COM 405: Mobile Networks – Fall 2025
Homework 1 Solutions
EPFL

1 Wireless Channel 20 points

Julie is standing in a large open agricultural field 1 km away from a cellular base station on a near by hill operating at 3 GHz. The direct line-of-sight path dominates that channel impulse response such that we can ignore multipath and assume it is a single path channel.

1. Julie sees an SNR of 21 dB on her phone. Her phone connection will not work below an SNR of 7 dB as the bit error rate will be too high. How much farther from the base station can Julie go before her phone connection will stop working?

> Julie's SNR need to drop 14 dB so that her connection drops. Hence, the amount of received power needs to go down by $10^{1.4} = 25.12$. Since received power is porportional to $1/d^2$, the distance needs to incearse by $\approx 5\times$. Hence, Julie can go up to 5 km before her connection breaks (i.e. go 4 km farther).

Julie is still 1 km away from the base station. A large tractor came and parked next to Julie creating a second strong path between Julie and the base station. Now the wireless channel consists of two paths, the direct line-of-sight path and the reflected path. Due to reflection, the reflected path is received with $9\times$ lower power and has a distance of 1.00135 km. Assuming that Julie uses a narrowband channel that can be approximated with a single complex value, compute the new SNR that Julie sees in dB. Assume signals travel at the speed of light $c = 3 \times 10^8 m/s$

> We have to compute the new power received at Julie relative to the pervious power.
>
> $$SNR = \frac{P_{RX}}{Noise} = \frac{|h|^2 \times P_{TX}}{Noise}$$
>
> Let $h_d = |h_d|e^{j\phi_d} = |h_d|e^{j2\pi d_d/\lambda}$ be the channel seen by the direct path where $d_d = 1$ km is the length of the direct path and $\lambda = c/f = 10$ cm.
>
> Let $h_r = |h_r|e^{j\phi_r} = |h_r|e^{j2\pi d_r/\lambda}$ be the channel seen by the reflected path where $d_r = 1.00135$ km is the length of the reflected path.

We have $|h_d|^2 = 9|h_r|^2 \rightarrow |h_r| = |h_d|/3$.

$$\begin{aligned} h = h_d + h_r = |h_d|e^{j2\pi d_d/\lambda} + \frac{1}{3}|h_d|e^{j2\pi d_r/\lambda} &= |h_d|e^{j2\pi d_d/\lambda}\left(1 + \frac{1}{3}e^{j2\pi(d_r - d_d)/\lambda}\right) \\ &= |h_d|e^{j2\pi d_d/\lambda}\left(1 + \frac{1}{3}e^{j2\pi(135cm)/(10cm)}\right) \\ &= |h_d|e^{j2\pi d_d/\lambda}\left(1 + \frac{1}{3}e^{j2\pi(13.5)}\right) \\ &= |h_d|e^{j2\pi d_d/\lambda}\left(1 + \frac{1}{3}e^{j\pi}\right) \\ &= |h_d|e^{j2\pi d_d/\lambda}\left(1 - \frac{1}{3}\right) = \frac{2}{3}|h_d|e^{j2\pi d_d/\lambda} \end{aligned}$$

SNR with direct path only:

$$SNR_d = \frac{|h_d|^2 \times P_{TX}}{Noise}$$

New SNR with direct and reflected path is:

$$SNR_{d+r} = \frac{|h|^2 \times P_{TX}}{Noise} = \frac{4}{9}\frac{|h_d|^2 \times P_{TX}}{Noise} = \frac{4}{9}SNR_d$$

In log scale:

$$SNR_{d+r} = 10\log_{10}\left(\frac{4}{9}\right) + SNR_d = -3.52 + 21 = 17.48\ dB$$

2. Did the SNR increase or decrease? What is this phenomenon called?

It decreased. It is called "Frequency Selective Fading" or "Destructive Interference".

3. Julie decides to switch to a different frequency to maximize her SNR. Find the set of possible frequencies that maximize the SNR and compute the maximium SNR?

- For simplicity, assume that the frequency does not affect the attenuation of the channel $|h|$ but only affects the phase $\angle h$.
- Hint: Channel Phase for single path $\angle h = \phi = 2\pi d/\lambda \mod 2\pi = 2\pi df/c + 2k\pi$ for $k \in \mathbb{Z}$ where d is the distance, λ is the wavelength, f is the frequency, and c is the speed of light.

