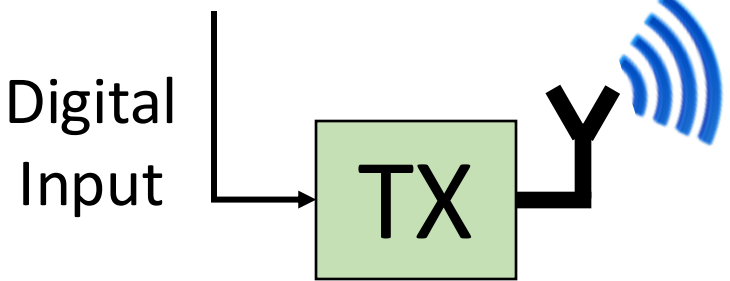


# COM-405: Mobile Networks

## Lecture 1.1: Wireless Channel Haitham Hassanieh



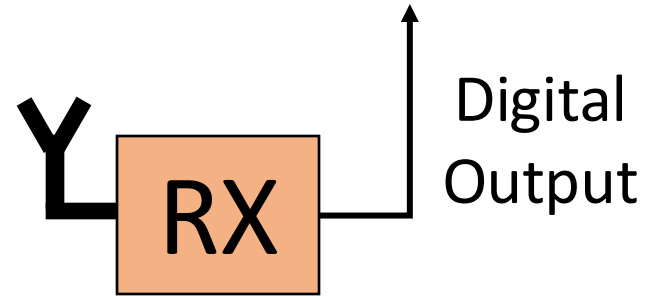
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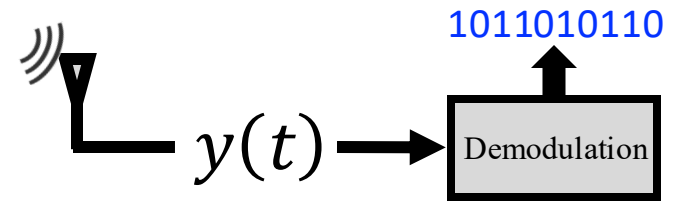
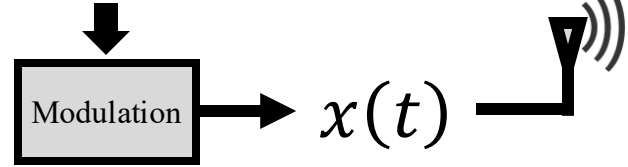
Wireless Channel



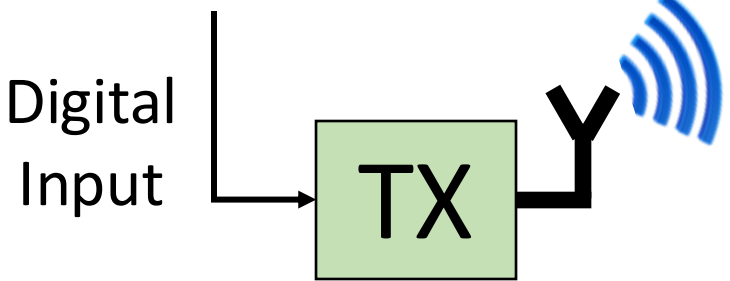
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1011010110



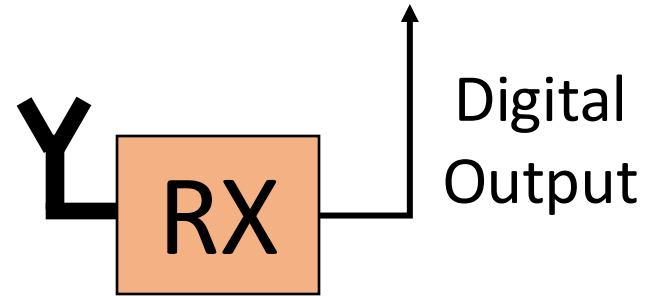
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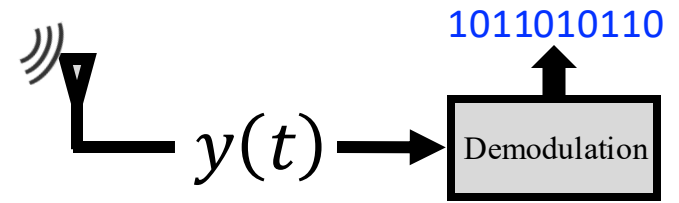
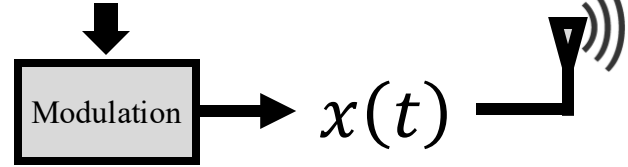
Wireless Channel



1011010110011001



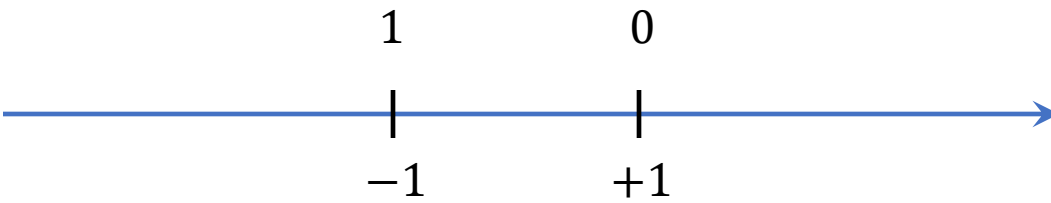
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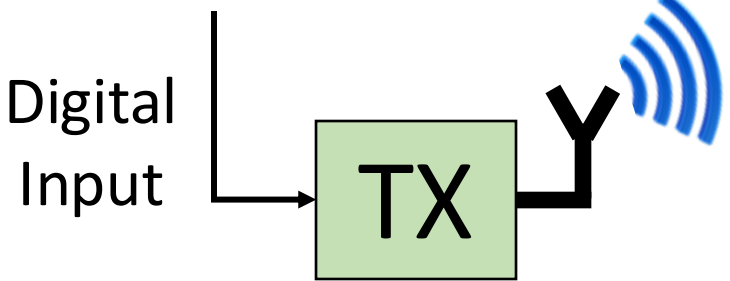
Modulation:

Mapping of Bits to Symbols

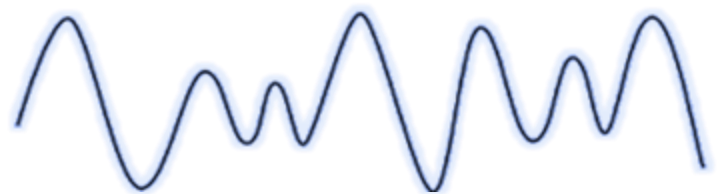
**BPSK: 1 bits/symbol**



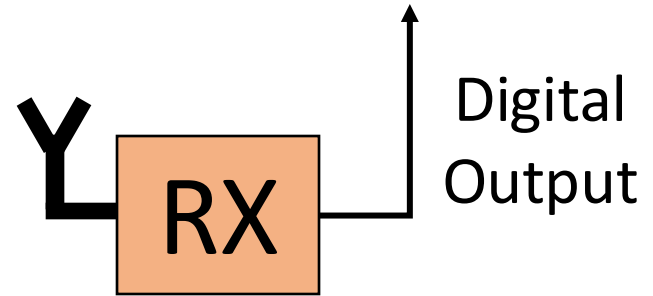
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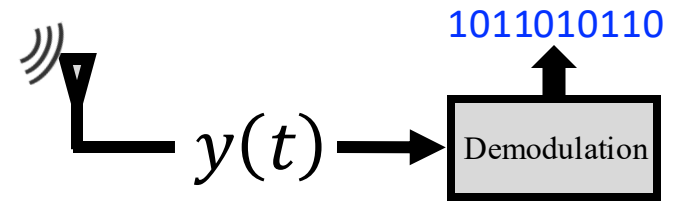
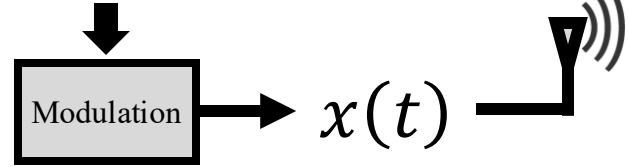
Wireless Channel



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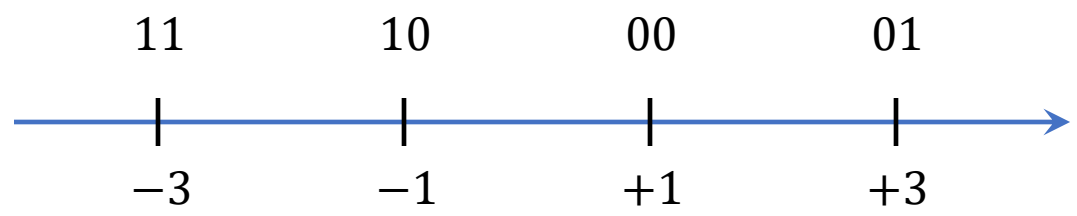


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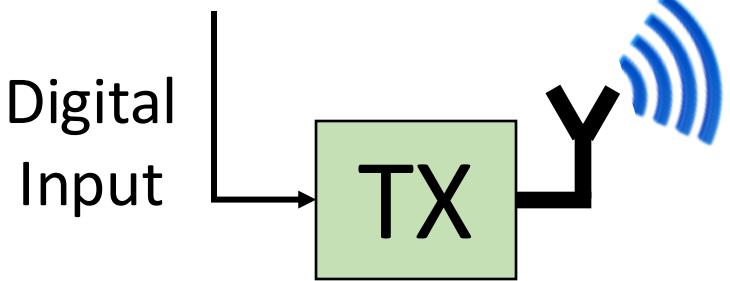


PAM: 2 bits/symbol

Modulation:  
Mapping of Bits to Symbols



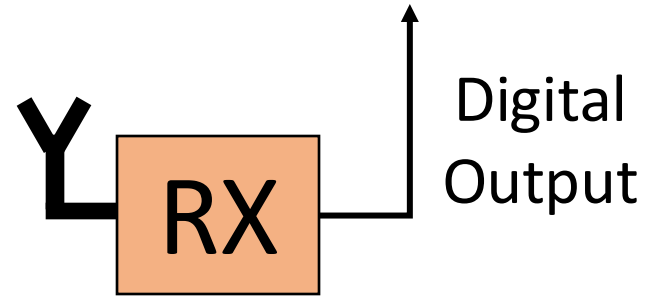
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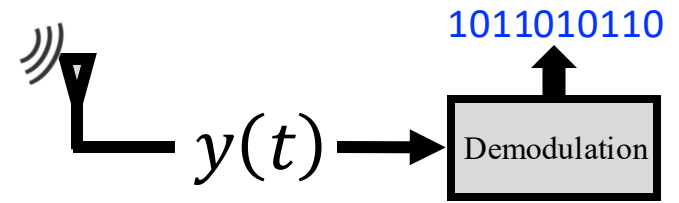
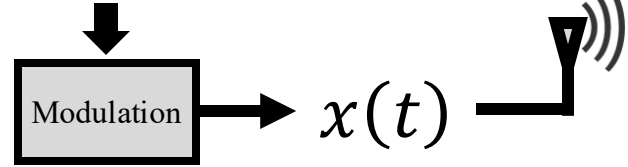
Wireless Channel



