Selected topics on advanced composites in engineering structures

Lecture II – Failure

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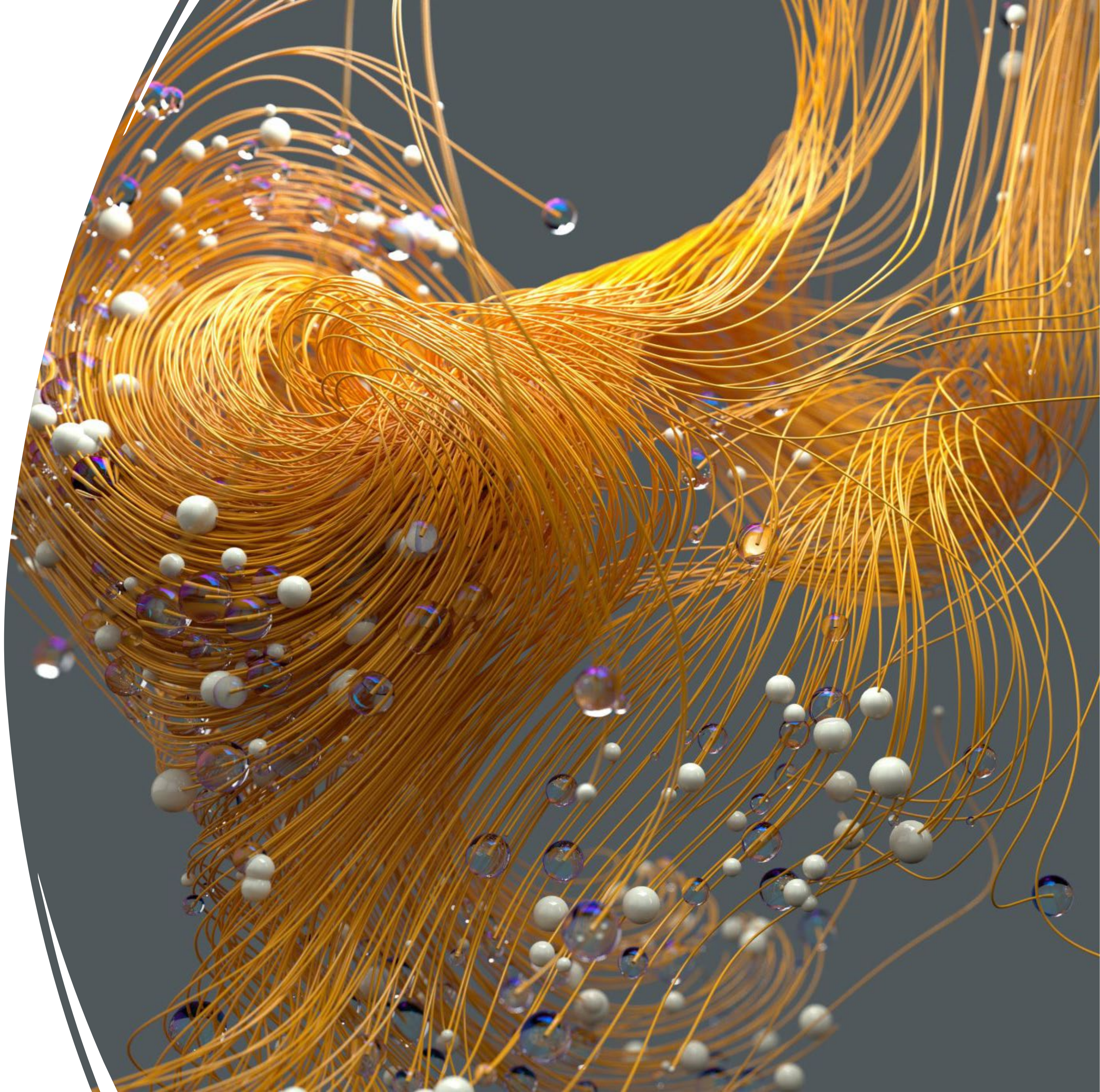
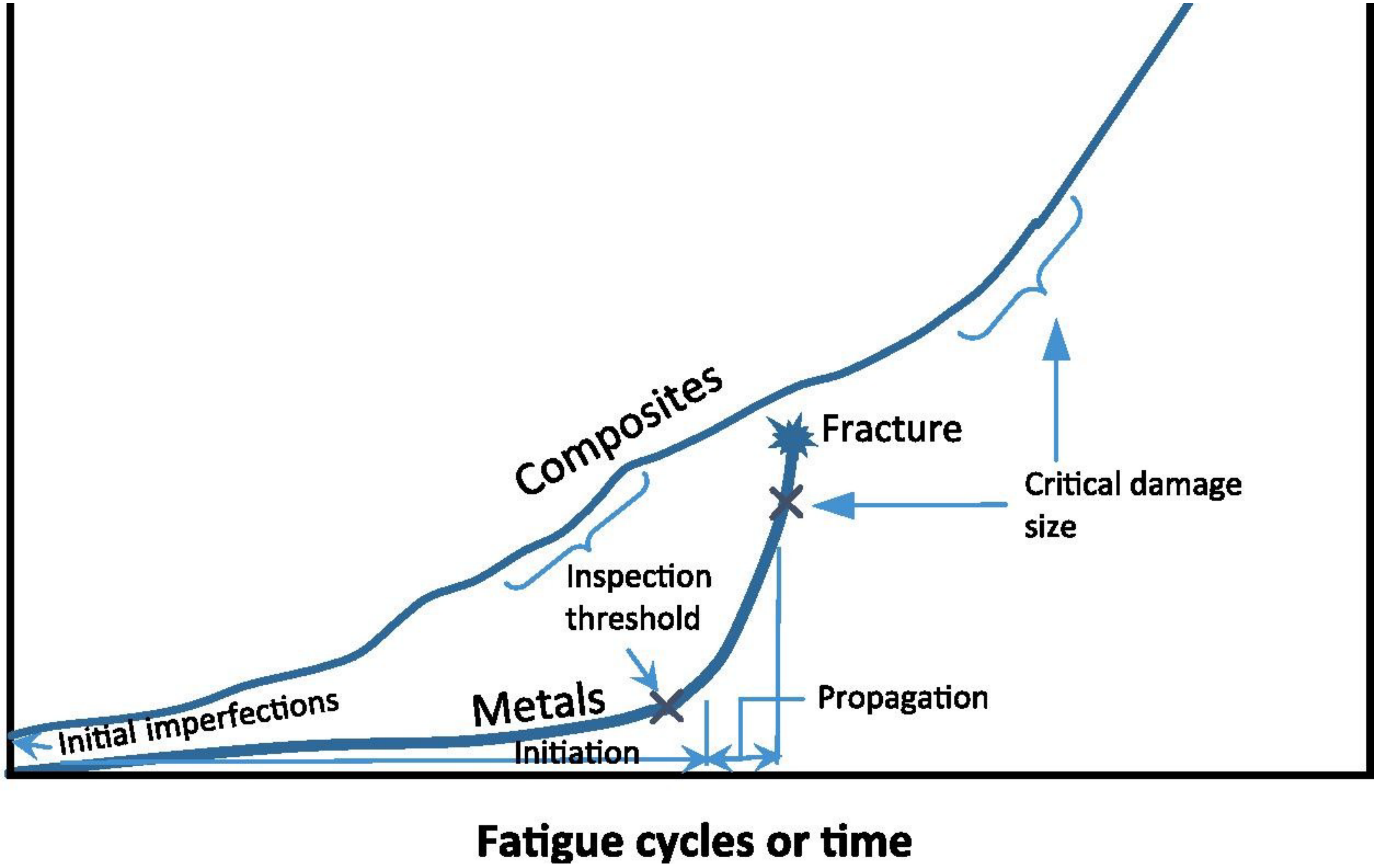
Composite failure

