COM 405: Mobile Networks – Fall 2025
Homework 3
EPFL

Due Friday December 12 at 8:00pm

Instructions

- Homework is due Friday December 12 at 8:00pm on Moodle.
- Homework can be done in groups of two or individually.
- Homework can be submitted handwritten or typed. If handwritten, please make sure to have good handwriting. Anything we do not understand, we do not correct. Scan handwritten homework and Submit as pdf.
- You must justify your answer or show your calculations to all questions even if it is not explicitly stated in the question. Simply giving the final answer without showing any work will not get you any credit.
- If you find any typos, please do not hesitate to let us know.
- Recall, you do not need to ask us to submit the HW late. You can simply take advantage of the following late submission policy:

  $0-24$ hrs late: -0 points

  $24-48$ hrs late: -20 points

  $48-72$ hrs late: -40 points

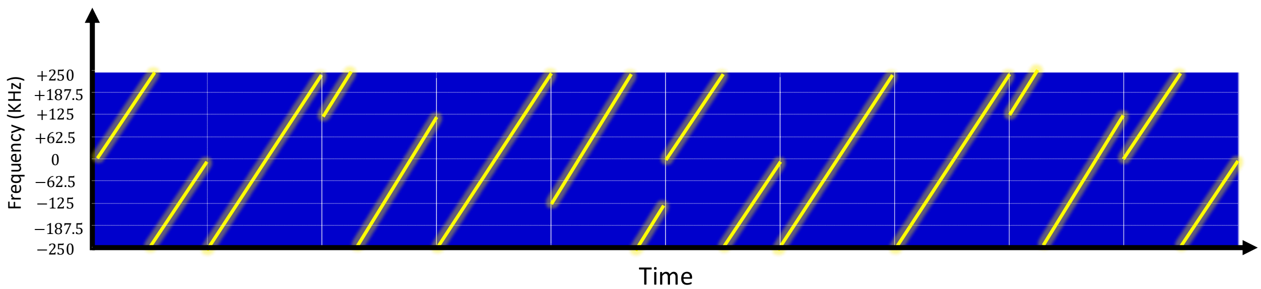
  >72 hrs late: -100 points

1 LoRA 20 points

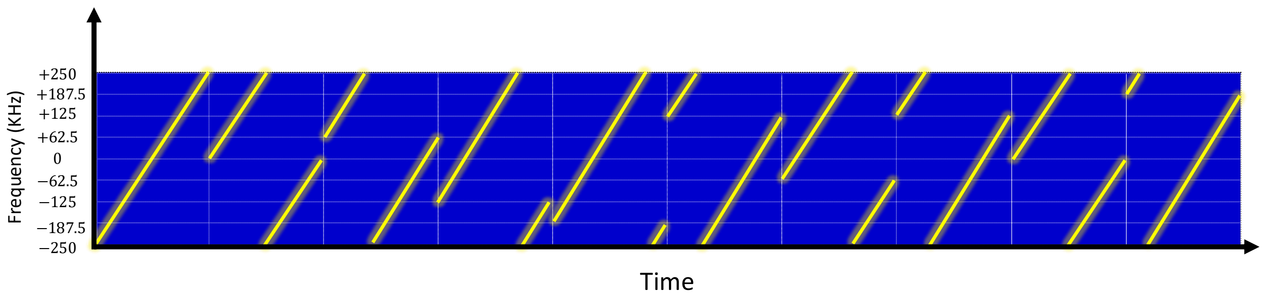
Consider the below spectrograms of LoRA signals. Assume LoRA uses a bandwidth of 500 kHz and the signal has the following equation with $\alpha = 200 MHz/sec$:

$$x(t) = \exp(j2\pi \frac{\alpha}{2} t^2 + j2\pi f_0 t)$$

(a)



(b)



Assume we use the following modulations in LoRA to map signals to bits:

f_0 (KHz)	-250	-187.5	-125	-62.5	0	$+62.5$	$+125$	$+187.5$
1st Order	'0'				'1'			
2nd Order	'00'		'01'		'10'		'11'	
3rd Order	'000'	'001'	'010'	'011'	'100'	'101'	'110'	'111'

1. Assuming all possible modulated chirps appear in the above spectrograms, what is the bit rate for each of the above LoRA signals?

2. Assume CFO and STO are both zero. What are the bits encoded in each of the above LoRA signals?

3. Assume the receiver has a $+12.4\ KHz$ CFO (Carrier Frequency Offset) from the transmitter but zero STO (Sampling Time Offset). After demodulating the chirp and taking the FFT, by how many FFT bins will the peak be shifted?

4. Assume that the CFO is zero but the receiver samples the spectrograms with an STO of 625 μs, i.e. the receiver start sampling 625 μs after the start of the signal. If the receiver does not correct for this offset, what will be the decoded bits for each spectrogram and how much bit errors will the receiver have for each signal.

