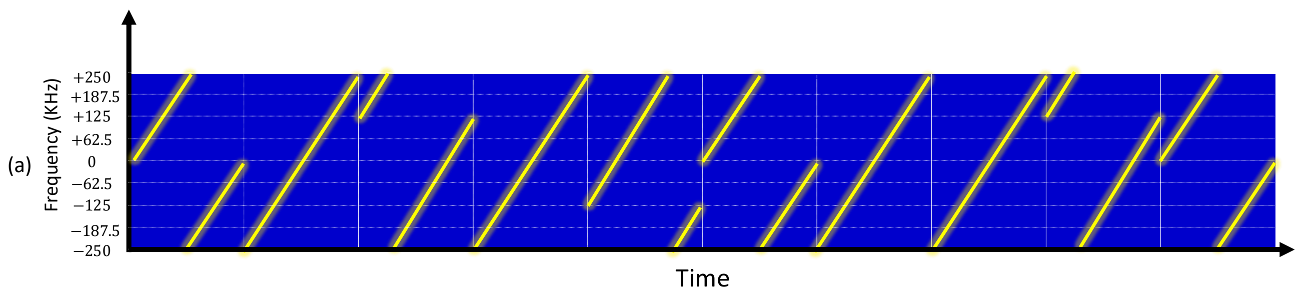
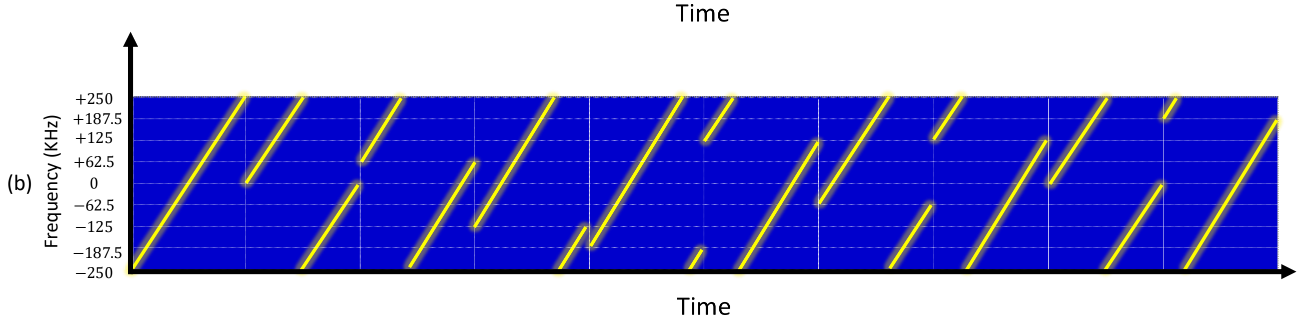
COM 405: Mobile Networks – Fall 2025
Homework 3 Solutions
EPFL

1 LoRA **20 points**

Consider the below spectrograms of LoRA signals. Assume LoRA uses a bandwidth of 500 kHz and the signal has the following equation with $\alpha = 200MHz/sec$:

$$x(t) = \exp(j2\pi \frac{\alpha}{2} t^2 + j2\pi f_0 t)$$





Assume we use the following modulations in LoRA to map signals to bits:

f_0 (KHz)	−250	−187.5	−125	−62.5	0	+62.5	+125	+187.5
1st Order	'0'				'1'			
2nd Order	'00'		'01'		'10'		'11'	
3rd Order	'000'	'001'	'010'	'011'	'100'	'101'	'110'	'111'

1. Assuming all possible modulated chirps appear in the above spectrograms, what is the bit rate for each of the above LoRA signals?

   Symbol Time:

   $$T_s = \frac{B}{\alpha} = \frac{500kHz}{200 \times 10^6} = 2.5ms$$

   Data Rate = Bits per Symbol / Symbol Time:

   (a) = 2/2.5ms = 800 Bits/s.

   (b) = 3/2.5ms = 1.2 kBits/s.