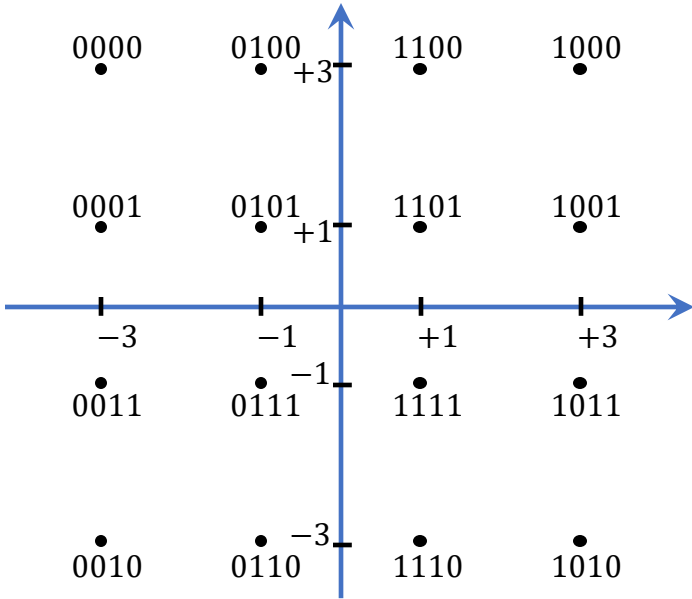
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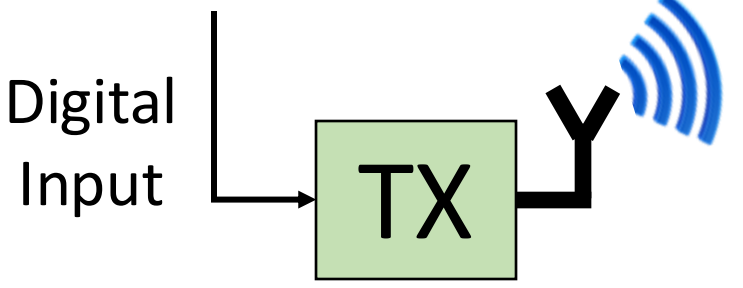


Modulation:  
Mapping of Bits to Symbols



**16-QAM:**  
**4 bits/symbol**  
**Complex Symbols!**

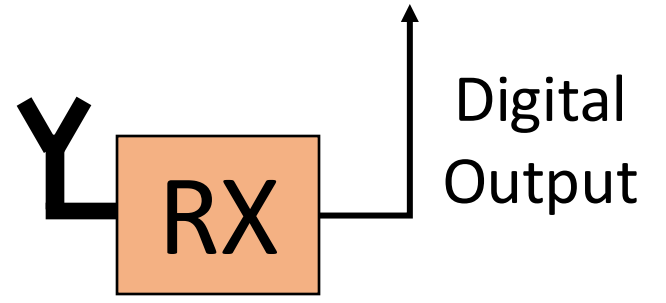
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Wireless Channel



1011010110011001



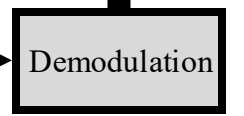
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$x(t)$



$y(t)$



1011010110



$$y(t) = x(t) + v(t)$$

Channel adds noise (AWGN)!

$$v(t) \sim N(0, \sigma)$$

# Noise

Circuit Components and wireless channel add noise to the wireless signal.

Transmit  
 $x(t)$



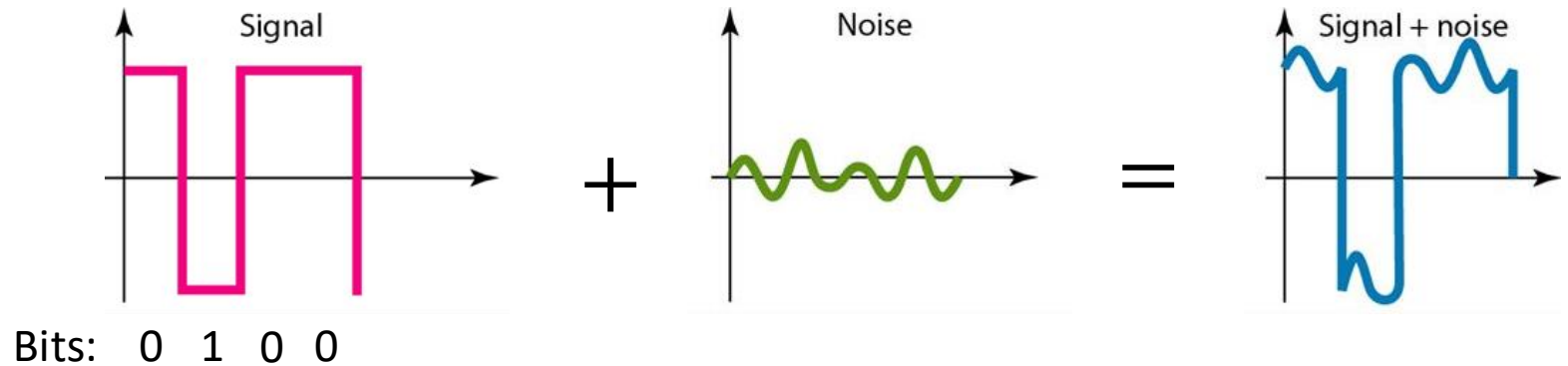
Receive

$$y(t) = x(t) + v(t)$$

$v(t)$  is Additive White Gaussian Noise (AWGN)

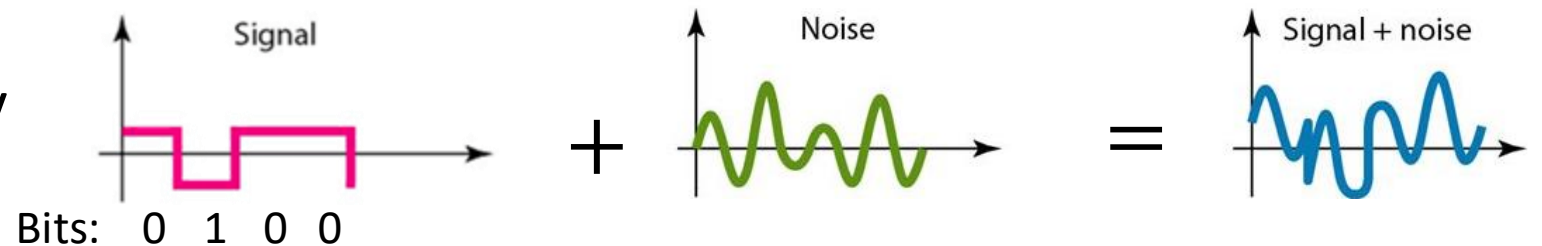
Low noise is good

→ Can decode Bits correctly



High noise is bad

→ Cannot decode Bits correctly



# Noise

Circuit Components and wireless channel add noise to the wireless signal.

$$\begin{array}{ccc} \text{Transmit} & & \text{Receive} \\ x(t) & \longrightarrow & y(t) = x(t) + v(t) \end{array}$$

$v(t)$  is Additive White Gaussian Noise (AWGN)

Low noise is good  $\rightarrow$  Can decode Bits correctly

High noise is bad  $\rightarrow$  Cannot decode Bits correctly

Signal-to-Noise Ratio (SNR)

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{E[|x(t)|^2]}{E[|v(t)|^2]} = \frac{E_s}{N_0}$$

# Noise

Circuit Components and wireless channel add noise to the wireless signal.

$$\begin{array}{ccc} \text{Transmit} & & \text{Receive} \\ x(t) & \longrightarrow & y(t) = x(t) + v(t) \end{array}$$

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Signal-to-Noise Ratio (SNR)

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{E[|x(t)|^2]}{E[|v(t)|^2]} = \frac{E_s}{N_0}$$

We typically measure and report SNR in dB scale:  $SNR_{dB} = 10 \log_{10} SNR$

e.g., if  $SNR = 100$ , then  $SNR_{dB} = 20 \text{ dB}$ ,

if  $SNR = 2$ , then  $SNR_{dB} \approx 3 \text{ dB}$ .



$$y(t) = x(t - \tau) + v(t)$$

Channel delays the signal!

$$\tau = \frac{d}{c}$$



$$y(t) = \mathbf{h} x(t - \tau) + v(t)$$

Channel attenuates the signal (Pathloss)



$$y(t) = \mathbf{h} x(t - \tau) + v(t)$$

Channel attenuates the signal (Pathloss)

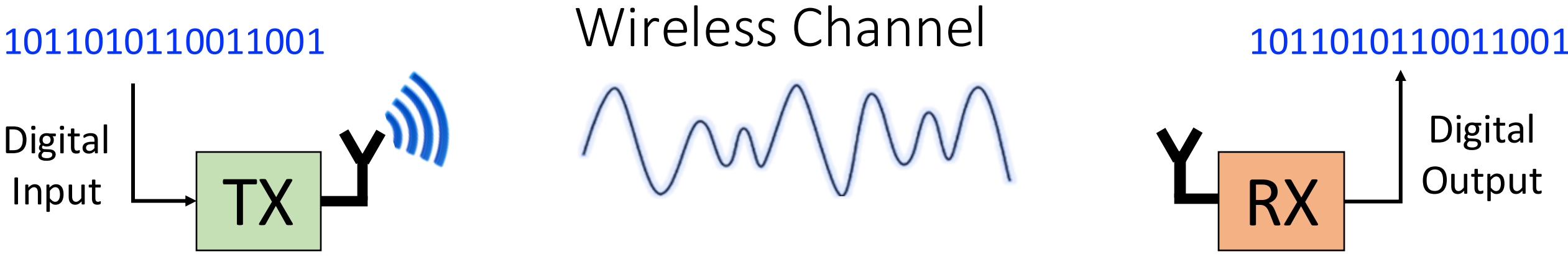
$$P_{RX} = G_{TX} G_{RX} \frac{\lambda^2}{(4\pi d)^2} P_{TX} \quad \Rightarrow \quad |h| \propto \frac{\lambda}{d}$$



$$y(t) = \mathbf{h} x(t - \tau) + v(t)$$

Channel rotates the signal (Adds Phase)

$$h \propto \frac{\lambda}{d} e^{j\phi} \rightarrow \phi = 2\pi \frac{d}{\lambda}$$



$$y(t) = \mathbf{h} x(t - \tau) + v(t)$$

$$x(t) \times e^{-j2\pi f_c t} \Rightarrow |h| x(t - \tau) e^{-j2\pi f_c (t - \tau)} \Rightarrow \times e^{j2\pi f_c t} \Rightarrow |h| x(t - \tau) e^{j2\pi f_c \tau}$$

$$h \propto \frac{\lambda}{d} e^{j\phi} \rightarrow \phi = 2\pi f_c \tau = 2\pi \frac{c}{\lambda} \frac{d}{c} = 2\pi \frac{d}{\lambda} \rightarrow h \propto \frac{\lambda}{d} e^{j2\pi d/\lambda}$$



$$y(t) = \mathbf{h} x(t - \tau) + v(t)$$

Channel:

- Adds Noise
- Delays the Signal
- Attenuates the Signal
- Rotates the Phase of the Signal

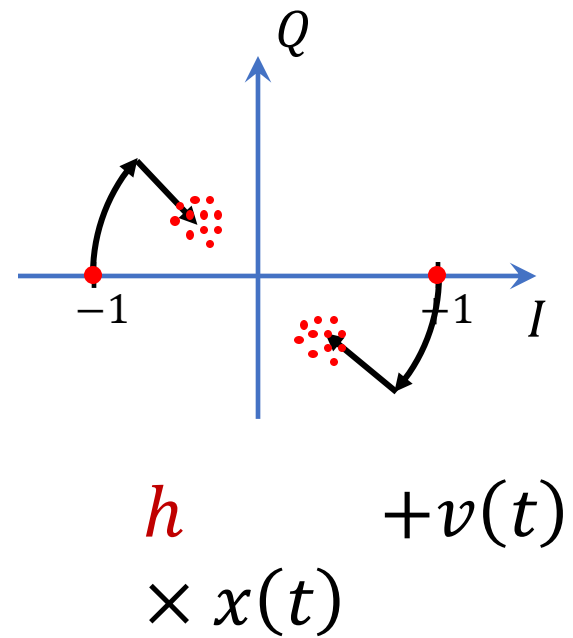
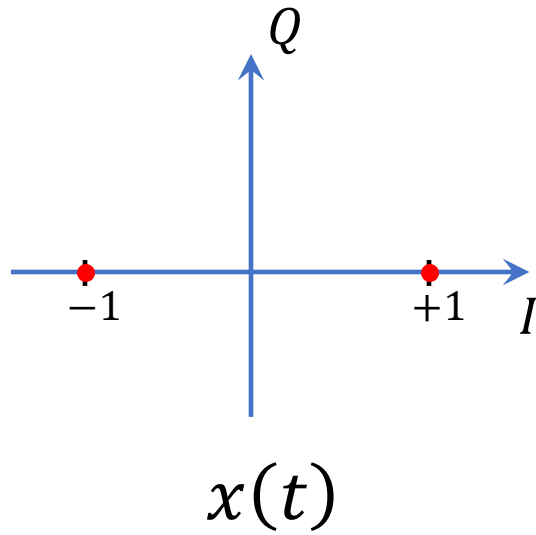
$$h \propto \frac{\lambda}{d} e^{j2\pi d/\lambda}$$

