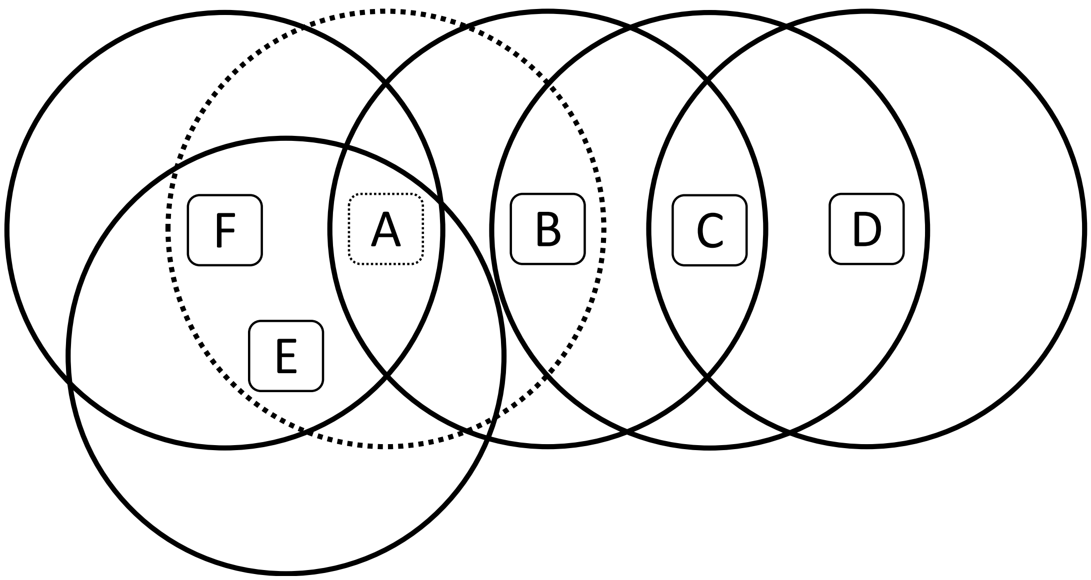
# COM 405: Mobile Networks – Fall 2024
# Homework 2 Solutions
# EPFL

## 1 Interference **22 points**

In the diagram below, each wireless node is shown along with its transmission radius. E.g., A's transmission radius is the circle with the dashed line.




1. List all the hidden terminals in the above wireless network?

   F and B, E and B, A and C, B and D.

2. The network uses CSMA/CA. When B wants to transmit to A it sends an RTS and A replies with a CTS to reserve the channel. Is this guaranteed to avoid collisions, explain why or why not?

   No, CTS or RTS might collide or get lost. Hence, a collision can still occur.

3. In 802.11, after node A receives the data packet successfully, it sends an ACK message. Why is it necessary to send an ACK message?

Because of hidden terminals, unreliable wireless channel with high loss rate, ...

4. Suppose the nodes F, A, B, C, D are equally spaced by a distance of $d$. Assume all nodes are identical and transmit at same power level on the same frequency. Also assume the signal attenuates based on free space pathloss equation presented in class.

   F is transmitting to A while C is transmitting to D. Compute the SINR of C's signal at D in the following cases?

   (a) The noise power at D is zero.

   $$SINR_{C-D} = \frac{RSSI_{C-D}}{Interference_{F-D} + Noise_D} = \frac{G(\lambda/(4\pi d))^2}{G(\lambda/(4\pi \times 4d))^2 + 0} = 16$$

   (b) The noise power at D is not zero and in the absence of any interference, the SNR of C's signal at D is 10.

   $$SNR_{C-D} = \frac{RSSI_{C-D}}{Noise_D} = \frac{G(\lambda/(4\pi d))^2}{Noise_D} = 10$$

   $$SINR_{C-D} = \frac{RSSI_{C-D}}{Interference_{F-D} + Noise_D} = \frac{G(\lambda/(4\pi d))^2}{G(\lambda/(4\pi \times 4d))^2 + G(\lambda/(4\pi d))^2/10} = \frac{80}{13}$$

## 2 WiFi MAC Fairness 10 points

Consider an 802.11 network with 2 clients connected to an AP. One client has a good channel to the AP that can sustain a bit rate of 54 Mb/s, while the second client has a bad channel to the AP that sustains only a bit rate of 2 Mb/s. Assume the MAC is perfectly fair and efficient, and has no overhead. Also, assume that bitrate adaptation is perfect and has converged to the optimal rates mentioned above.