The SNR is maximized when the two paths sum up constructively, i.e., their phases are aligned.

$$h = h_d + h_r = |h_d|e^{j2\pi d_d/\lambda}\left(1 + \frac{1}{3}e^{j2\pi(d_r - d_d)/\lambda}\right)$$

Hence, we need

$$2\pi(d_r - d_d)/\lambda = 0 \mod 2\pi$$
$$\Rightarrow 2\pi(d_r - d_d)f/c = 2k\pi \quad k \in \mathbb{Z}$$
$$\Rightarrow f = ck/(d_r - d_d) = \frac{3 \times 10^8}{1.35}k = 0.22\bar{2}k \; GHz \quad k \in \mathbb{Z}$$

Hence, the frequencies that maximize the SNR are 222.2 MHz, 444.4 MHz, 666.6 MHz, 888.8 MHz, 1.111 GHz, 1.333 GHz, 1.555 GHz, ...
At these frequencies,

$$h = |h_d|e^{j2\pi d_d/\lambda}\left(1 + \frac{1}{3}e^{j2\pi(d_r - d_d)/\lambda}\right) = \frac{4}{3}|h_d|e^{j2\pi d_d/\lambda}$$

Maximum SNR:

$$SNR_{d+r} = 10\log_{10}\left(\frac{16}{9}\right) + SNR_d = 2.5 + 21 = 23.5 \; dB$$

4. Julie's hardware only supports frequencies in the range 2.9 GHz - 3.2 GHz. Which frequency should Julie use to maximize SNR?

We need:

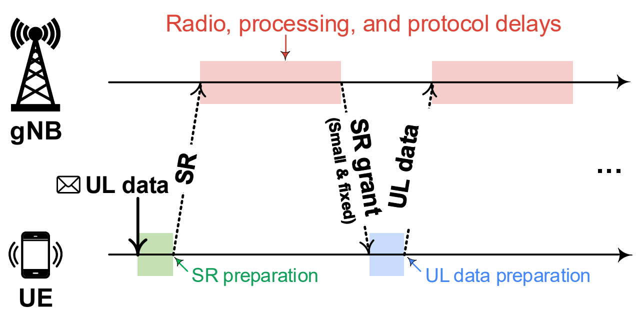
$$2.9 \; GHz \leq 0.22\bar{2}k \; GHz \leq 3.2 \; GHz \Rightarrow 13.05 \leq k \leq 14.4$$

Since $k \in \mathbb{Z}$, then $k = 14$ and Julie should use 3.11 GHz.

2 5G Uplink Scheduling **30 points**

Consider a 5G network that uses TDD (Time Division Duplexing) with numerology 1 where the time slot length 0.5 ms. The base station can use one of the below TDD patterns where UL stands for uplink slot, and DL stands for downlink slot.

Recall that for uplink transmission, the UE needs to send a scheduling request (SR) and once the basestation sends the UE an SR grant, the UE can start transmitting uplink data as shown in the figure below. For simplicity, assume that the amount of uplink data that the UE needs to send is small and fits in the initial resources that the base station allocates to the UE in the SR grant. Also assume that the SR can be sent in any uplink time slot, i.e., ignore the SR period.



Slot Number:	1	2	3	4	5	6	7	8	9	10	11	12
TDD Pattern 1	DL	DL	DL	DL	DL	UL	DL	DL	DL	DL	DL	UL
TDD Pattern 2	DL	DL	DL	UL	DL	DL	DL	UL	DL	DL	DL	UL
TDD Pattern 3	DL	DL	UL	UL	DL	DL	UL	UL	DL	DL	UL	UL
TDD Pattern 4	DL	UL	UL	UL	DL	UL	UL	UL	DL	UL	UL	UL
TDD Pattern 5	DL	UL	DL	UL	DL	UL	DL	UL	DL	UL	DL	UL

t = 0ms 1ms 2ms 3ms 4ms 5ms 6ms

Time (ms)

1. Compute the length of the TDD period in ms for each of the 5 TDD patterns.

> TDD Period 1 $= 6 \times 0.5$ ms $= 3$ ms.
>
> TDD Period 2 $= 4 \times 0.5$ ms $= 2$ ms.
>
> TDD Period 3 $= 4 \times 0.5$ ms $= 2$ ms.
>
> TDD Period 4 $= 4 \times 0.5$ ms $= 2$ ms.
>
> TDD Period 5 $= 2 \times 0.5$ ms $= 1$ ms.