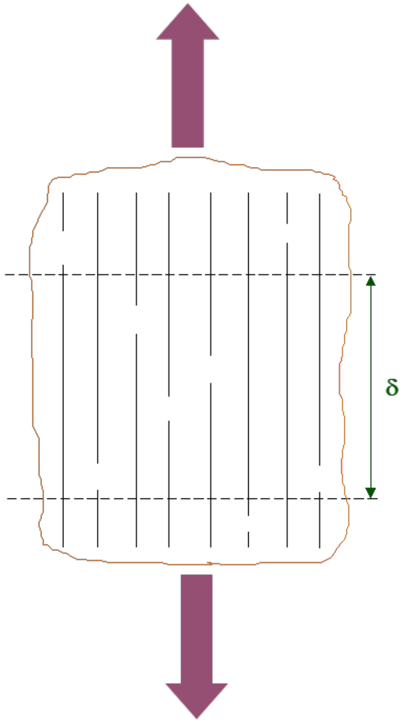
- **Composites** fail in a different way compared to metals
- Exhibit different failure modes
- Brittle fibers in a ductile matrix
  - Sudden failure
  - Crazing and matrix cracking might occur
  - Delamination and debonding



Metals
- Crack length

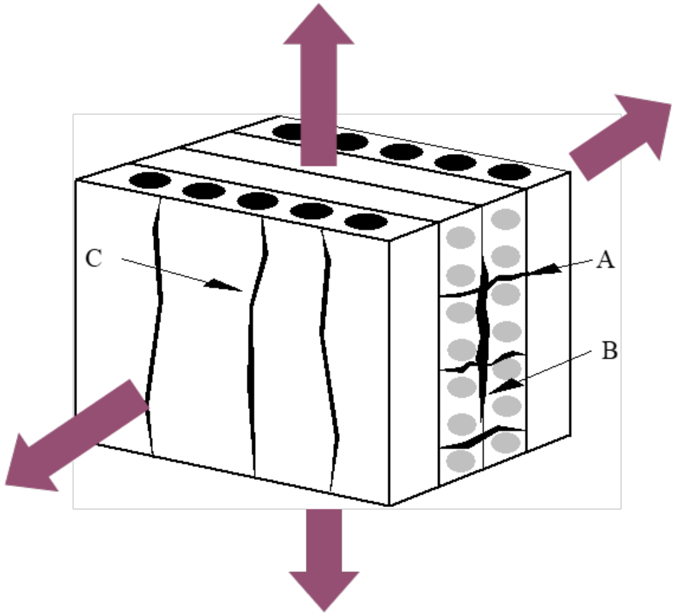
Composites
- Broken fibers
- Delamination
- Matrix cracking
- Composite cracking
- Debonding
- Voids