# The Channel

Consider BPSK Modulation.

$$0 \rightarrow -1$$

$$1 \rightarrow +1$$

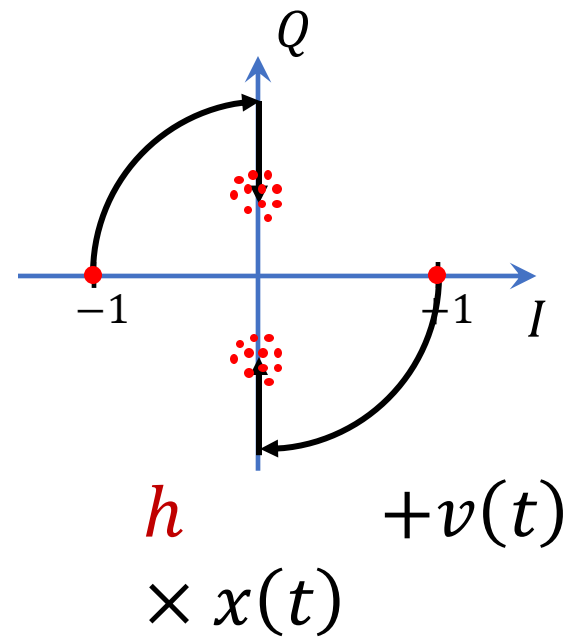
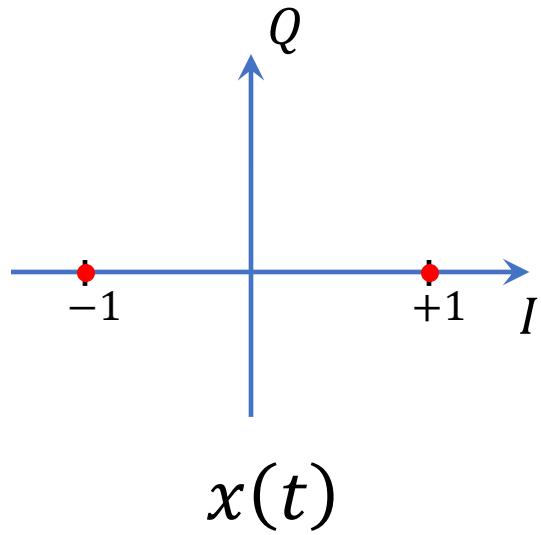


# The Channel

Consider BPSK Modulation.

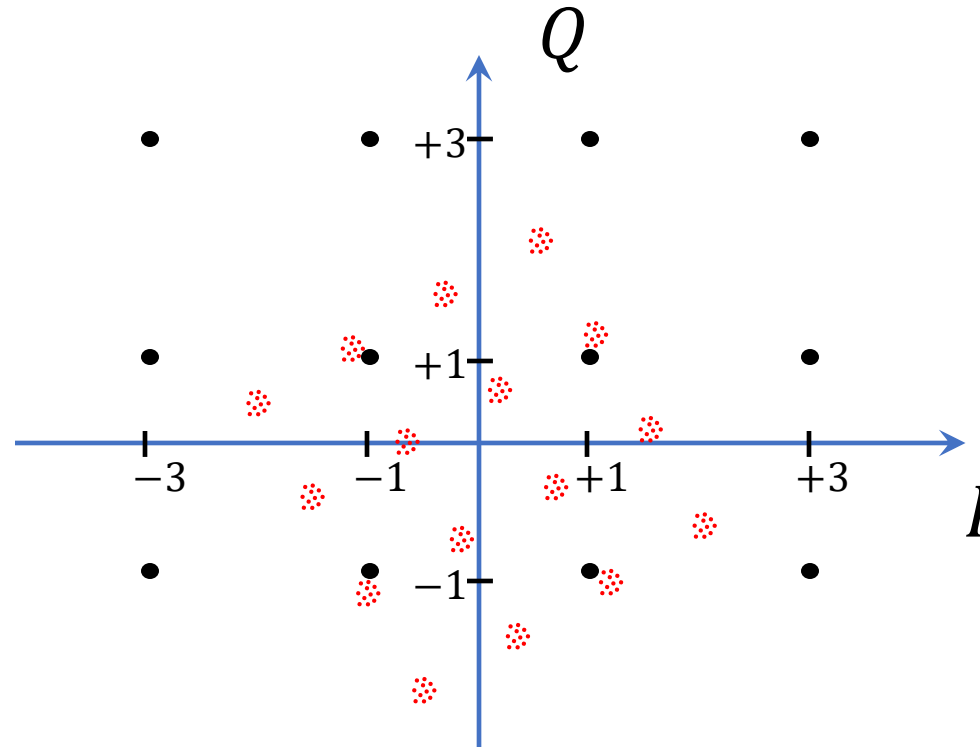
$$0 \rightarrow -1$$

$$1 \rightarrow +1$$



# The Channel

Consider QAM Modulation



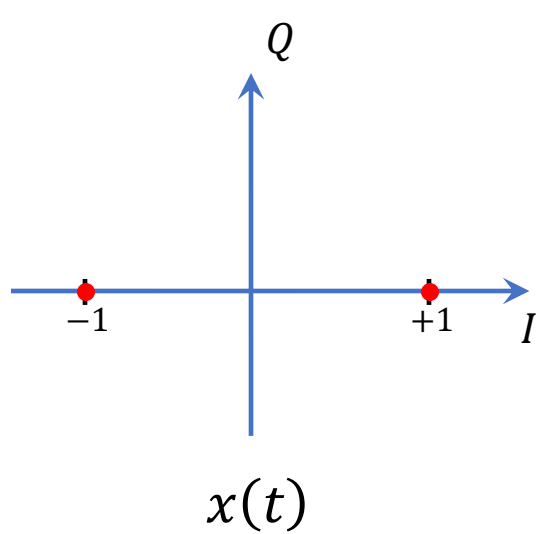
Demodulating correctly requires COHERENCE!  
i.e., Need to estimate & correct for the channel  $h$



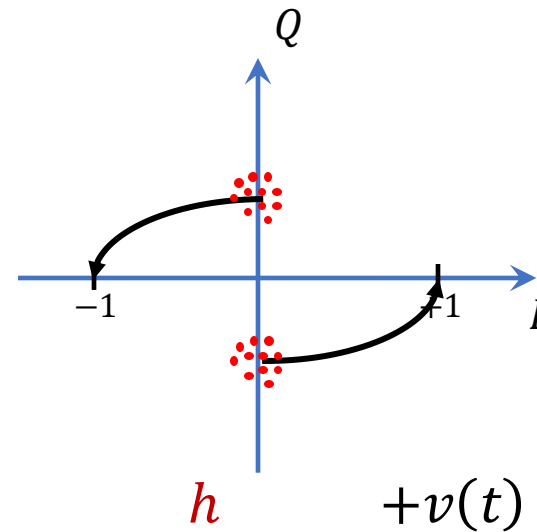
Channel Estimation & Correction

# Channel Estimation & Correction

Consider BPSK Modulation.



$0 \rightarrow -1$   
 $1 \rightarrow +1$



Send Training Sequence (Preamble Bits): Known Bits

$$x(0) = 1 \longrightarrow y(0) = h + v(0)$$

$$x(1) = 1 \longrightarrow y(1) = h + v(1)$$

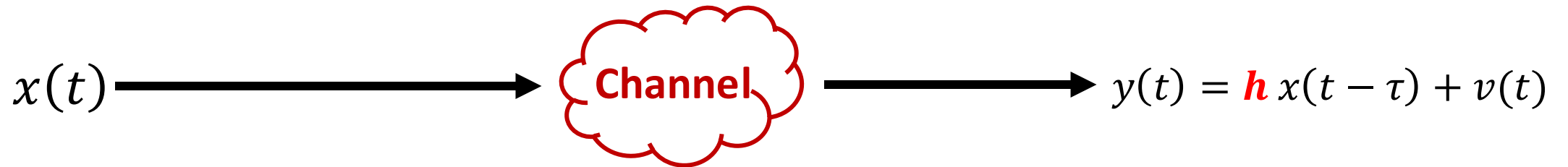
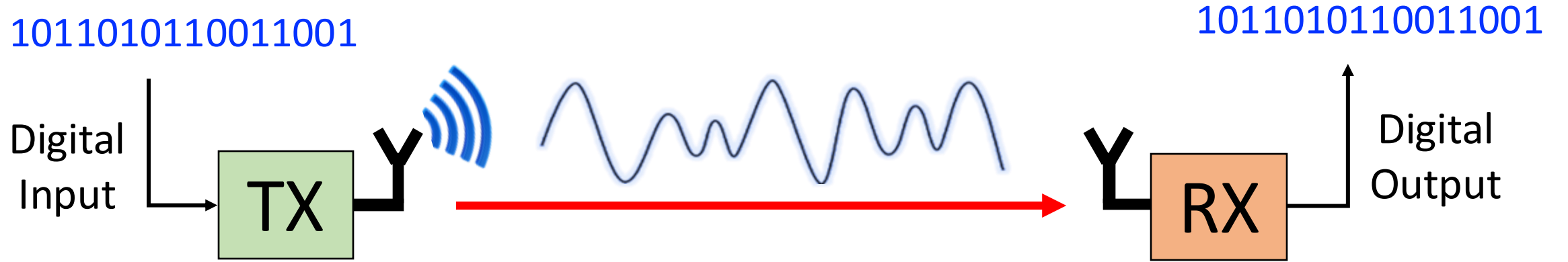
$$x(2) = -1 \longrightarrow y(2) = -h + v(2)$$

$\vdots$

$$\text{Estimate channel: } \tilde{h} = \frac{1}{K} \sum_{k=1} \frac{y(k)}{x(k)}$$

$$\text{Correct channel: } \tilde{x}(t) = \frac{y(t)}{\tilde{h}}$$

# Single Path Channel



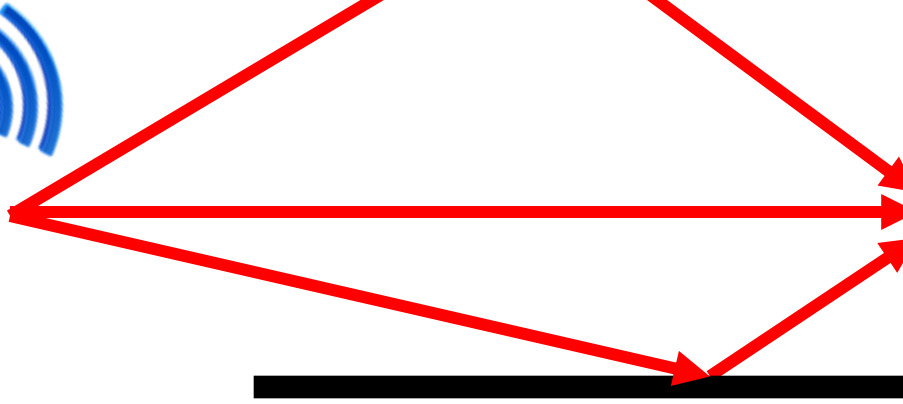
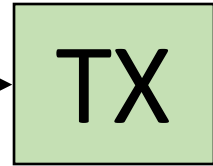
$$h \propto \frac{\lambda}{d} e^{j2\pi d/\lambda}$$

Assumes single path!

