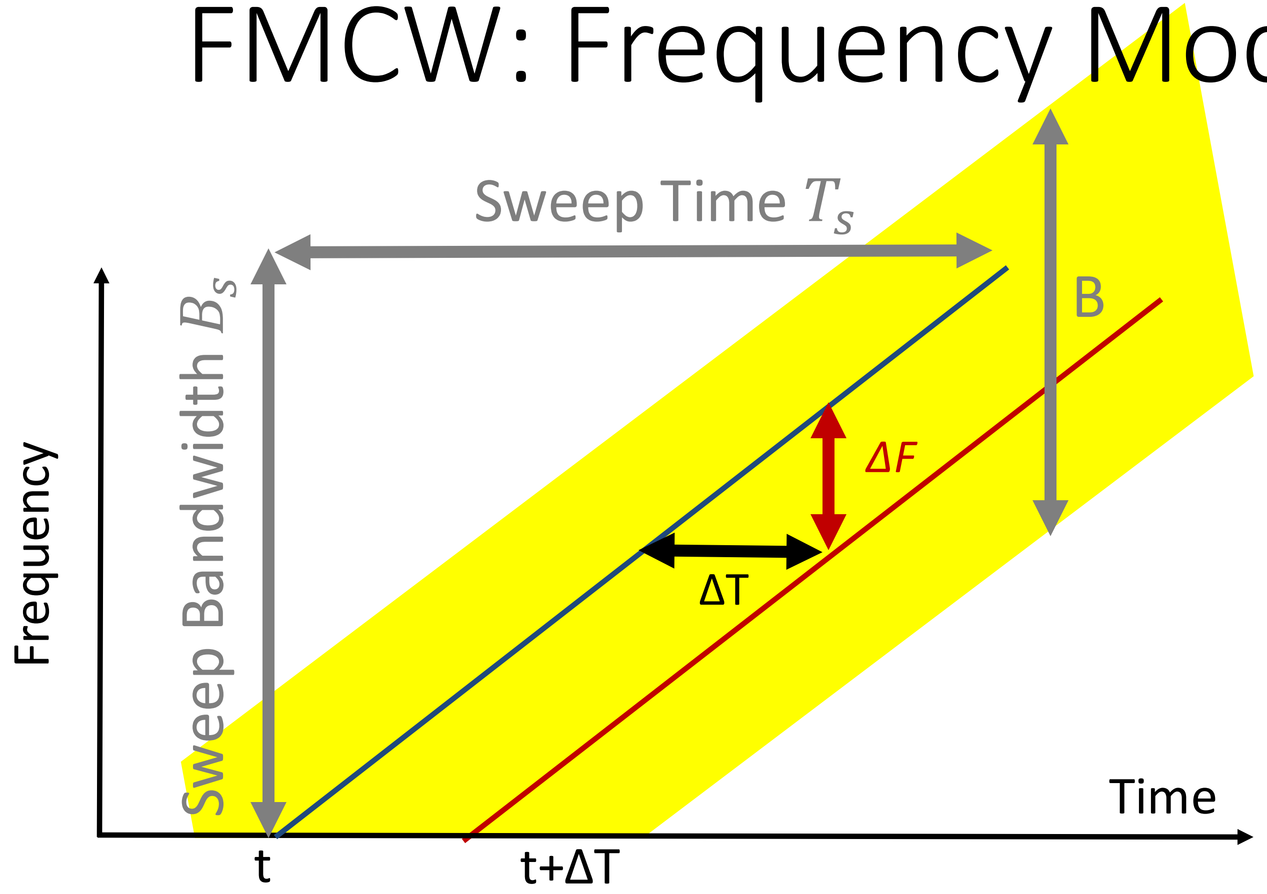
Slope: $\alpha = \frac{B_s}{T_s}$

$\Delta F = \alpha \Delta T = \alpha \tau_i$

Sampling Rate: $B \ll B_s$

FFT Window: $T = T_s$

FMCW: Frequency Modulated Carrier Wave



Slope: $\alpha = \dfrac{B_s}{T_s}$

$\Delta F = \alpha \Delta T = \alpha \tau_i$

Sampling Rate: $B \ll B_s$

FFT Window: $T = T_s$

Max Range: $\Delta F < B \quad \Rightarrow \tau_{max} < B/\alpha \quad = B\, T_s/B_s \quad \Rightarrow d_{max} < c\, B\, T_s/2B_s$

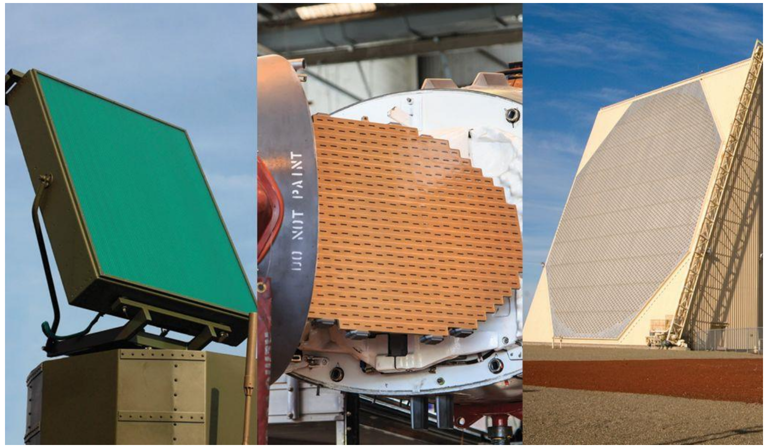
Resolution: $\delta F = 1/T = 1/T_s \; \Rightarrow \tau_{min} = 1/\alpha T_s = 1/B_s \; \Rightarrow d_{min} = c/2B_s$

Radar: Interested in Detecting

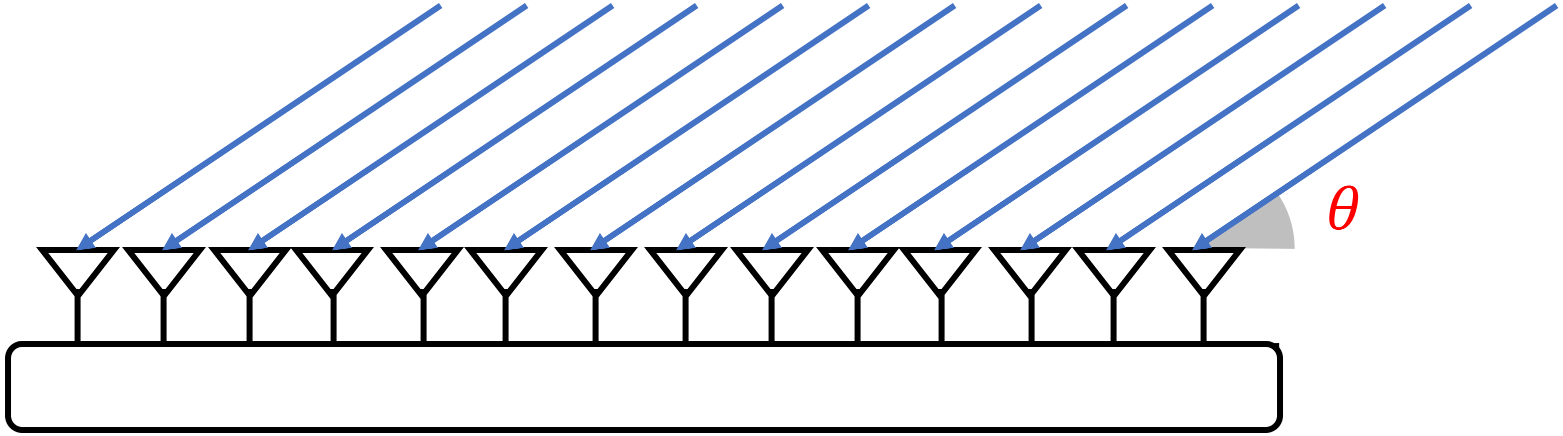
- Range: Distance from the radar
  - UWB: Ultra Wide Band
  - FMCW: Frequency Modulated Carrier Wave
- Direction: Angle from which the object is coming

Directional antenna that is rotating.

Phased Antenna Array that is always Scanning.