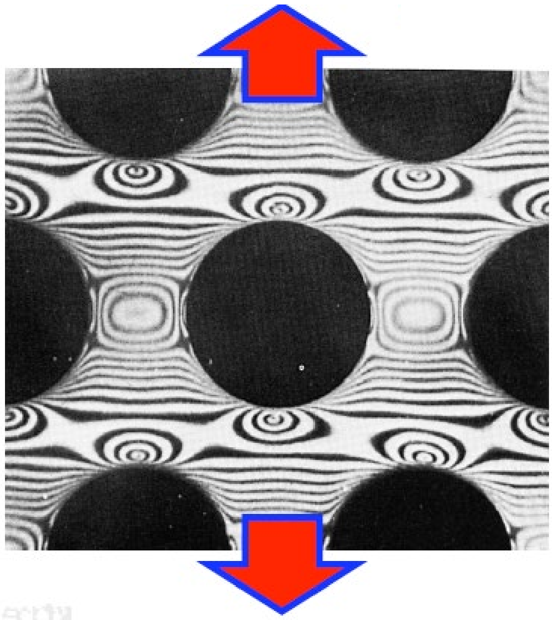
Tensile Failure

Compression Failure

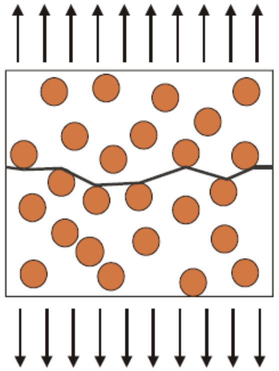


Matrix Cracking
& Delamination

Damage Modes

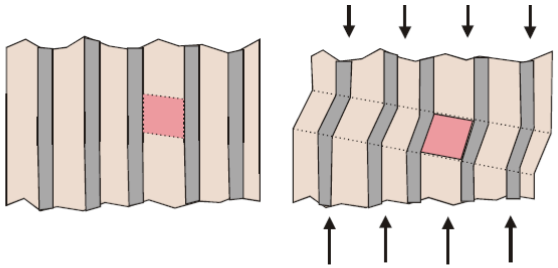


Transverse tension

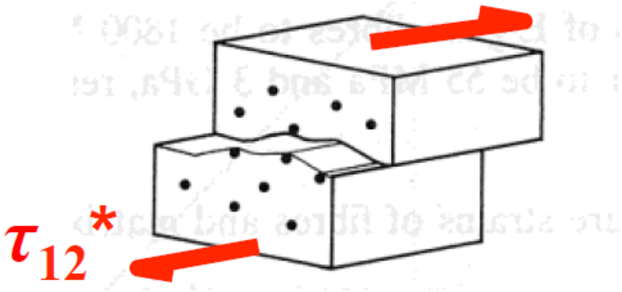


Damage Modes

Axial compression

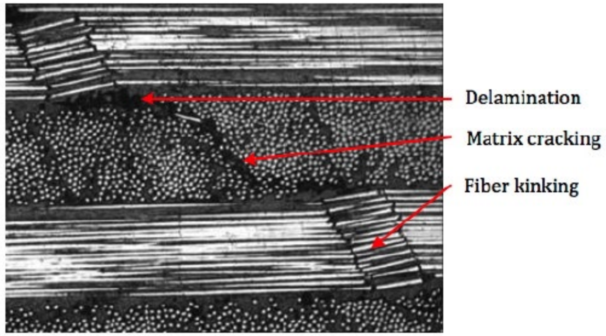


Axial shear

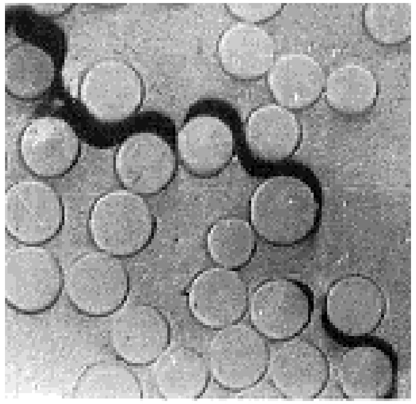


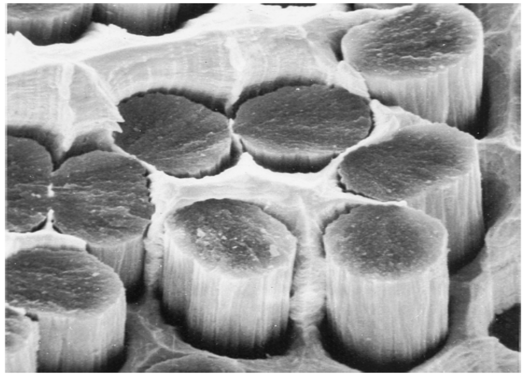
Typical failures of composites (damage mode)

- Matrix failure (brittle matrices)
- Fiber-matrix debonding (soft matrices)
- Delamination



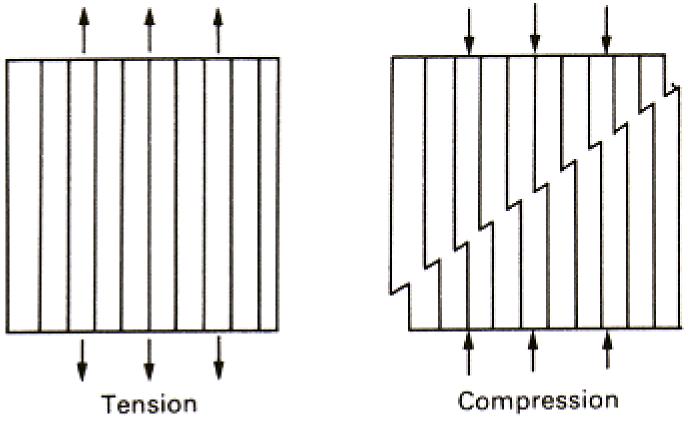
S. Pinho. Modelling failure of laminated composites using physically-based failure models. PhD thesis, Imperial College London, 2005.

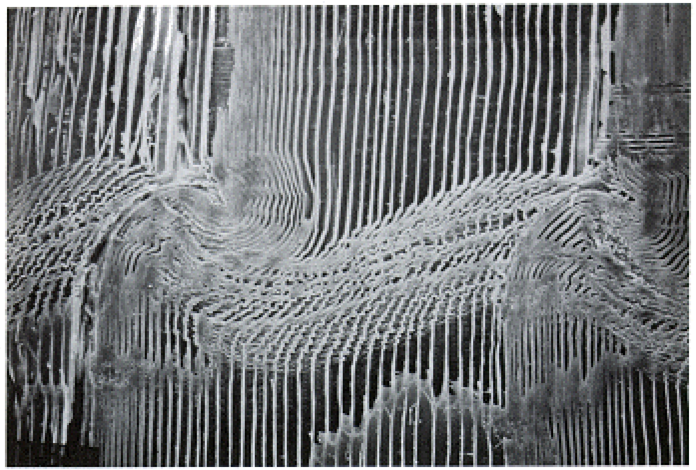




Typical (compressive) failure of composites

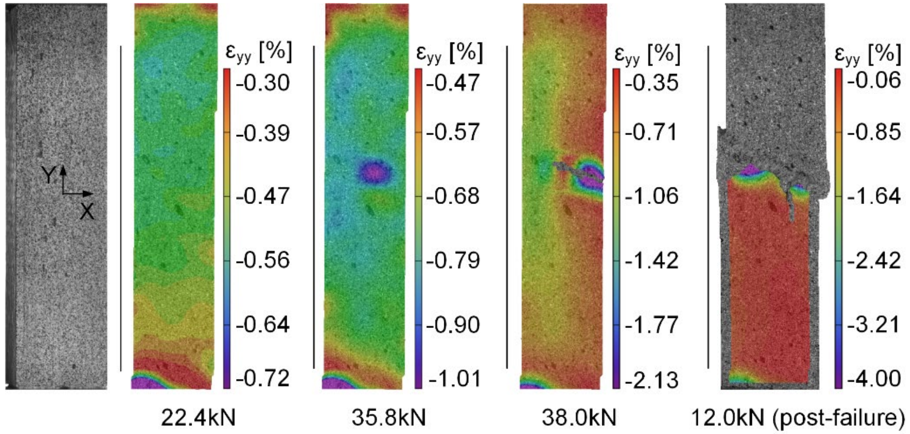
- Kinking & crushing





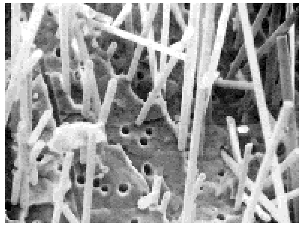
Typical (compressive) failure of composites