2. Assume CFO and STO are both zero. What are the bits encoded in each of the above LoRA signals?

   (a) f_0 = 0, -250, +125, -250, -125, 0, -250, -250, +125, 0
   ⇒ Bits = 10, 00, 11, 00, 01, 10, 00, 00, 11, 10

   (b) f_0 =-250, 0, +62.5, -125, -187.5, +125, -62.5, +125, 0, +187.5
   ⇒ Bits = 000, 100, 101, 010, 001, 110, 011, 110, 100, 111

3. Assume the receiver has a +12.4 KHz CFO (Carrier Frequency Offset) from the transmitter but zero STO (Sampling Time Offset). After demodulating the chirp and taking the FFT, by how many FFT bins will the peak be shifted?

   FFT Bin width = $1/T_s = 400Hz$.
   Peak shifts by 12400/400 = 31 bins.

4. Assume that the CFO is zero but the receiver samples the spectrograms with an STO of 625 μs, i.e. the receiver start sampling 625 μs after the start of the signal. If the receiver does not correct for this offset, what will be the decoded bits for each spectrogram and how much bit errors will the receiver have for each signal.

Given the STO and the linear relation between time and frequency, the receiver will not measure f_0 correctly but will instead measure it as $f_0 + \alpha \times STO = f_0 + 125KHz$ wrapped around 250 KHz.

(a) f_0 = +125, -125, -250, -125, 0, +125, -125, -125, -250, +125
$\Rightarrow$ Bits = 11, 01, 00, 01, 10, 11, 01, 01, 00, 11

Correct Bits:	10001100011000001110
Error Bits:	11010001101101010011

13 out of 20 bits in error.

(b) f_0 =-125, +125, +187.5, 0, -62.5, -250, +62.5, -250, +125, -187.5
$\Rightarrow$ Bits = 010, 110, 111, 100, 011, 000, 101, 000, 110, 001

Correct Bits:	000100101010001110011110100111
Error Bits:	010110111110001100010100011000 1

15 out of 30 bits in error.

2 Wireless Imaging — 30 points

Dave wants to build a wireless imaging system using an FMCW transmitter and a 2D antenna array receiver. He needs a distance resolution of 3 cm and an angular resolution of 1° along both azimuth and elevation. He also needs the system to work for a maximum range of 60 m. To average noise, $K = 10$ FMCW sweeps are used and averaged together to create a single image frame. Dave needs a frame rate of $F = 40$ frames/sec. Assume, the speed of light is $c = 3 \times 10^8$ m/s.

1. What are the FMCW sweep bandwidth B_s and sweep time T_s needed to achieve the above range resolution and frame rate?

$$\delta = \frac{c}{2B_s} \rightarrow B_s = \frac{c}{2\delta} = \frac{3 \times 10^8}{2 \times 0.03} = 5 \text{ GHz}$$

$$\frac{1}{K \times T_s} = F \text{ frames/sec} \rightarrow T_s = \frac{1}{400} = 2.5ms$$

2. What is the sampling bandwidth B needed to achieve an imaging range of 60 m assuming the FFT window is taken over on FMCW sweep?

$$\text{Max Rang} = 60\ m = \frac{cBT_s}{2B_s} \rightarrow B = \frac{60\ m \times 2B_s}{cT_s} = \frac{60 \times 2 \times 5 \times 10^9}{3 \times 10^8 \times 2.5 \times 10^{-3}} = 800 \text{ kHz}$$

3. For a linear uniform antenna array, the angular resolution needed can be approxmiated as $0.89\frac{\lambda}{D}$ radians where λ is the wavelenght of the signal and D is the length of the array. Assuming, the

antennas in the array are uniformly spaced by $\lambda/2$ and the 2D antenna array has $N \times N$ antennas, what is the value of N needed to achieve an angular resolution of $1°$ along both azimuth and elevation? (Hint: You do not need to know λ.)

$$D = N \times \frac{\lambda}{2}$$

$$0.89\frac{\lambda}{D} = 1° \times \frac{\pi}{180°} \rightarrow 0.89\frac{2}{N} = \frac{\pi}{180} \rightarrow N = \left\lceil 0.89\frac{360}{\pi} \right\rceil = 102$$

4. Assume that every antenna in the receiver is connected to its own receiver front-end and ADC. How many samples/sec in total does this system generate i.e. aggregated across all the receivers?