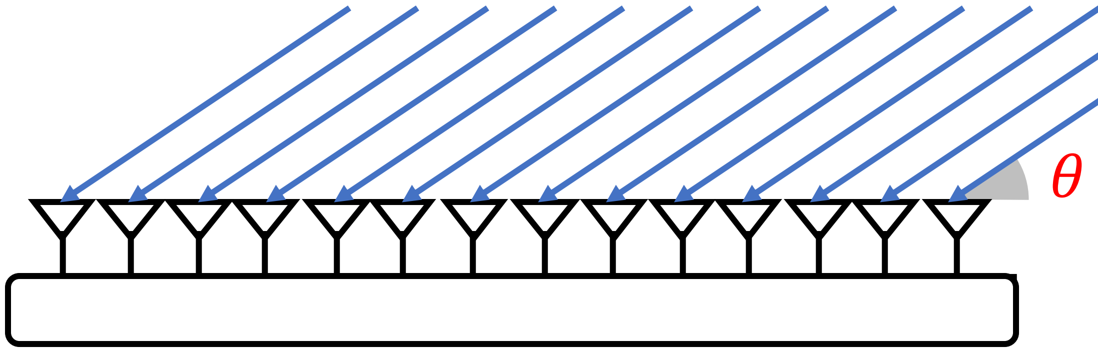
Phased Arrays Primer



$$h_k = \alpha_1 e^{-j2\pi \frac{d_1 - k\ s \cos \theta_1}{\lambda}}$$

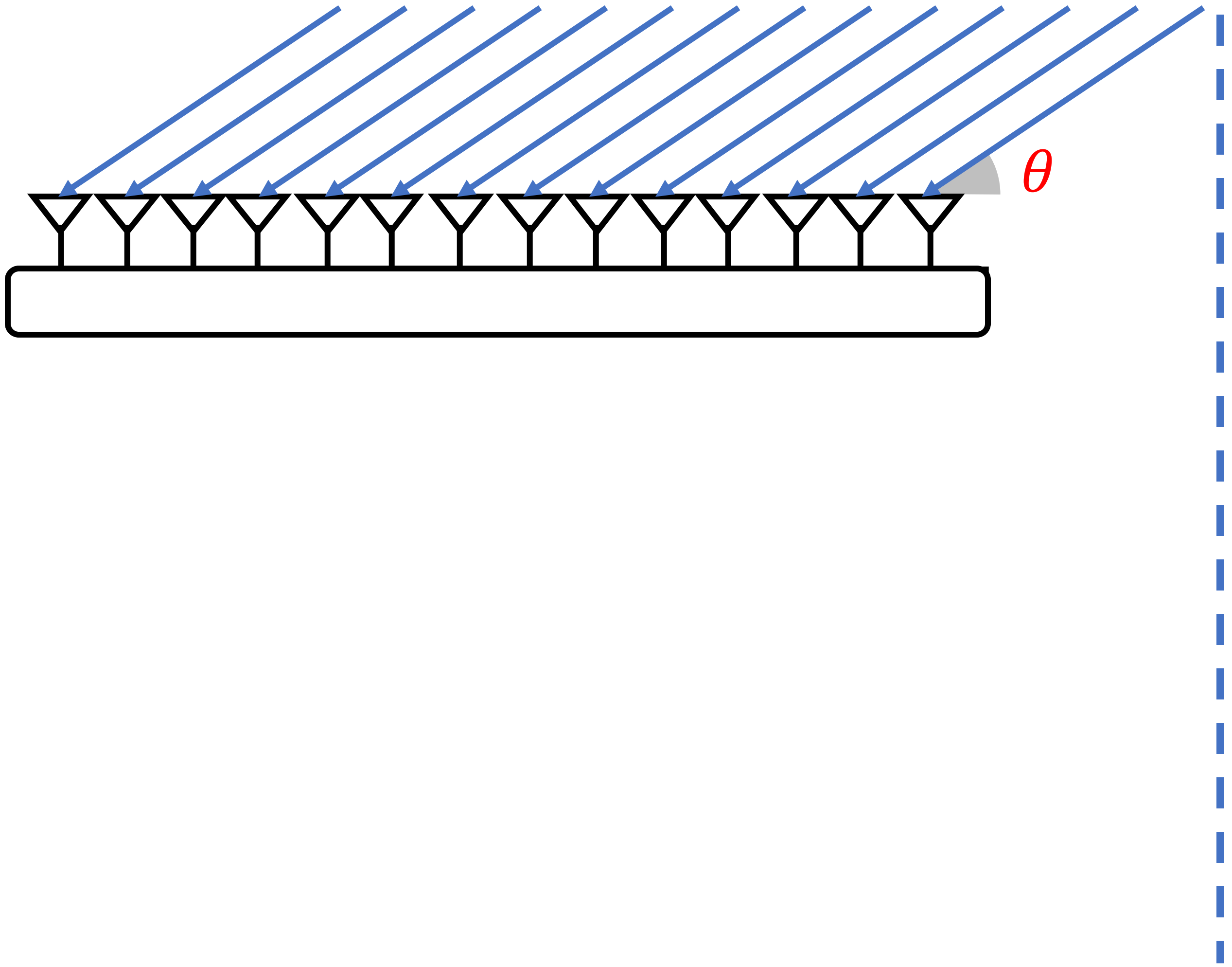
Phased Arrays Primer

Digital Phased Arrays



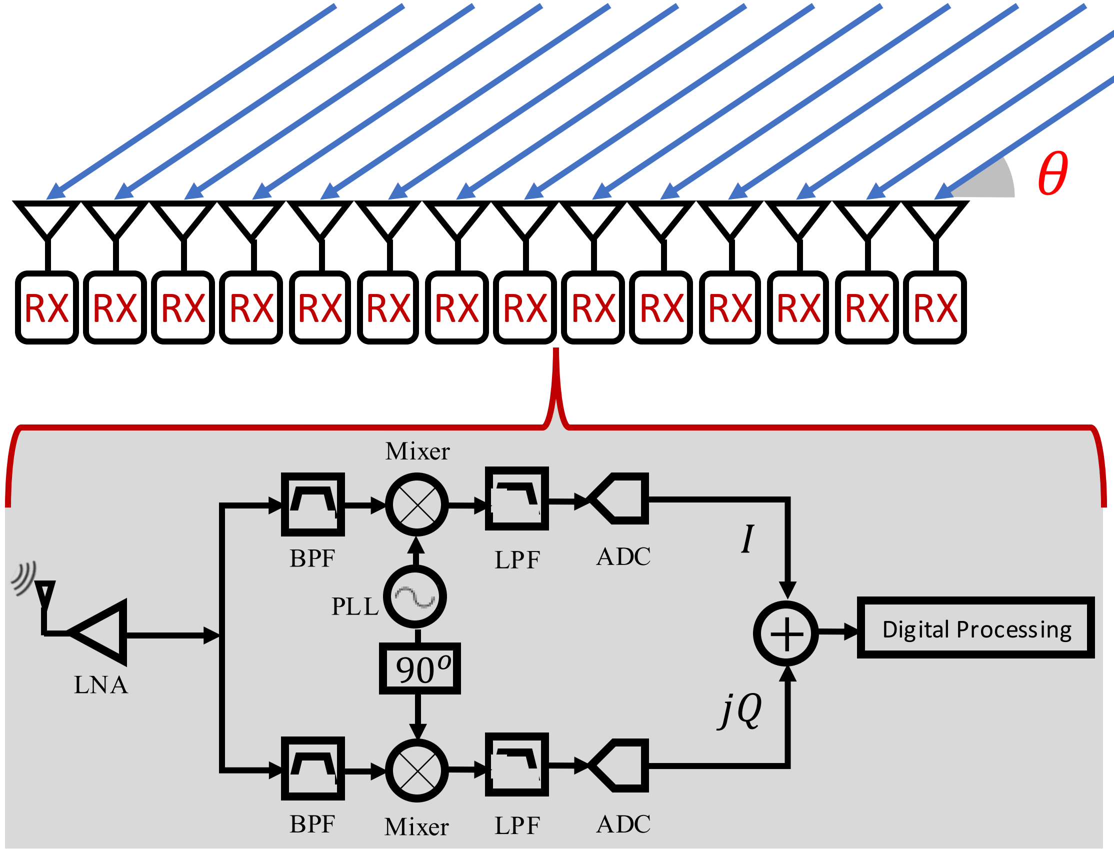
Phased Arrays Primer

Digital Phased Arrays



Phased Arrays Primer

Digital Phased Arrays

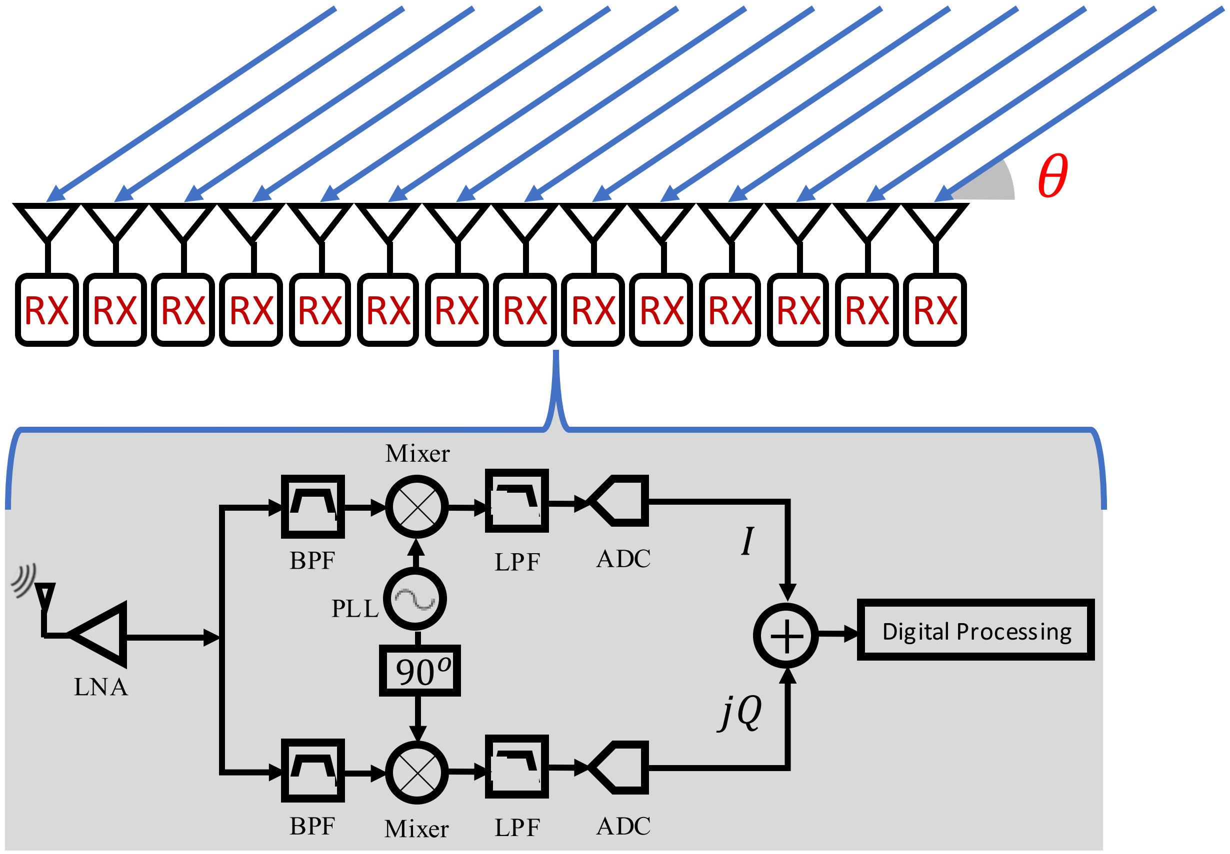


Every antenna connected to full fledged RX

Sample & Process Signals in Digital

Phased Arrays Primer

Digital Phased Arrays

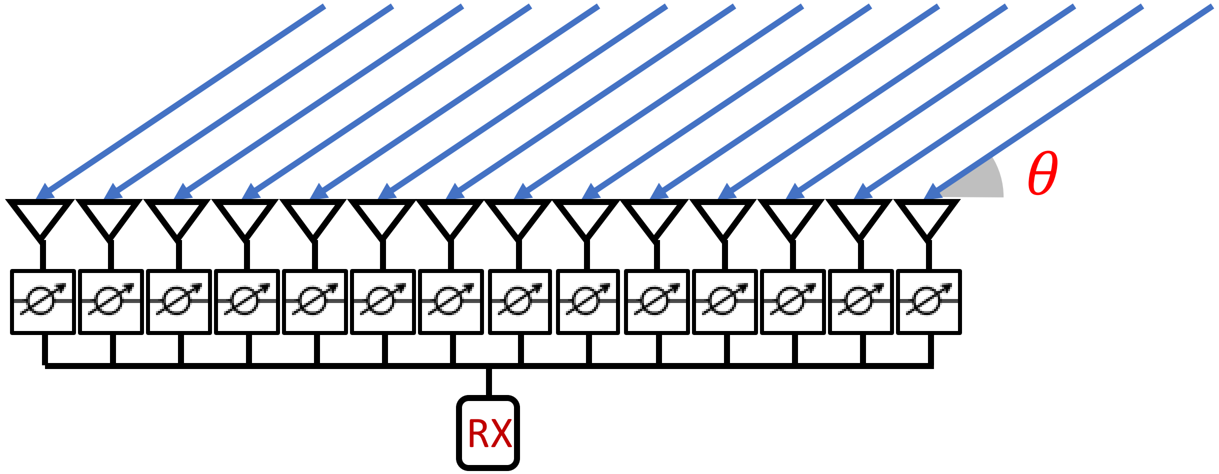


Every antenna connected to full fledged RX

Sample & Process Signals in Digital

Very expensive and high power for large arrays & mmWave

Analog Phased Arrays

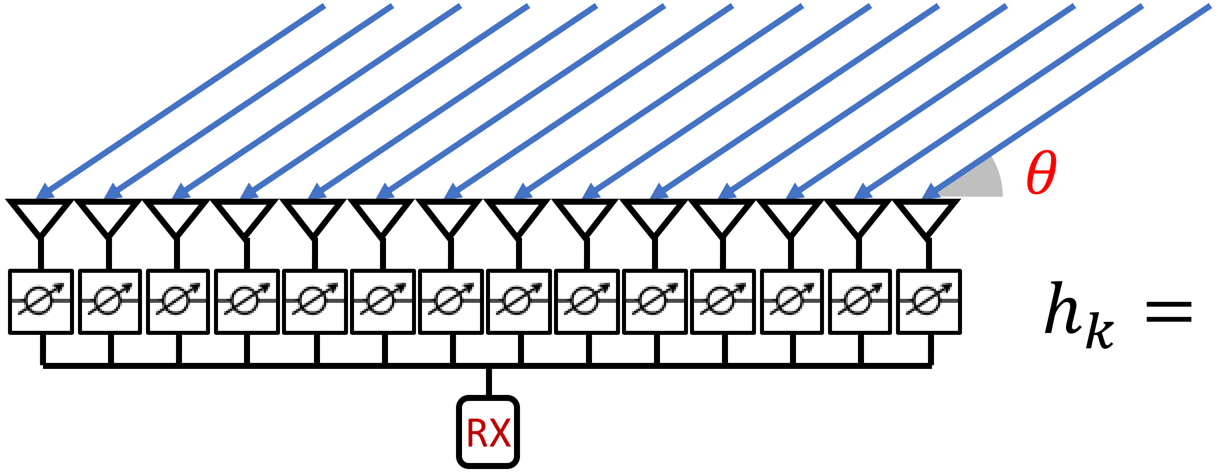


All antennas connected to a single receiver

- Each antenna connected to a phase shifter.
- Phase shifter changes the phase of the signal on each antenna by multiplying with $e^{j\phi}$.
- Steer the beam electronically by changing the phases of the signals.
- Get the sum along a certain direction.

Cheap & low power but requires scanning

Analog Phased Arrays



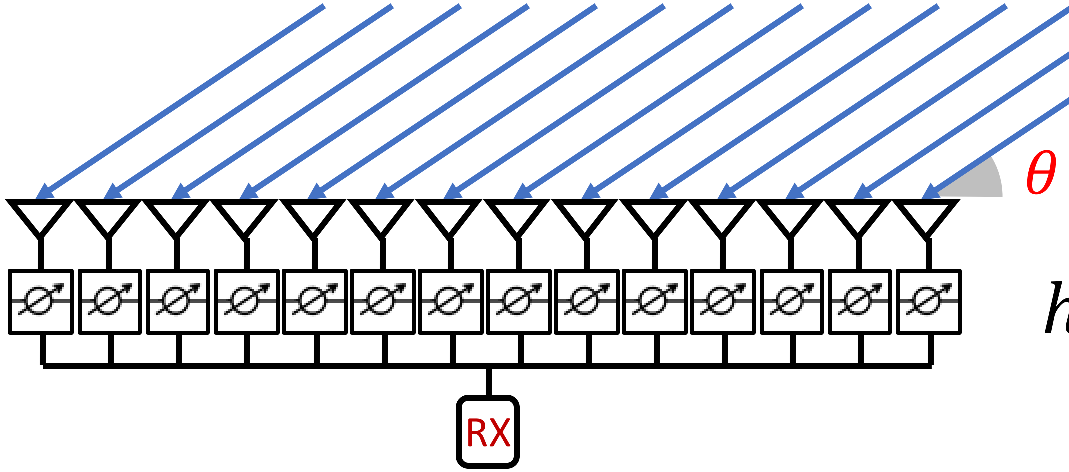
$$h_k = \alpha e^{-j2\pi\frac{d-k\ s\cos\theta}{\lambda}}$$

$$y(t) = \sum_{1}^{N} y_k(t)e^{j\phi_k} = \sum_{1}^{N} h_k x(t)e^{j\phi_k} = \sum_{1}^{N} \alpha e^{-j2\pi\frac{d-k\ s\cos\theta}{\lambda}} x(t)e^{j\phi_k}$$

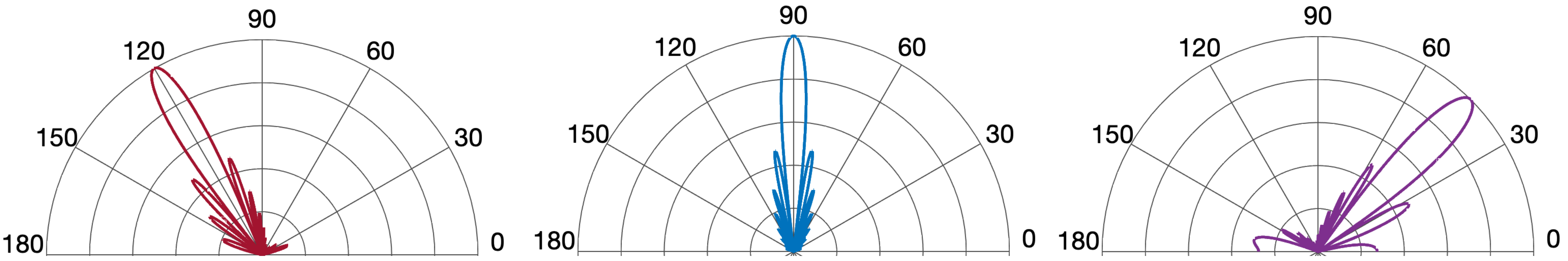
$$= x(t)\alpha e^{-j2\pi\frac{d}{\lambda}} \sum_{1}^{N} e^{j\pi k\ \cos\theta} e^{j\phi_k}$$

To get signal along direction θ_1, set the phases on the phase shifters to $\phi_k = -\pi k \cos\theta$

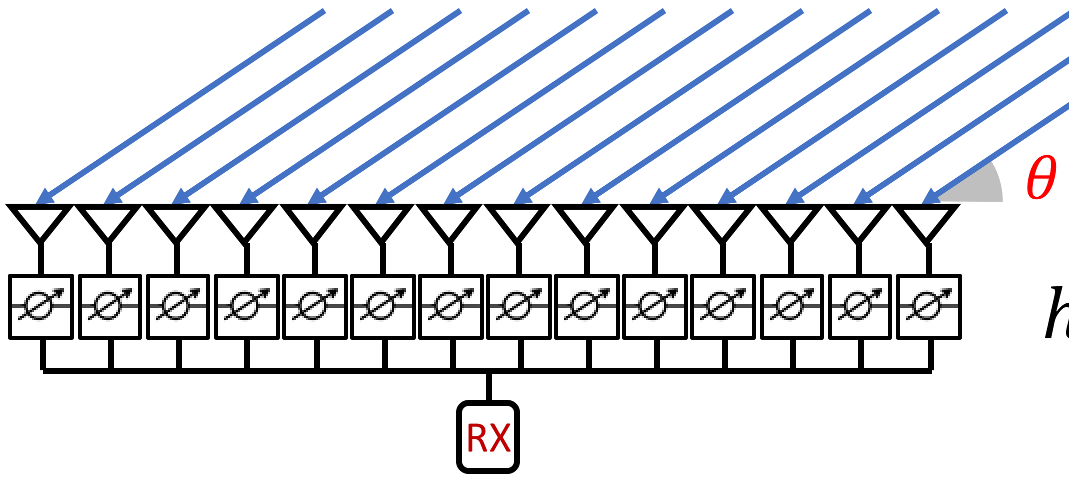
Analog Phased Arrays



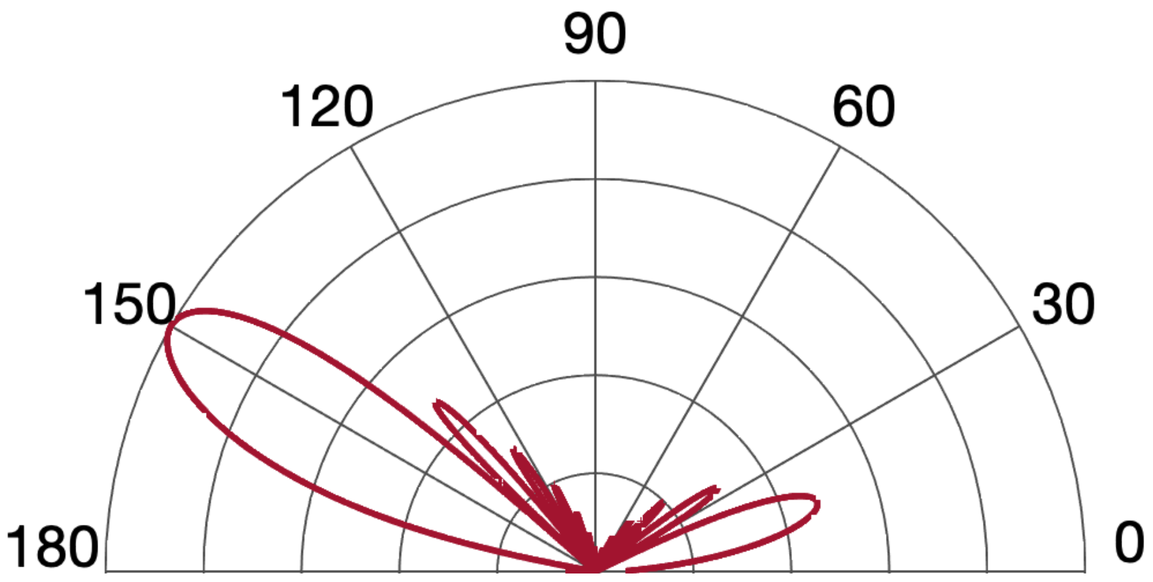
$$h_k = \alpha e^{-j2\pi \frac{d - k\, s \cos\theta}{\lambda}}$$



Analog Phased Arrays



$$h_k = \alpha e^{-j2\pi\frac{d-k\,s\cos\theta}{\lambda}}$$



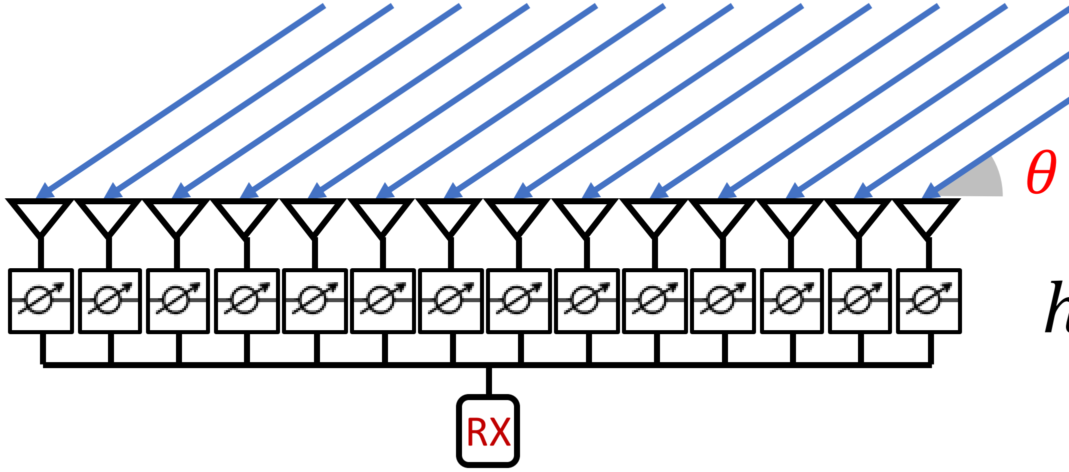
(1) Set the phase shifters to receive signals from a given direction.

(2) Send FMCW signals.

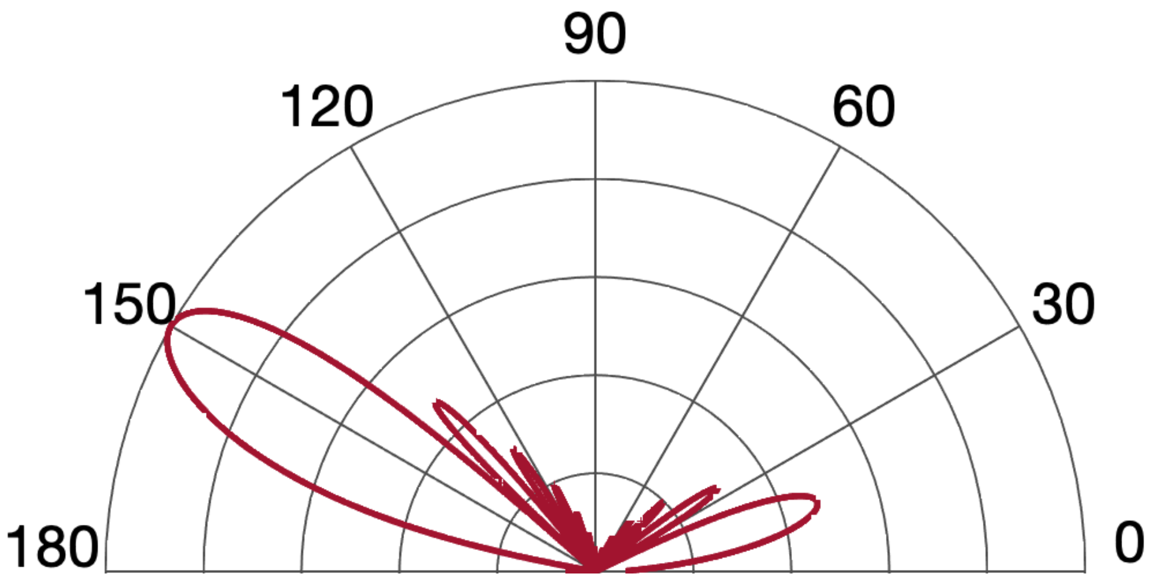
(3) Receiver FMCW reflections, down convert, sample and compute range FFT.