- Kinking & crushing

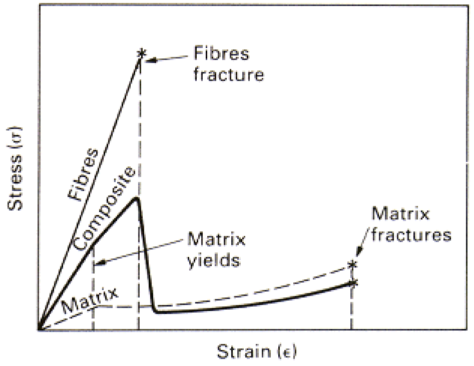


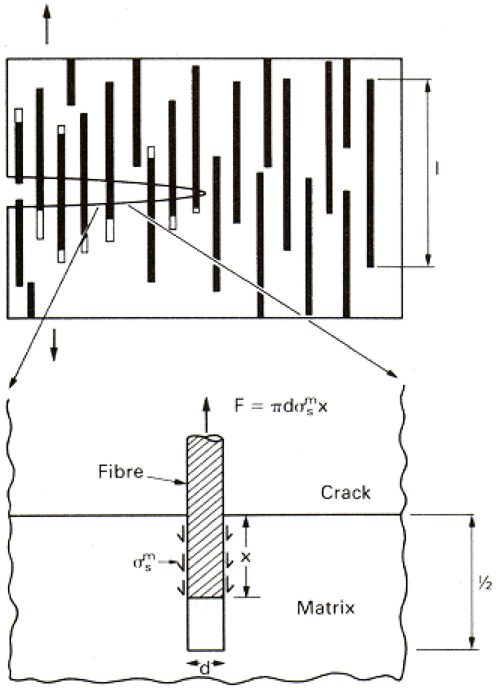
Failure at the micro- and macroscale

Fiber pull-out & fiber failure

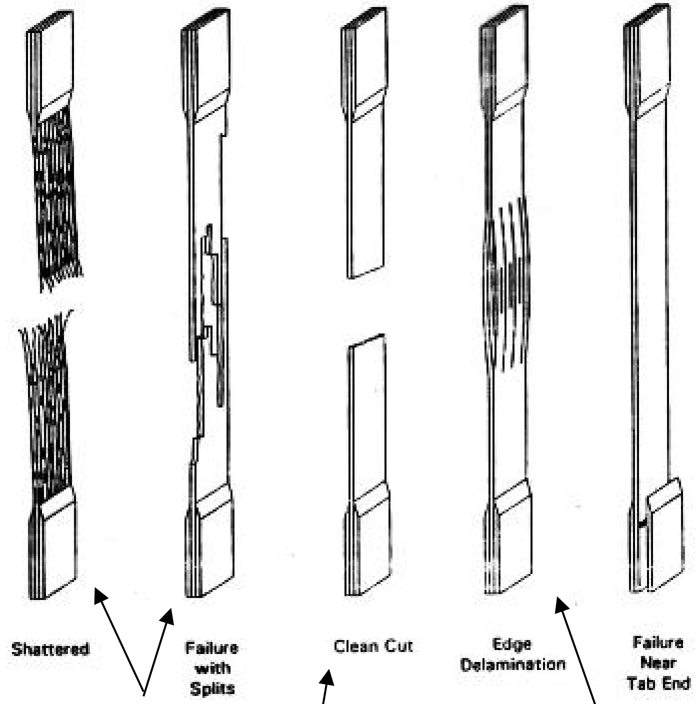


Stress strain curve of a composite





Typical Failure Modes in Straight-Sided Coupons



(Unacceptable)

(Acceptable & common in unidirectional specimens)

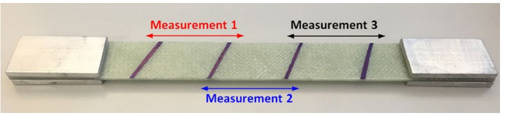
(Acceptable & common in 90° or 90° dominated layups)

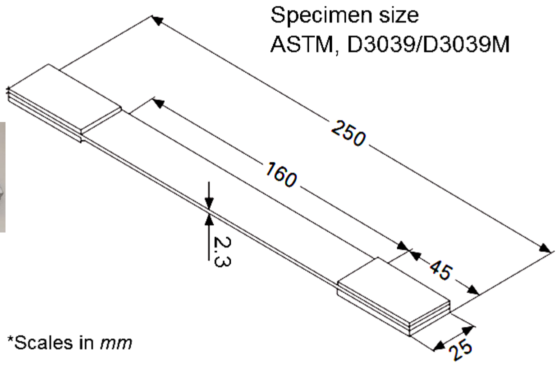
(May be found in cross-ply layups; unacceptable)

Specimens and manufacturing

Fabrication:

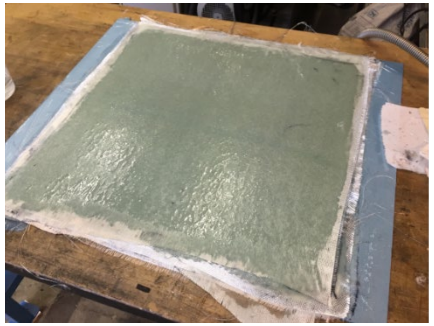
- ❖ Used material: Glass/epoxy (GFRP)
- ❖ Specimen layup $(\pm 45)_{2s}$
- ❖ Vacuum assisted hand-lay up technique