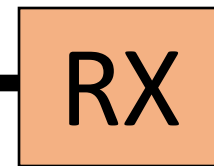
# Multipath Channel

1011010110011001

Digital  
Input



1011010110011001



Digital  
Output

*Multipath Propagation:* radio signal reflects off objects ground, arriving at destination at slightly different times

$$y(t) = \alpha_1 e^{j\phi_1} x(t - \tau_1) + \alpha_2 e^{j\phi_2} x(t - \tau_2) + \alpha_3 e^{j\phi_3} x(t - \tau_3) \dots$$

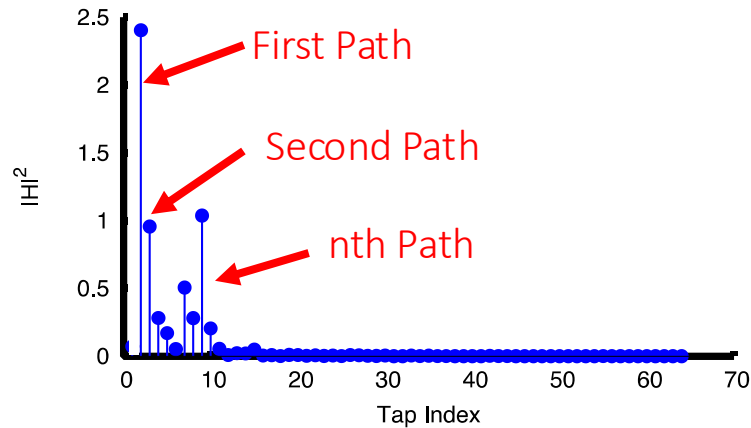
$$y(t) = \sum_k \alpha_k e^{j\phi_k} x(t - \tau_k) = \sum_k h(\tau_k) x(t - \tau_k) = h(t) * x(t)$$

*$h(t)$  is channel impulse response.*

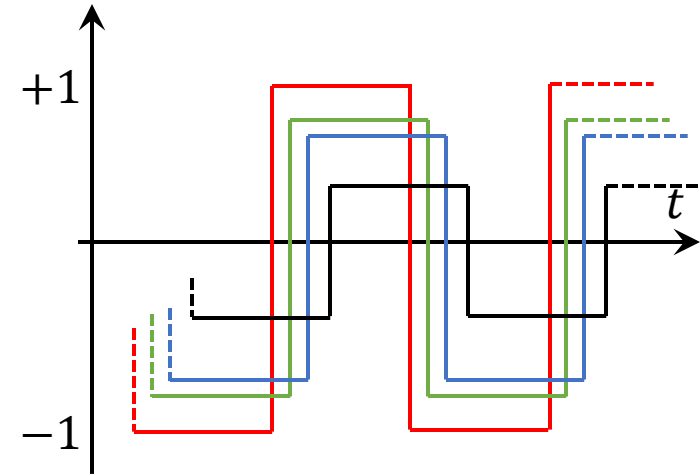
# Multipath Channel

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Multi-tap Channel



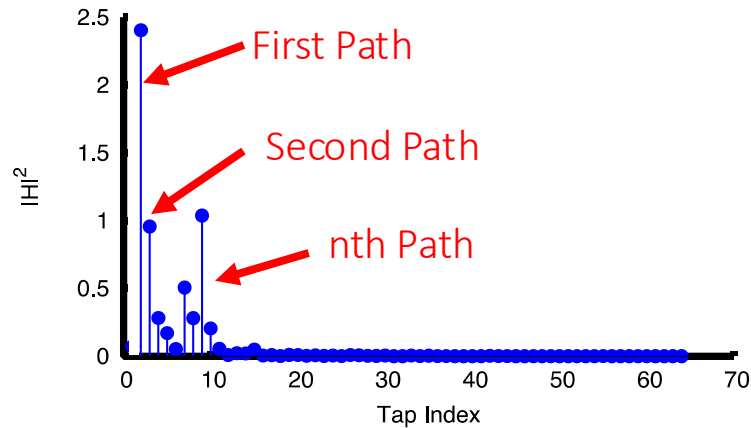
ISI: Inter-Symbol-Interference

Symbols arriving along late paths interfere with following symbols.

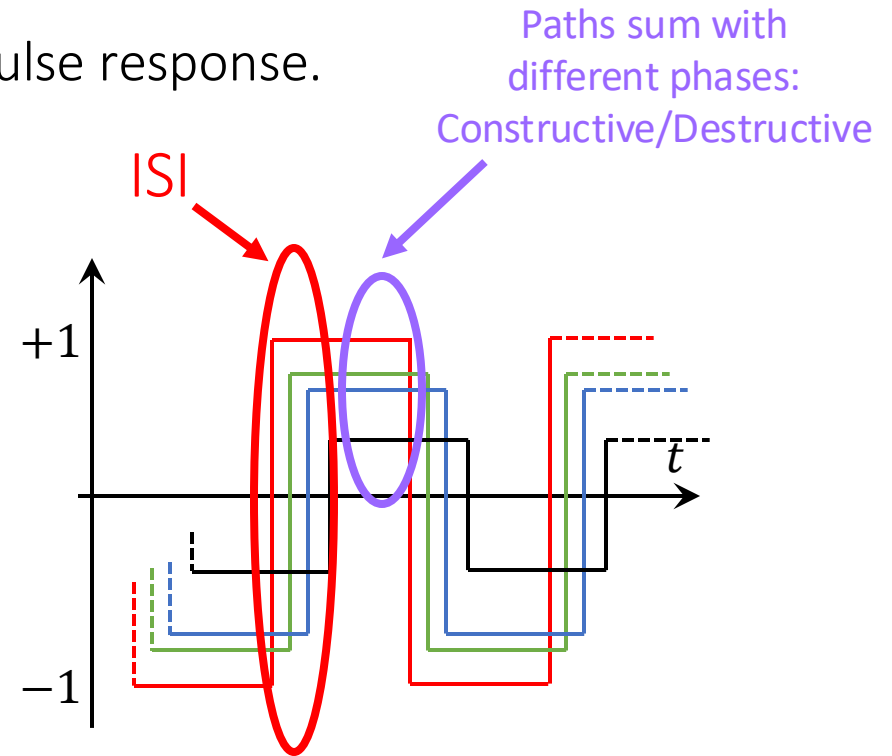
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Multi-tap Channel



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Symbols arriving along late paths interfere with following symbols.



Channel Fading

Symbols arriving along different paths sum up destructively

# Multipath Channel

$h(t)$  is channel impulse response.

$$y(t) = \sum_k h(\tau_k) x(t - \tau_k) = h(t) * x(t)$$

**Channel Fading:** Symbols arriving along different paths sum up destructively

*Example 2 paths with distance  $d_1$  and  $d_2$*

$$\begin{aligned} h &= h_1 + h_2 = \frac{\lambda}{d_1} e^{j2\pi d_1/\lambda} + \frac{\lambda}{d_2} e^{j2\pi d_2/\lambda} \\ &= \frac{\lambda}{d_1} e^{j2\pi d_1/\lambda} \left( 1 + \frac{d_1}{d_2} e^{j2\pi(d_2-d_1)/\lambda} \right) \quad \frac{d_1}{d_2} \approx 1 \end{aligned}$$

$$\text{if } \frac{d_2 - d_1}{\lambda} \approx \frac{1}{2} \rightarrow h = \frac{\lambda}{d_1} e^{j2\pi d_1/\lambda} (1 + e^{j\pi}) = 0 \quad \text{Destructive Interference!}$$

# Multipath Channel

Channel Fading: Symbols arriving along different paths sum up destructively

Suppose the 2 paths have distance  $d_1 = 1m, d_2 = 1.06m$ :


$$h = h_1 + h_2 = \frac{\lambda}{d_1} e^{j2\pi d_1/\lambda} + \frac{\lambda}{d_2} e^{j2\pi d_2/\lambda}$$

Compute  $h$  if the signal is transmitted at 2.5 GHz and 5 GHz. How much will power of the receiver signal differ if it is transmitted at 5 GHz instead of 2.5 GHz?

@ $f_1 = 2.5GHz$  ( $\lambda = 12\text{ cm}$ ):  $|h| = |0.12 e^{j\frac{2\pi}{3}} + 0.113 e^{j\frac{5\pi}{3}}| \approx 0.0068$

@ $f_2 = 5GHz$  ( $\lambda = 6\text{ cm}$ ):  $|h| = |0.06 e^{j\frac{4\pi}{3}} + 0.056 e^{j\frac{4\pi}{3}}| \approx 0.116$

$17^2 \times$   
(24.6dB)

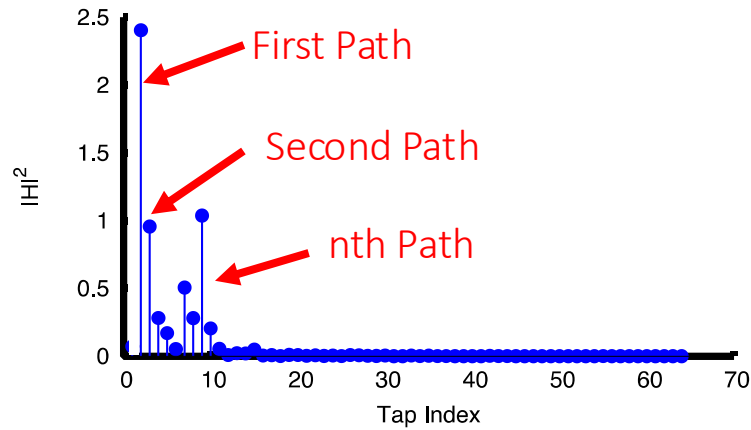


## Frequency Selective Fading

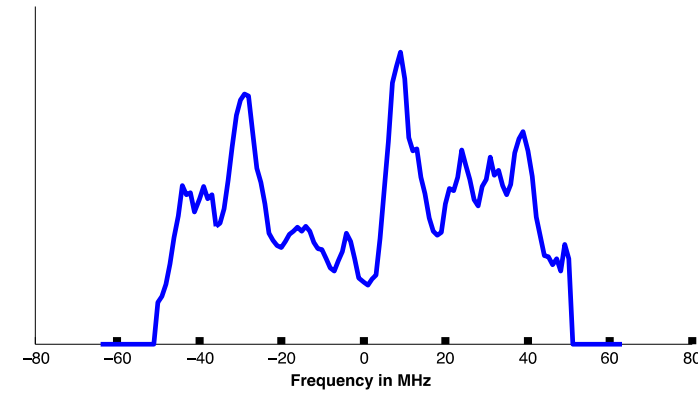
# Multipath Channel

$h(t)$  is channel impulse response.

$$y(t) = \sum_k h(\tau_k) x(t - \tau_k) = h(t) * x(t) \quad \Leftrightarrow \quad H(f)X(f)$$