System generates $102 \times 102 \times 800\ KHz = 8.3232$ GigaSample/sec.

5. Dave's machine can only handle 1 GigaSample/sec. Which system parameters do we need to change to ensure that we do not exceed 1 GS/s while maintaining the same distance resolution, angular resolution, maximum range, and frame rate? What are the values of these parameters?

We need change B, K, and T_s.

$$B = \frac{1\ GS/s}{N \times N} = 96.11\ kHz$$

$$\text{Max Rang} = 60\ m = \frac{cBT_s}{2B_s} \rightarrow T_s = \frac{60\ m \times 2B_s}{cB} = \frac{60 \times 2 \times 5 \times 10^9}{3 \times 10^8 \times 96.11 \times 10^3} = 20.8\ ms$$

$$F = \frac{1}{K \times T_s} \rightarrow K = \left\lfloor \frac{1}{F \times T_s} \right\rfloor = \left\lfloor \frac{1}{40 \times 20.8 \times 10^{-3}} \right\rfloor = 1$$

6. Dana suggests to Dave that he can significantly reduce the cost of the system by using a MIMO radar with N transmitters and N receivers instead of his radar which has 1 transmitter and $N \times N$ receivers. In the MIMO radar, the N transmitters will use TDMA to avoid interfering i.e. they will take turns transmitting their FMCW chirps. Dana tells Dave that by using this MIMO radar, he can also reduce the sweep time T_s and hence, average more sweeps within each frame (i.e. increase K) while maintaining the same resolution, max range, and frame rate. Dave agrees that he will be able to reduce the sweep time T_s but he will not be able to average more sweeps (i.e. cannot increase K). Compute the new values of B, T_s, and K if Dave uses Dana's MIMO radar.

$$B = \frac{1\ GS/s}{N} = 9.8\ MHz$$

$$\text{Max Rang} = 60\ m = \frac{cBT_s}{2B_s} \rightarrow T_s = \frac{60\ m \times 2B_s}{cB} = \frac{60 \times 2 \times 5 \times 10^9}{3 \times 10^8 \times 9.8 \times 10^6} = 204\ \mu s$$

However, since the N transmitters have to take turns transmitting, each frame will take $K \times N \times T_s$.

$$F = \frac{1}{K \times N \times T_s} \rightarrow K = \left\lfloor \frac{1}{F \times N \times T_s} \right\rfloor = \left\lfloor \frac{1}{40 \times 102 \times 204 \times 10^{-6}} \right\rfloor = 1$$

7. Who was right Dave or Dana? Justify your answer.

Dave was right. While the sweep time can be shorter, the MIMO transmitters have to take turns transmitting and hence there is no change in the time it takes to transmit a frame and i.e. we cannot average more sweeps if we want to maintain the same frame rate.

8. Susan suggests that Dave can still increase the number of sweeps he averages i.e. K by using a technique called Mult-shift FMCW where the each transmitter does not wait the entire T_s of the previous transmitter to finish before starting. It rather starts transmitting immediatel after the maximum reflection time i.e. the FMCW chirps are shifted from each other by the maximum reflection time rather than the sweep time. What is the maximum reflection time for this system?

Max Reflection Time = $2\times$ Max Range $/c = \frac{2\times60}{3\times10^8} = 0.4\mu s$

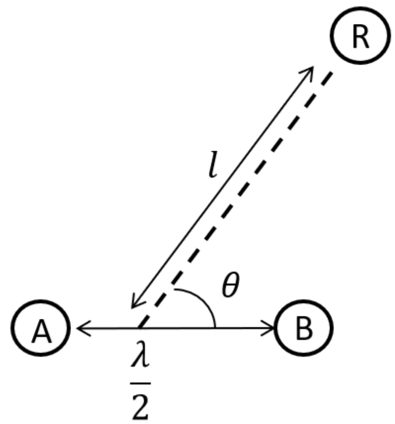
9. How many chirps can be averaged if Dave uses MIMO radar with Multi-Shift FMCW i.e. compute the new value of K?