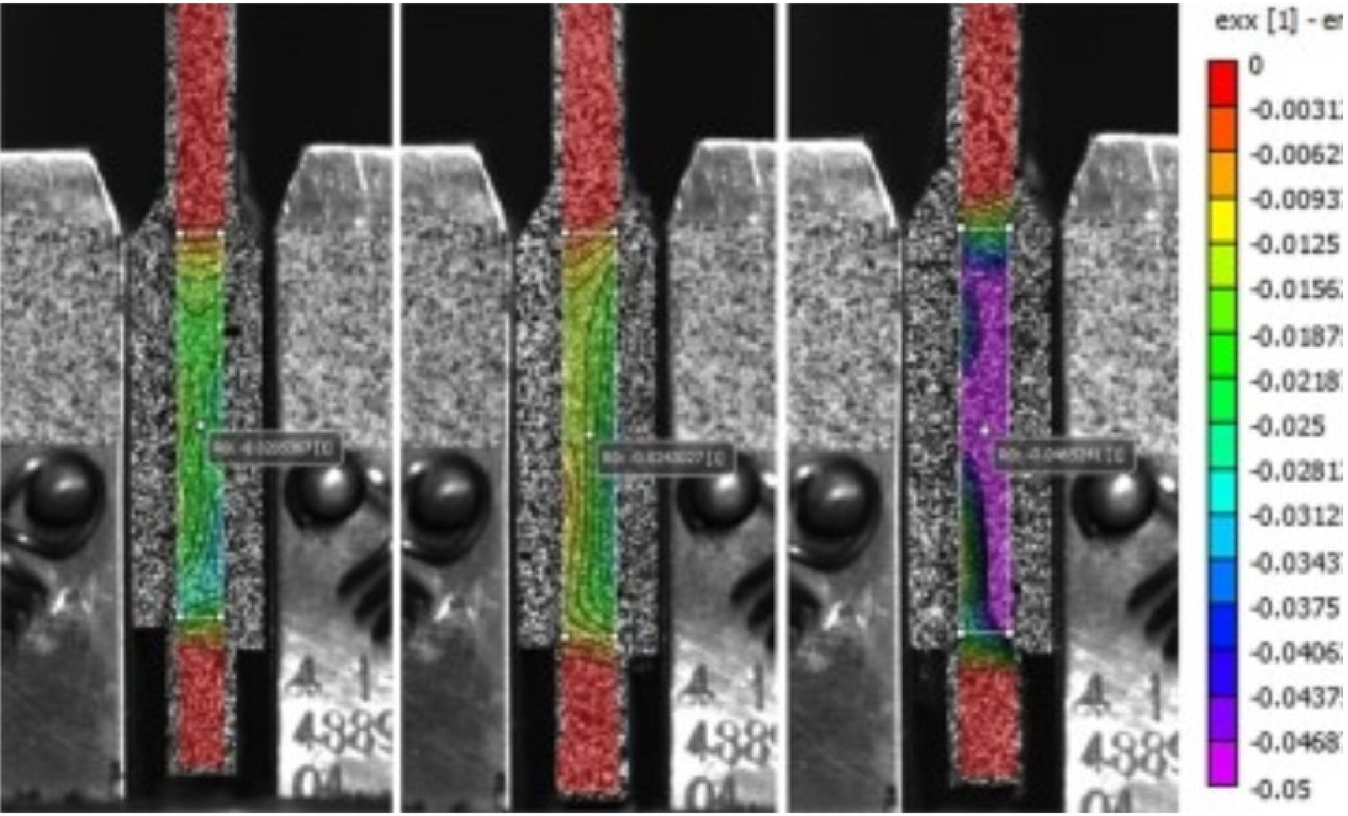


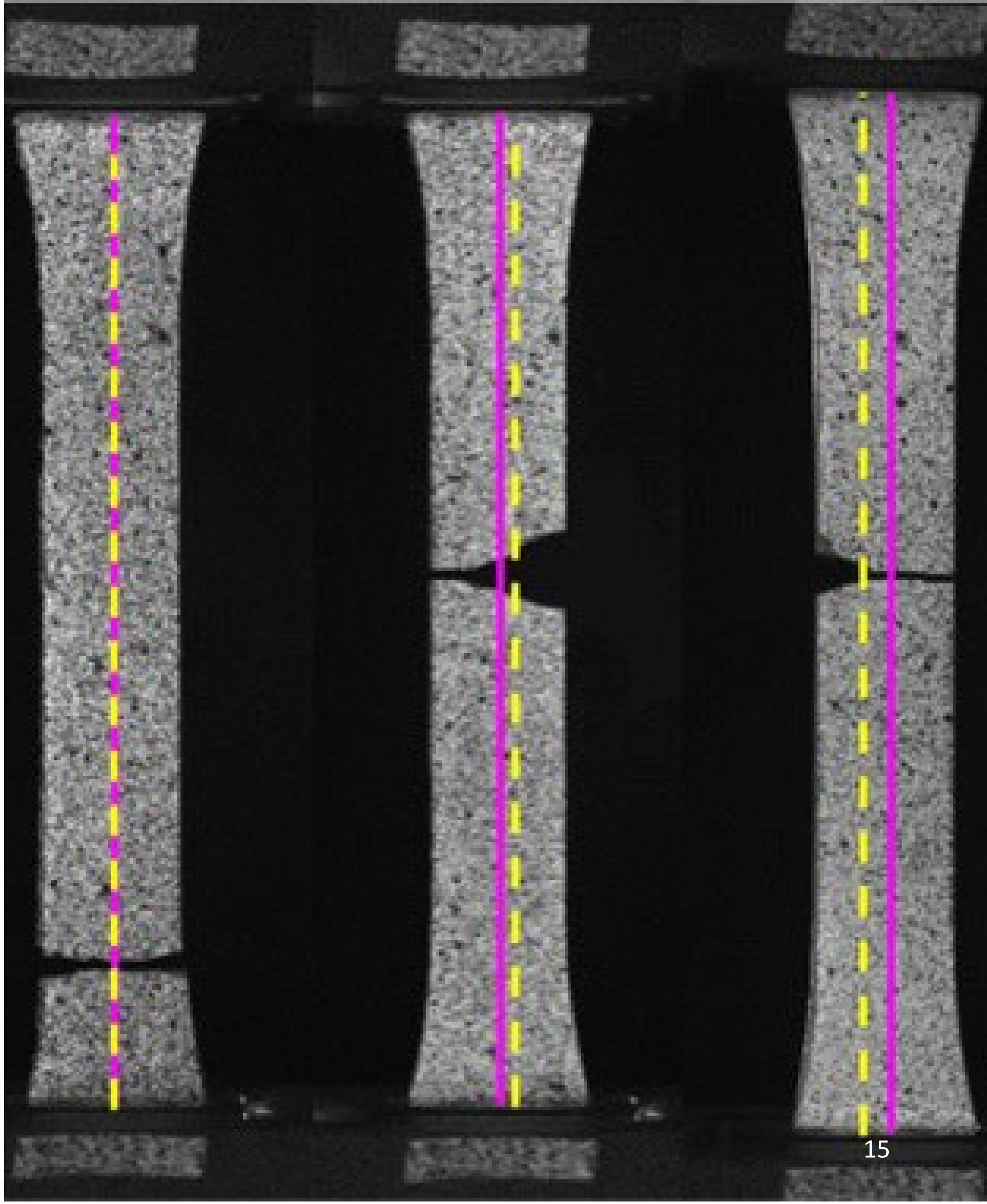
Burn off test:

- ❖ 62% Fiber content
- ❖ 38% Matrix content

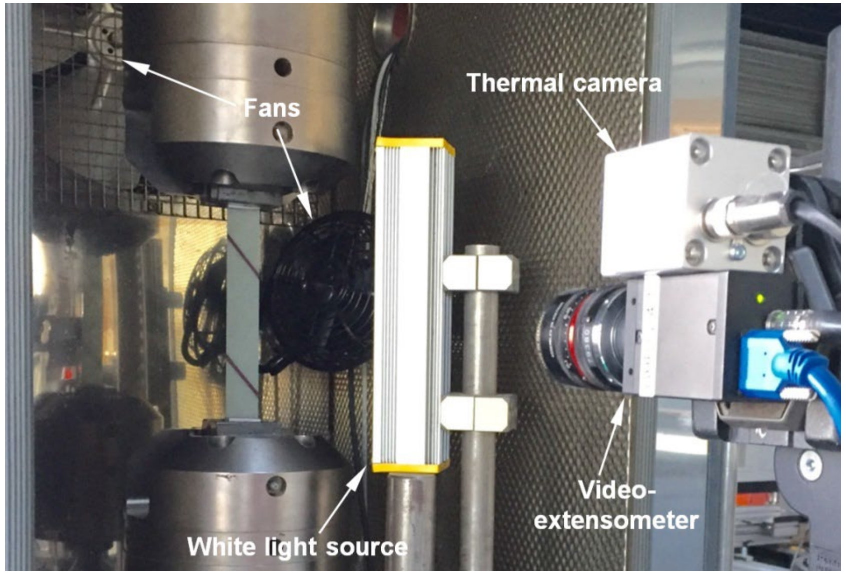


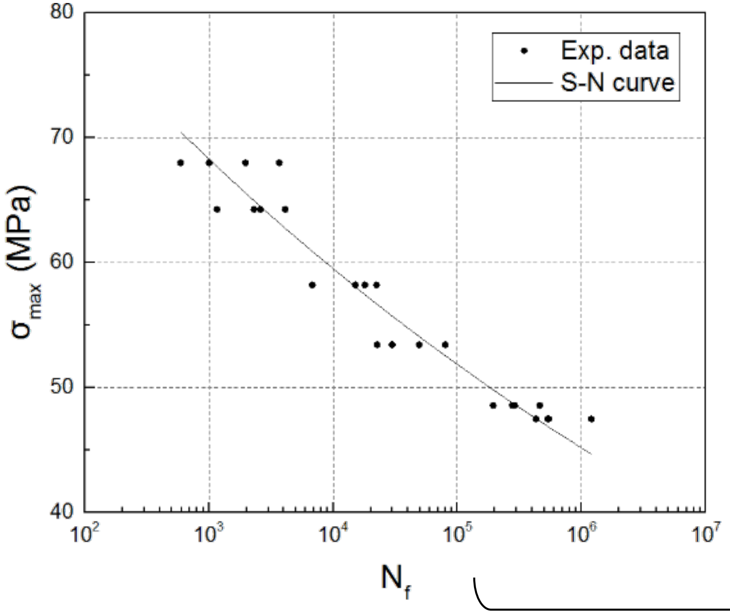
- Sensitivity in preparation and loading