2. Assume all radio, processing, and protocol delays are zero at both the UE and the gNB unlike the figure above. Hence, once a data packet arrives from the application at the UE, the SR request is sent in the immediate next uplink slot. Similarly, once an SR request is granted, the UE sends the data in the immediate next uplink slot. Finally, once the SR request arrives at the gNB, the SR grant is sent in the immediate next downlink slot. Note, however, if the data packet arrives at the UE in an uplink slot, it has to wait for the next uplink slot to send the SR and cannot send it in the current uplink slot.

   Define the latency as the time between when the packet arrives at the UE and the time the UE starts transmitting it on the air. For each of the five TDD patterns, compute the latency in ms for transmitting an uplink packet assuming the packet arrives at the UE at the beginning of time slot 1, $t = 0$ ms in the figure above. Which pattern exhibits the lowest latency?

TDD Pattern 1: The SR request is sent in slot 6 and the data is sent in slot 12. Hence, the latency is 11 time slots = 5.5 ms.

TDD Pattern 2: The SR request is sent in slot 4 and the data is sent in slot 8. Hence, the latency is 7 time slots = 3.5 ms.

TDD Pattern 3: The SR request is sent in slot 3 and the data is sent in slot 7. Hence, the latency is 6 time slots = 3 ms.

TDD Pattern 4: The SR request is sent in slot 2 and the data is sent in slot 6. Hence, the latency is 5 time slots = 2.5 ms.

TDD Pattern 5: The SR request is sent in slot 2 and the data is sent in slot 4. Hence, the latency is 3 time slots = 1.5 ms.

TDD Pattern 5 has the lowest latency.

3. Now assume that the packet can arrive at any time t within the first TDD period of each of the patterns below. Derive a formula as a function of t for the latency in each pattern in ms.

If the packet arrives during the downlink part of the first TDD period, then the SR can be transmitted in the first uplink slot of the first TDD period, otherwise the SR has to wait for the next uplink. Hence, we have to consider multiple cases for each of the TDD patterns.

TDD Pattern 1:
If packet arrives in slots 1 to 5: $5 \times 0.5\ ms - t + 6 \times 0.5\ ms$
If packet arrives in slot 6: $6 \times 0.5\ ms - t + (6 + 5) \times 0.5\ ms$

$$\text{Latency 1} = \begin{cases} 5.5\ ms - t \text{ if } t \leq 2.5\ ms \\ 8.5\ ms - t \text{ if } 2.5\ ms < t \leq 3\ ms \end{cases}$$

TDD Pattern 2:
If packet arrives in slots 1 to 3: $3 \times 0.5\ ms - t + 4 \times 0.5\ ms$
If packet arrives in slot 4: $4 \times 0.5\ ms - t + (4 + 3) \times 0.5\ ms$

$$\text{Latency 2} = \begin{cases} 3.5\ ms - t \text{ if } t \leq 1.5\ ms \\ 5.5\ ms - t \text{ if } 1.5\ ms < t \leq 2\ ms \end{cases}$$

TDD Pattern 3:
If packet arrives in slots 1 to 2: $2 \times 0.5\ ms - t + 4 \times 0.5\ ms$
If packet arrives in slot 3: $3 \times 0.5\ ms - t + 3 \times 0.5\ ms$
If packet arrives in slot 4: $4 \times 0.5\ ms - t + 6 \times 0.5\ ms$

$$\text{Latency 3} = \begin{cases} 3\ ms - t \text{ if } t \leq 1.5\ ms \\ 5\ ms - t \text{ if } 1.5\ ms < t \leq 2\ ms \end{cases}$$