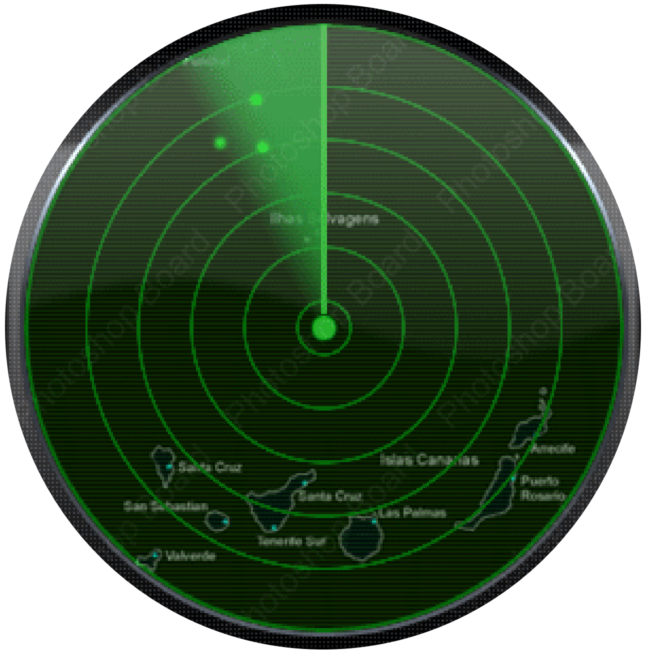
(4) Repeat until you get 2D range image: AoA + Range

Analog Phased Arrays

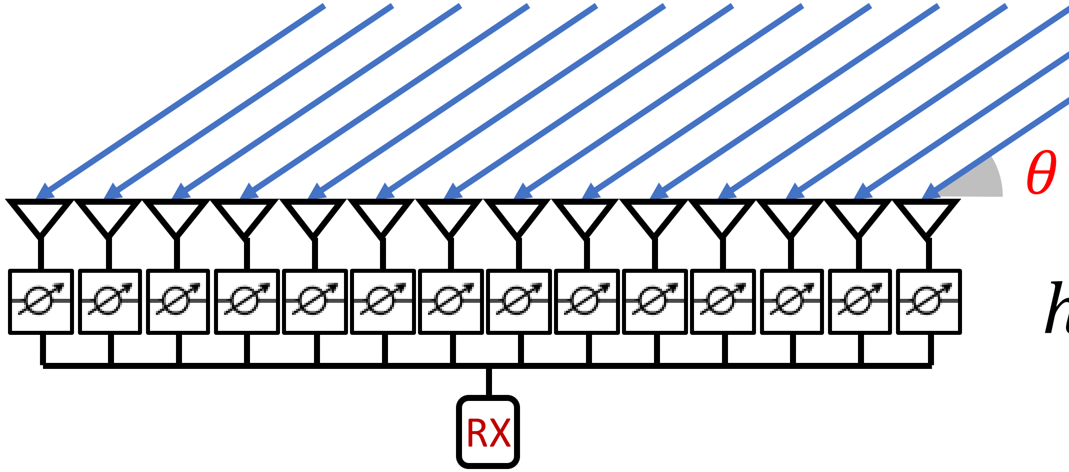


$$h_k = \alpha e^{-j2\pi\frac{d-k\,s\cos\theta}{\lambda}}$$

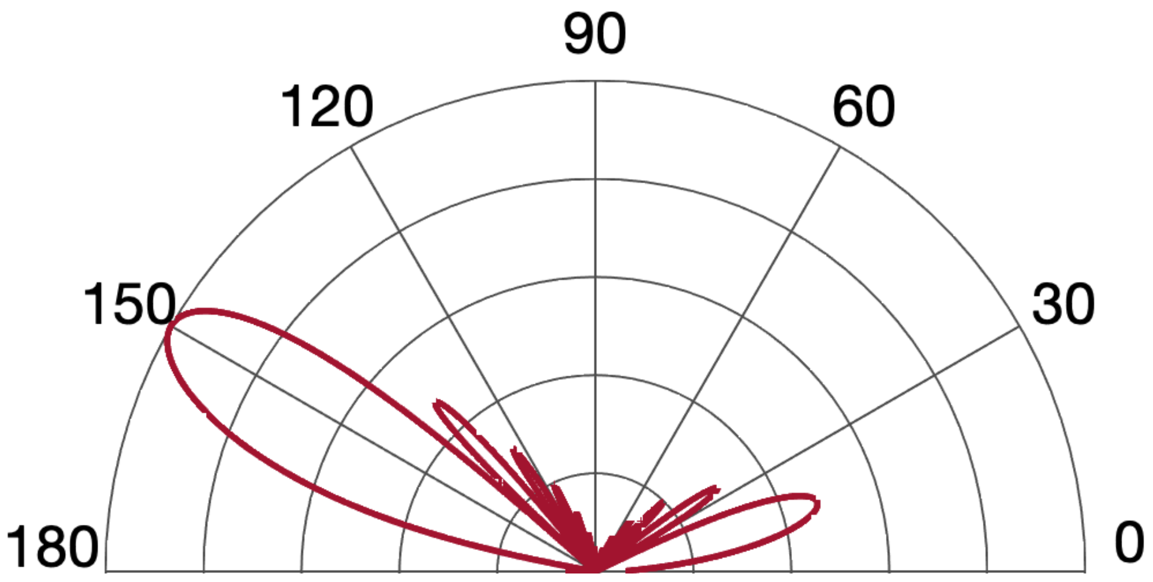


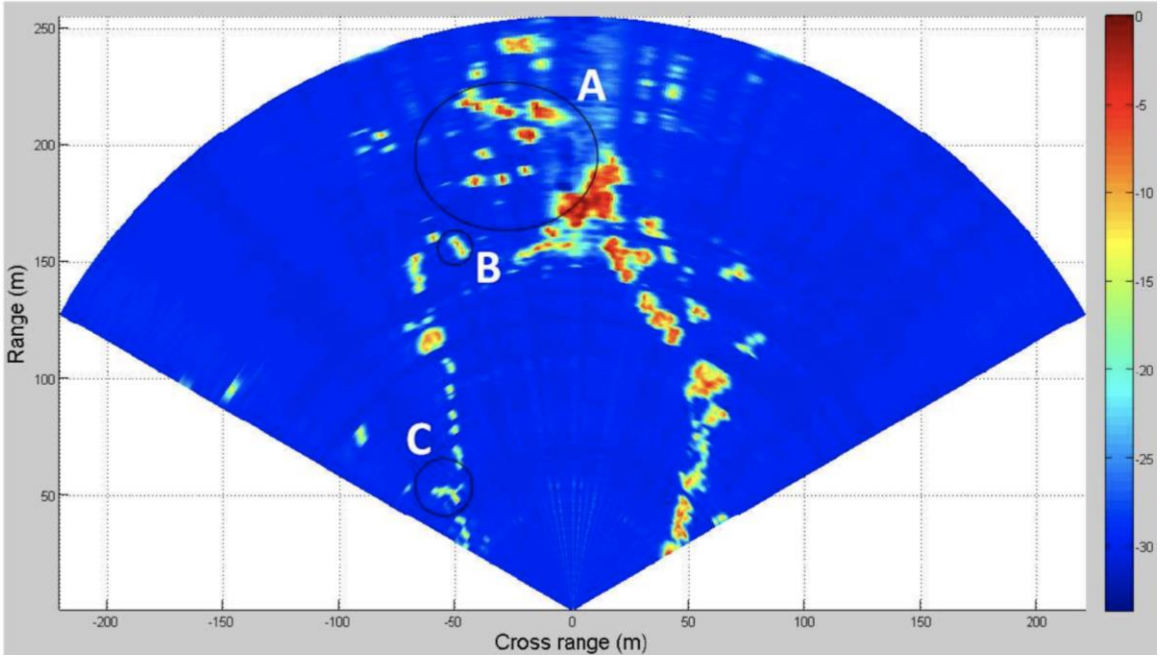


Analog Phased Arrays

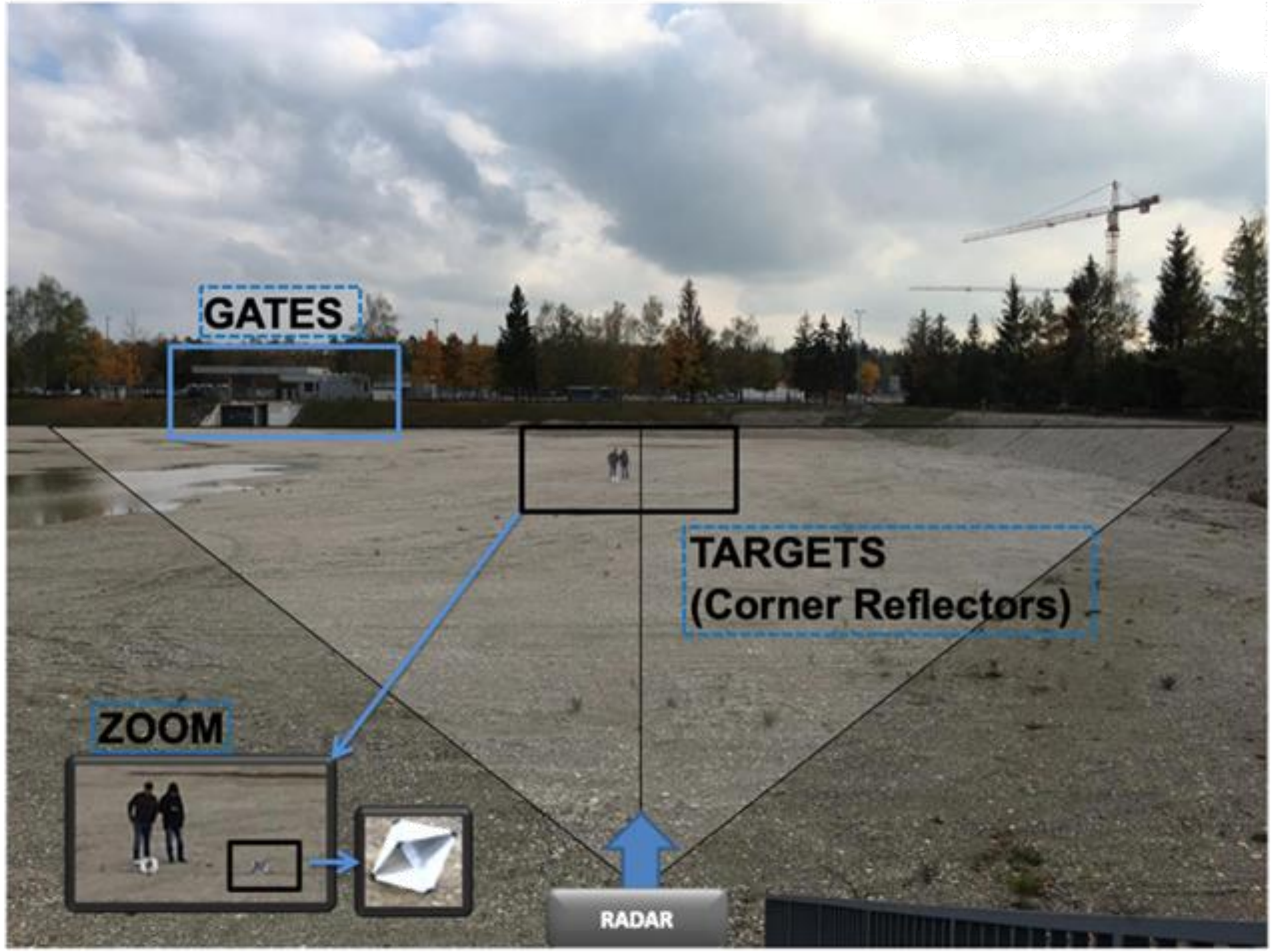


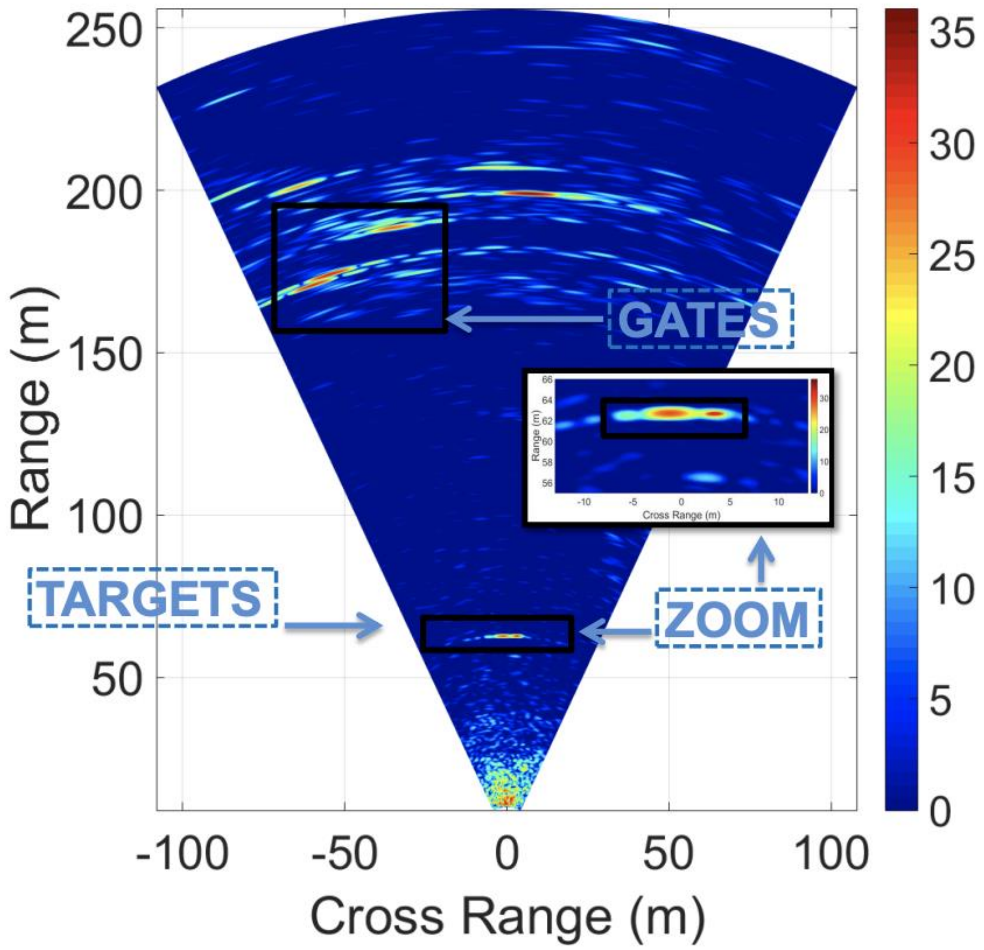
$$h_k = \alpha e^{-j2\pi \frac{d - k\, s \cos\theta}{\lambda}}$$





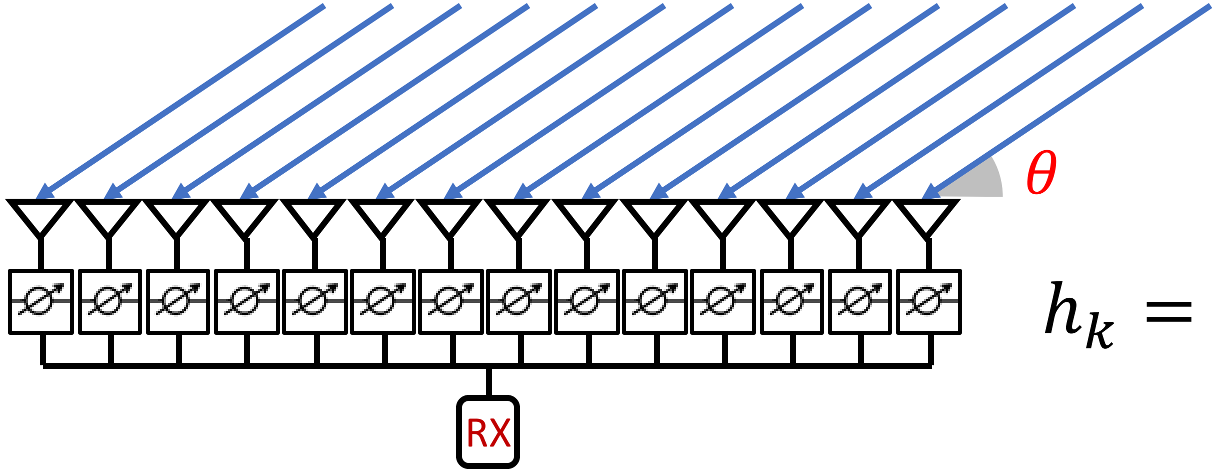
Analog Phased Arrays



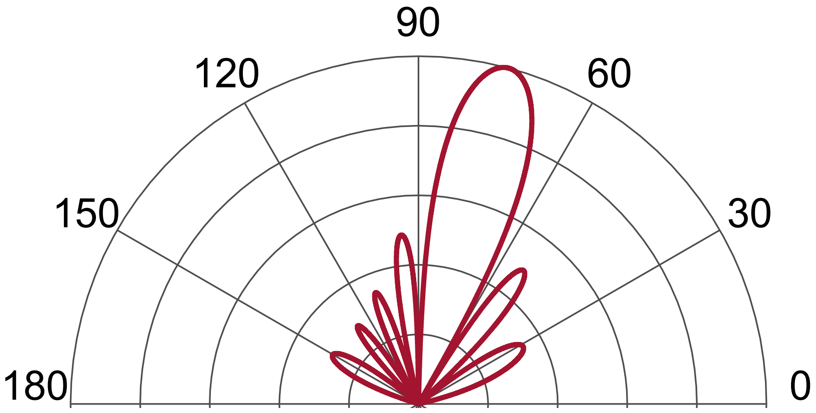


Ganis, A.; Miralles-Navarro, E.; Schoenlinner, B.; Prechtel, U.; Meusling, A.; Heller, C.; Spreng, T.... (2018). A portable 3D Imaging FMCW MIMO Radar Demonstrator with a 24x24 Antenna Array for Medium Range Applications. IEEE Transactions on Geoscience and Remote Sensing. 56(1):298-312. https://doi.org/10.1109/TGRS.2017.2746739

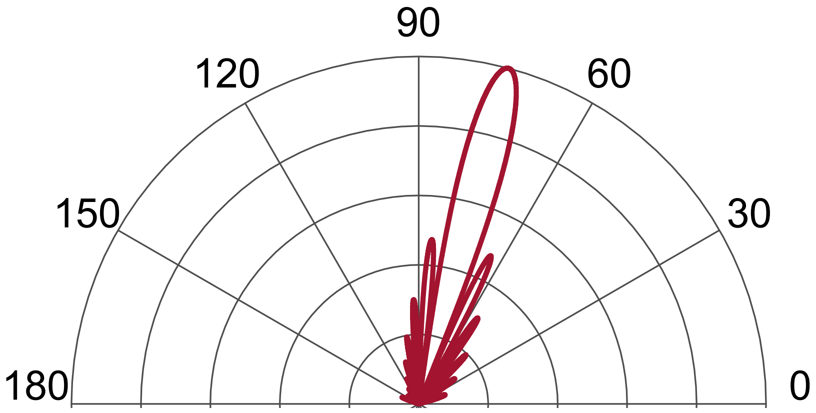
Analog Phased Arrays



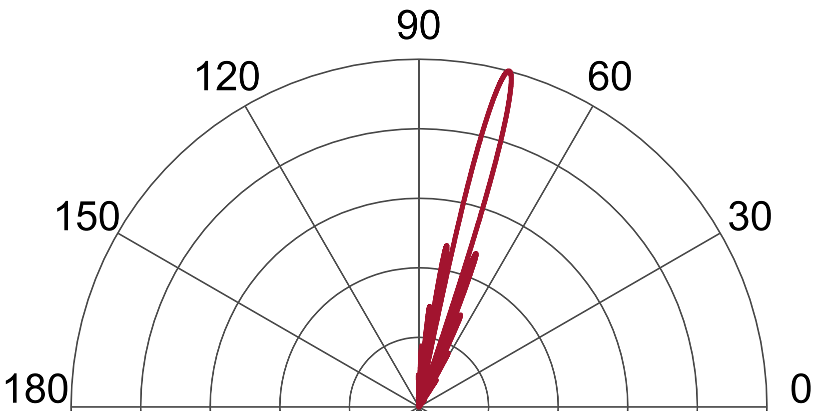
$$h_k = \alpha e^{-j2\pi \frac{d-k\, s \cos\theta}{\lambda}}$$