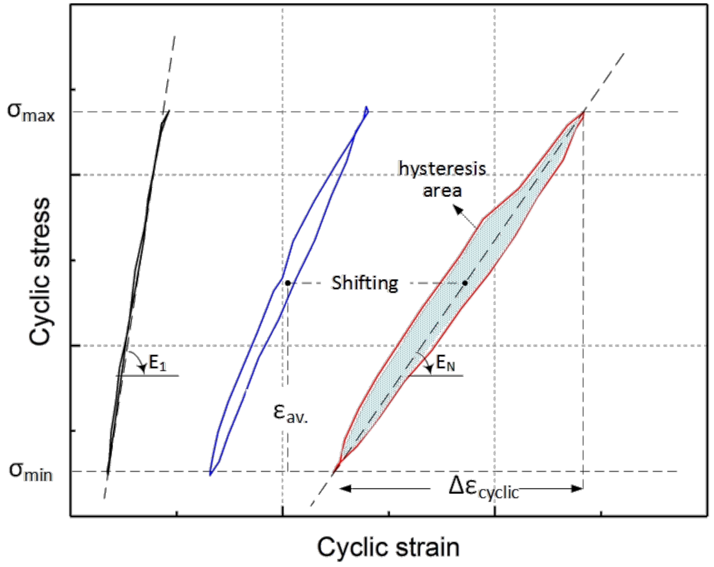


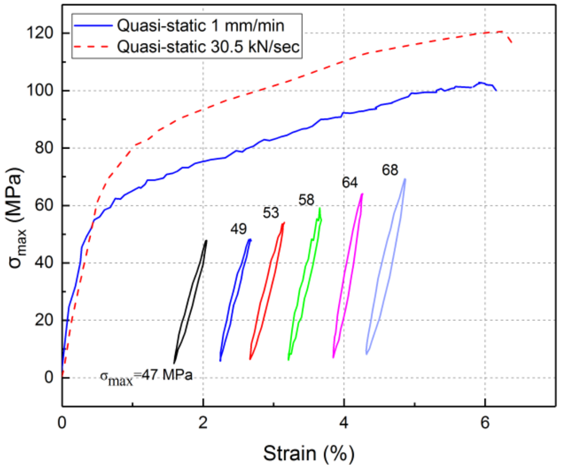


Experimental results: S-N curves









Fatigue behavior

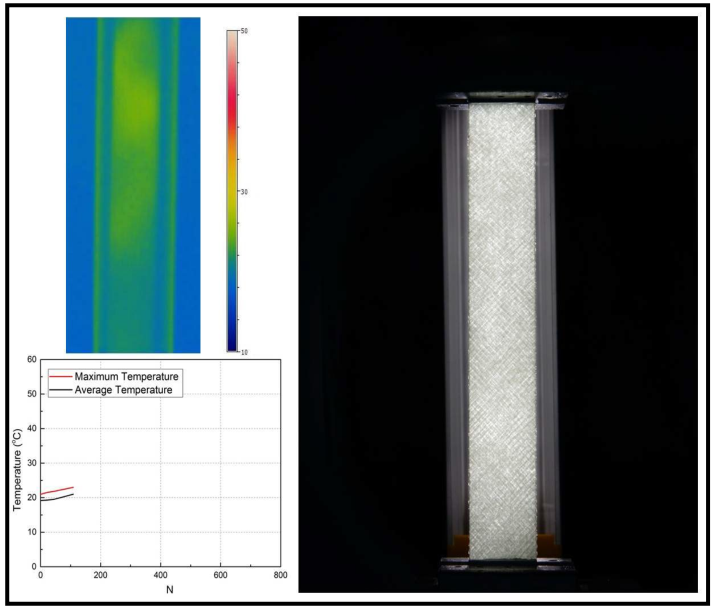
Fatigue life

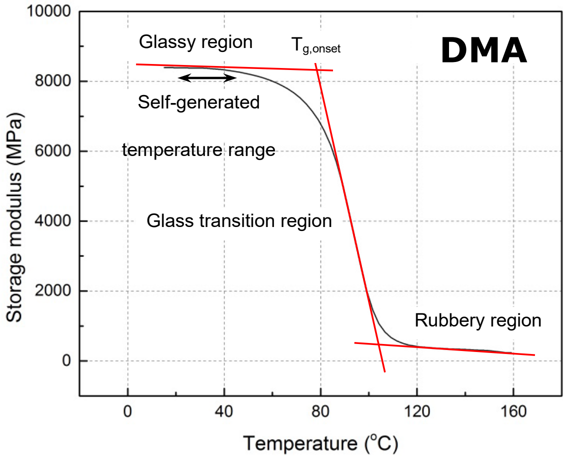
Fatigue stiffness

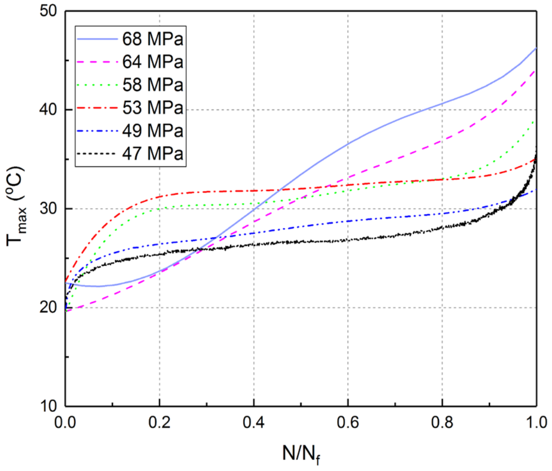
Hysteresis loop area

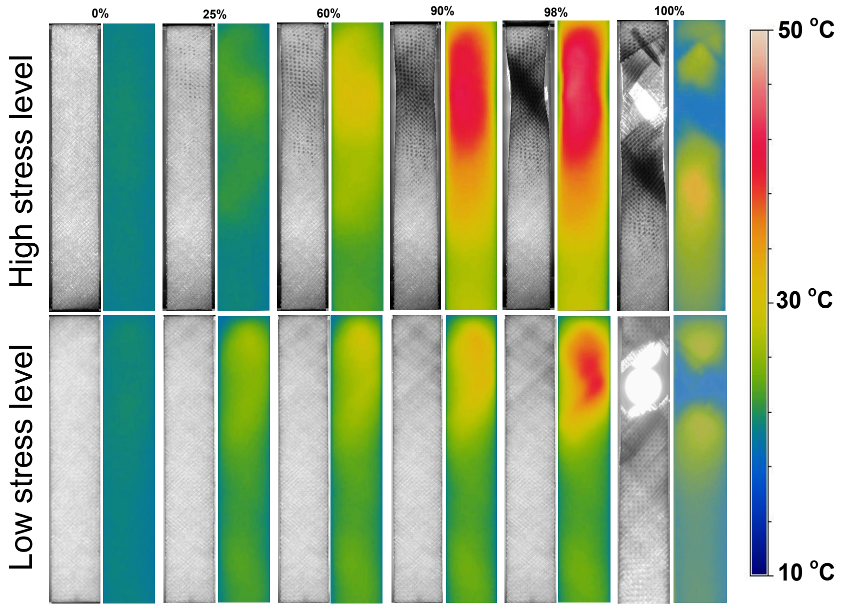
Cyclic creep: $(\varepsilon_{max}+\varepsilon_{min})/2$

Thermal behavior/observation (R=0.1)