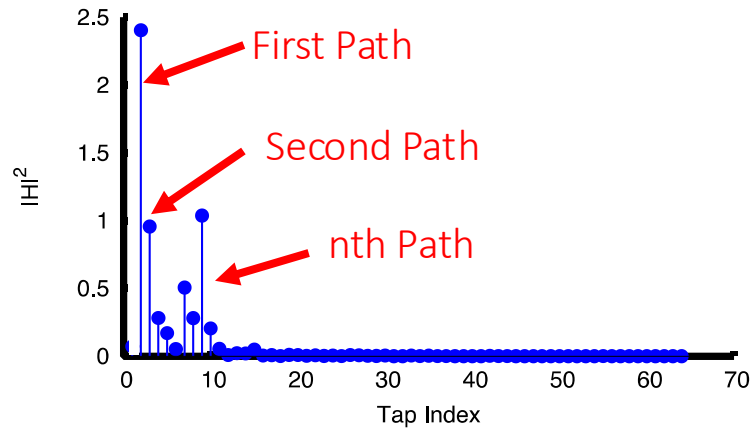
Multi-tap Channel



# Multipath Channel

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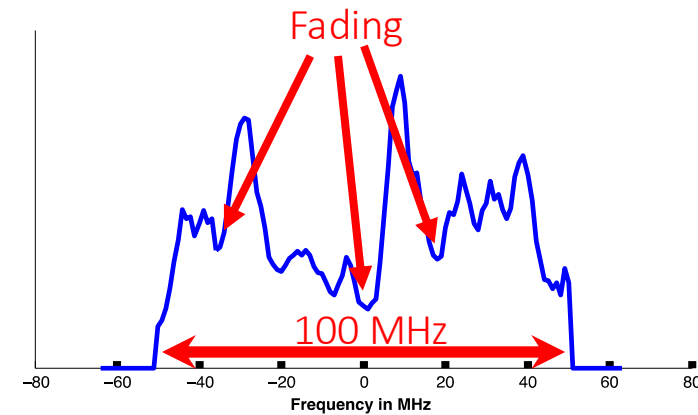


Multi-tap Channel



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Frequency Selective Fading

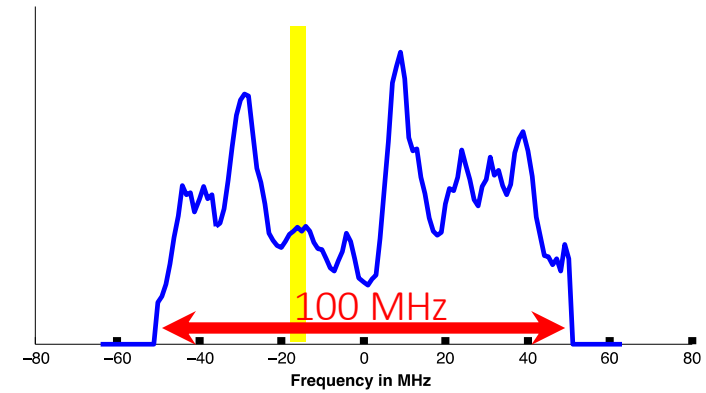
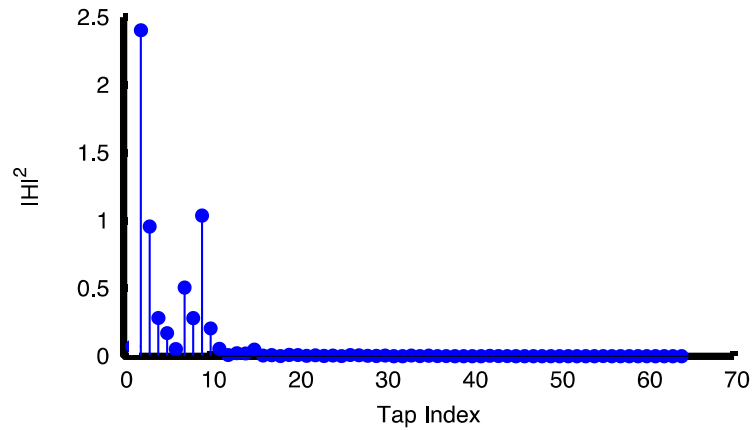
Symbols arriving along different paths sum up destructively

Problematic in  
Wideband Channel!

# Narrowband Channel

$h(t)$  is channel impulse response.

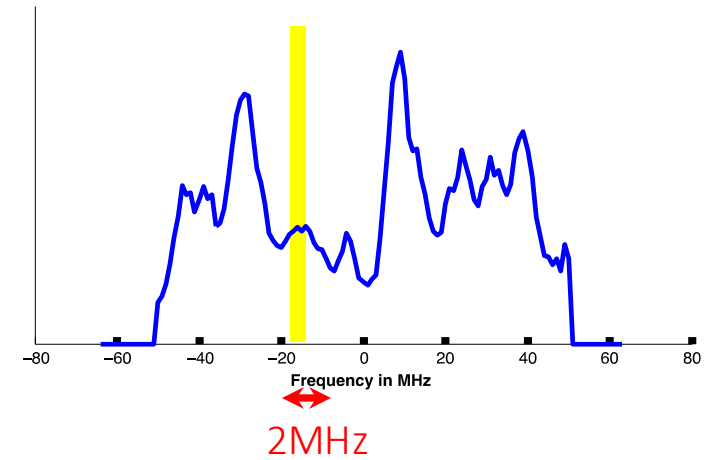
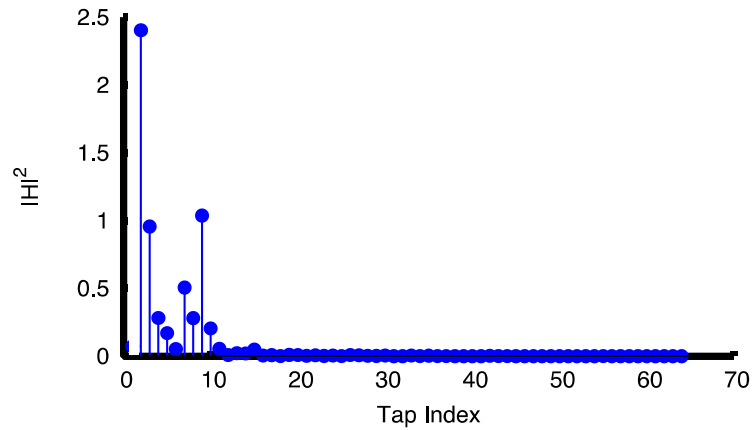
$$y(t) = \sum_k h(\tau_k) x(t - \tau_k) = h(t) * x(t) \quad \Leftrightarrow \quad H(f)X(f)$$



# Narrowband Channel

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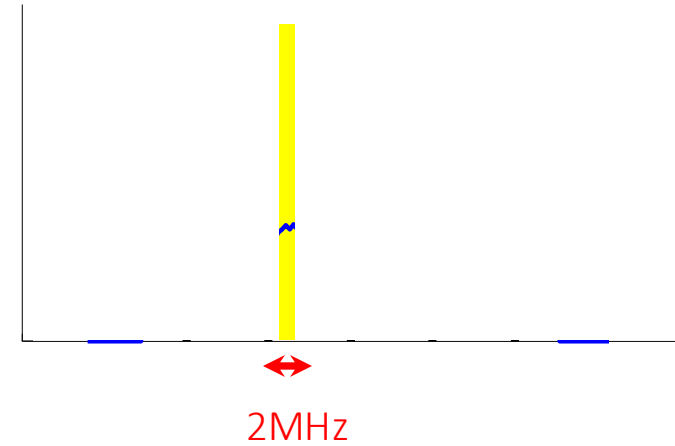
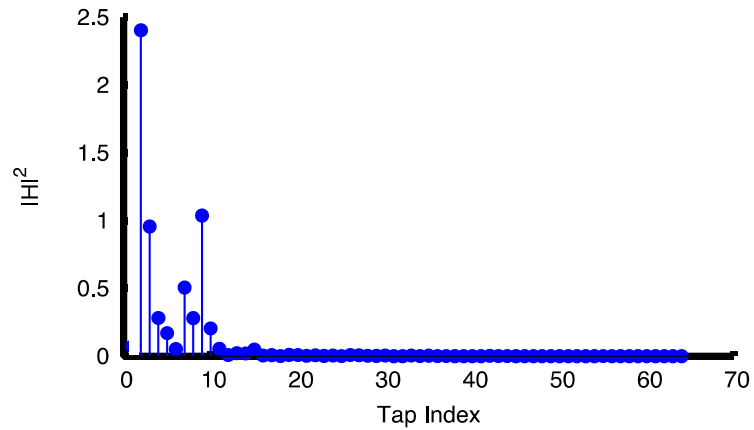
$$y(t) = \sum_k h(\tau_k) x(t - \tau_k) = h(t) * x(t) \quad \Leftrightarrow \quad H(f)X(f)$$



# Narrowband Channel

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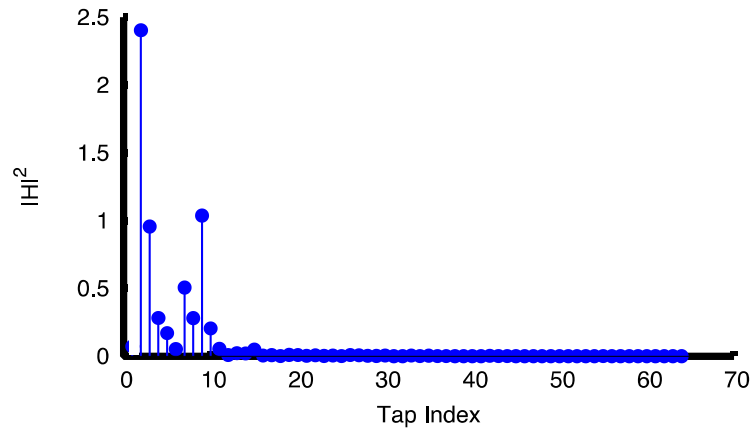
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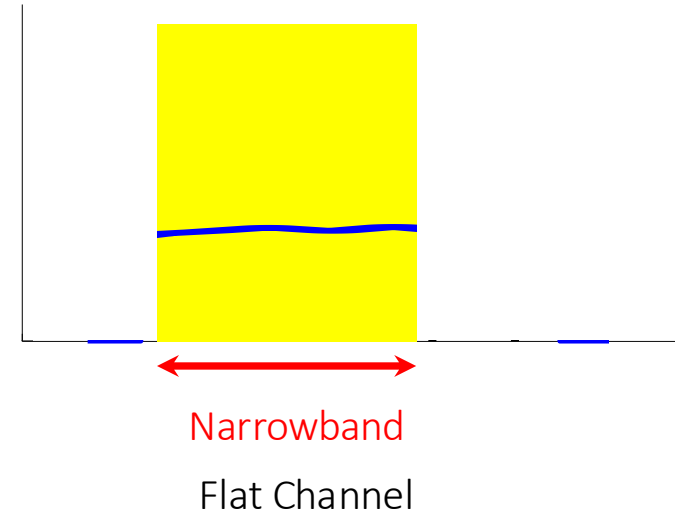
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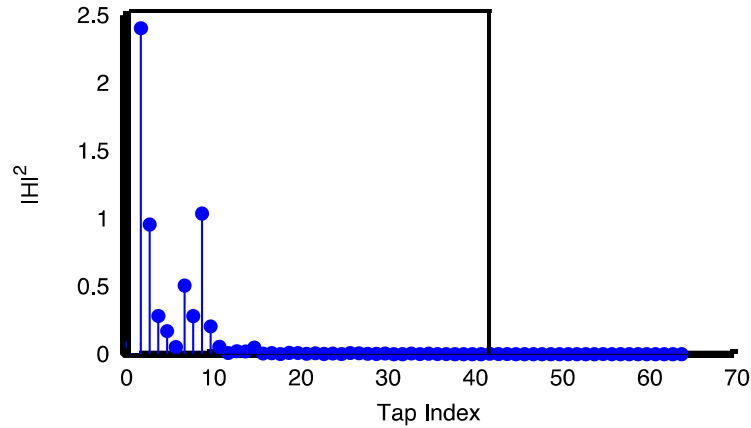
Symbol time:  $T \propto \frac{1}{\text{Bandwidth}}$   $\gg \tau_k$