8 Antenna



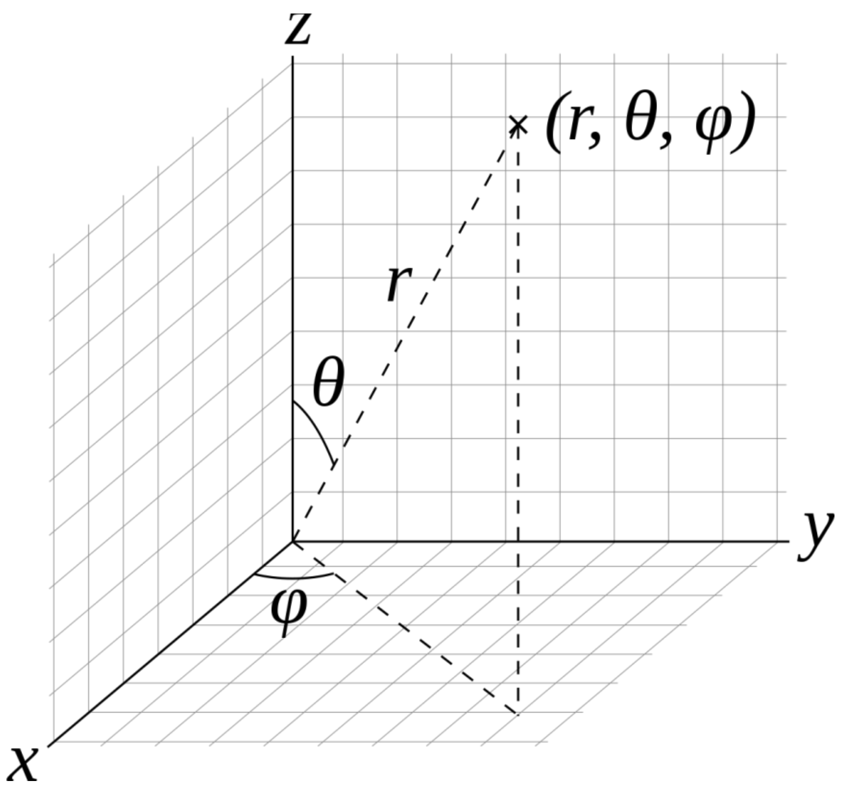
16 Antenna

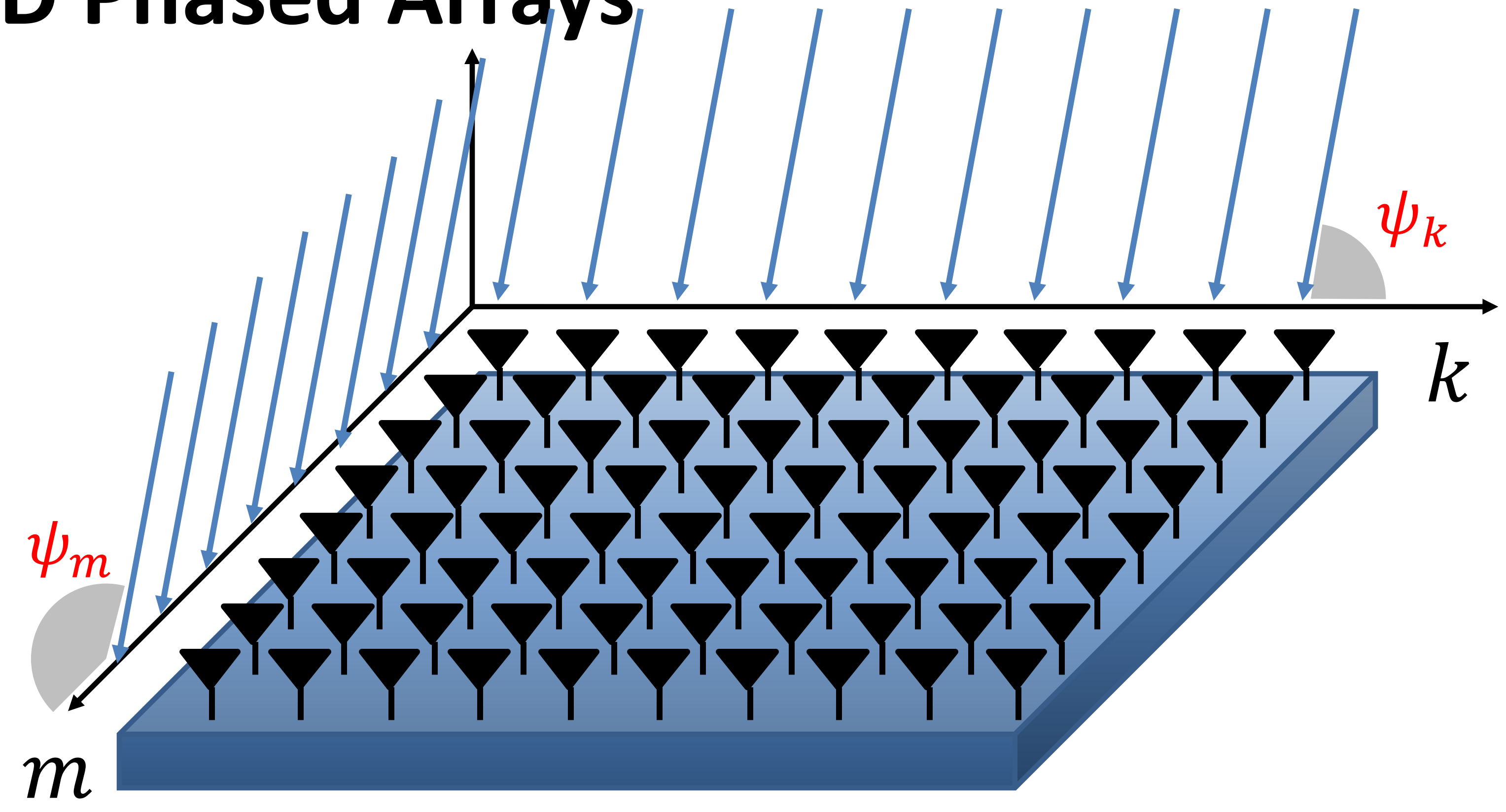


32 Antenna

Larger Array → Narrower Beams → Higher Resolution

2D Phased Arrays





$$h_{m,k} = \alpha e^{-j\frac{2\pi}{\lambda}(r+f(m)+f(k))}$$

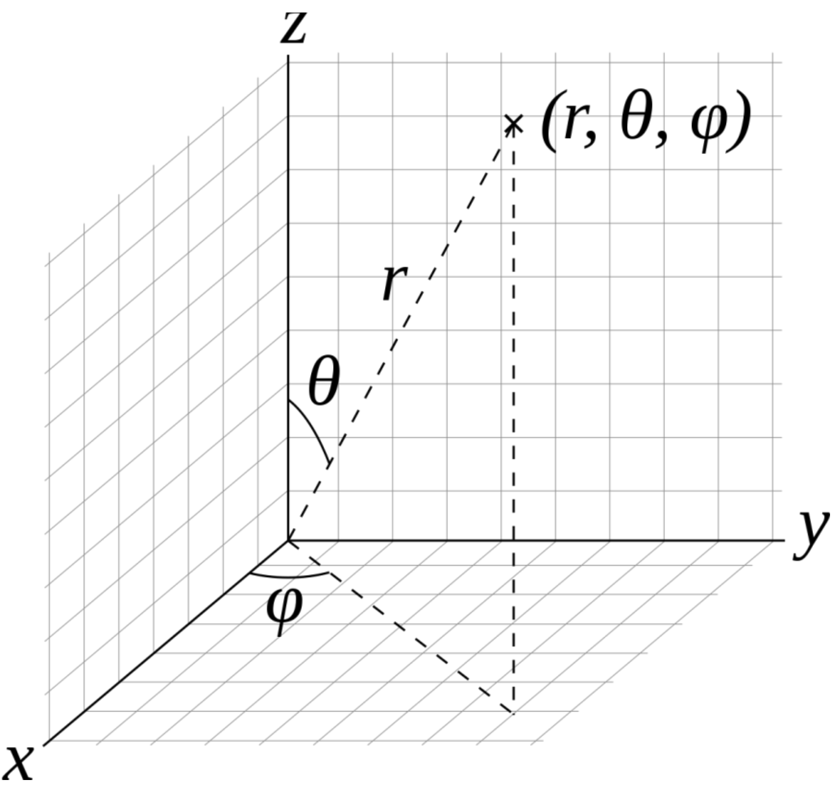
Fix m**:** $$h_{m,k} = \alpha e^{-j\frac{2\pi}{\lambda}(r+f(m)+ks\cos\psi_k)}$$

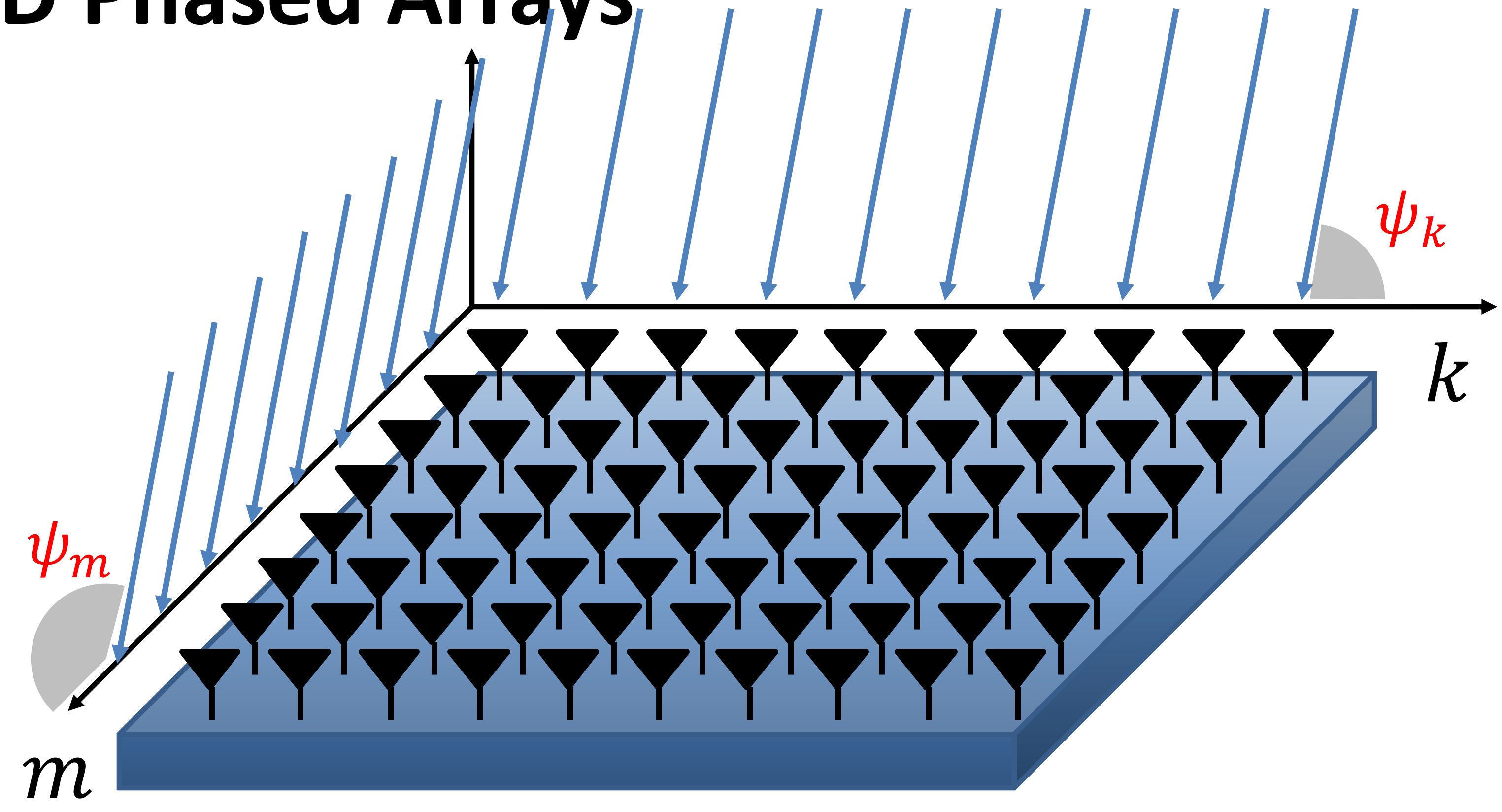
Fix k**:** $$h_{m,k} = \alpha e^{-j\frac{2\pi}{\lambda}(r+ms\cos\psi_m+ks\cos\psi_k)}$$

Can recover:

- Range: r
- Azimuth AoA: φ
- Elevation AoA: θ

2D Phased Arrays





Can recover:

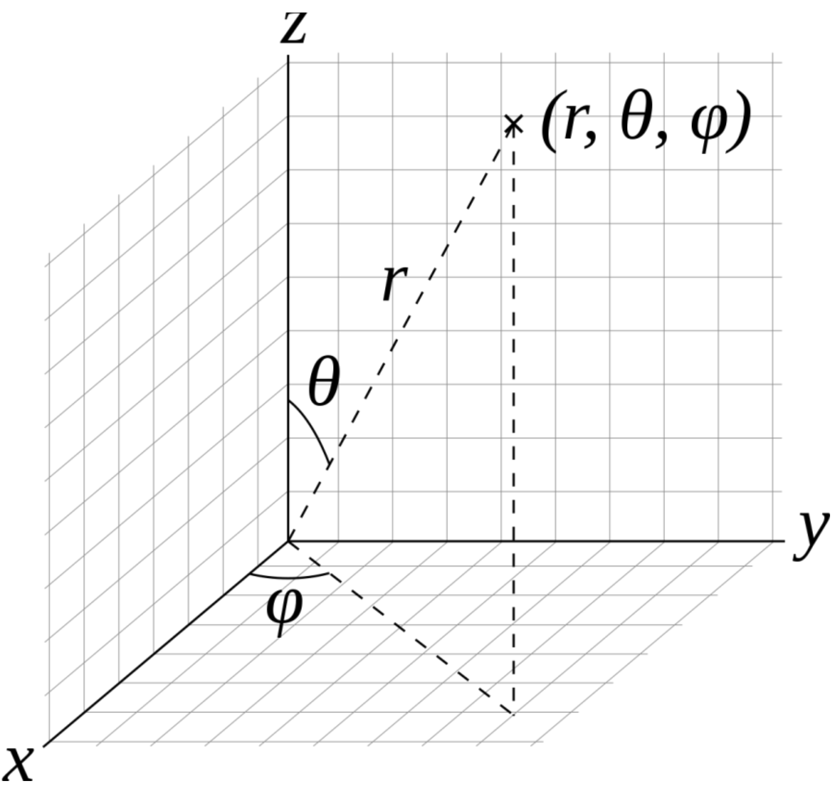
- Range: r
- Azimuth AoA: φ
- Elevation AoA: θ

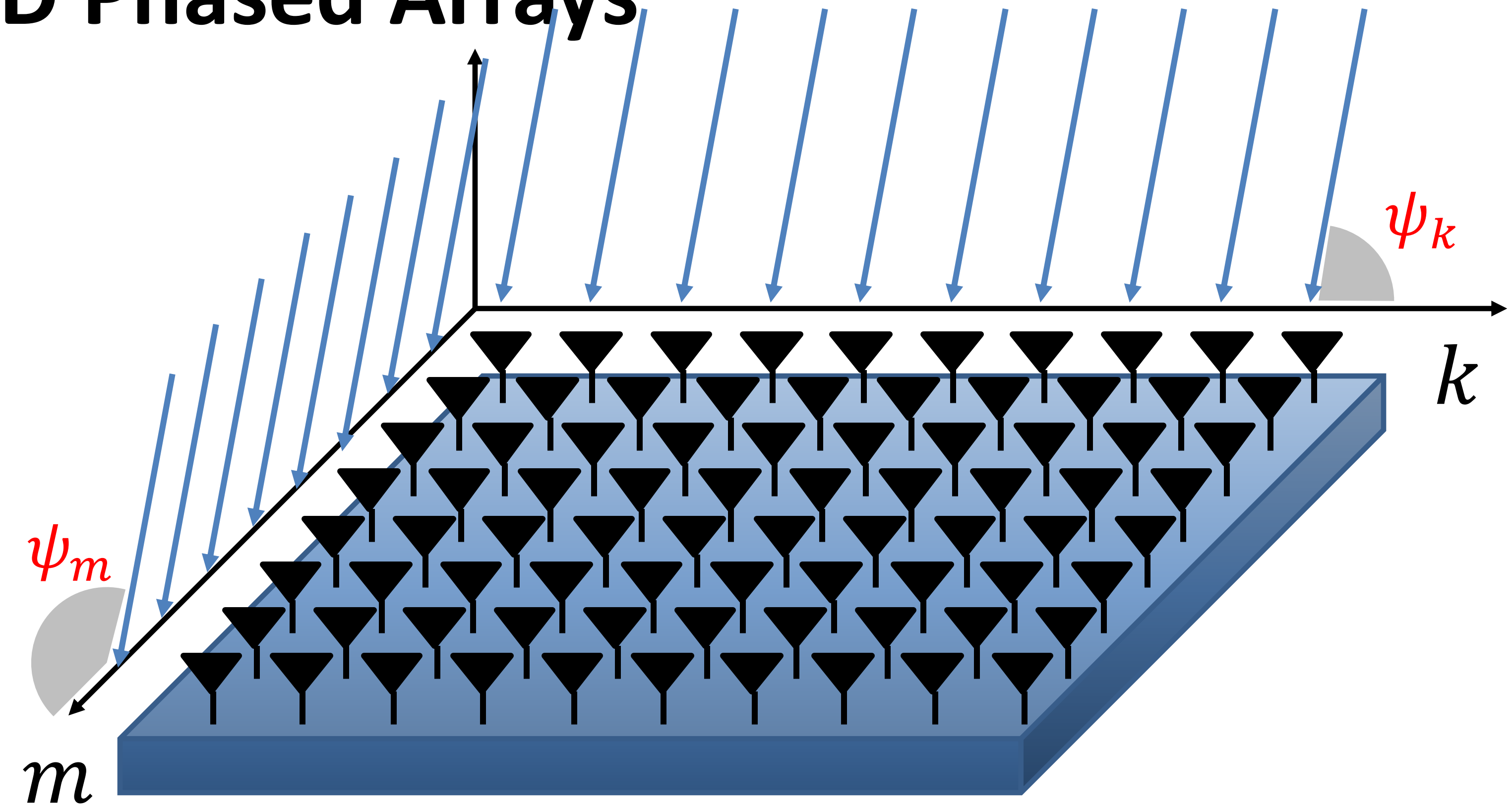
$$h_{m,k} = \alpha e^{-j\frac{2\pi}{\lambda}(r + ms\cos\psi_m + ks\cos\psi_k)}$$