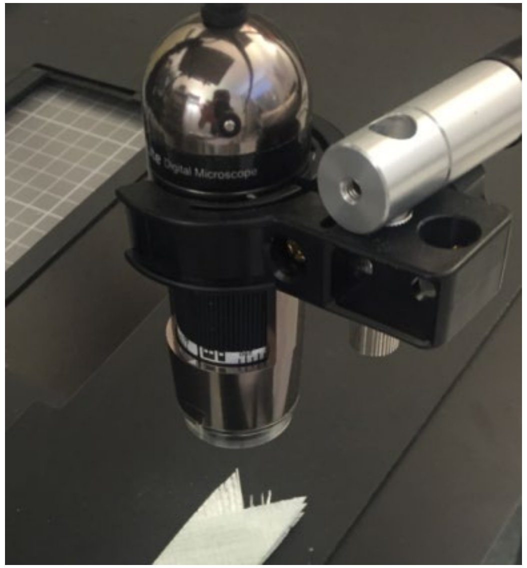




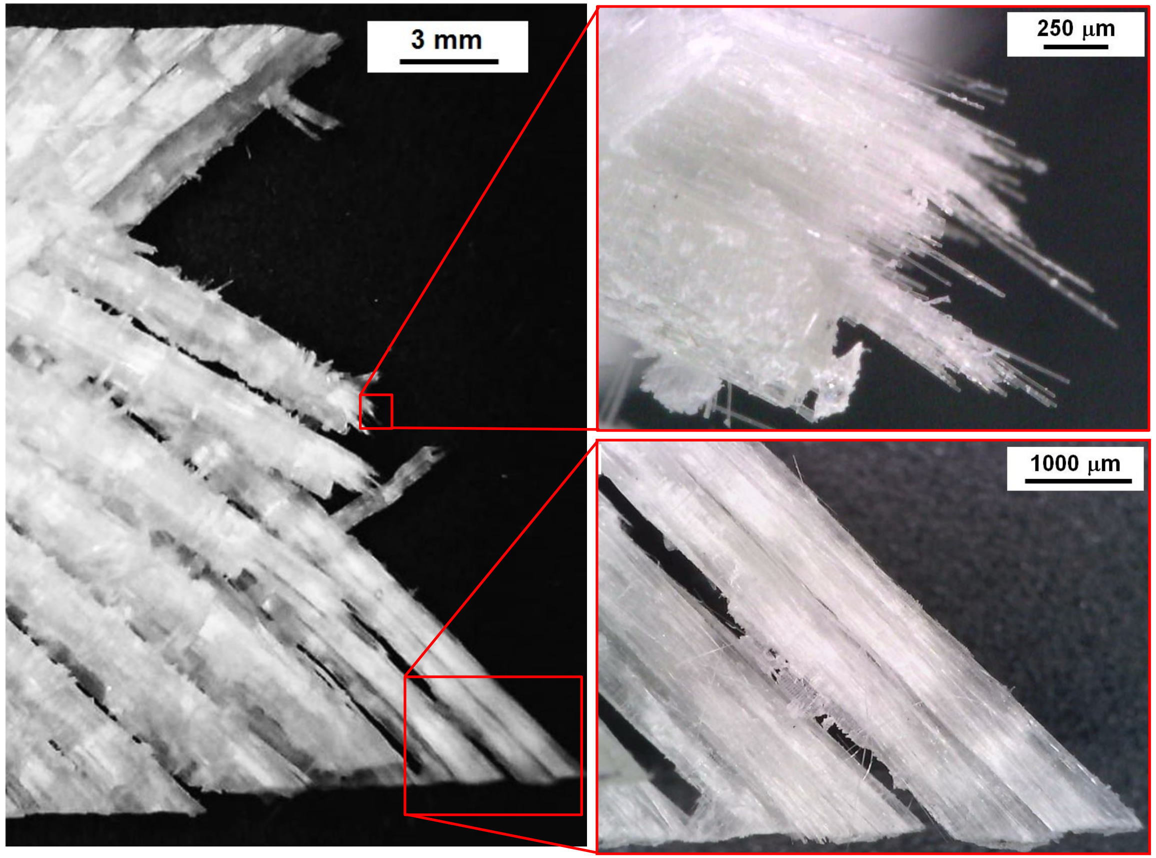


Damage development with stress level

Failure mechanisms



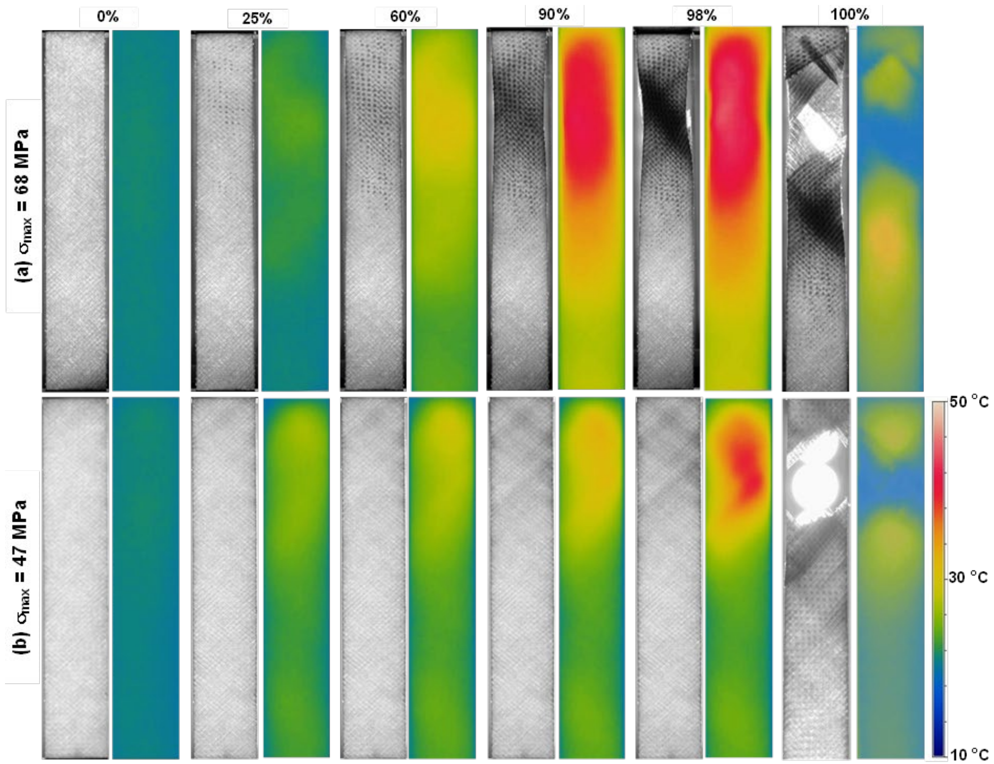
Digital handheld microscope



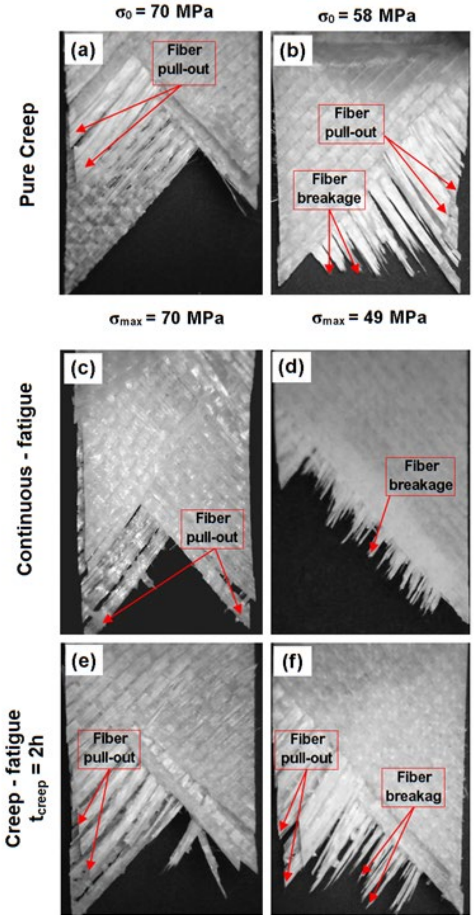
1. Fiber breakage