1. What is the throughput that each client achieves to the AP when operating individually i.e. only one of the clients is present?

   **Answer:** In principle, they achieve throughputs of 54 Mb/s and 2 Mb/s respectively.

2. What is the throughput that each client achieves to the AP when the two clients operate jointly i.e. both clients contend and get to transmit the same number of packets on the channel?

   **Answer:** 802.11 MAC gives each node a fair chance to access the channel and then send their packets i.e. it has packet-based fairness. Assuming packet sizes are the same, since the MAC is fair, each node has a rate of

   $$\frac{1}{\frac{1}{54} + \frac{1}{2}} = 1.93Mbps.$$

## 3 CDMA Codes & GPS **26 points**

As mentioned in class, the most popular use cases of CDMA codes are 3G and GPS. Since 3G is being phased out, we will focus on GPS. GPS uses a CDMA code of length 1023 that is transmitted with 1.023 MHz bandwidth at a center frequency of 1.57542 GHz. The GPS receiver correlates with the codes of known satellites to lock on to each satellite i.e., decode the data bits and determine the relative delay between satellites.

1. Calculate the data rate of the data bits transmitted by the GPS satellite given the fact that each bit is repeated 20 times before encoding it using the CDMA code? Show your calculation.

   Symbol length $= 1023 \times (1/1.023 \text{ MHz}) = 1$ ms.
   Data bit length $= 1 \times 20 = 20$ ms.
   Data rate $= 1/(20 \times 10^{-3}) = 50$ bits/sec.

2. For BPSK, the BER $< 10^{-3}$ for SNR above 7 dB. The noise floor of the GPS receiver is -110 dBm.

   (a) At what power in (dBm) should the GPS signal be received to ensure that the BPSK coded bits are decoded with a BER less than $10^{-3}$? (Hint: GPS signals are typically received below the noise floor and correlating with the CDMA code brings the signal above the noise floor.)

   After correlating with the CDMA code, the SNR should be larger than 7 dB. Hence, the power should be larger than $-103$ dBm.

   The CDMA code increases the power by $10\log_{10}(1023) = 30.1$ dB. Thus, the GPS signal should be received with power larger than $-133.1$ dBm.

   (b) Assuming a GPS satellite is right above the receiver at a distance of 20200 km. Assume that the signal loss mainly comes from free space propagation pathloss. The GPS satellite uses an antenna with a gain of 12 dB and the GPS receivers uses an antenna with a gain of 4 dB. An additional 5 dB loss is incurred due to atmospheric absorption. What is the minimum transmit power in Watts of the GPS satellite that ensures the BPSK bits are decoded with BER less than $10^{-3}$?

   $$P_{RX} = \frac{G_{TX} G_{RX} \lambda^2}{(4\pi d)^2} \times Loss_{Atmosphere} \times P_{TX}$$

   $$P_{TX} = \frac{(4\pi d)^2}{G_{TX} G_{RX} \lambda^2 \times Loss_{Atmosphere}} \times P_{RX}$$

   $P_{RX} = 4.89 \times 10^{-17}$ Watts
   $Loss_{Atmospher} = 10^{-0.5} = 0.316$
   $G_{TX} = 10^{1.2} = 15.84$
   $G_{RX} = 10^{0.4} = 2.5$
   $d = 20.2 \times 10^6$ m
   $\lambda = 3 \times 10^8 / (1.57542 \times 10^9) = 0.19$ m

   $$P_{TX} = \frac{(4\pi \times 20.2 \times 10^6)^2}{15.84 \times 2.5 \times 0.19^2 \times 0.316} \times 4.89 \times 10^{-17} = 6.97 \text{ Watts}$$

(c) GPS satellites typically transmit at 44.8 Watts. Does this match the answer computed in the previous questions? Explain why or why not.