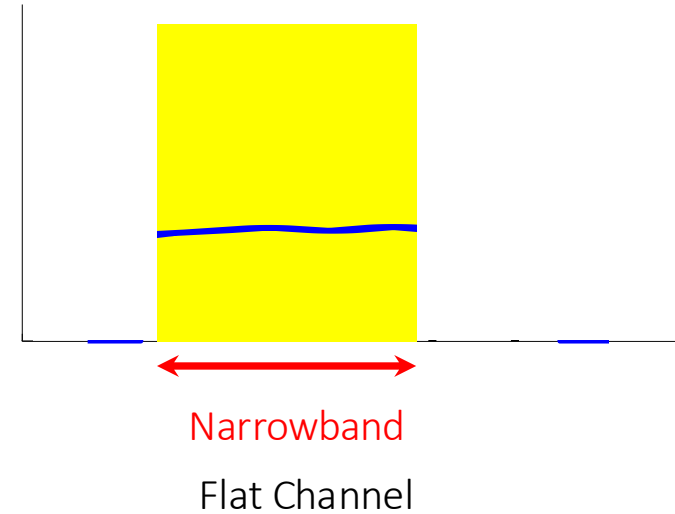
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$h(t)$  is channel impulse response.

$$y(t) = \sum_k h(\tau_k) x(t - \tau_k) = h(t) * x(t) \quad \Leftrightarrow \quad H(f)X(f)$$



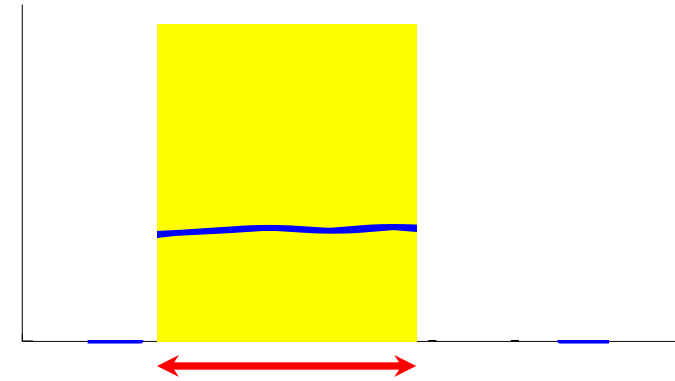
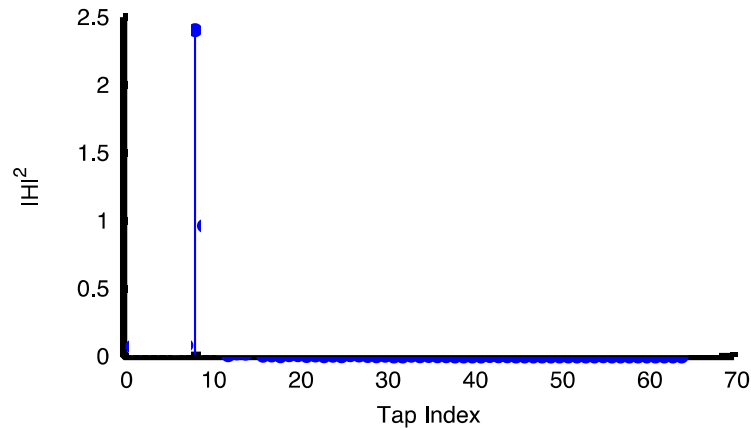
Symbol time:  $T \propto \frac{1}{\text{Bandwidth}}$   $\gg \tau_k$



# Narrowband Channel

$h(t)$  is channel impulse response.

$$y(t) = \sum_k h(\tau_k) x(t - \tau_k) = h(t) * x(t) \quad \Leftrightarrow \quad H(f)X(f)$$



Symbol time:  $T \propto \frac{1}{\text{Bandwidth}}$   $\gg \tau_k$

Narrowband

Flat Channel

$$y(t) = \sum_k h(\tau_k) x(t - \tau_k) \approx \sum_k h(\tau_k) x(t) = \left( \sum_k h(\tau_k) \right) x(t) = hx(t)$$

Narrowband Channel is Approximated by a Single Tap Channel

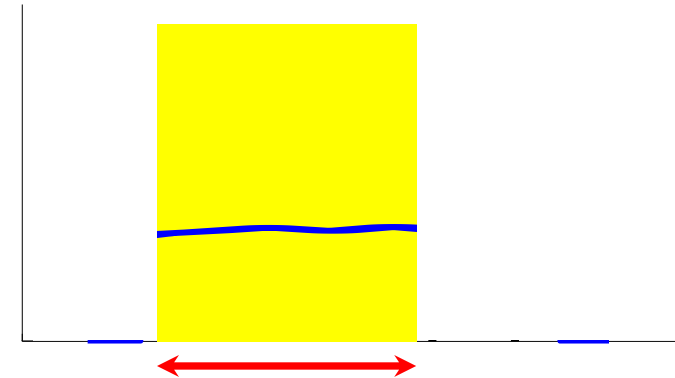
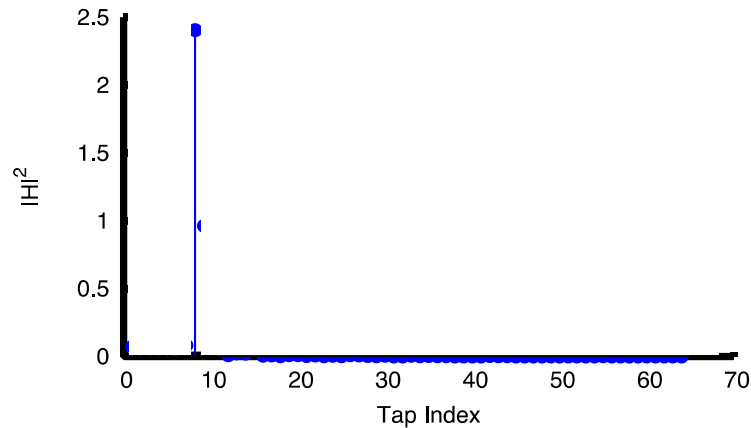
# Narrowband Channel

$h(t)$  is channel impulse response.

$$y(t) = \sum_k h(\tau_k) x(t) = h x(t)$$

$\Leftrightarrow$

$$h X(f)$$



Symbol time:  $T \propto \frac{1}{\text{Bandwidth}} \gg \tau_k$

Narrowband

Flat Channel

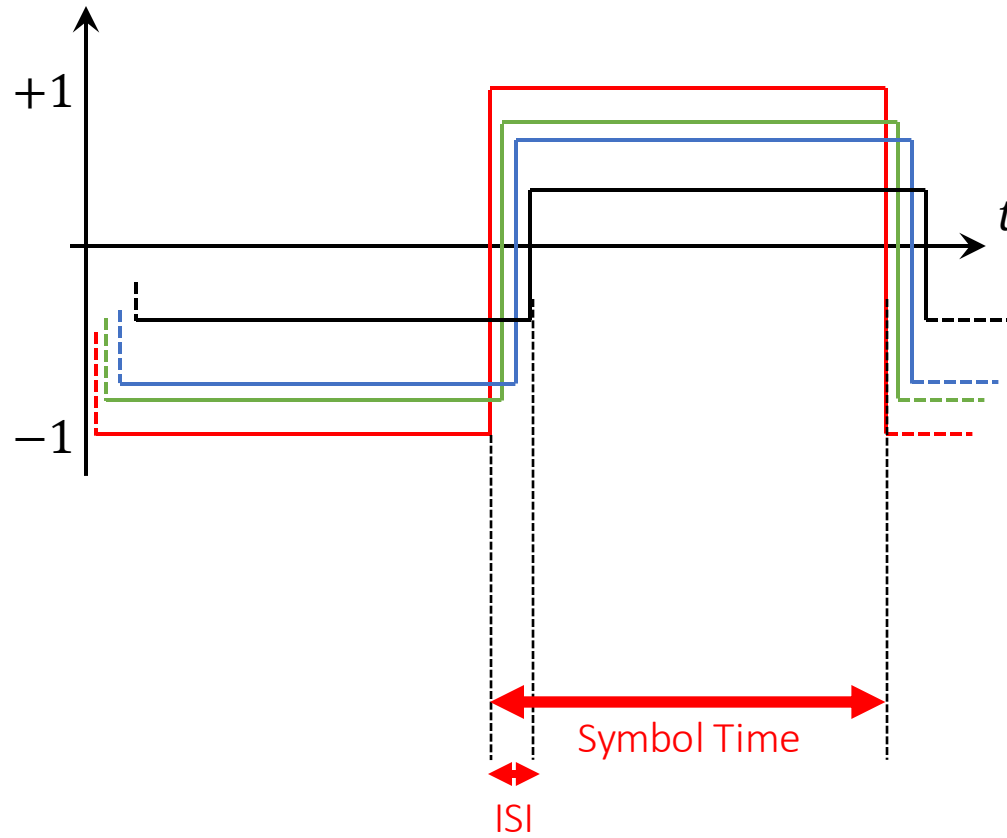
$$y(t) = \sum_k h(\tau_k) x(t - \tau_k) \approx \sum_k h(\tau_k) x(t) = \left( \sum_k h(\tau_k) \right) x(t) = h x(t)$$

Narrowband Channel is Approximated by a Single Tap Channel

# Narrowband vs. Wideband Channel

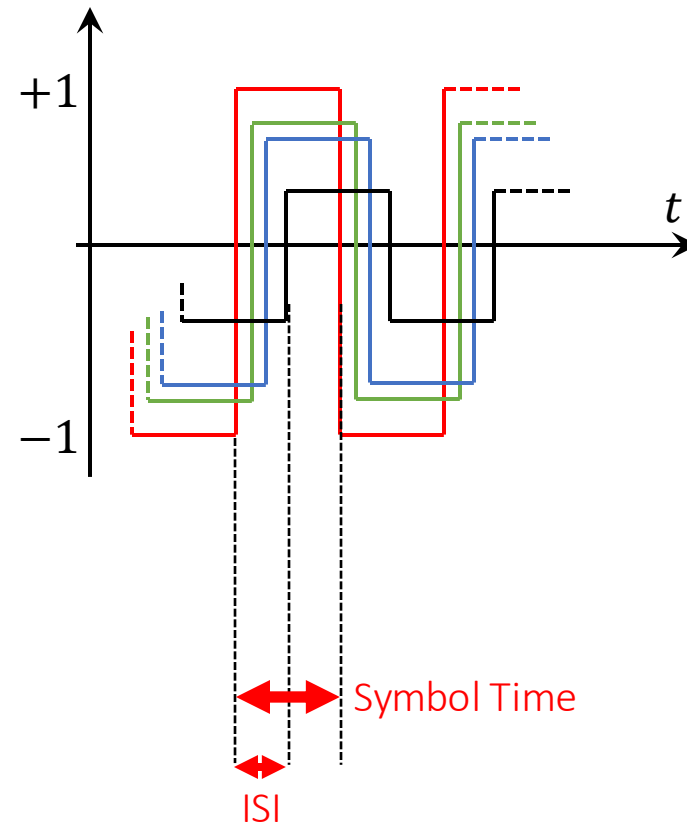
Narrowband Channel

$$h x(t)$$



Wideband Channel

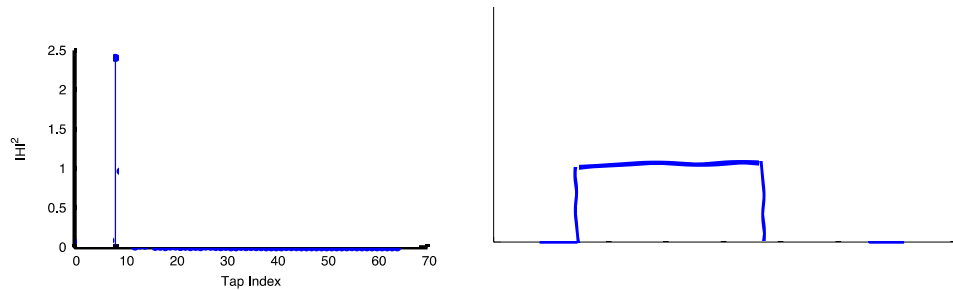
$$h(t) * x(t)$$



# Narrowband vs. Wideband Channel

## Narrowband Channel

$$h x(t) \iff h X(f)$$

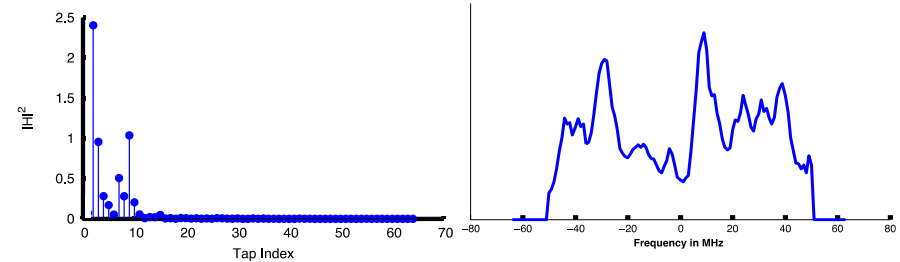


$\approx$  Single Tap

$\approx$  Flat Channel

## Wideband Channel

$$h(t) * x(t) \iff H(f) X(f)$$



Multi-tap  
Channel  
(ISI)

Frequency  
Selective Channel

Need to correct for ISI to  
be able to decode  
correctly!