2 Wireless Imaging 30 points

Dave wants to build a wireless imaging system using an FMCW transmitter and a 2D antenna array receiver. He needs a distance resolution of 3 cm and an angular resolution of 1° along both azimuth and elevation. He also needs the system to work for a maximum range of 60 m. To average noise, $K = 10$ FMCW sweeps are used and averaged together to create a single image frame. Dave needs a frame rate of $F = 40$ frames/sec. Assume, the speed of light is $c = 3 \times 10^8$ m/s.

1. What are the FMCW sweep bandwidth B_s and sweep time T_s needed to achieve the above range resolution and frame rate?

2. What is the sampling bandwidth B needed to achieve an imaging range of 60 m assuming the FFT window is taken over on FMCW sweep?

3. For a linear uniform antenna array, the angular resolution needed can be approxmiated as $0.89\frac{\lambda}{D}$ radians where λ is the wavelenght of the signal and D is the length of the array. Assuming, the antennas in the array are uniformly spaced by $\lambda/2$ and the 2D antenna array has $N \times N$ antennas, what is the value of N needed to achieve an angular resolution of 1° along both azimuth and elevation? (Hint: You do not need to know λ.)

4. Assume that every antenna in the receiver is connected to its own receiver front-end and ADC. How many samples/sec in total does this system generate i.e. aggregated across all the receivers?

5. Dave's machine can only handle 1 GigaSample/sec. Which system parameters do we need to change to ensure that we do not exceed 1 GS/s while maintaining the same distance resolution, angular resolution, maximum range, and frame rate? What are the values of these parameters?

6. Dana suggests to Dave that he can significantly reduce the cost of the system by using a MIMO radar with N transmitters and N receivers instead of his radar which has 1 transmitter and $N \times N$ receivers. In the MIMO radar, the N transmitters will use TDMA to avoid interfering i.e. they will take turns transmitting their FMCW chirps. Dana tells Dave that by using this MIMO radar, he can also reduce the sweep time T_s and hence, average more sweeps within each frame (i.e. increase K) while maintaining the same resolution, max range, and frame rate. Dave agrees that he will be able to reduce the sweep time T_s but he will not be able to average more sweeps (i.e. cannot increase K). Compute the new values of B, T_s, and K if Dave uses Dana's MIMO radar.

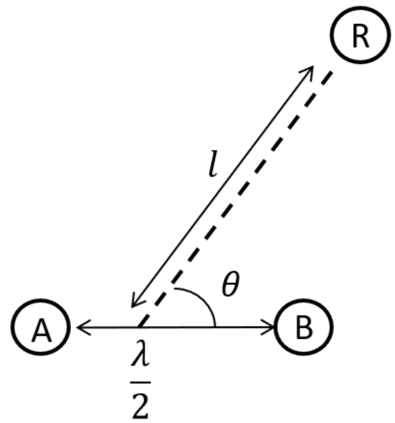
7. Who was right Dave or Dana? Justify your answer.

8. Susan suggests that Dave can still increase the number of sweeps he averages i.e. K by using a technique called Mult-shift FMCW where the each transmitter does not wait the entire T_s of the previous transmitter to finish before starting. It rather starts transmitting immediatel after the maximum reflection time i.e. the FMCW chirps are shifted from each other by the maximum reflection time rather than the sweep time. What is the maximum reflection time for this system?

9. How many chirps can be averaged if Dave uses MIMO radar with Multi-Shift FMCW i.e. compute the new value of K?

3 Antenna Arrays 24 points

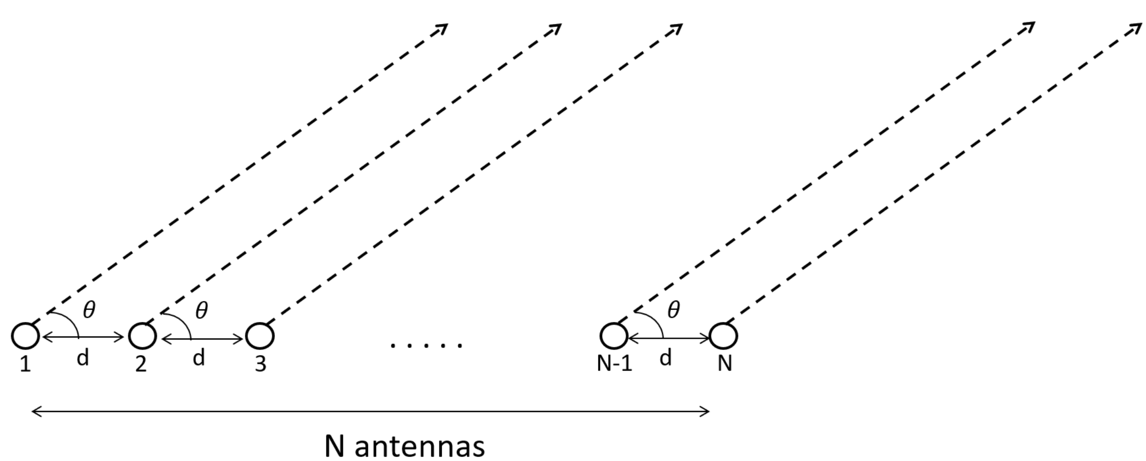
1. Consider two antennas A and B separated by a distance of $\frac{\lambda}{2}$, and transmitting in-phase signals of wavelength λ. A receiver R is placed on a line passing through the mid-point of the two antennas and making an angle θ relative to them as shown in the figure below. The receiver is at a distance $\ell = k\lambda$ from the midpoint from some integer k. Determine the exact formula for the phase difference between signals of A and B that arrive at R as a function of k and θ.