> This is answer is larger since the above calculation assumes perfect conditions and no blockage by building, no multipath, .... Hence, GPS transmits at roughly 6 times higher power than what was computed and the SNR after correlating with the CDMA code is typically larger than 7 dB.

3. The motion of the satellites creates a Doppler shift that can corrupt the correlation between the CDMA code and the received signal.

(a) Mathematically explain why the Doppler shift prevents us from finding the peak of the correlation.

> For simplicity, let us consider a single satellite. When we correlate with the code with a shift of $\tau$, we get:
>
> $$R(t) = \sum_{i=1}^{N} hdc(i-\tau)c(i-t)$$
>
> Where $d$ is the data bit, $h$ is the wireless channel, and $c$ is the CDMA code. When $t = \tau$, the correct shift, we get:
>
> $$R(\tau) = \sum_{i=1}^{N} hd|c(i-\tau)|^2 = Nhd$$
>
> The signal gets amplified by $N$ times and we get a peak. However, when there is a Doppler shift $\Delta f_D$, we get:
>
> $$R(t) = \sum_{i=1}^{N} hdc(i-\tau)e^{j2\pi\Delta f_D i}c(i-t)$$
>
> and at the correct shift, we get
>
> $$R(\tau) = hd\sum_{i=1}^{N} e^{j2\pi\Delta f_D i}$$
>
> which sums up to a small number and does not amplify the signal or create a peak.

(b) Assuming no CFO (carrier frequency offset) and satellites travel at a speed of 14000 km/hr, what is the maximum Doppler shift that the receiver sees.

> Doppler Shift $= v\cos\theta/\lambda \leq v/\lambda = 14000 \times 1000/3600 \times 1/0.19 = 20.46$ kHz.

(c) To deal with Doppler shift, GPS receivers typically do a search over Doppler shift. The receiver tries to correct for the Doppler shift and then does the correlation. The receiver tries 41 Doppler shifts between -10 kHz and +10 kHz. What is the resolution at which the Doppler shift is corrected for?

> Resolution: $20kH/40 = 500Hz$

(d) Give one advantage and one disadvantage of using higher resolution in correcting for the Doppler shit.

> Advantage of Higher Resolution: Higher SNR and better alignment.
>
> Disadvantage of Higher Resolution: Computational overhead of repeating the correlation from more Doppler shits.

## 4 Scheduling — 18 points

In a downlink single-cell cellular network (using TDD), all clients experience free space line-of-sight channels to the base station. In each time slot, only one client can be scheduled and the client will send a packet across the whole system bandwidth $B = 100$ MHz. The SNR gap is $\theta = 1.2$. The transmission power of the base station $P_{TX}$ is 1 Watt. The noise floor $N_0$ is -192.5 dBm/Hz. Five clients are located at 10, 50, 100, 150, and 250 meters aways from the base station. The base station and clients are operating at a carrier frequency $f_c = 2$ GHz.

Recall that the expected data rate of a terminal can be computed as:

$$S = B \times \log_2 \left(1 + \frac{\Gamma}{\theta}\right)$$

where $\Gamma$ is the SNR and can be calculated as:

$$\Gamma = \frac{P_{RX}}{N}$$

We have:

- $P_{RX} = \frac{\lambda^2}{(4\pi d)^2} P_{TX}$ where $d$ is the distance between the client and the base station, $P_{RX}$ is received power, and $\lambda = c/f_c$ with $c$ being the speed of light at $3 \times 10^8$ m/s.
- $N = N_0 \times B$ where $N$ is the total noise power, $N_0$ is the noise spectral desnity (noise floor), and $B$ is the bandwith. Remember to convert dBm/Hz to mW/Hz when you preform your calculations.