$$\cos\psi_m = \sin\theta\cos\varphi$$

$$\cos\psi_k = \sin\theta\sin\varphi$$

2D Phased Arrays



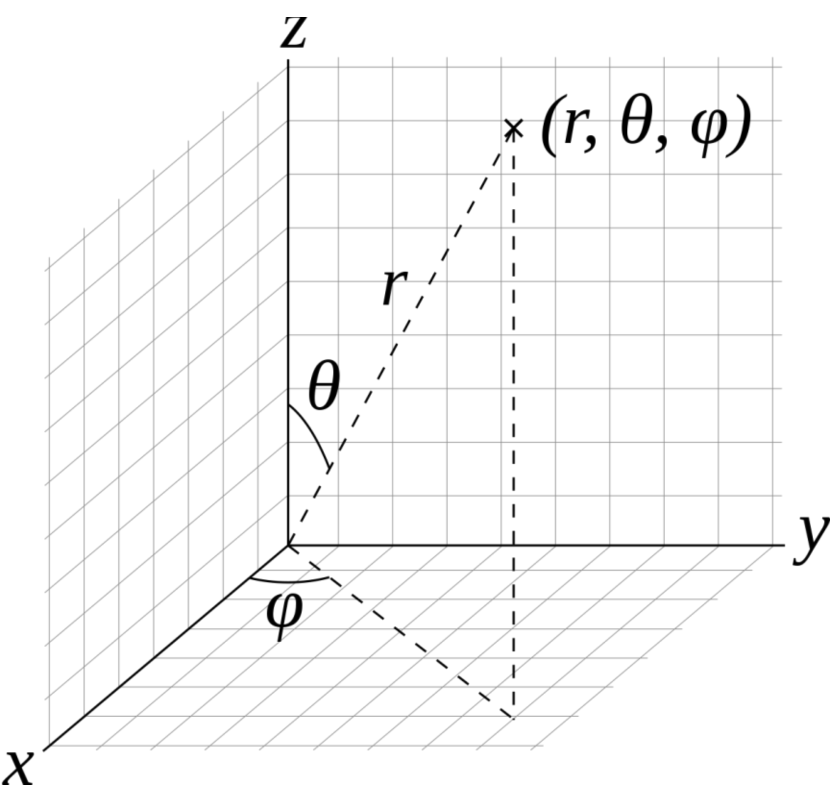


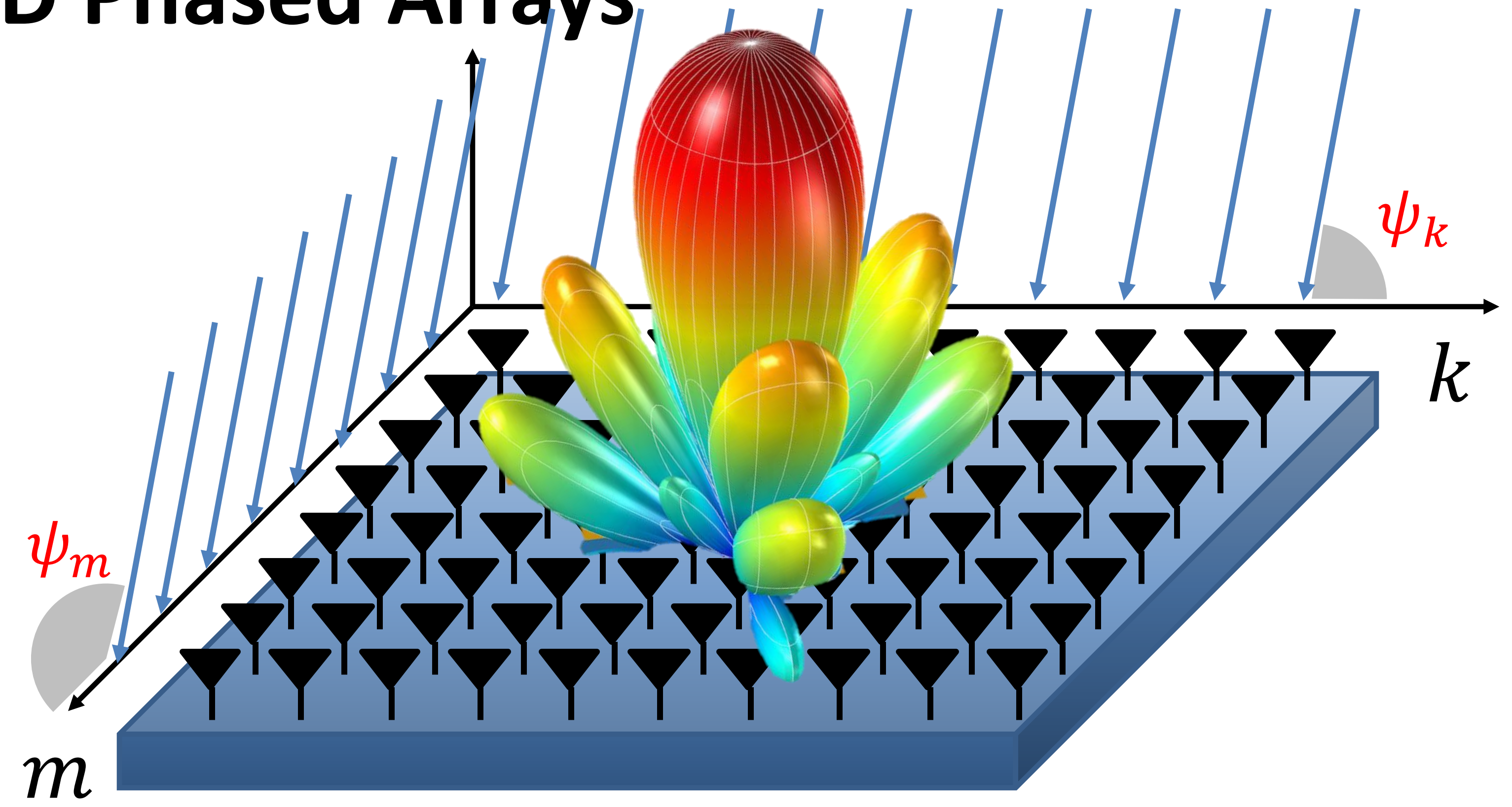
Can recover:

- Range: r
- Azimuth AoA: φ
- Elevation AoA: θ

$$h_{m,k} = \alpha e^{-j\frac{2\pi}{\lambda}(r+ms\sin\theta\cos\varphi+ks\sin\theta\sin\varphi)}$$

2D Phased Arrays





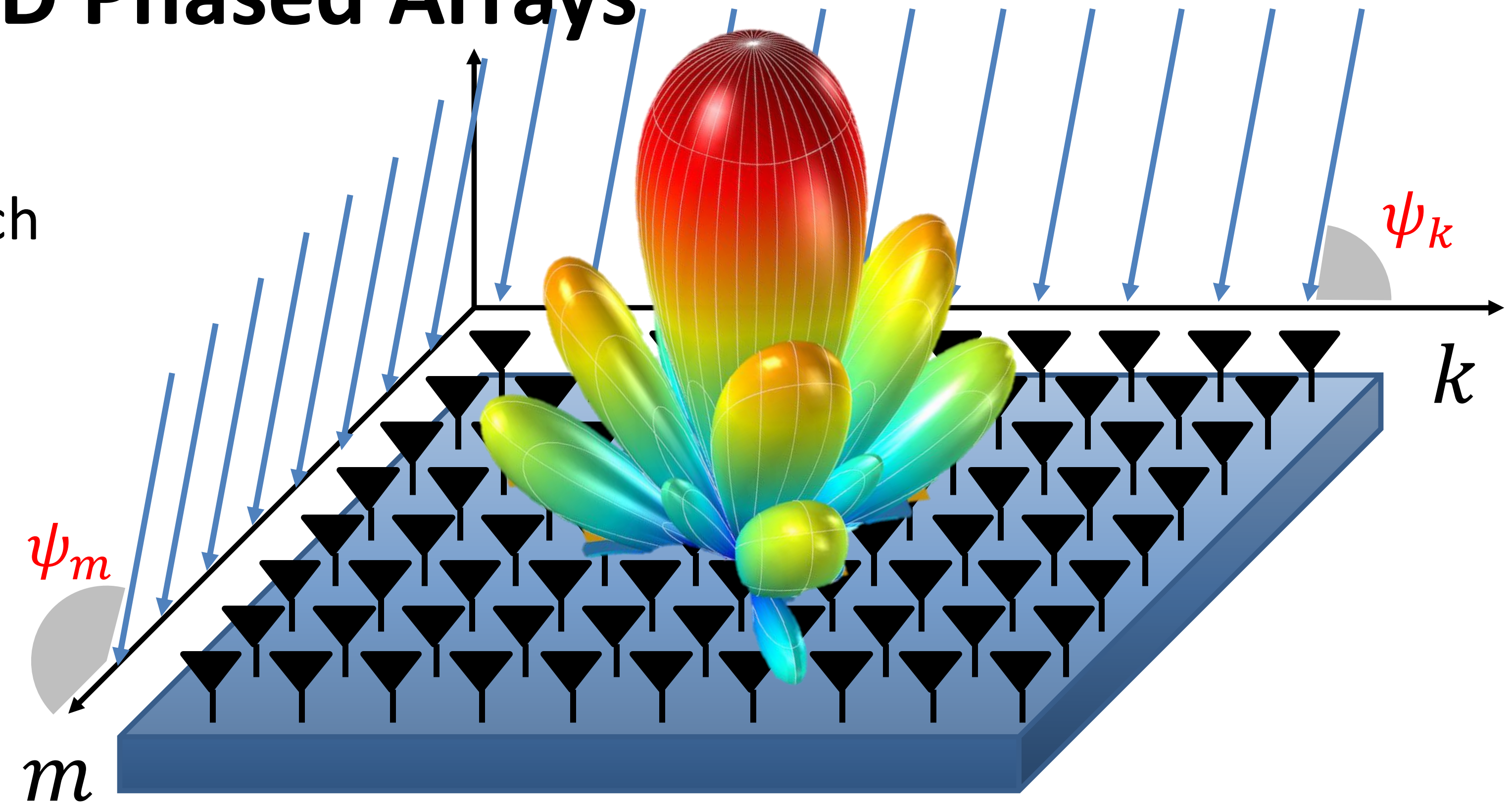
Can recover:

- Range: r
- Azimuth AoA: φ
- Elevation AoA: θ

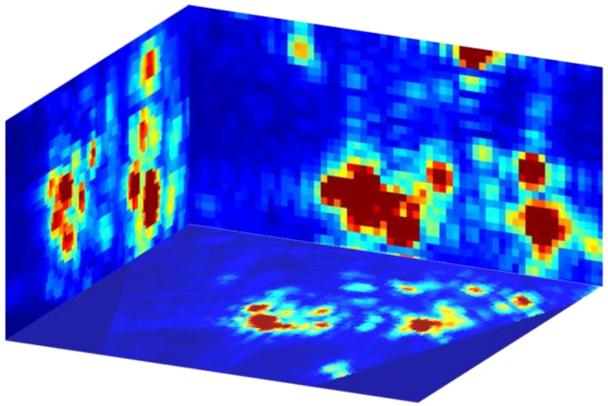
$$h_{m,k} = \alpha e^{-j\frac{2\pi}{\lambda}(r+ms\sin\theta\cos\varphi+ks\sin\theta\sin\varphi)}$$

2D Phased Arrays

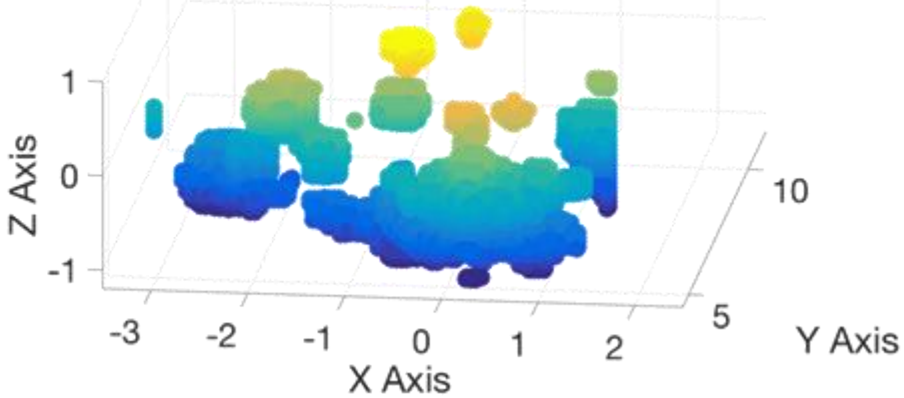
1) Pick the phase shift on each antenna to create a beam in each 3D direction.
2) Transmit FMCW signals and receive reflections.
3) Mix RX signal with TX and take range FFT .
4) Repeat in every direction.



→ **3D Heatmap Image**

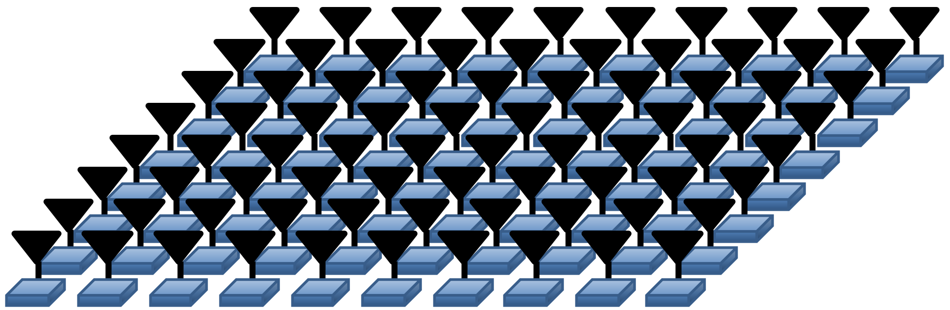


3D Point Cloud



Phased Arrays Primer

Digital Phased Arrays



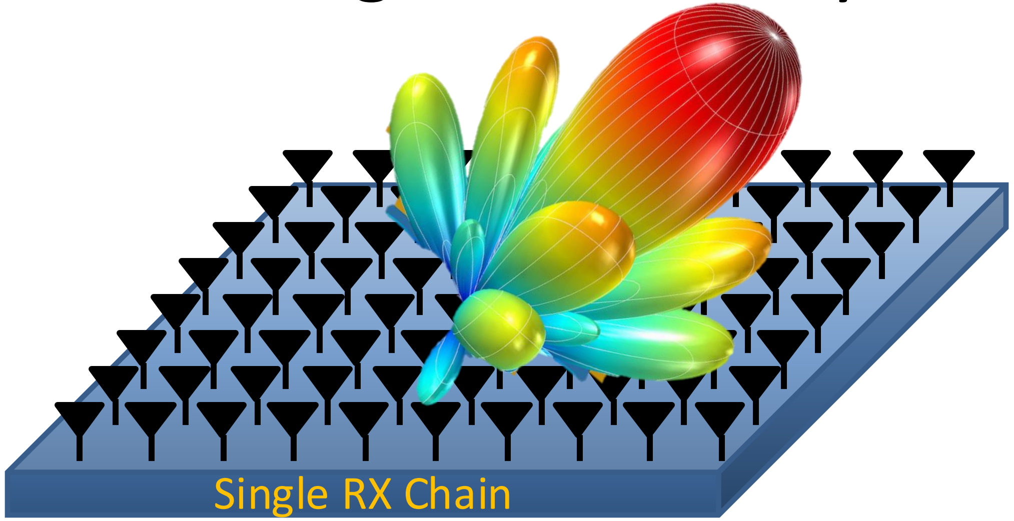
$N \times N$ **RX Chains**

Could Potentially Do the Same thing:

1) Mix the RX signal with TX.
2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.
3) Compute Range FFT.
4) Repeat in every direction.

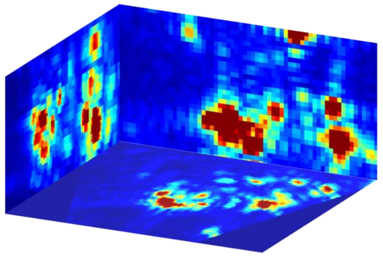
$$(N \times N \times T + T\log T) \times N \times N = O(N^4 T + N^2 T \log T)$$

Analog Phased Arrays



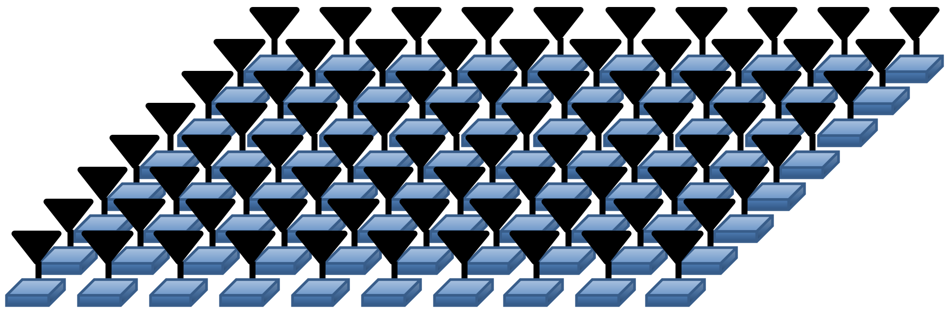
1) Pick the phase shift on each antenna to create a beam in each 3D direction.
2) Transmit FMCW signals and receive reflections.
3) Mix RX signal with TX and take range FFT .
4) Repeat in every direction.

3D Heatmap Image



Phased Arrays Primer

Digital Phased Arrays



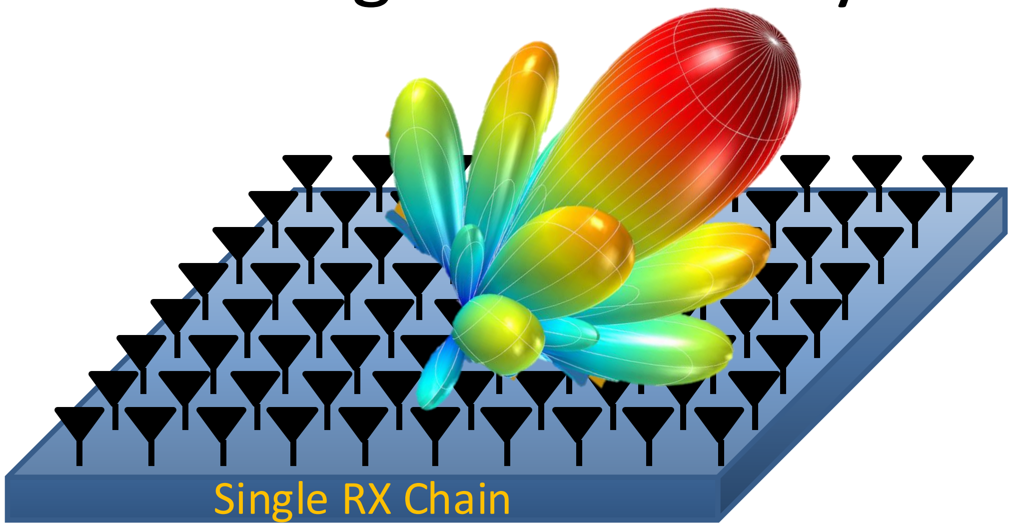
$N \times N$ **RX Chains**

Could Potentially Do the Same thing:

1) Mix the RX signal with TX.
3) Compute Range FFT.
2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.
4) Repeat in every direction.

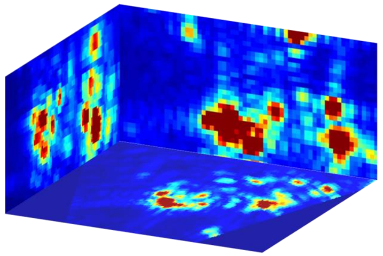
$$N \times N \times T \log T + N \times N \times T \times N \times N = O(N^4 T + N^2 T \log T)$$

Analog Phased Arrays



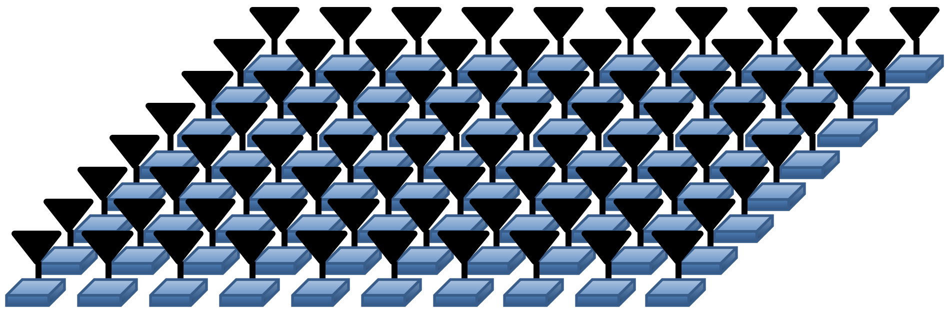
1) Pick the phase shift on each antenna to create a beam in each 3D direction.
2) Transmit FMCW signals and receive reflections.
3) Mix RX signal with TX and take range FFT .
4) Repeat in every direction.