TDD Pattern 4:
If packet arrives in slots 1: $1 \times 0.5\ ms - t + 4 \times 0.5\ ms$
If packet arrives in slot 2: $2 \times 0.5\ ms - t + 3 \times 0.5\ ms$
If packet arrives in slot 3: $3 \times 0.5\ ms - t + 2 \times 0.5\ ms$
If packet arrives in slot 4: $4 \times 0.5\ ms - t + 5 \times 0.5\ ms$

$$\text{Latency 4} = \begin{cases} 2.5\ ms - t \text{ if } t \leq 1.5\ ms \\ 4.5\ ms - t \text{ if } 1.5\ ms < t \leq 2\ ms \end{cases}$$

TDD Pattern 5:
If packet arrives in slots 1: $1 \times 0.5\ ms - t + 2 \times 0.5\ ms$
If packet arrives in slot 2: $2 \times 0.5\ ms - t + 3 \times 0.5\ ms$

$$\text{Latency 5} = \begin{cases} 1.5\ ms - t \text{ if } t \leq 0.5\ ms \\ 2.5\ ms - t \text{ if } 0.5\ ms < t \leq 1\ ms \end{cases}$$

4. Assume the packet is equally likely to arrive anytime during the first TDD period, i.e., uniform distribution of probability of arrival in the first TDD pattern. For each TDD patern, compute the max, min, and mean latency in ms.

TDD Pattern 1:
Min Latency 1 = 3 ms
Max Latency 1 = 6 ms
Mean Latency 1 = $(5/6) \times (5.5 + 3)/2 + (1/6) \times (5.5 + 6)/2 = 4.5$ ms

TDD Pattern 2:
Min Latency 2 = 2 ms
Max Latency 2 = 4 ms
Mean Latency 2 = $(3/4) \times (3.5 + 2)/2 + (1/4) \times (3.5 + 4)/2 = 3$ ms

TDD Pattern 3:
Min Latency 3 = 1.5 ms
Max Latency 3 = 3.5 ms
Mean Latency 3 = $(3/4) \times (3 + 1.5)/2 + (1/4) \times (3 + 3.5)/2 = 2.5$ ms

TDD Pattern 4:
Min Latency 4 = 1 ms
Max Latency 4 = 3 ms
Mean Latency 4 = $(3/4) \times (2.5 + 1)/2 + (1/4) \times (3 + 2.5)/2 = 2$ ms

TDD Pattern 5:
Min Latency 5 = 1 ms
Max Latency 5 = 2 ms
Mean Latency 5 = $(1/2) \times (1.5 + 1)/2 + (1/2) \times (2 + 1.5)/2 = 1.5$ ms

5. Which pattern minimizes uplink latency distribution? What is a drawback of using such pattern in practice if any?

> TDD Pattern 5 minimizes latency. The drawback is the equal amount of resources are dedicated to uplink and downlink which in many cases does not match the users needs are today users still require more downlink bandwidth than uplink.

6. Now assume that the transmission, radio, processing, and protocol delays are 0.7 ms on the gNB and 0.9 ms on the UE. Recompute the latency in ms for each of the 5 patterns assuming the packet arrives at the UE at the beginning of time slot 1, $t = 0$ in the figure above. Note, that the gNB and the UE cannot transmit in the same slot when the delays due to radio, processing, and protocol are done as resources for this slot have already been allocated. They always need to wait for the next slot.

TDD Pattern 1:

(a) Data arrives at $t = 0$

(b) SR request is ready at $t = 0.9$ ms (slot 2)

(c) SR request is sent at $t = 2.5$ ms (slot 6)

(d) SR grant is ready at $t = 3.2$ ms (slot 7)

(e) SR grant is transmitted at $t = 3.5$ ms (slot 8)

(f) UE is ready to transmit the data at $t = 4.4$ ms (slot 9)

(g) UE tranmists the data at 5.5 ms (slot 12).

Hence, the latency is 5.5 ms.

TDD Pattern 2:

(a) Data arrives at $t = 0$

(b) SR request is ready at $t = 0.9$ ms (slot 2)

(c) SR request is sent at $t = 1.5$ ms (slot 4)

(d) SR grant is ready at $t = 2.2$ ms (slot 5)

(e) SR grant is transmitted at $t = 2.5$ ms (slot 6)

(f) UE is ready to transmit the data at $t = 3.4$ ms (slot 7)

(g) UE tranmists the data at 3.5 ms (slot 8).