1. What are the expected downlink data rates of the clients that are at 10, 50, 100, 150, and 250 meters away from the base station? Indicate the steps that you are taking to find the solution and mention the unit of the computed values.

We have $\lambda = c/f_c = \frac{3\times10^8 m/s}{2\times10^9 Hz} = 0.15$m.

The noise power $N = N_0 \times B = 10^{-19.25} mW/Hz \times 100 \times 10^6 Hz = 5.62 \times 10^{-15}$ W.

For client at 10m:
$P_{RX,10} = \frac{0.15^2}{16\times\pi^2\times100} \times 1W = 1.42 \times 10^{-6} W. \Rightarrow \Gamma_{10} = 2.52 \times 10^8$
$\Rightarrow S_{10} = B\log_2(1 + \Gamma_{10}/1.2) = 2.764\ Gbps$

For client at 50m:
$P_{RX,50} = \frac{0.15^2}{16\times\pi^2\times2500} \times 1W = 5.86 \times 10^{-8} W. \Rightarrow \Gamma_{50} = 1.01 \times 10^7$
$\Rightarrow S_{50} = B\log_2(1 + \Gamma_{10}/1.2) = 2.3\ Gbps$

For client at 100m:
$P_{RX,100} = \frac{0.15^2}{16\times\pi^2\times10000} \times 1W = 1.42 \times 10^{-8} W. \Rightarrow \Gamma_{100} = 2.52 \times 10^6$
$\Rightarrow S_{100} = B\log_2(1 + \Gamma_{10}/1.2) = 2.1\ Gbps$

For client at 100m:
$P_{RX,150} = \frac{0.15^2}{16\times\pi^2\times10000} \times 1W = 6.31 \times 10^{-9} W. \Rightarrow \Gamma_{150} = 1.12 \times 10^6$
$\Rightarrow S_{150} = B\log_2(1 + \Gamma_{10}/1.2) = 1.983\ Gbps$

For client at 250m:
$P_{RX,250} = \frac{0.15^2}{16\times\pi^2\times62500} \times 1W = 2.27 \times 10^{-9} W. \Rightarrow \Gamma_{250} = 4.03 \times 10^5$
$\Rightarrow S_{250} = B\log_2(1 + \Gamma_{10}/1.2) = 1.835\ Gbps$

2. Assuming the constant from lecture 4.1, $\tau = 4$ and the throughput $S_i$ of all clients at $t = 0$ what is the average network data rate over 30 time slots if the scheduling algorithm is:

   (i) *Round Robin*

   (ii) *Max-Throughput*

(i) *Round Robin:* the network data rate will be equal to the sum of the clients' data rate divided by the number of terminals. Hence,

$$S_{Net} = \frac{1}{5}\sum S_i = 2.196\ Gbps$$

(ii) *Max-Throughput:* the network data rate will be equal to the max client data rate. Hence,

$$S_{Net} = \max S_i = 2.764\ Gbps$$

3. A sixth client moves from another cell to this cell's center and stops for communications at 250 meters away from the base station. Answer the following questions for both *Round Robin* and *Max-Throughput* scheduling:

(i) What is the (down-link) data rate of this client when it stops for communications?

(ii) What is the new average network data rate at that moment?

> (i) As the data rate for a terminal is determined by its position with respect to the base station, we directly obtain, from question 1, that $S_{250} = 1.835$ Gbps.
>
> (ii) For max-throughput, the network data rate does not change. For round robin, the average network data rate becomes:
>
> $$S_{Net} = \frac{1}{6} \sum S_i = 2.136\ Gbps$$

4. The client that was at 150 meters away from the base station has now moved and is at 37 meters away from the base station. Answer the following questions for both *Round Robin* and *Max-Throughput* scheduling:

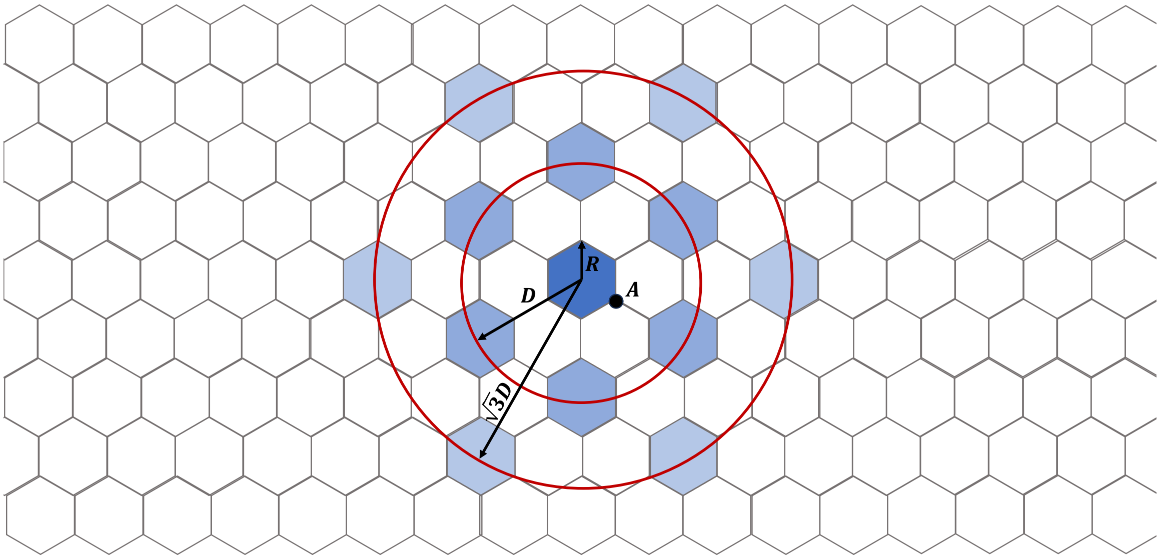
(i) What effect does this move have on the other clients' data rates?

(ii) How does the average network throughput change?

> (i) The other clients data rates are not affected.
>
> (ii) For max-throughput, the network data rate does not change. For round robin, the average network data rate increases as the number of clients remains constant and oneclient's data rate increases.

## 5 Cellular Networks 24 points

Consider a cellular system where the cell plan used is a symmetric hexagonal plan. The system under consideration is illustrated in the figure below. Note here that not all cells are colored in the picture. The blue colored hexagons in the figure represent co-channel cells (i.e., the cells using the same set of frequencies). These cells cause so called co-channel interference to any blue hexagon. The distance between the centers of the nearest co-channel cells denoted with $D$, and the radius of a cell with $R$.




In a symmertic hexagonal cell plan, each cell has exactly 6 co-channel cells at distance $D$ as shown in the figure above. In addition there are 6 co-channel cells at distance $\sqrt{3}D$, 6 at $\sqrt{4}D$, 6 at $\sqrt{7}D$, $\cdots$, $\sqrt{N}D$, and so forth, where $N = (i+j)^2 - ij$, $i, j = 0, 1, 2, 3, \cdots$. Recall that for the hexagonal cell plan, we have the following: $K = \frac{1}{3} \cdot (\frac{D}{R})^2$, where $K$ is the number of cells that divide the available channels in unique and disjoint channel groups of approcimately the same size.

Assume that the propagation model is a power law distance dependence model, that is, $P_r = cP_t d^{-\alpha}$, where $P_t$ is the transmitted power, $P_r$ is the received power, $d$ is the transmitter-receiver separation distance, $\alpha$ is the propagation loss exponent and $c$ is a constant. Also assume that the network is of infinite size, and that all base stations transmit at the same power $P$.

1. Find an expression for the co-channel interference (SIR) on the downlink channel (from a base station to a user) that a user at point $A$ in the above figure experiences. Express the SIR as a function of $\frac{D}{R}$ for the following two values of the propagation loss exponent $\alpha = 2$ and $\alpha = 4$. Assume that all interferers at the same interference tier are at the same distance from point $A$ i.e., all the first tier interferers are at distance $D$ from $A$, the second tier are at distance $\sqrt{3}D$, $\cdots$.