3D Heatmap Image



Phased Arrays Primer

Digital Phased Arrays



$N \times N$ **RX Chains**

Algorithm 1: $O(N^4 T + N^2 T \log T)$

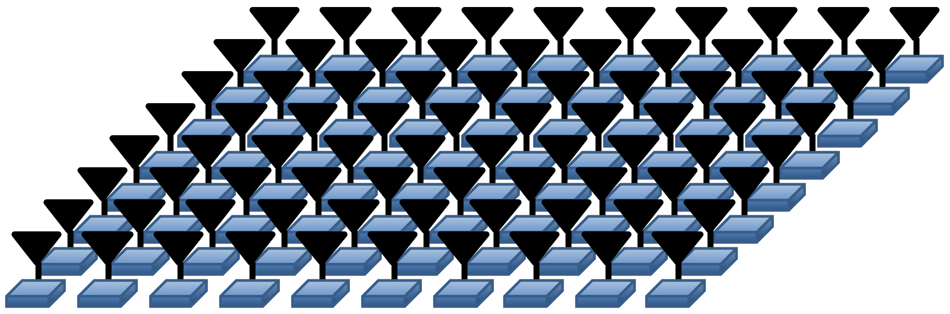
1) Mix the RX signal with TX.
3) Compute Range FFT.
2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.
4) Repeat in every direction.

Algorithm 2: (Faster) **IDEA: antenna arrays are Fourier Transforms**

$$h_{m,k} = \alpha e^{-j\frac{2\pi}{\lambda}(r + ms \sin\theta \cos\varphi + ks \sin\theta \sin\varphi)}$$

Phased Arrays Primer

Digital Phased Arrays



$N \times N$ RX Chains

Algorithm 1: $O(N^4 T + N^2 T \log T)$

1) Mix the RX signal with TX.
3) Compute Range FFT.
2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.
4) Repeat in every direction.

Algorithm 2: (Faster) **IDEA: antenna arrays are Fourier Transforms**

$$h_{m,k} = \sum_{\theta_l} \sum_{\varphi_l} \alpha_l e^{-j\frac{2\pi}{\lambda}(r_l + ms\sin\theta_l\cos\varphi_l + ks\sin\theta_l\sin\varphi_l)}$$

$$h(x,y) = \sum_{f_x} \sum_{f_y} P(f_x, f_y) e^{-j2\pi(xf_x + yf_y)}$$

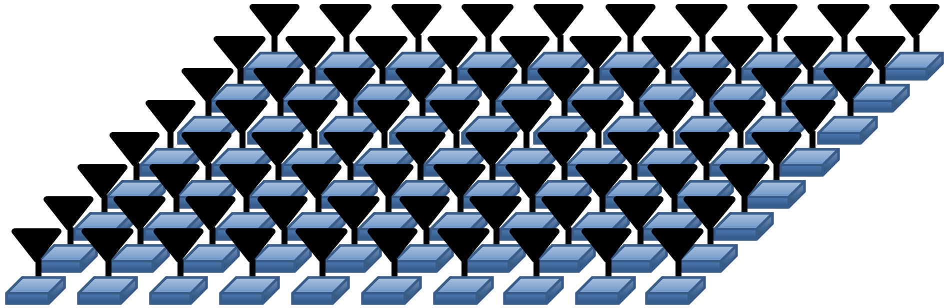
→ **Use 2D FFT**

$f_x = sin(\theta_l)cos(\phi_l)$,
$f_y = sin(\theta_l)sin(\phi_l)$,
$x = ms/\lambda$,
$y = ks/\lambda$

$$P(f_x, f_y) = \alpha_l e^{-j\frac{2\pi r_l}{\lambda}}$$

Phased Arrays Primer

Digital Phased Arrays



$N \times N$ RX Chains

Algorithm 1: $O(N^4T + N^2T \log T)$

1) Mix the RX signal with TX.

3) Compute Range FFT.

2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.

4) Repeat in every direction.

Algorithm 2: (Faster) **IDEA: antenna arrays are Fourier Transforms**

1) Mix the RX signal with TX.

2) Compute 2D FFT across antennas

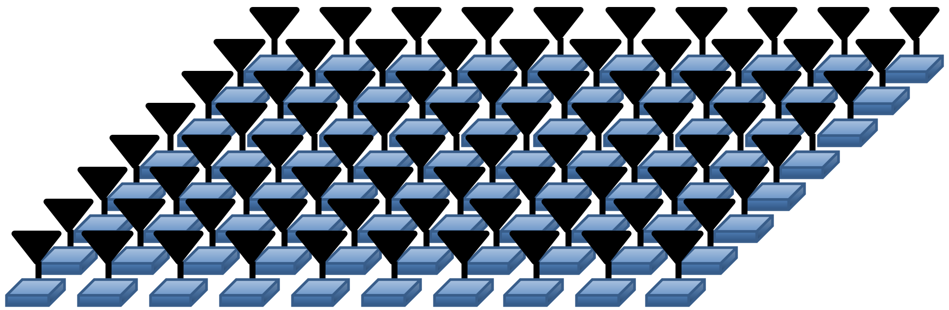
3) Compute Range FFT.

3D FFT

$$N^2 \log N^2 \times T + N \times N \times T \log T = O(N^2T \log N^2T)$$

Phased Arrays Primer

Digital Phased Arrays



$N \times N$ **RX Chains**

Algorithm 3: (More Accurate)

Algorithm 1: $O(N^4 T + N^2 T \log T)$

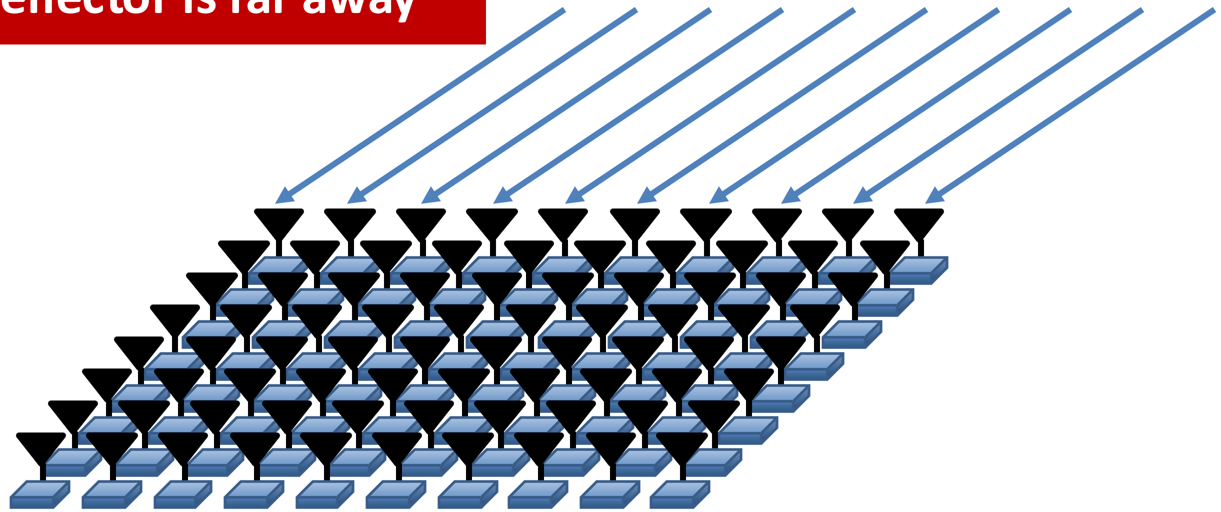
1) Mix the RX signal with TX.
3) Compute Range FFT.
2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.
4) Repeat in every direction.

Algorithm 2: (Faster) $O(N^2 T \log NT)$ **3D FFT**

1) Mix the RX signal with TX.
2) Compute 2D FFT across antennas
3) Compute Range FFT.

Phased Arrays Primer

Assumes parallel waves
Reflector is far away

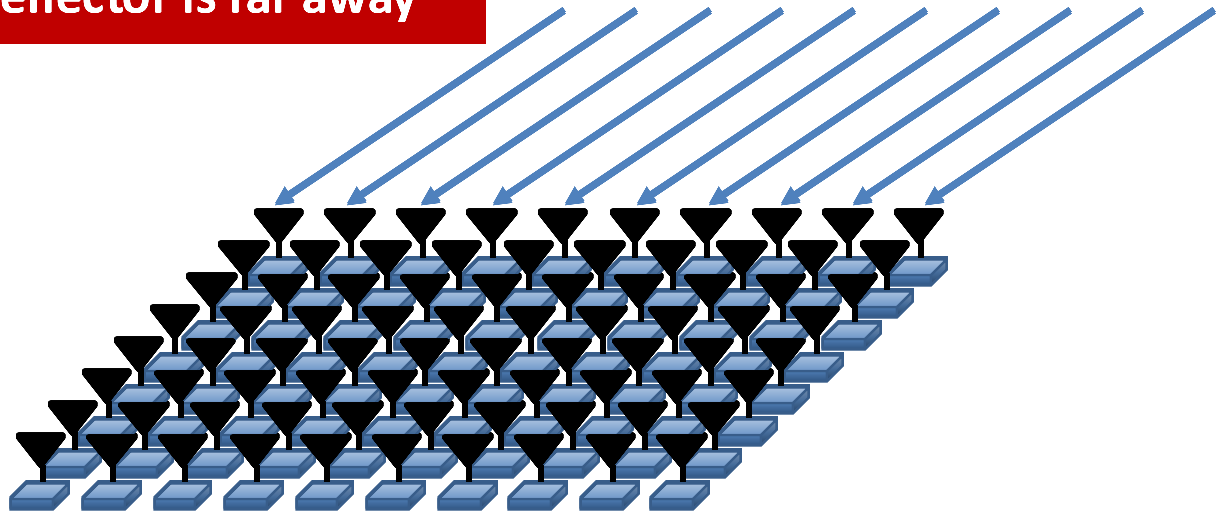


$N \times N$ **RX Chains**

Algorithm 3: (More Accurate)

Phased Arrays Primer

Assumes parallel waves
Reflector is far away

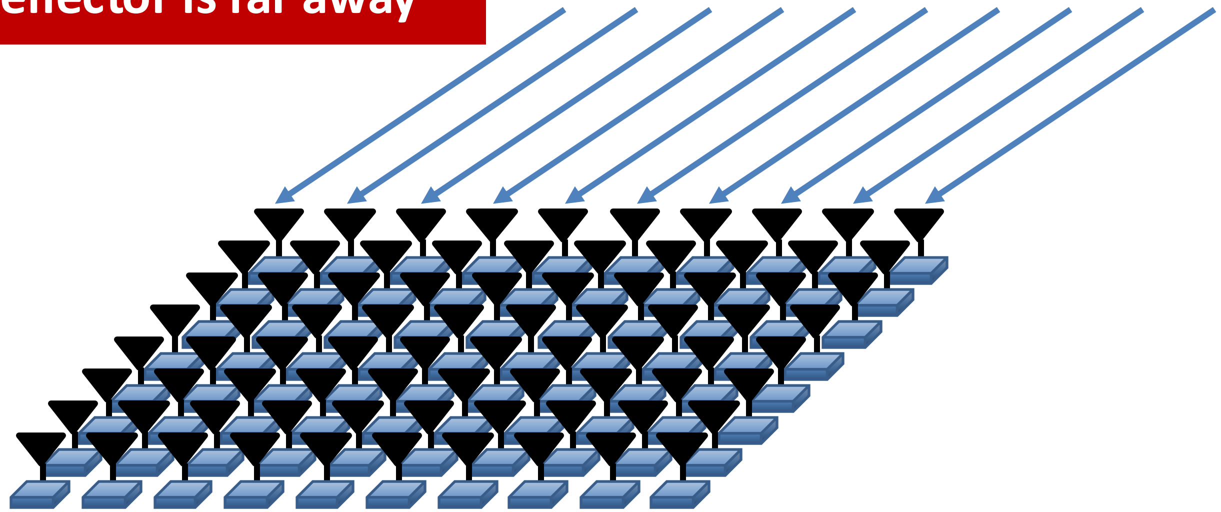


$N \times N$ RX Chains

Algorithm 3: (More Accurate)

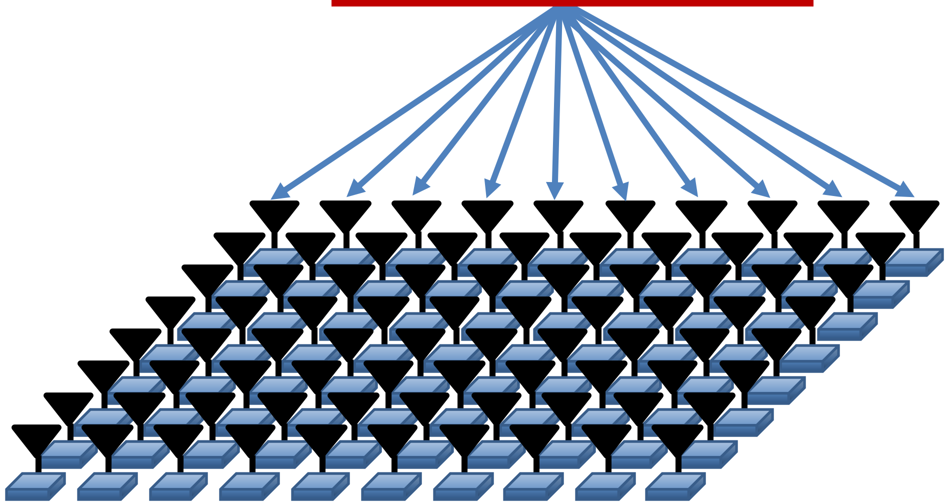
Phased Arrays Primer

Assumes parallel waves
Reflector is far away



$N \times N$ **RX Chains**

Reflector is not far



$N \times N$ **RX Chains**

Algorithm 3: (More Accurate) **IDEA: use the exact equation**

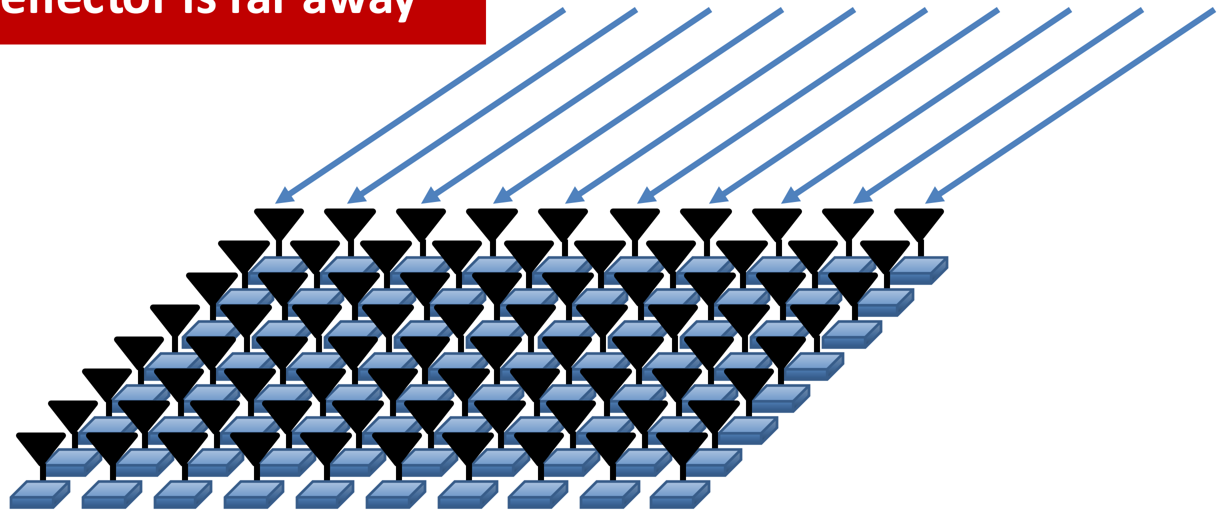
p: **FMCW Slope**

$$s_{m,k}(t) = \alpha_l e^{-j2\pi(p\tau_l t + f_0 \tau_l)} = \alpha_l e^{-j2\pi(p2d_l t/c + 2d_l/\lambda)}$$