Hence, the latency is 3.5 ms.

TDD Pattern 3:

(a) Data arrives at $t = 0$

(b) SR request is ready at $t = 0.9$ ms (slot 2)

(c) SR request is sent at $t = 1$ ms (slot 3)

(d) SR grant is ready at $t = 1.7$ ms (slot 4)

(e) SR grant is transmitted at $t = 2$ ms (slot 5)

(f) UE is ready to transmit the data at $t = 2.9$ ms (slot 6)

(g) UE tranmists the data at 3 ms (slot 7).

Hence, the latency is 3 ms.

TDD Pattern 4:

(a) Data arrives at $t = 0$

(b) SR request is ready at $t = 0.9$ ms (slot 2)

(c) SR request is sent at $t = 1$ ms (slot 3)

(d) SR grant is ready at $t = 1.7$ ms (slot 4)

(e) SR grant is transmitted at $t = 2$ ms (slot 5)

(f) UE is ready to transmit the data at $t = 2.9$ ms (slot 6)

(g) UE tranmists the data at 3 ms (slot 7).

Hence, the latency is 3 ms.

TDD Pattern 5:

(a) Data arrives at $t = 0$

(b) SR request is ready at $t = 0.9$ ms (slot 2)

(c) SR request is sent at $t = 1.5$ ms (slot 4)

(d) SR grant is ready at $t = 2.2$ ms (slot 5)

(e) SR grant is transmitted at $t = 3$ ms (slot 7)

(f) UE is ready to transmit the data at $t = 3.9$ ms (slot 8)

(g) UE tranmists the data at 4.5 ms (slot 10).

Hence, the latency is 4.5 ms.

7. Which TDD pattern yields the lowest latency in question 6? Is it the same as the one in question 2? What does this tell us about which pattern is best?

   TDD patterns 3 and 4 have the lowest latency in question 6 but 5 has the lowest in question 2 so it is not the same. This tells us that the latency heavily depends on the the radio, processing, and protocol delays as well as the packet arrival time.

3 OFDM **30 points**

Consider a wireless system using OFDM with a bandwidth of 80 MHz, $N = 512$ and the subcarriers are used as follows:

- DC Bins: $-1, 0, +1$
- Guard Bins: -256 to -246 and $+245$ to $+255$
- Pilot Bins: $\pm 25, \pm 53, \pm 89, \pm 117, \pm 139, \pm 167, \pm 203, \pm 231$
- Data Bins: All other subcarriers.

The system uses 8 preamble symbols as shown in the figure below: 4 for packet detection, 2 for CFO estimation, and 2 for channel estimation. The cyclic prefix is set to 0.8 μs.

The transmitter can choose between 3 modulation and coding schemes:

- MCS 0: BPSK with coding rate 1/2
- MCS 1: 16 QAM with coding rate 3/4
- MCS 2: 64 QAM with coding rate 2/3
- MCS 3: 1024 QAM with coding rate 3/4

1. Identify an error in the above figure that will prevent decoding the OFDM packet properly? Explain why is it an error and how to fix it.

   There should be a cyclic prefix after the last preamble. If packet detection identifies the start of the packet as few samples into the packet, then the FFT of the last preamble will contain samples from the first data symbol. Moreover, the ISI from the last preamble will impact the first data symbol. Hence, we should add a cyclic prefix of the preamble before starting data symbols.

2. Compute the data rate as described in lecture 3 for MCS0, MCS1, MCS2, and MCS3.

   Data Rate = Bandwidth $\times$ Modulation Rate $\times$ Coding Rate $\times(1-$ Overhead)

   Overhead $= \frac{\#GuardBins + \#PilotBins + \#DCBins + CP}{N + CP} = \frac{22+16+3+64}{512+64} = \frac{105}{576}$

   MCS 0: Data Rate $= 80 \times 10^6 \times 1 \times 1/2 \times (471/576) = 32.708$ Mbps.

   MCS 1: Data Rate $= 80 \times 10^6 \times 4 \times 3/4 \times (471/576) = 196.25$ Mbps.

   MCS 2: Data Rate $= 80 \times 10^6 \times 6 \times 2/3 \times (471/576) = 261.66$ Mbps.