Each frame will take $K \times N \times$ Max Reflection time.

$$F = \frac{1}{K \times N \times 0.4\ \mu s} \rightarrow K = \left\lfloor \frac{1}{F \times N \times 0.4\mu s} \right\rfloor = \left\lfloor \frac{1}{40 \times 102 \times 0.4 \times 10^{-6}} \right\rfloor = 612$$

3 Antenna Arrays **24 points**

1. Consider two antennas A and B separated by a distance of $\frac{\lambda}{2}$, and transmitting in-phase signals of wavelength λ. A receiver R is placed on a line passing through the mid-point of the two antennas and making an angle θ relative to them as shown in the figure below. The receiver is at a distance $\ell = k\lambda$ from the midpoint from some integer k. Determine the exact formula for the phase difference between signals of A and B that arrive at R as a function of k and θ.



$$\Delta\phi = 2\pi\frac{\Delta d}{\lambda} = \frac{2\pi}{\lambda}(d_{AR} - d_{BR})$$

$$d_{AR} = \sqrt{\ell^2 + (\lambda/4)^2 - 2\ell(\lambda/4)\cos(\pi - \theta)}$$

$$d_{BR} = \sqrt{\ell^2 + (\lambda/4)^2 - 2\ell(\lambda/4)\cos(\theta)}$$

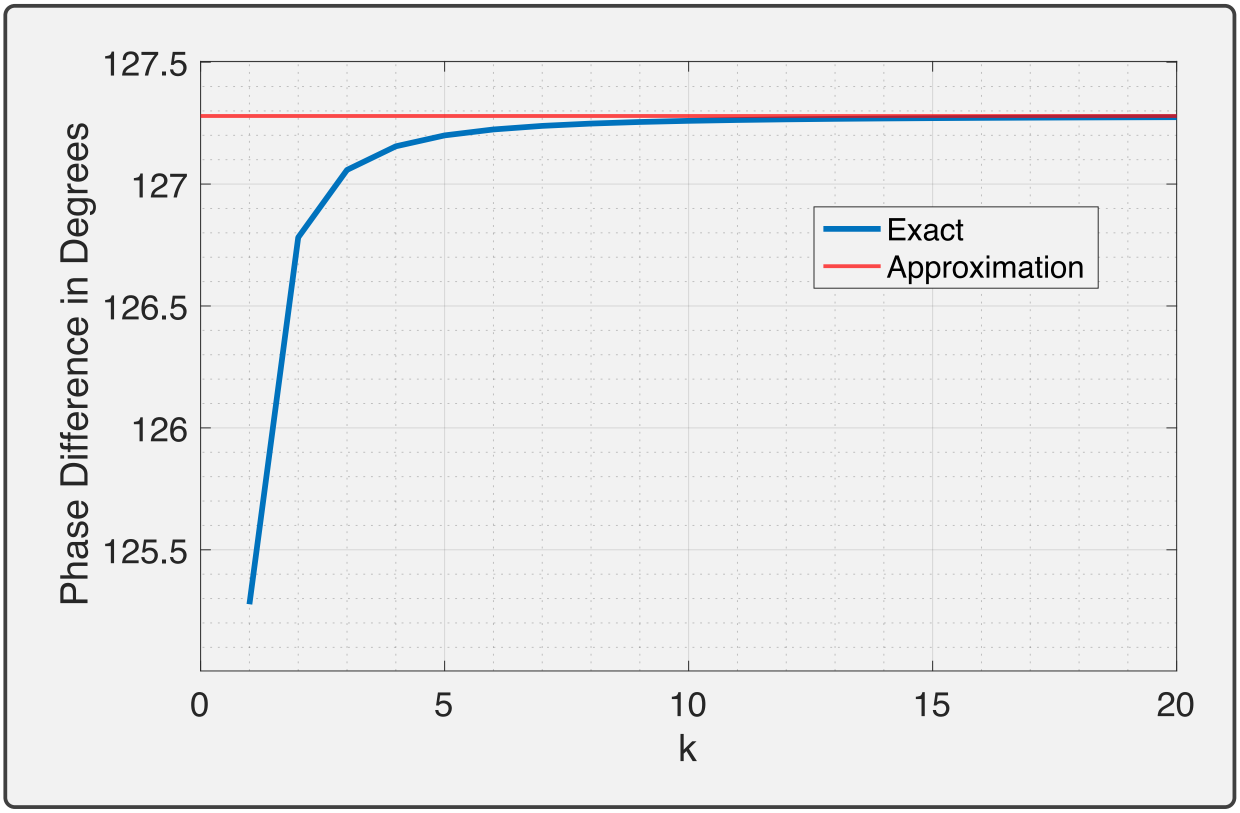
$$\Delta\phi = 2\pi\left(\sqrt{(\ell/\lambda)^2 + 1/16 + (\ell/2\lambda)\cos\theta} - \sqrt{(\ell/\lambda)^2 + 1/16 - (\ell/2\lambda)\cos\theta}\right)$$