Received FMCW signal from reflector l after mixing with TX

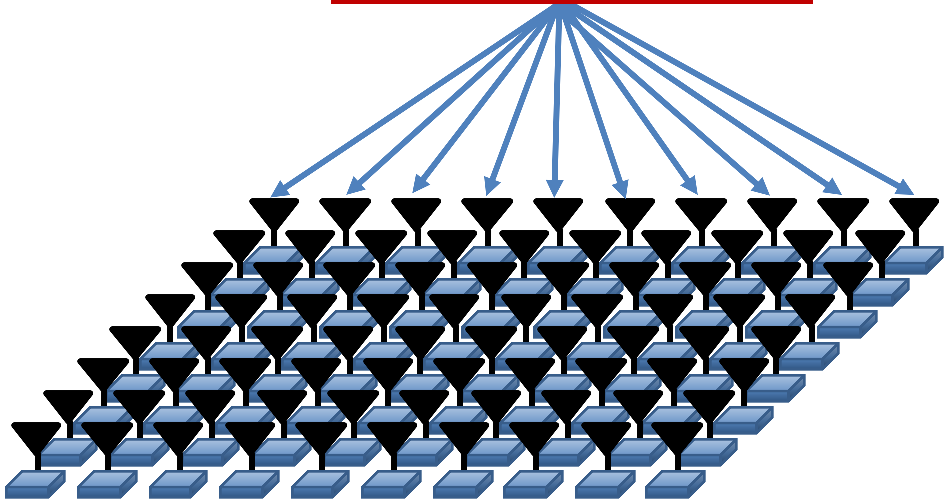
Phased Arrays Primer

Assumes parallel waves
Reflector is far away



$N \times N$ **RX Chains**

Reflector is not far



$N \times N$ **RX Chains**

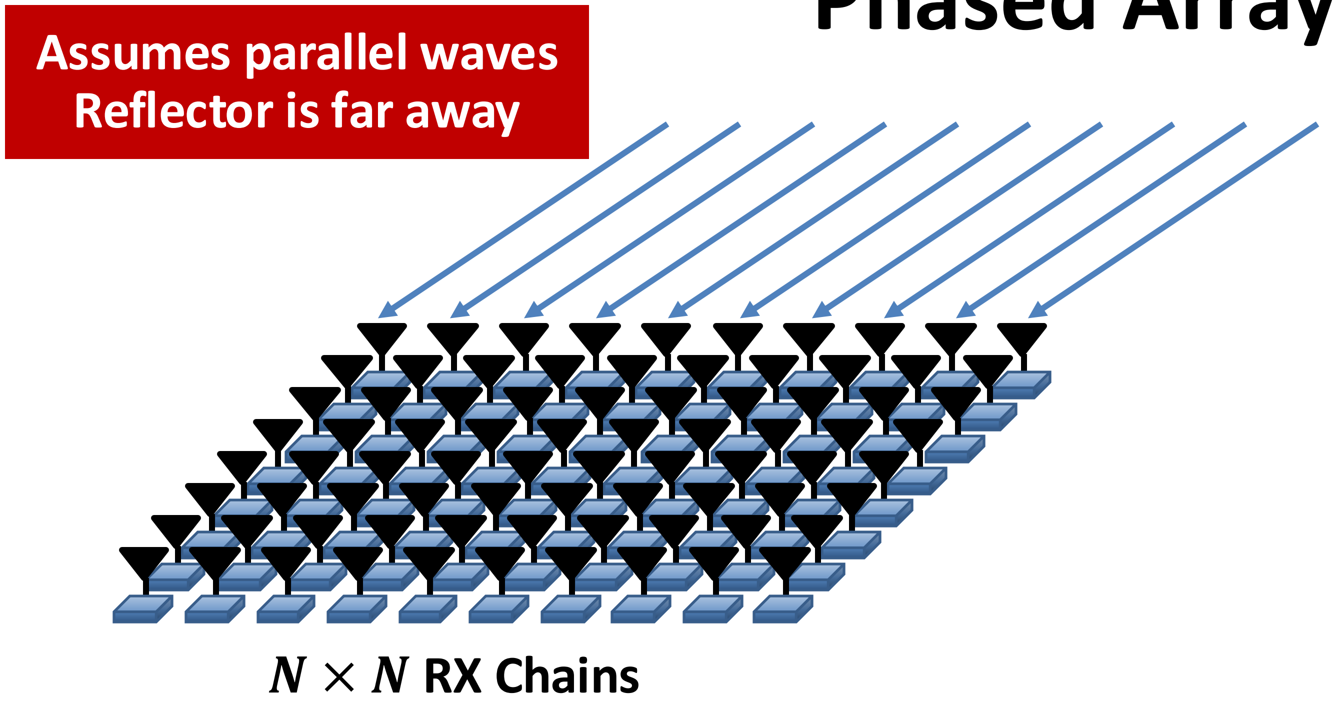
Assume antenna 0,0 at origin
$x_{m,k} = ms, y_{m,k} = ks, z_{m,k} = z_c$

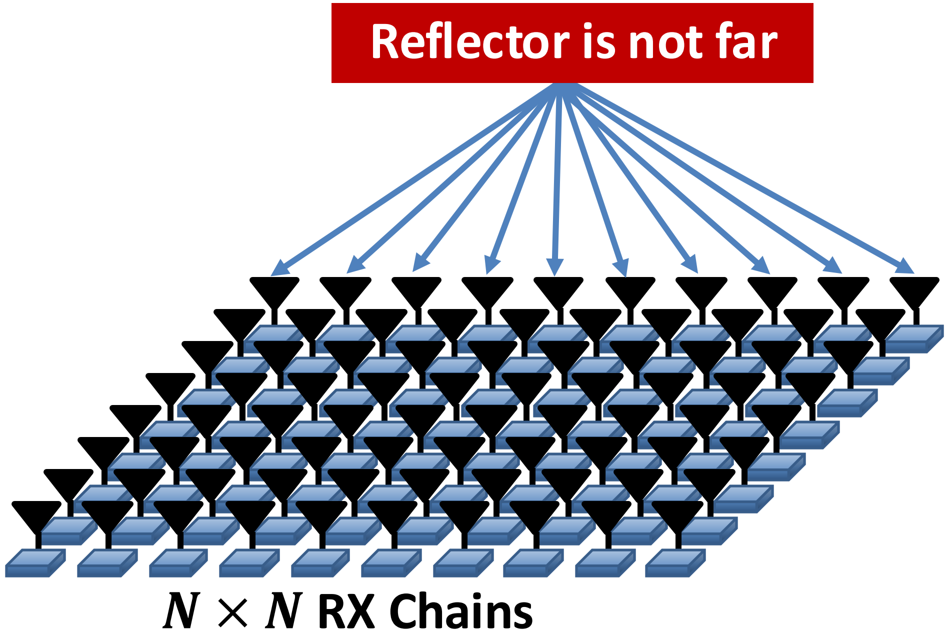
Algorithm 3: (More Accurate) **IDEA: use the exact equation**

$$s_{m,k}(t) = \alpha_l e^{-j2\pi(p\tau_l t + f_0 \tau_l)} = \alpha_l e^{-j2\pi(p2d_l t/c + 2d_l/\lambda)}$$

$$d_l = \sqrt{\left(x_l - x_{m,k}\right)^2 + \left(y_l - y_{m,k}\right)^2 + \left(z_l - z_{m,k}\right)^2} = \sqrt{(x_l - ms)^2 + (y_l - ks)^2 + (z_l - z_c)^2}$$

Phased Arrays Primer





Assume antenna 0,0 at origin
$x_{m,k} = ms, y_{m,k} = ks, z_{m,k} = z_c$

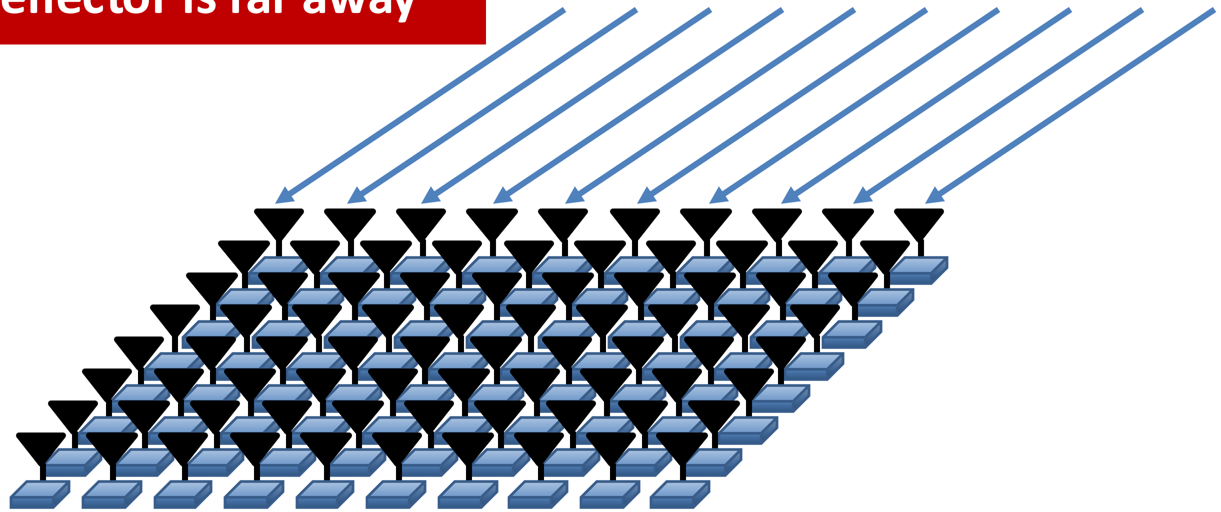
Algorithm 3: (More Accurate) **IDEA: use the exact equation**

$$s_{m,k}(t) = \alpha_l e^{-j2\pi(p\tau_l t + f_0 \tau_l)} = \alpha_l e^{-j2\pi(p2d_l t/c + 2d_l/\lambda)}$$

$$= \alpha_l e^{-j4\pi(pt/c+1/\lambda)\sqrt{(x_l - ms)^2 + (y_l - ks)^2 + (z_l - z_c)^2}}$$

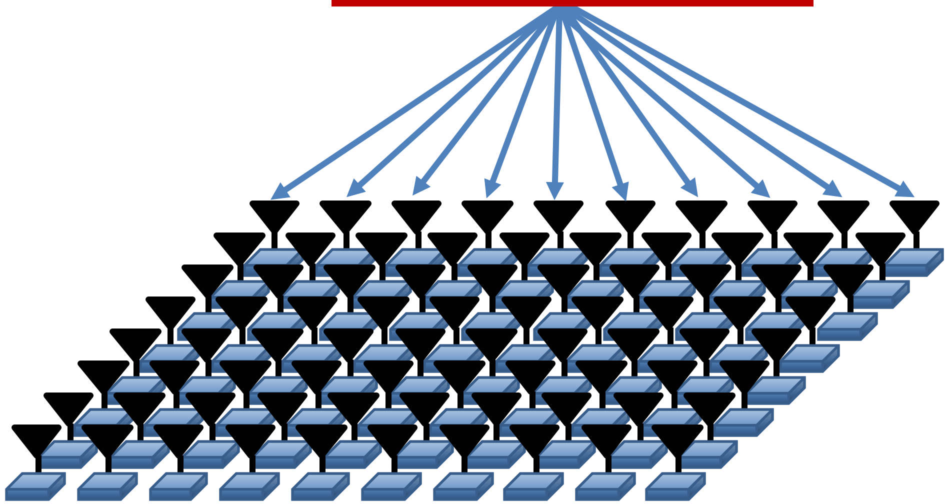
Phased Arrays Primer

Assumes parallel waves
Reflector is far away



$N \times N$ **RX Chains**

Reflector is not far



$N \times N$ **RX Chains**

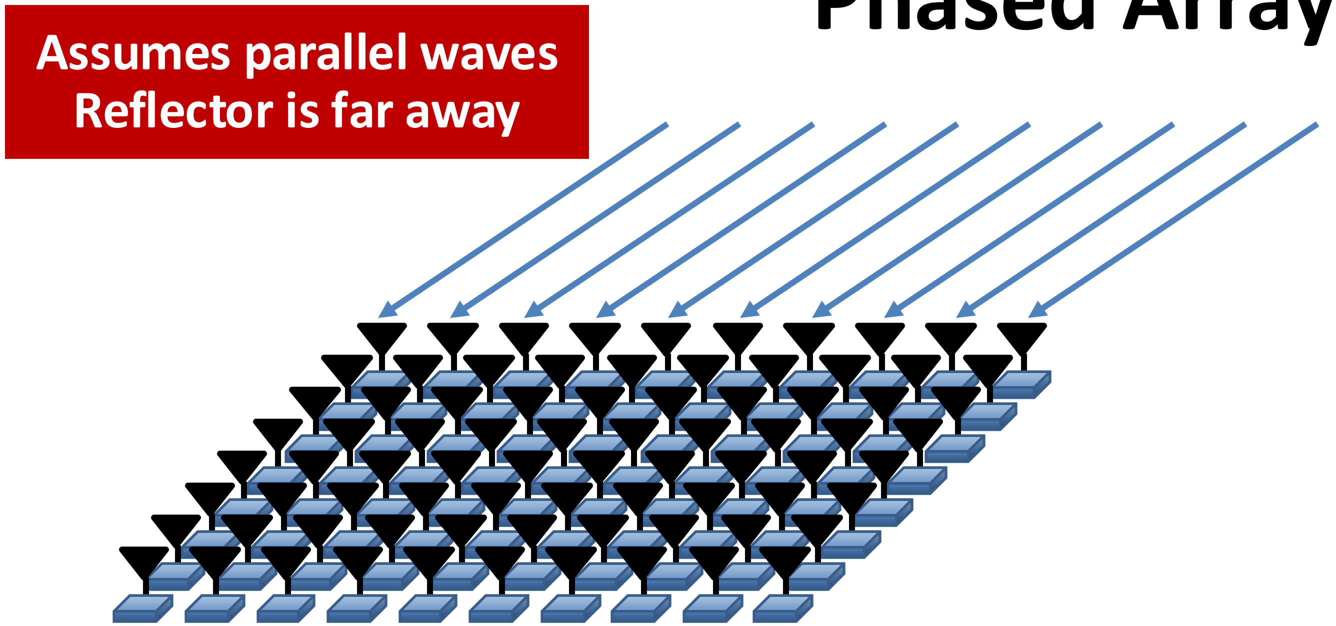
Assume antenna 0,0 at origin
$x_{m,k} = ms, y_{m,k} = ks, z_{m,k} = z_c$

Algorithm 3: (More Accurate) **IDEA: use the exact equation**

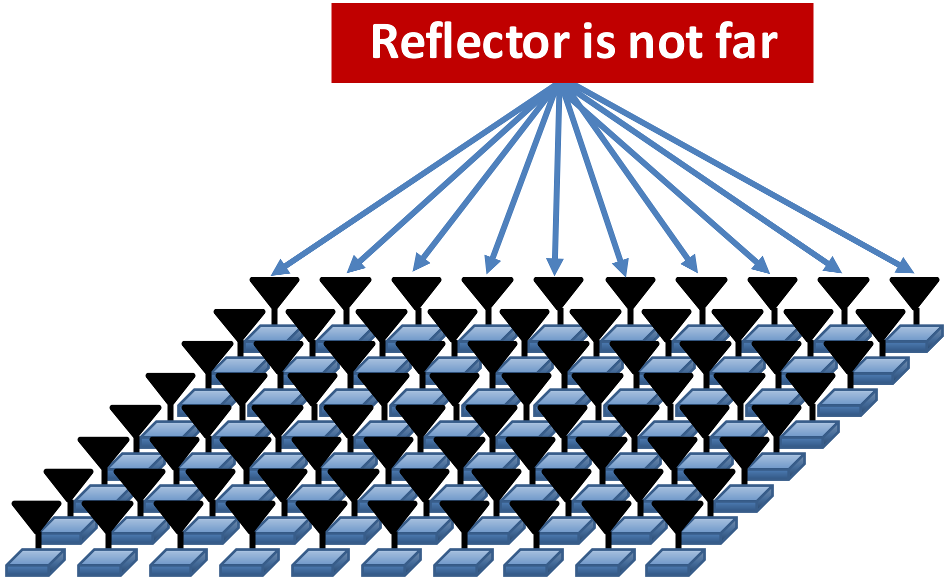
$$s_{m,k}(t) = \sum_l \alpha_l e^{-j2\pi(p\tau_l t + f_0 \tau_l)} = \sum_l \alpha_l e^{-j2\pi(p2d_l t/c + 2d_l/\lambda)}$$

$$= \sum_l \alpha_l e^{-j4\pi(pt/c + 1/\lambda)\sqrt{(x_l - ms)^2 + (y_l - ks)^2 + (z_l - z_c)^2}}$$

Phased Arrays Primer



$N \times N$ **RX Chains**



$N \times N$ **RX Chains**

Assume antenna 0,0 at origin

$x_{m,k} = ms, y_{m,k} = ks, z_{m,k} = z_c$

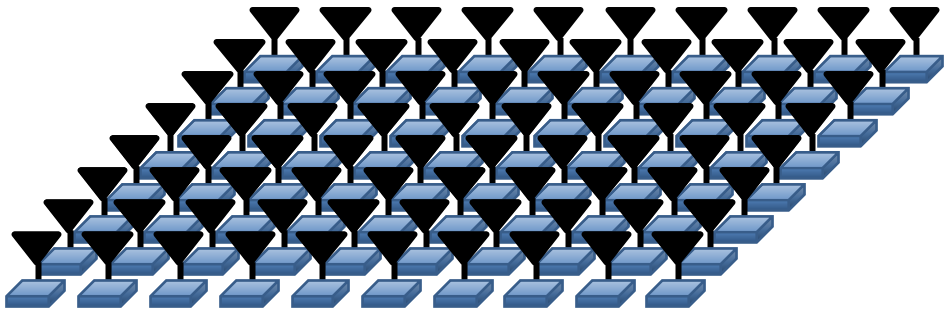
Algorithm 3: (More Accurate) **IDEA: use the exact equation**

$$s_{m,k}(t) = \sum_{l} \alpha_l e^{-j4\pi(pt/c+1/\lambda)\sqrt{(x_l - ms)^2 + (y_l - ks)^2 + (z_l - z)^2}}$$

$$P(x,y,z) = \sum_{m}^{N} \sum_{k}^{N} \sum_{t}^{T} s_{m,k}(t) \times e^{j4\pi(pt/c+1/\lambda)\sqrt{(x - ms)^2 + (y - ks)^2 + (z - z_c)^2}} = N^2 T \alpha_l$$

Phased Arrays Primer

Digital Phased Arrays



$N \times N$ **RX Chains**

Algorithm 3: (More Accurate) $O(L^3N^2T) = o(N^5T)$

1) Descritize space into $L \times L \times L$ grid.
2) For each point in space compute the received signal using the below equation.

$$P(x,y,z) = \sum_m \sum_k \sum_t s_{m,k}(t) \times e^{j4\pi(kt/c+1/\lambda)\sqrt{(x-ms)^2+(y-ks)^2+(z-z_c)^2}}$$

Algorithm 1: $O(N^4T + N^2T\log T)$

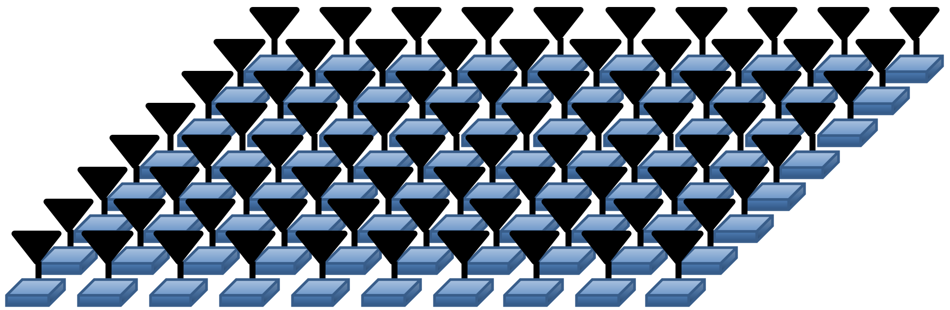
1) Mix the RX signal with TX.
3) Compute Range FFT.
2) Multiply the resulting signal on each antenna with $e^{j\phi_{m,k}}$ and sum the signals.
4) Repeat in every direction.

Algorithm 2: (Faster) $O(N^2T\log NT)$ **3D FFT**

1) Mix the RX signal with TX.
2) Compute 2D FFT across antennas
3) Compute Range FFT.