   MCS 3: Data Rate $= 80 \times 10^6 \times 10 \times 3/4 \times (471/576) = 490.62$ Mbps.

3. Suppose each transmitted packet must contain 1500 bytes of data bits. How long does it take to transmit the entire packet including preambles for each of MCS0, MCS1, MCS2, and MCS3.

Packet Time
$= 8\times$Preamble Time $+CP +$ #Data Symbols$\times$(Symbol Time $+CP$)
$= 8 \times 512/(80 \times 10^6) + 0.8\mu s +$ #Data Symbols$\times(512/(80 \times 10^6) + 0.8\mu s)$
$= 8 \times 512/(80 \times 10^6) + 0.8\mu s +$ #Data Symbols$\times(512/(80 \times 10^6) + 0.8\mu s)$
$= 52\mu s +$ #DataSymbols$\times 7.2\mu s$

$$\#\text{DataSymbols} = \left\lceil \frac{\#\text{Coded Bits}}{\#\text{Bits in OFDM Symbol}} \right\rceil = \left\lceil \frac{\#\text{Data Bits/Coding Rate}}{\text{Modulation Rate} \times \#\text{Data Bins}} \right\rceil$$

MCS 0: #DataSymbols $= \left\lceil \frac{1500\times8/(1/2)}{1\times471} \right\rceil = 51 \rightarrow$ Packet Time$= 52\mu s + 51\times7.2\mu s = 419.2\mu s$

MCS 1: #DataSymbols $= \left\lceil \frac{1500\times8/(3/4)}{4\times471} \right\rceil = 9 \rightarrow$ Packet Time $= 52\mu s + 9 \times 7.2\mu s = 116.8\mu s$

MCS 2: #DataSymbols $= \left\lceil \frac{1500\times8/(2/3)}{6\times471} \right\rceil = 7 \rightarrow$ Packet Time $= 52\mu s + 7 \times 7.2\mu s = 102.4\mu s$

MCS 3: #DataSymbols $= \left\lceil \frac{1500\times8/(3/4)}{10\times471} \right\rceil = 4 \rightarrow$ Packet Time $= 52\mu s + 4 \times 7.2\mu s = 80.8\mu s$

Notes:
-2pts if CP of the preamble is not accounted for.
-4pts if # data bits is used instead of coded bits.

4. We can compute the data rate by dividing the number of data bits transmitted in the packet by the total time to transmit the packet. Compute the data rate using this method for each of MCS0, MCS1, MCS2, and MCS3.

Data Rate $= 1500 \times 8/$Packet Time

MCS 0: Data Rate $= 12000/419.2\mu s = 28.6$ Mbps

MCS 1: Data Rate $= 12000/116.8\mu s = 102.7$ Mbps

MCS 2: Data Rate $= 12000/102.4\mu s = 117.18$ Mbps

MCS 3: Data Rate $= 12000/80.8\mu s = 148.51$ Mbps

5. Do the data rates computed in questions 2 and 4 match? If not, explain why and which one is more accurate?

> They do not match because the original computation does not account for the overhead of the preamble. The data rates computed in question 4 are more accurate.

6. Compute the relative error between the data rates computed in questions 2 and 4 for MCS0, MCS1, MCS2, and MCS3 (relative to the more accurate values)? How does the error vary with the MCS? Explain why the error varies this way.

> MCS 0: $100 \times (32.708 - 28.6)/28.6 = 14.36\%$
>
> MCS 1: $100 \times (196.25 - 102.7)/102.7 = 91.09\%$
>
> MCS 2: $100 \times (261.66 - 117.18)/117.18 = 123.29\%$
>
> MCS 3: $100 \times (490.62 - 148.51)/117.18 = 230.35\%$
>
> The error increases for larger MCS because the overhead of the preamble becomes larger as we go to higher order modulation and have higher data rates. This is because the number of data symbols needed becomes smaller whereas the number of preamble symbols remains constant.

7. The preamble is sent with BPSK modulation. Would it make sense to transmit the preamble at higher order modulation to reduce its overhead?

> No, the number of preamble symbols does not decrease if we send it at higher modulation as the preamble is used for synchronization, channel estimation, etc. which do not change with irrespective of the order of modulation.