$$= 2\pi\left(\sqrt{k^2 + 1/16 + k/2\cos\theta} - \sqrt{k^2 + 1/16 - k/2\cos\theta}\right)$$

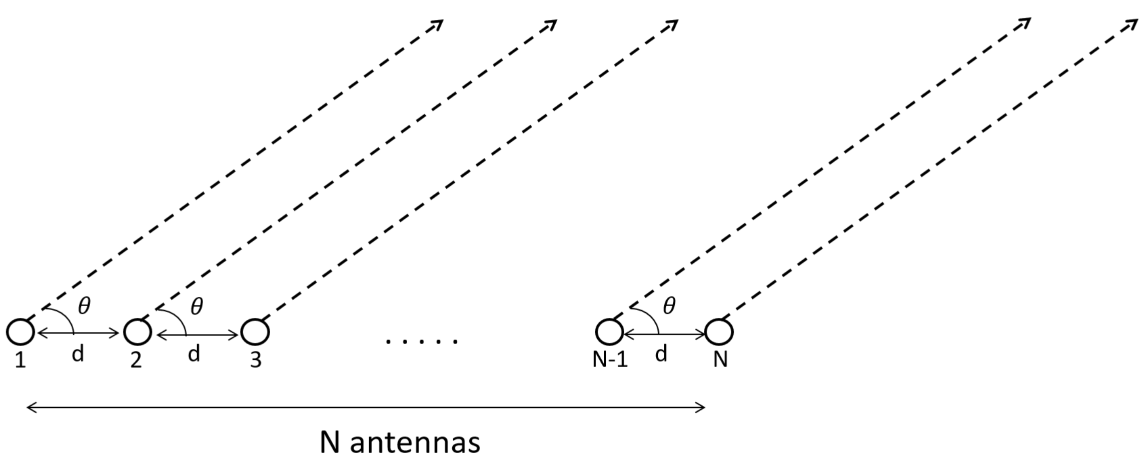
2. Assuming ℓ is very large, what is the approximate phase difference between signals from A and B that arrive at R.

$$\Delta\phi = 2\pi\frac{\Delta d}{\lambda} = 2\pi\frac{\lambda/2\cos\theta}{\lambda} = \pi\cos\theta$$

3. Plot the phase difference in degrees as a function of k when k varies from 1 to 20 and $\theta = 45°$. Also plot the phase difference using the approximation in part 2. (You can use your favorite plotting tool: Matlab, GNUPlot, Oracle, Python, ... but recall that the phase wraps around every 2π):



4. Consider a uniform linear array of N antennas as shown in figure below. The antenna separation is $d = \lambda/2$ as shown, and all antennas transmit signals in phase, at a wavelength of λ. Consider a receiver at an angle θ very far away from the antenna array. Since the receiver is far, we can assume that lines joining the antennas to the receiver are parallel to each other, making an angle θ above the horizontal line. Find all angles θ_{null} where the sum of arriving signals from all antennas add up to 0, creating a perfect null. (Hint: You can plot the beam pattern to get an idea of where the nulls are.)



$$P(\theta) = x(t)e^{j\phi}\sum_{k=1}^{N} e^{-j\pi k\cos\theta}$$

Goal: Find θ such that $P(\theta) = 0$.