Phased Arrays Primer

Digital Phased Arrays



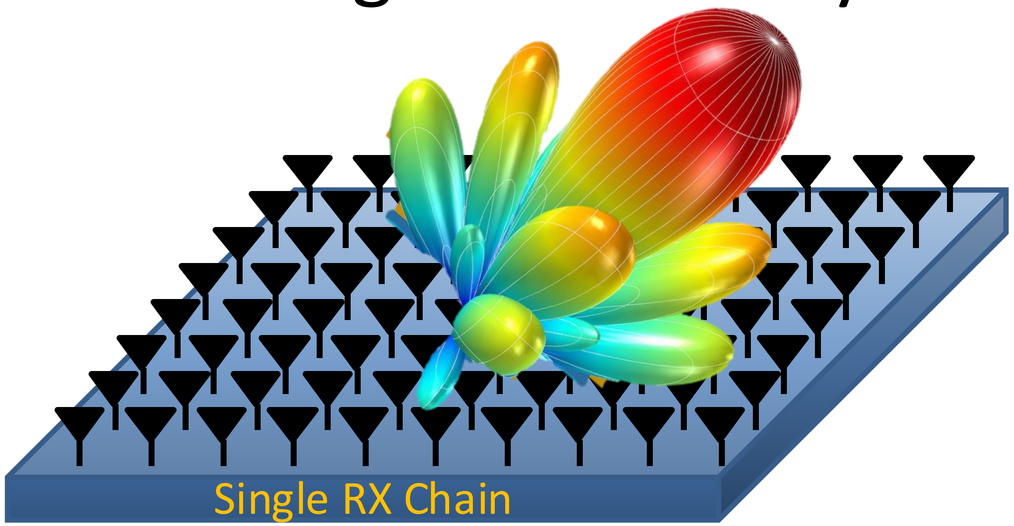
$N \times N$ **RX Chains**

Algorithm 1: $O(N^4 T + N^2 T \log T)$ **Similar to Analog**

Algorithm 2: (Faster) $O(N^2 T \log NT)$ **3D FFT**

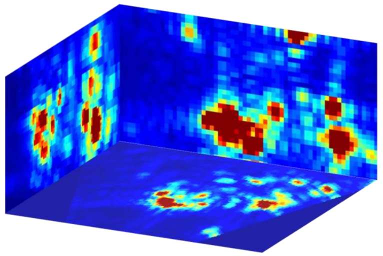
Algorithm 3: (More Accurate) $O(L^3 N^2 T) = o(N^5 T)$

Analog Phased Arrays

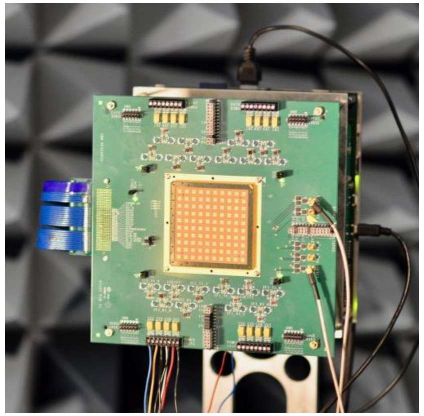


1) Pick the phase shift on each antenna to create a beam in each 3D direction.
2) Transmit FMCW signals and receive reflections.
3) Mix RX signal with TX and take range FFT .
4) Repeat in every direction.

3D Heatmap Image



2D Phased Arrays

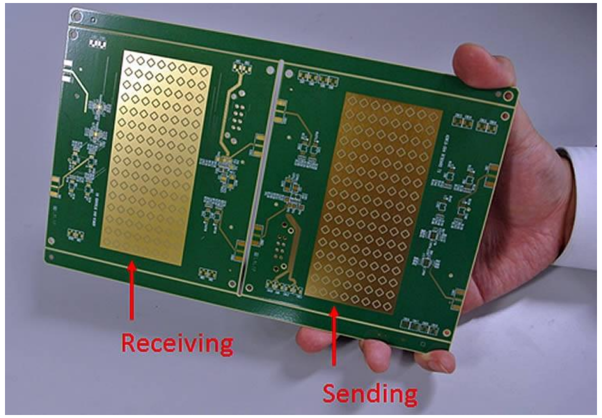


IBM

64 elements

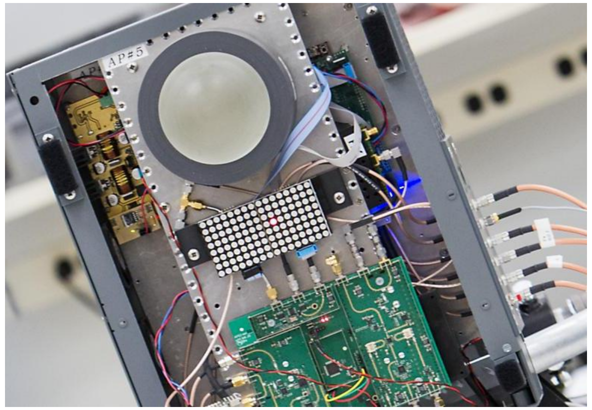
Anokiwave

256 elements

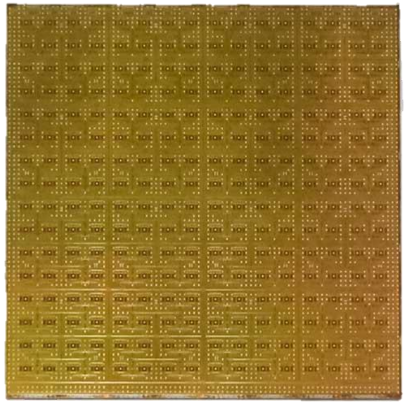


Fujitsu

64 elements



Nokia & National Instruments

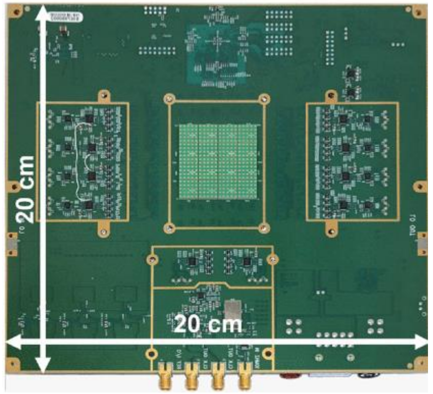


UCSD

256 elements

UCSD

64 elements

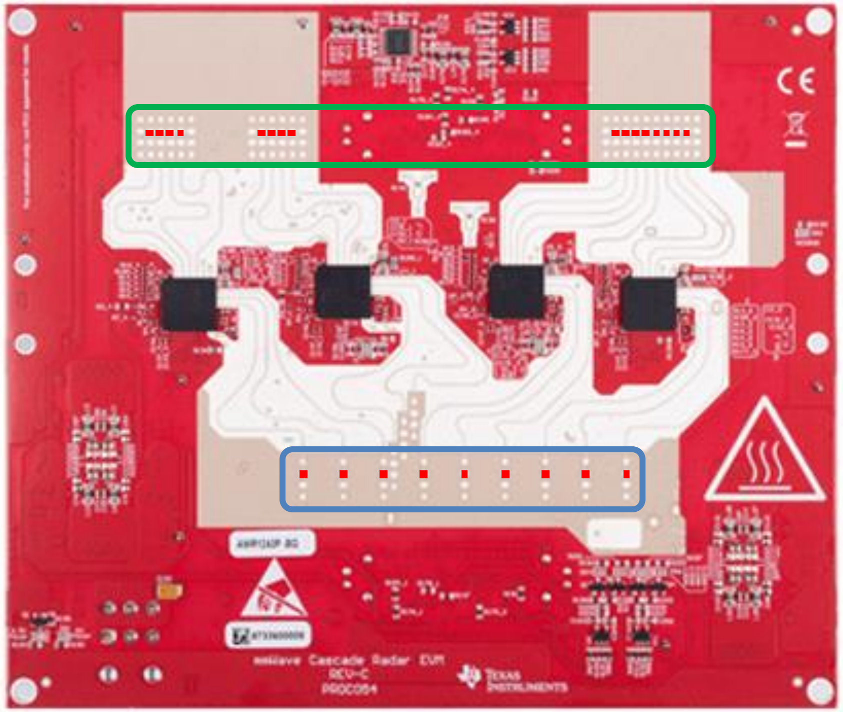


Bell Labs

384 elements

MIMO Radar

Receiver
Array
(16 elements)

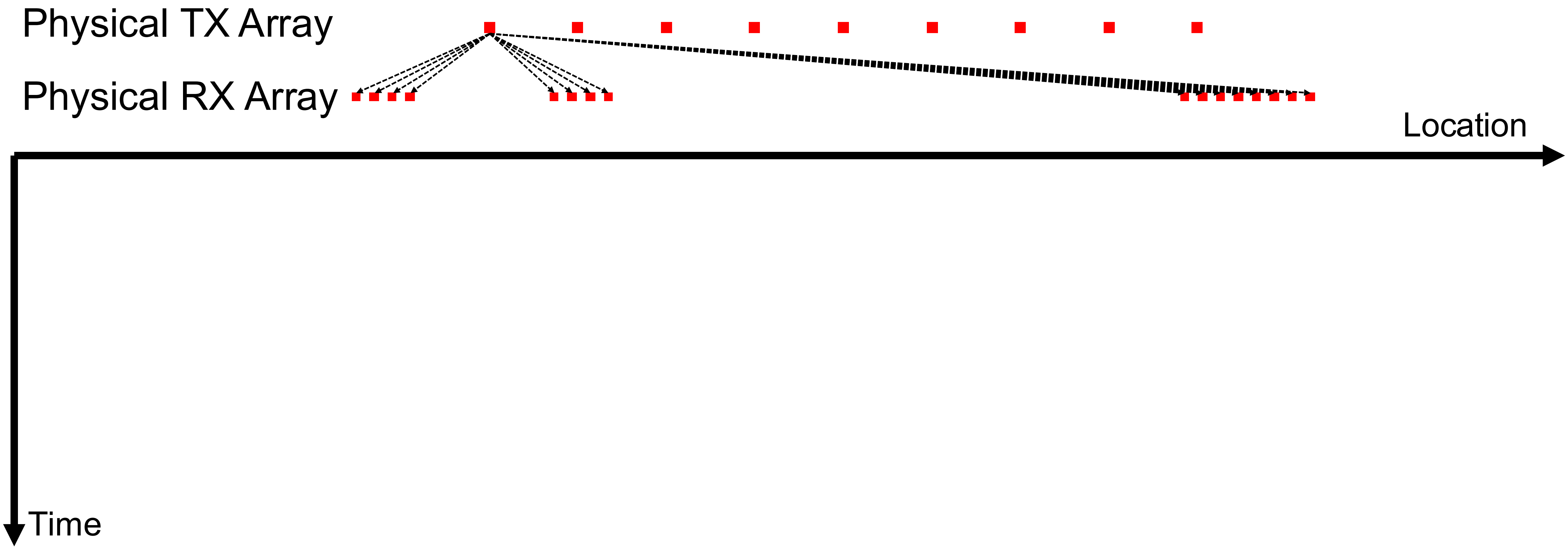


Transmitter
Array
（9 elements）

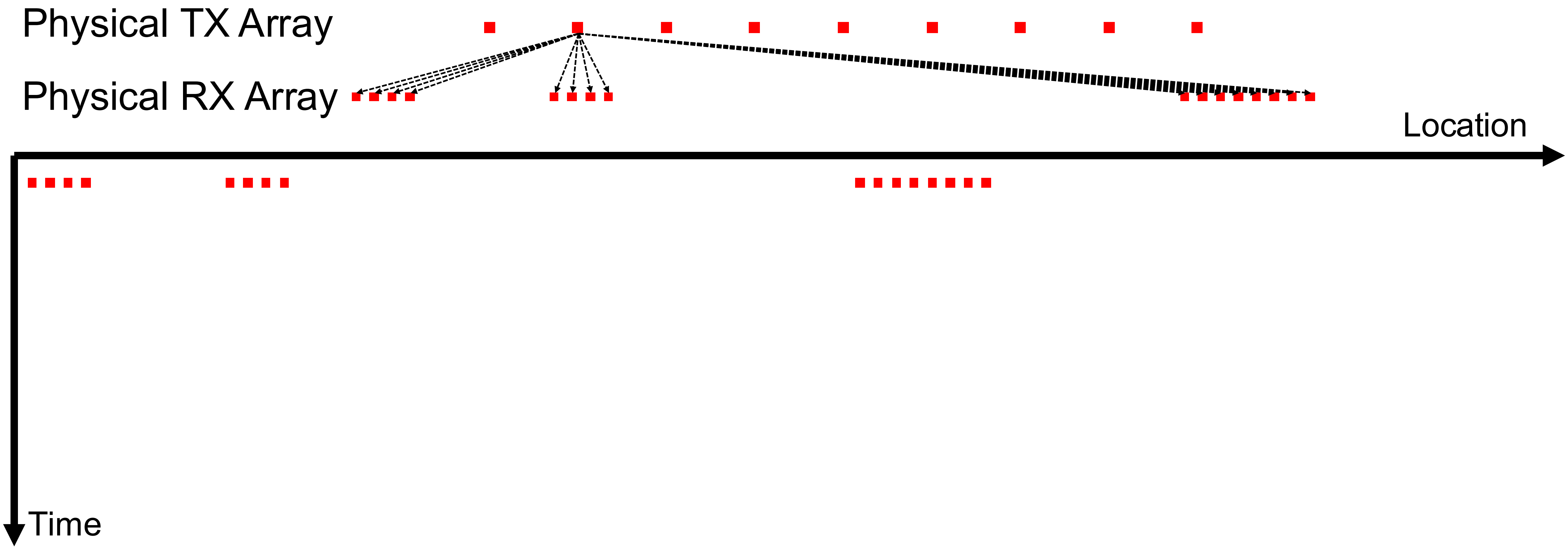
TI
MIMO Radar

Angular resolution of radar is proportional to antenna array size.

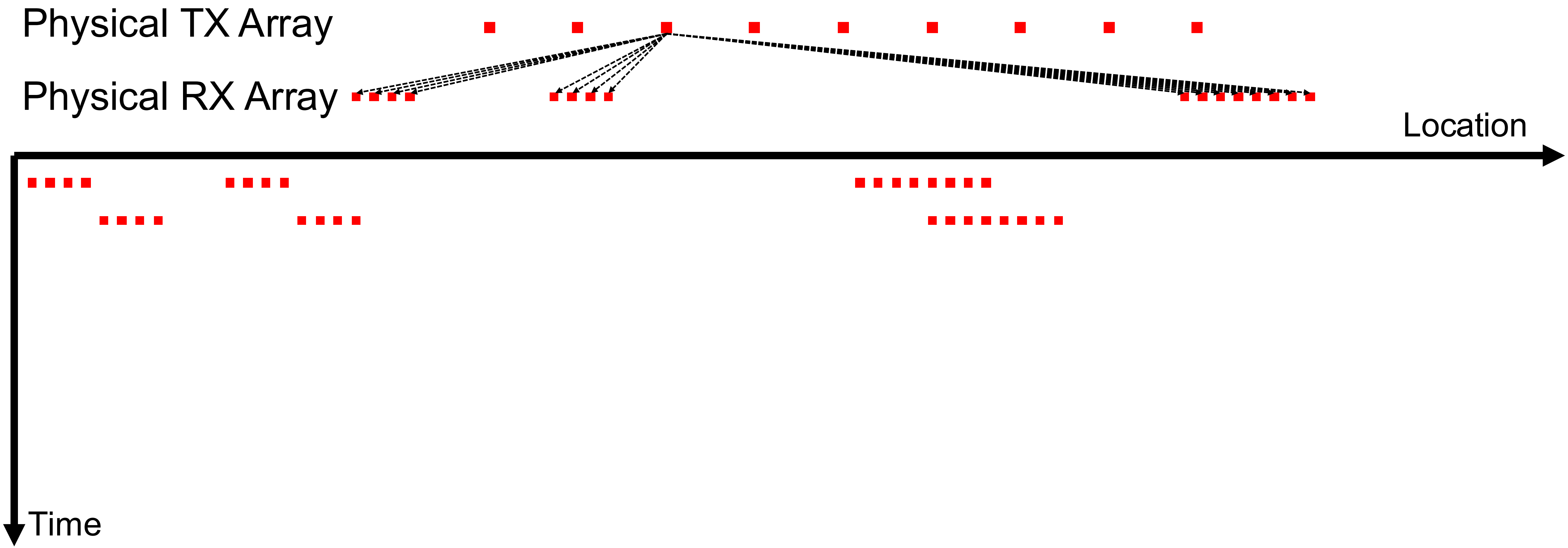
Virtual Antenna Array



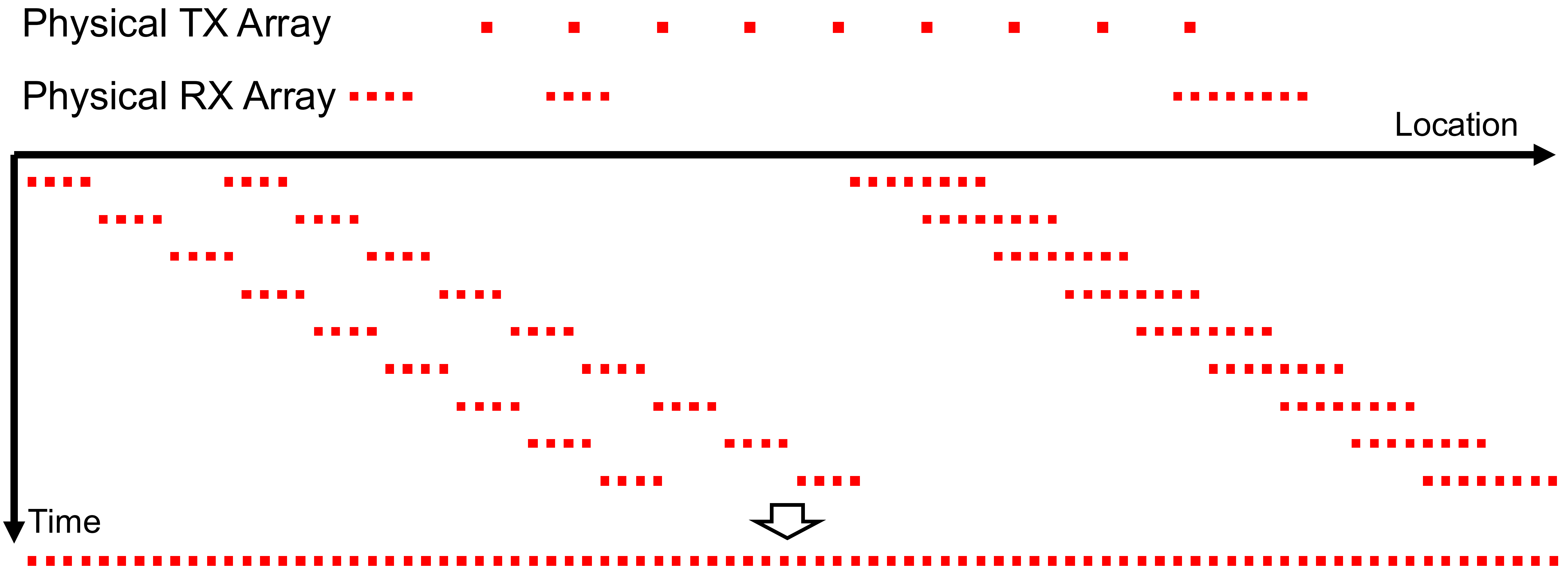
Virtual Antenna Array



Virtual Antenna Array



Virtual Antenna Array



86 element array

1.8º azimuth resolution 10x higher than single-chip radars

MIMO Radar

Receiver Array (20 elements)

Transmitter Array (20 elements)