8. Propose one possible solution to reduce the overhead of the preamble.

> Aggregate packets together or increase the number of data bits in the packet.

4 MIMO 20 points

For this problem, the following assumptions hold:

- All the participating nodes are in the radio range of each other
- The frequency offset between transmitter i and receiver j, Δf_{ij}, is known only at receiver j. The transmitters do not know the frequency offsets and each receiver knows only the frequency offsets between itself and the transmitters.
- The channel between any transmit-receive antenna pair is known to all transmitters, i.e., all h_{ij} are known to all transmitter nodes.
- assume the h_{ij}'s are all non-zero.

1. Consider Figure 1, where a 2-antenna AP transmits to two one-antenna clients A and B. The AP has two symbols p_1 and p_2, and wants to deliver p_1 to client A, and p_2 to client B.

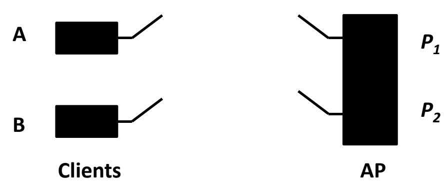


Figure 1: A 2-antenna AP transmitting to 2 clients, each with one antenna.

Assume there are no frequency offsets, i.e., $\Delta f_{ij} = 0$ for any i and j. Let n_i be the noise, then the symbols received at the clients y_A and y_B are given by:

$$\begin{bmatrix} y_A \\ y_B \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

If the transmitting AP does not zero-force by precoding the transmitted symbols p_1 and p_2, neither client can decode its symbol. Find a precoding matrix M such that if the AP multiplies the transmitted symbols by M before transmitting, i.e.:

$$[M] \begin{bmatrix} p_1 \\ p_2 \end{bmatrix},$$

then each client receives its respective symbol without interference. If no such matrix exist, then explain why this is the case.

$$M = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} = \frac{1}{h_{11}h_{22} - h_{12}h_{21}} \begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix}$$

2. Again, consider the scenario in Figure 1, but assume now that there are **different and nonzero** frequency offsets Δf_{11} and Δf_{21} between the transmitting AP and the two receivers. The symbols at the receivers are now given by

$$\begin{bmatrix} y_A \\ y_B \end{bmatrix} = \begin{bmatrix} h_{11}e^{j2\pi\Delta f_{11}t} & h_{12}e^{j2\pi\Delta f_{11}t} \\ h_{21}e^{j2\pi\Delta f_{21}t} & h_{22}e^{j2\pi\Delta f_{21}t} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Can the AP still zero-force at the transmitter by precoding with a matrix M such that both clients can successfully decode their respective symbols without interference? If "yes", what is M and how do the clients decode, if "no", then why?

Yes.

$$M = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} = \frac{1}{h_{11}h_{22} - h_{12}h_{21}} \begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} y_A \\ y_B \end{bmatrix} &= \begin{bmatrix} e^{j2\pi\Delta f_{11}t} & 0 \\ 0 & e^{j2\pi\Delta f_{21}t} \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \\ &= \begin{bmatrix} e^{j2\pi\Delta f_{11}t} & 0 \\ 0 & e^{j2\pi\Delta f_{21}t} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \end{aligned}$$

By tracking their own frequency offset, each receiver can correctly decode their respective symbols.

3. Let us now consider the scenario shown in Figure 2, where we have two single-antenna APs instead. AP1 has symbol p_1, which it wants to deliver to client A, and AP2 has symbol p_2, which it wants to deliver to client B.

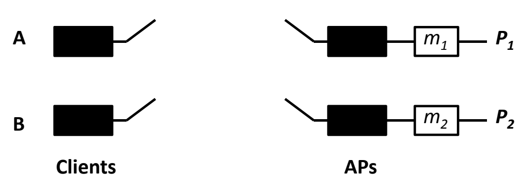


Figure 2: Two one-antenna APs transmitting to two one-antenna clients

Assume there is no frequency offset, i.e., $\Delta f_{ij} = 0$. Does there exist two multipliers m_1 and m_2 such that each transmitted symbol is received at its intended receiver without any interference? If so, what are these multipliers? If not, why not? (Note that both APs know all channels h_{ij} between all senders and receivers.)