Estimating and Correcting for multi-tap channel is hard

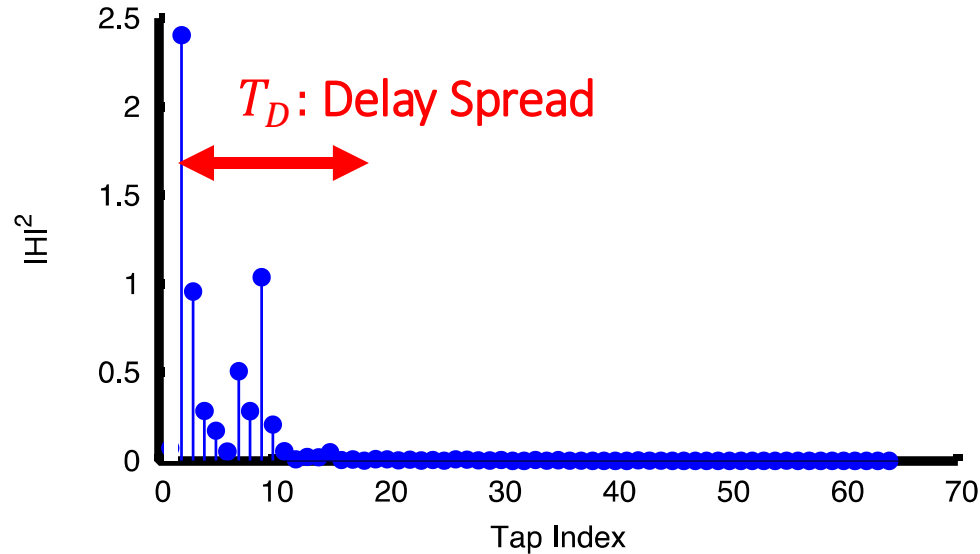
# Multipath Wideband Channel

$$h(t) * x(t)$$

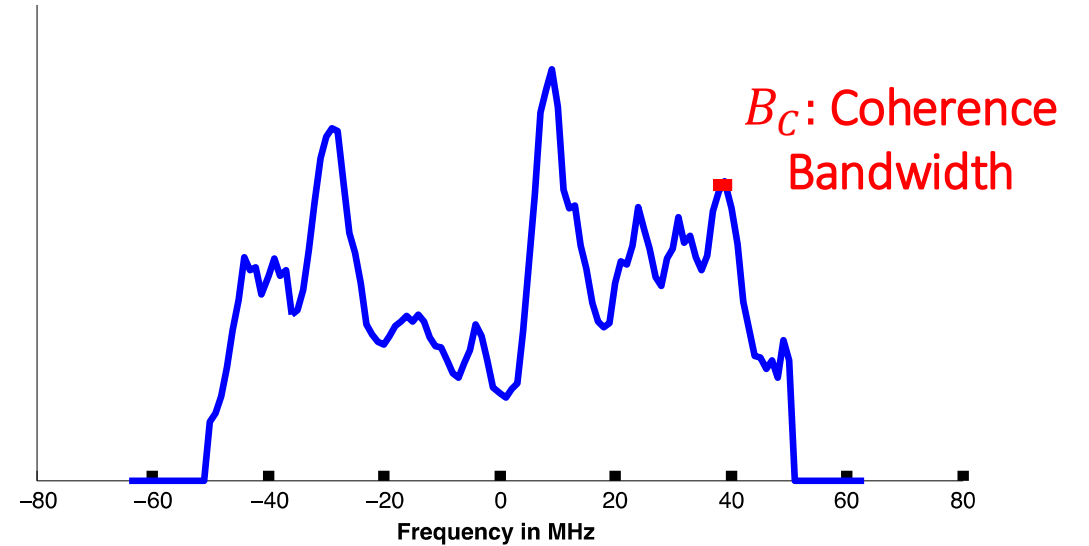
$\Leftrightarrow$

$$H(f) X(f)$$

Multi-tap Channel (ISI)



Frequency Selective Channel



$$T_D = 1/B_C$$

**Delay Spread:** Spread of the channel in time i.e. time delay between first and last path received above noise floor

**Coherence Bandwidth:** region where the channel frequency response is flat (coherent).

# Time Varying Channel

So far, considered only static channels, i.e. the channel does not change.  
But the channel  $h(t)$  cannot be the same forever

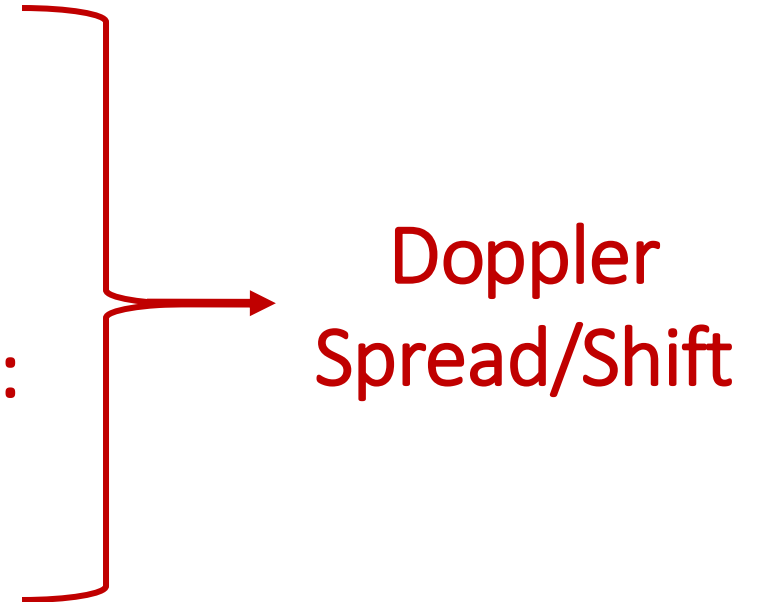
Reasons why  $h(t)$  changes:

- **Environment changes:**

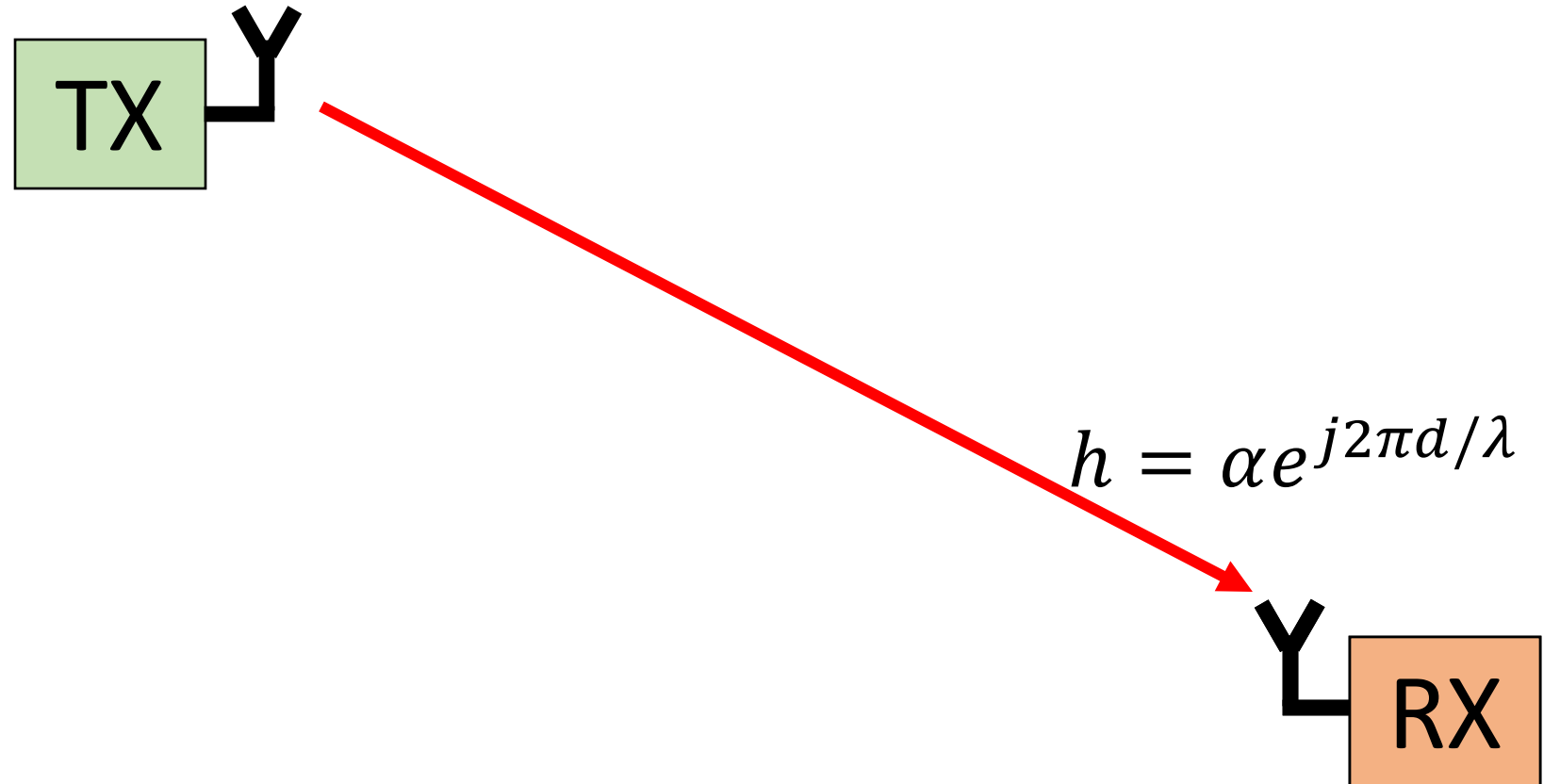
- Reflectors move
- People moving around