   *Hint 1:* Compute the $SIR$ at $A$ as the ratio between the received power at $A$ from the center cell (i.e., the signal) and the total received power at $A$ from the other cells (i.e., the interference).

   *Hint 2:* $1 + \frac{1}{3} + \frac{1}{4} + \frac{1}{7} + \cdots = \infty$

   *Hint 3:* $1 + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{7^2} + \cdots = 1.233$

Interference at terminal $A$ is caused by the co-channel cells of the cell where terminal A is located. Thus, the co-channel SIR at terminal $A$ is given by:

$$\begin{aligned}\Gamma_A = & \frac{\frac{cP}{R^\alpha}}{6\frac{cP}{D^\alpha} + 6\frac{cP}{(\sqrt{3}D)^\alpha} + 6\frac{cP}{(\sqrt{4}D)^\alpha} + 6\frac{cP}{(\sqrt{7}D)^\alpha} + \cdots} \\ = & \left(\frac{D}{R}\right)^\alpha \frac{1}{6} \frac{1}{1 + \frac{1}{3^{\alpha/2}} + \frac{1}{4^{\alpha/2}} + \frac{1}{7^{\alpha/2}} + \cdots} \\ = & \begin{cases} \left(\frac{D}{R}\right)^2 \frac{1}{6} \frac{1}{1+\frac{1}{3}+\frac{1}{4}+\frac{1}{7}+\cdots}, & \text{if } \alpha = 2 \\ \left(\frac{D}{R}\right)^4 \frac{1}{6} \frac{1}{1+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{7^2}+\cdots}, & \text{if } \alpha = 4 \end{cases} \\ = & \begin{cases} 0, & \text{if } \alpha = 2 \\ \left(\frac{D}{R}\right)^4 \frac{1}{6\times 1.233}, & \text{if } \alpha = 4 \end{cases}\end{aligned}$$

2. What is the problem with $\alpha = 2$? Why is this not a problem in practice?

For $\alpha = 2$, the co-channel interference at $A$ is infinite and the SIR is zero $\Gamma_A = 0$. Consequently, this cellular system can't work.

This is not a problem in practice because networks are finite while in thie problem we assume the network is infinite.

3. Consider that this celluar system has $C = 500$ channels, and a minimum signal-to-interference raio ($SIR$) of 19 dB is required in order to provide acceptable signal quality at a receiver. The radio capacity $\eta$ of a cellular system is defined as the number of users that can be supported in a single cell, that is, $\eta = \lfloor \frac{C}{K} \rfloor$ radio channels/cell (for fixed channel allocation). What is the radio capacity $\eta$ of this cellular system, assuming $\alpha = 4$?

The radio capacity can be obtrained from the following expression $\eta = \lfloor \frac{C}{K} \rfloor$. In our case, $C = 500$, and we still have to find $K$. We calculate $K$ as follows. Recall that $K = \frac{1}{3} \cdot (\frac{D}{R})^2$. We have:

$$\Gamma_A \geq 79.43 = 10^{1.9} \ (19 \ dB) \ \Rightarrow \left(\frac{D}{R}\right)^4 \frac{1}{6 \times 1.233} \geq 79.43$$

$$\Rightarrow \left(\frac{D}{R}\right)^2 \geq \sqrt{79.43 \times 6 \times 1.233} \Rightarrow \frac{1}{3} \cdot \left(\frac{D}{R}\right)^2 \geq \frac{24.24}{3} \Rightarrow K \geq 8.08$$

However, given the symmetric hexagonal cell plan, $K$ has to satisfy $K = (i+j)^2 - ij$, where $i, j = 0, 1, 2, 3, \cdots$ Hence, we set $K$ to 9. Finally, the radio capacity of this cellylar system is:

$$\eta = \left\lfloor \frac{C}{K} \right\rfloor = \left\lfloor \frac{500}{9} \right\rfloor = 55 \text{ radio channels /cell}$$