No. Assume there exist two such multipliers. Then,

$$\begin{bmatrix} y_A \\ y_B \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} m_1 p_1 \\ m_2 p_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$
$$= \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$
$$= \begin{bmatrix} m_1 h_{11} & m_2 h_{12} \\ m_1 h_{21} & m_2 h_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

According to the above equation, for each transmitted symbol to be received at the corresponding receiver with no interference, the off-diagonal terms in the matrix

$$\begin{bmatrix} m_1 h_{11} & m_2 h_{12} \\ m_1 h_{21} & m_2 h_{22} \end{bmatrix}$$

should be zero $\Rightarrow m_1 = m_2 = 0$. But this will prevent delivering any signal to the receiver. Hence, no such m_1 and m_2 exist.

4. Now let's assume that the APs are connected by a back-end Ethernet and can communicate the transmitted symbols to each other as shown in Figure 3. Hence, both of them now know p_1 and p_2. Still we need to deliver p_1 to client A and p_2 to client B.

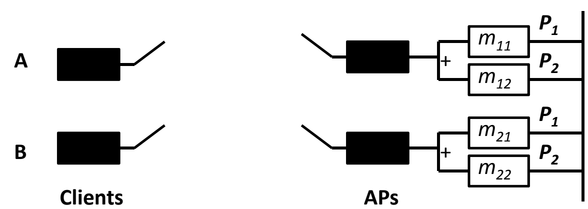


Figure 3: Two one-antenna APs transmitting to two one-antenna clients. The APs however can exchange symbols over a backend Ethernet

Does there exist multipliers m_{11}, m_{12}, m_{21} and m_{22} such that each client can receive the symbol intended for it without any interference? Assume that there is no frequency offsets (i.e., $\Delta f_{ij} = 0$ for all i and j).

Yes,

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1}$$

5. Consider again the scenario shown in Figure 3. Let's now assume that there are non-zero frequency offests, i.e., $\Delta f_{11}, \Delta f_{12}, \Delta f_{21}$ and Δf_{22} are nonzero and different. Does there exist m_{11}, m_{12}, m_{21} and m_{22} such that client A will receive symbol p_1 without any interference, and client B will receive symbol p_2 without any interference?

$$\begin{bmatrix} y_A \\ y_B \end{bmatrix} = \begin{bmatrix} e^{j2\pi\Delta f_{11}t}h_{11} & e^{j2\pi\Delta f_{12}t}h_{12} \\ e^{j2\pi\Delta f_{21}t}h_{21} & e^{j2\pi\Delta f_{22}t}h_{22} \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$
$$= \begin{bmatrix} m_{11}e^{j2\pi\Delta f_{11}t}h_{11} + m_{21}e^{j2\pi\Delta f_{12}t}h_{12} & m_{12}e^{j2\pi\Delta f_{11}t}h_{11} + m_{22}e^{j2\pi\Delta f_{12}t}h_{12} \\ m_{11}e^{j2\pi\Delta f_{21}t}h_{21} + m_{21}e^{j2\pi\Delta f_{22}t}h_{22} & m_{11}e^{j2\pi\Delta f_{21}t}h_{21} + m_{21}e^{j2\pi\Delta f_{22}t}h_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

According to the above equation, for each transmitted symbol to be received at the corresponding receiver with no interference, the off-diagonal terms in the matrix

$$\begin{bmatrix} m_{11}e^{j2\pi\Delta f_{11}t}h_{11} + m_{21}e^{j2\pi\Delta f_{12}t}h_{12} & m_{12}e^{j2\pi\Delta f_{11}t}h_{11} + m_{22}e^{j2\pi\Delta f_{12}t}h_{12} \\ m_{11}e^{j2\pi\Delta f_{21}t}h_{21} + m_{21}e^{j2\pi\Delta f_{22}t}h_{22} & m_{11}e^{j2\pi\Delta f_{21}t}h_{21} + m_{21}e^{j2\pi\Delta f_{22}t}h_{22} \end{bmatrix}$$

should be zero $\Rightarrow$ m_i depend on Δf_{ij}. But the transmitters don't know Δf_{ij} and hence cannot make the transmission interference free at the receivers.