2. Assuming ℓ is very large, what is the approximate phase difference between signals from A and B that arrive at R.

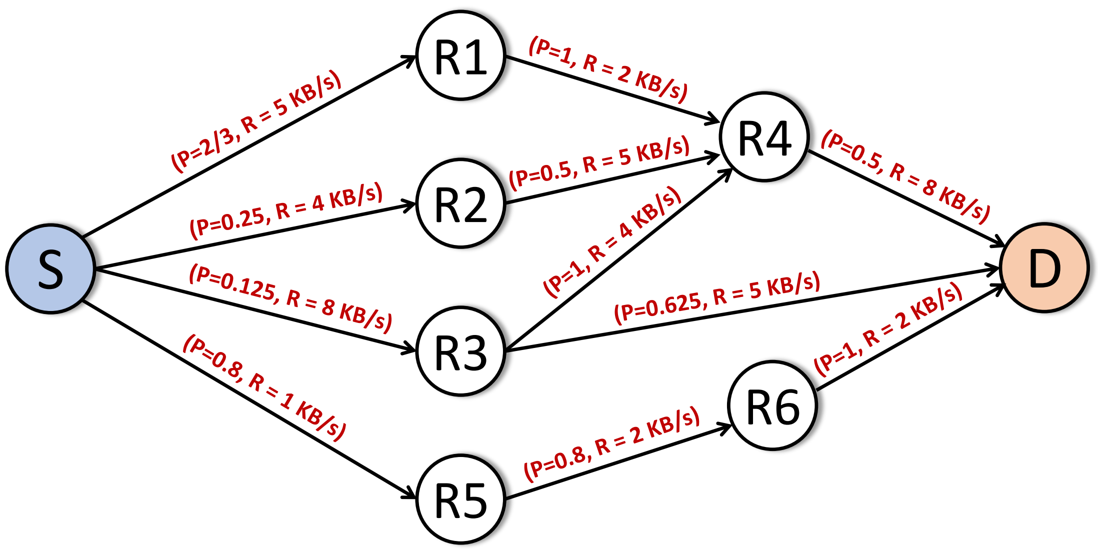
3. Plot the phase difference in degrees as a function of k when k varies from 1 to 20 and $\theta = 45°$. Also plot the phase difference using the approximation in part 2. (You can use your favorite plotting tool: Matlab, GNUPlot, Oracle, Python, ... but recall that the phase wraps around every 2π):

4. Consider a uniform linear array of N antennas as shown in figure below. The antenna separation is $d = \lambda/2$ as shown, and all antennas transmit signals in phase, at a wavelength of λ. Consider a receiver at an angle θ very far away from the antenna array. Since the receiver is far, we can assume that lines joining the antennas to the receiver are parallel to each other, making an angle θ above the horizontal line. Find all angles θ_{null} where the sum of arriving signals from all antennas add up to 0, creating a perfect null. (Hint: You can plot the beam pattern to get an idea of where the nulls are.)



4 Multihop Networks **26**

Consider the wireless network shown below which is formed for multihop links from source S to destination D. Each link is labeled with two numbers. P is The delivery probability which is the combined delivery probabilities (i.e., the product of the forward and reverse link delivery probabilities). R is the data rate of each link in Kilobytes per second. If no arrow is shown between two nodes, then the nodes are out of range. Assume all packets are of equal size of 1000 bytes.



We will consider the following routing metrics:

1. Hop Count: Number of hops (links) along the path from S to D.
2. Bottleneck Probability: The minimum delivery probability along the path from S to D.
3. Bottleneck Data Rate: The minimum data rate along the path from S to D.
4. ETX: Expected Transmission Count.
5. ETT: Expected Transmission Time.
6. PSP: Packet Survival Probability: the probability that a packet is successful transmitted on all the hops from S to D i.e., it is not lost and no retransmissions are needed.

1. For each of the above metrics, specify whether we should pick the routing path that maximizes or minimizes the metric?
2. There are 5 possible paths between S and D. For each path, compute the above metric and fill in the below table. You do not need to justify your answer or show your calculations.

Route Number	Route	Hop Count	Bottleneck Probability	Bottleneck Data Rate	ETX	ETT	PSP
1							
2							
3							
4							
5							

3. For each of the above metrics, which path should the packet be routed on?
4. Which of the above 5 path delivers the highest average throughput? Justify your answer and compute this throughput in KB/s?
5. Is it possible to achieve even higher throughput than what is computed in part 4? Justify your answer.