Solution 1: Let $w = e^{-j\pi\cos\theta}$, then we have $P(\theta) = 0$ when

$$\sum_{k=1}^{N} w^k = 0$$

Since,$w \neq 0$, we have

$$1 + w + w^2 + w^3 + \cdots + w^{N-1} = 0$$

Using the geometric series formula, we get:

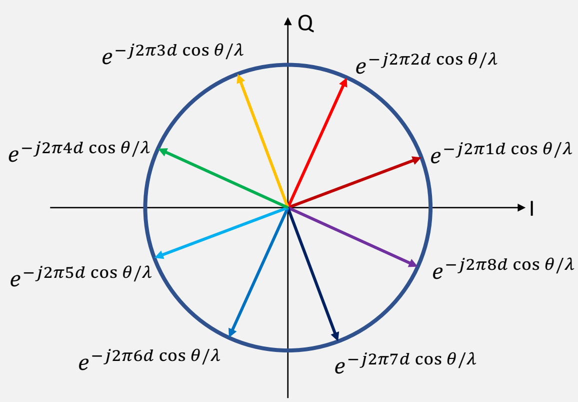
$$\frac{1-w^N}{1-w} = 0 \iff w^N = 1 \text{ and } w \neq 1 \iff e^{-j\pi N\cos\theta} = 1 \text{ and } e^{-j\pi\cos\theta} \neq 1$$

Hence, $\theta \neq \pi/2$ or $3\pi/2$ and

$$\pi N\cos\theta = 0 \mod 2\pi \Rightarrow \cos\theta = 2z/N \text{ for } z = 0, 1, 2, \cdots$$

$$\Rightarrow \theta = \cos^{-1}(2z/N) \text{ for } z = 0, 1, 2, \cdots$$

Solution 2:
We can think of the complex coefficients as vectors in the complex plane as shown in the figure below. Since all the complex coefficients have unit amplitude, they all lie on a unit circle. They are also equally spaced by an angle of $\pi\cos\theta$. They will clearly sum up to zero if they are equally distributed across the circle as show in the figure below.



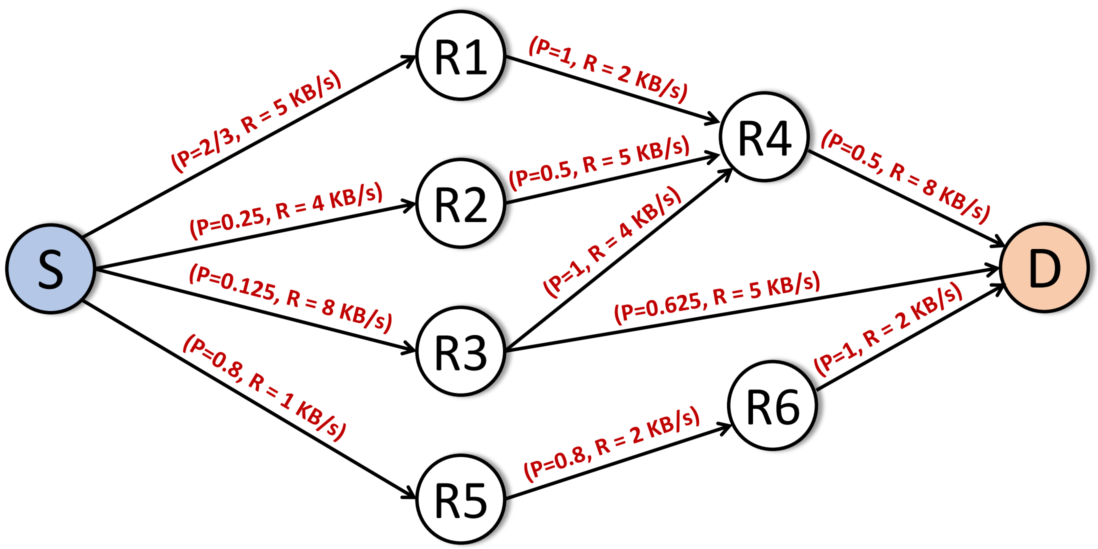
Hence, the vectors will sum up to zero when they form a complet 2π rotation i.e.