- **Transmitter or Receiver Moves:**

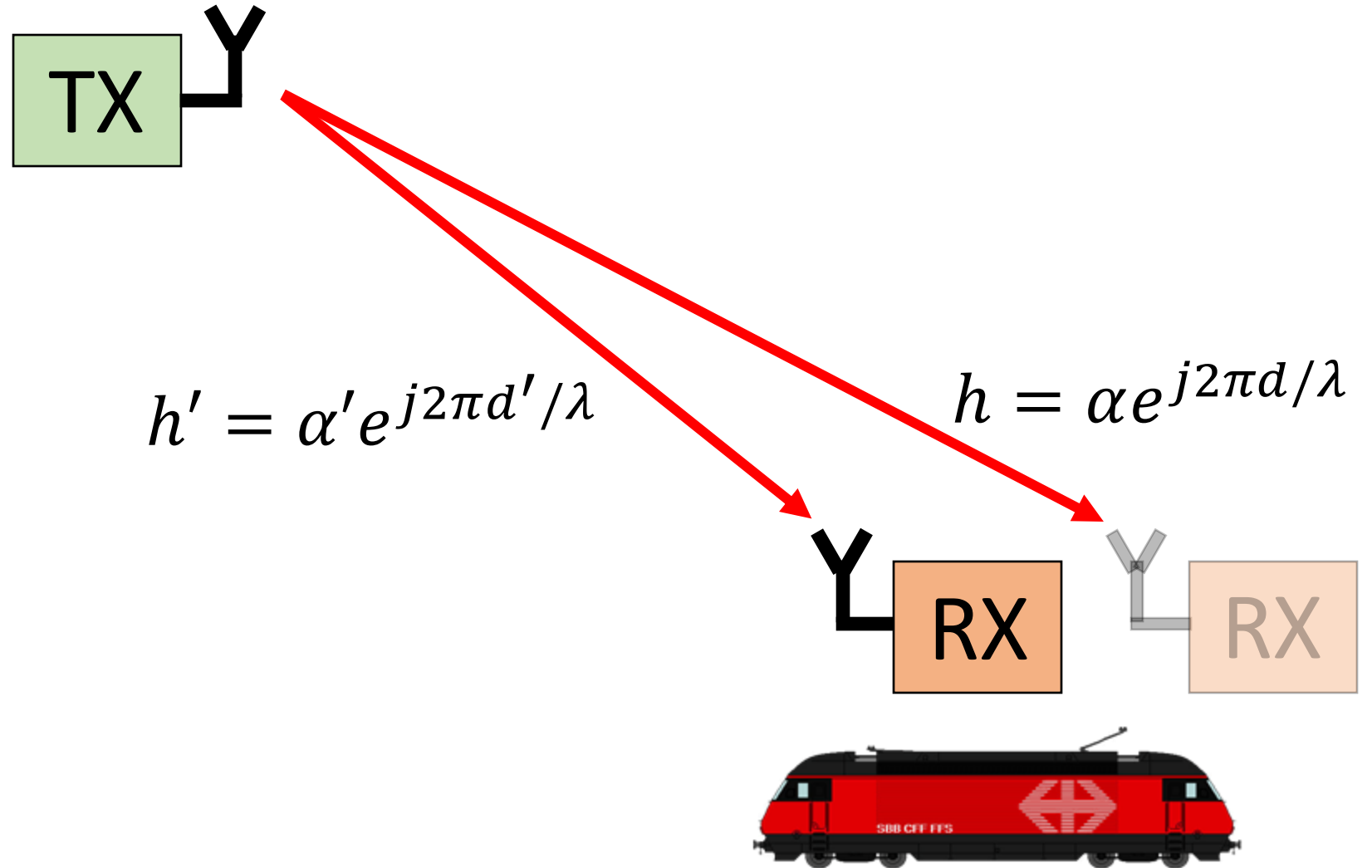
- Reflectors move
- People moving around



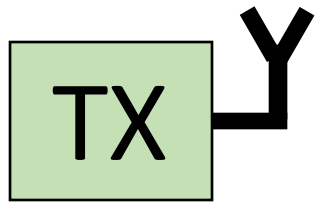
# Doppler Shift



# Doppler Shift



# Doppler Shift



$$d - d' = p \cos \theta$$

$$h' = \alpha e^{j2\pi(d - p \cos \theta) / \lambda}$$

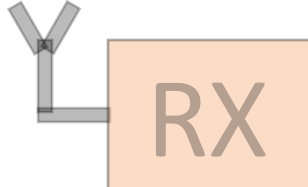
$$h' = \alpha e^{j2\pi d' / \lambda}$$

$p$

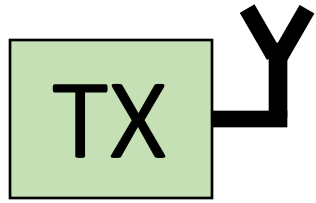
$$h = \alpha e^{j2\pi d / \lambda}$$

$\theta$

$\theta$

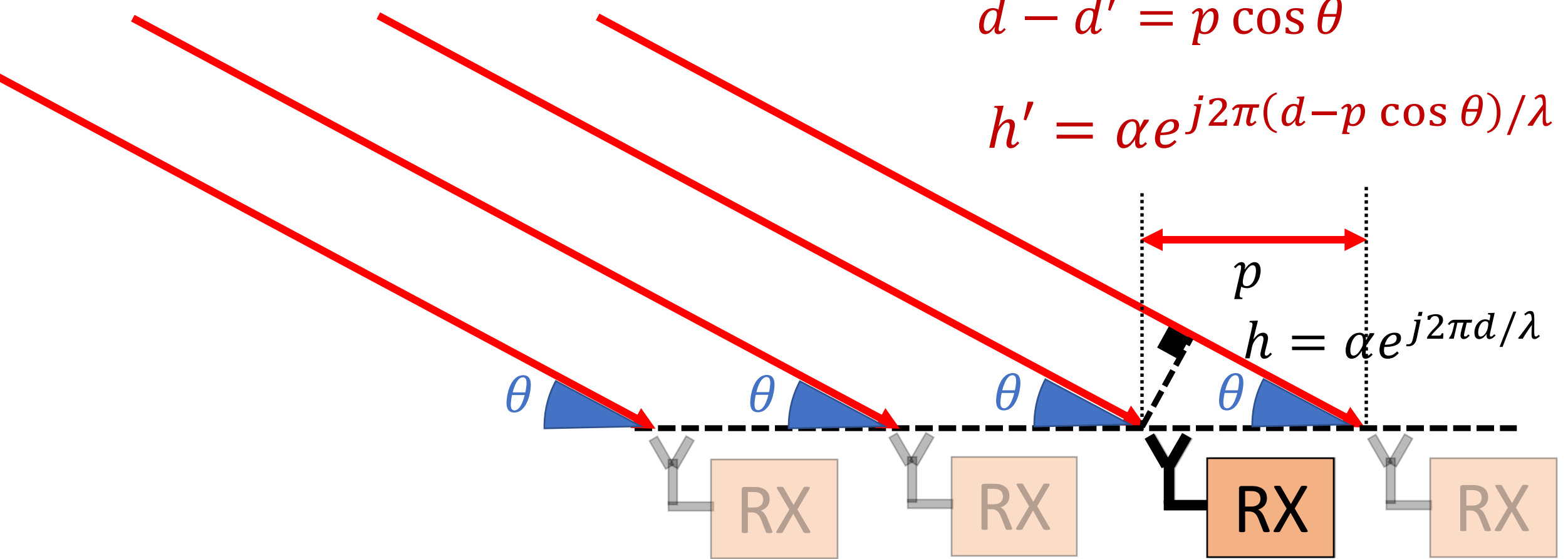


# Doppler Shift



$$d - d' = p \cos \theta$$

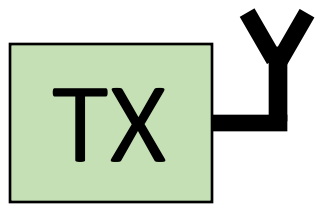
$$h' = \alpha e^{j2\pi(d - p \cos \theta) / \lambda}$$



$$h = \alpha e^{j2\pi d / \lambda}$$



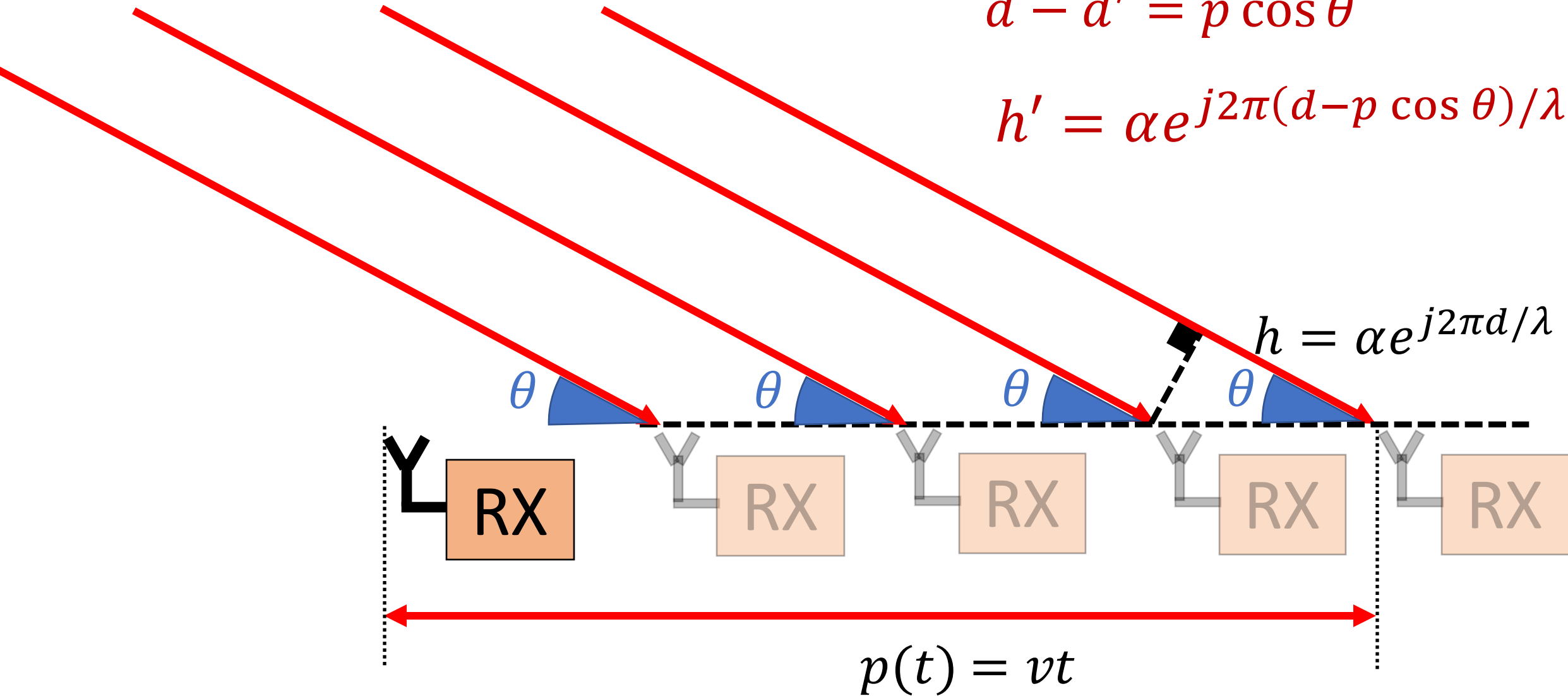
# Doppler Shift



$$d - d' = p \cos \theta$$

$$h' = \alpha e^{j2\pi(d - p \cos \theta) / \lambda}$$

$$h = \alpha e^{j2\pi d / \lambda}$$



# Doppler Shift

$$\begin{aligned}h' &= \alpha e^{j2\pi(d - vt \cos \theta) / \lambda} \\ &= \alpha e^{j2\pi d / \lambda} e^{-j2\pi(v \cos \theta / \lambda)t} \\ &= h e^{-j2\pi \Delta f t}\end{aligned}$$

**Doppler Shift:** Frequency shift caused by the movement in TX/RX or reflectors

$$B_D = \frac{v \cos \theta}{\lambda}$$

# Time Varying Channel

$$h(\tau, t)$$

Delay of  
different paths

Time  
Changing

$h(\tau, 0)$ : Channel Impulse Response at Time  $t = 0$ .

$h(0, t)$ : Channel of the first path over time.

$h(t)$ : Channel Impulse Response of static channel.

# Time Varying Channel

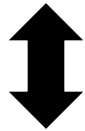
$$h(\tau, t)$$

Delay of  
different paths

Time  
Changing

Delay Spread

$$T_D = \Delta\tau$$



Coherence Bandwidth

$$B_C = 1/T_D$$

Doppler Spread

$$B_D = \Delta f$$



Coherence Time

$$T_C = 1/B_D$$