$$N\pi\cos\theta = 2\pi \mod 2\pi \Rightarrow \cos\theta = \frac{2z}{N} \text{ for } z = 0, 1, 2, \cdots \Rightarrow \theta = \cos^{-1}(2z/N) \text{ for } z = 0, 1, 2, \cdots$$

Solution 3: Derive the antenna array beam pattern as the discrete sinc function equation and find the zeros of the sinc function.

4 Multihop Networks **26 points**

Consider the wireless network shown below which is formed for multihop links from source S to destination D. Each link is labeled with two numbers. P is The delivery probability which is the combined delivery probabilities (i.e., the product of the forward and reverse link delivery probabilities). R is the data rate of each link in Kilobytes per second. If no arrow is shown between two nodes, then the nodes are out of range. Assume all packets are of equal size of 1000 bytes.



We will consider the following routing metrics:

1. Hop Count: Number of hops (links) along the path from S to D.
2. Bottleneck Probability: The minimum delivery probability along the path from S to D.
3. Bottleneck Data Rate: The minimum data rate along the path from S to D.
4. ETX: Expected Transmission Count.
5. ETT: Expected Transmission Time.
6. PSP: Packet Survival Probability: the probability that a packet is successful transmitted on all the hops from S to D i.e., it is not lost and no retransmissions are needed.

1. For each of the above metrics, specify whether we should pick the routing path that maximizes or minimizes the metric?

(a) Minimize Hop Count

(b) Maximize Bottleneck Probability

(c) Maximize Bottleneck Data Rate

(d) Minimize ETX

(e) Minimize ETT

(f) Maximize PSP

2. There are 5 possible paths between S and D. For each path, compute the above metric and fill in the below table. You do not need to justify your answer or show your calculations.

Route Number	Route	Hop Count	Bottleneck Probability	Bottleneck Data Rate	ETX	ETT	PSP
1	$S \rightarrow R1 \rightarrow R4 \rightarrow D$	3	0.5	2 KB/s	4.5	1.05	1/3
2	$S \rightarrow R2 \rightarrow R4 \rightarrow D$	3	0.25	4 KB/s	8	1.65	1/16
3	$S \rightarrow R3 \rightarrow R4 \rightarrow D$	3	0.125	4 KB/s	11	1.5	1/16
4	$S \rightarrow R3 \rightarrow D$	2	0.125	5 KB/s	9.6	1.32	5/16
5	$S \rightarrow R5 \rightarrow R6 \rightarrow D$	3	0.8	1 KB/s	3.5	2.375	16/25

3. For each of the above metrics, which path should the packet be routed on?

(a) Hop Count: 4

(b) Bottleneck Probability: 5

(c) Bottleneck Data Rate: 4

(d) ETX: 5

(e) ETT: 1

(f) PSP: 5

4. Which of the above 5 path delivers the highest average throughput? Justify your answer and compute this throughput in KB/s?

Minimizing ETT leads to maximizing the highest average throughput since it is equal to the transmitted data over the average transmission time. Hence, path 1 maximizes the average throughput which is 1000 bytes/ 1.05 = 0.952 KB/s.

5. Is it possible to achieve even higher throughput than what is computed in part 4? Justify your answer.

Yes, by using opportunistic routing or network coding, it is possible to achieve even higher throughput.