2. Fiber pull-out

Damage monitoring



Optical microscopy



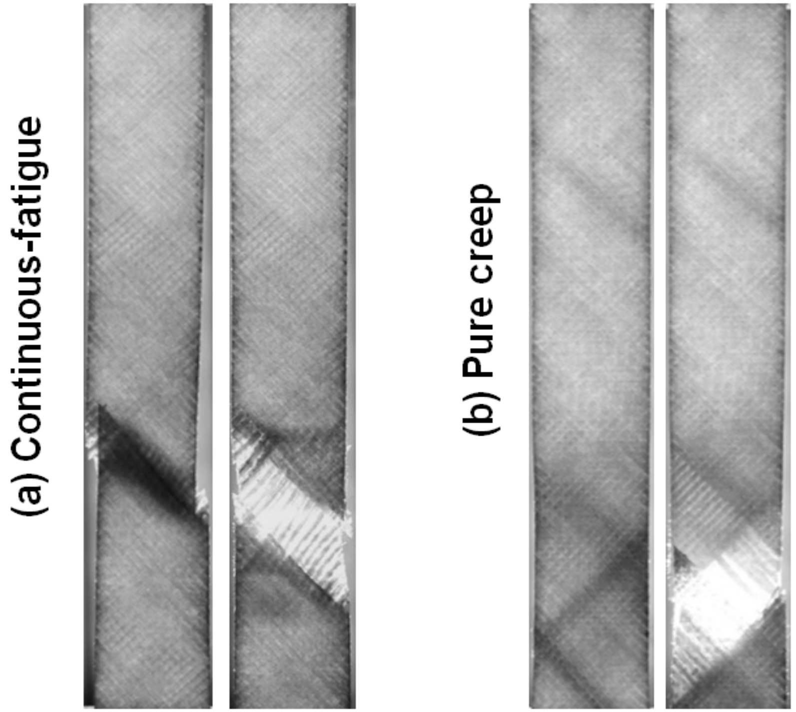
Lateral view of

90^o off-axis specimen

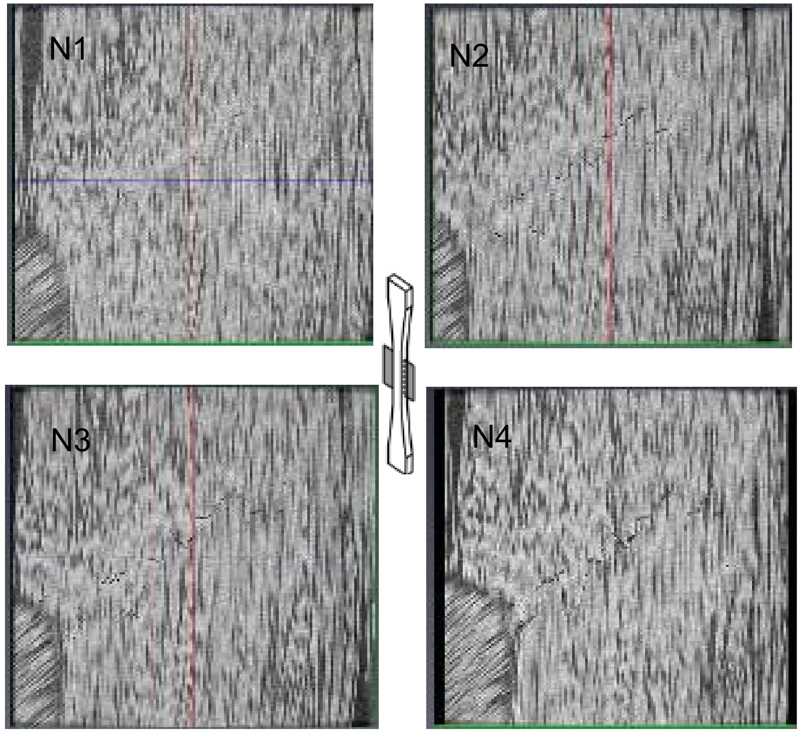
under R = –1 at 38 MPa

Damage monitoring (smaller scale)

Optical inspection

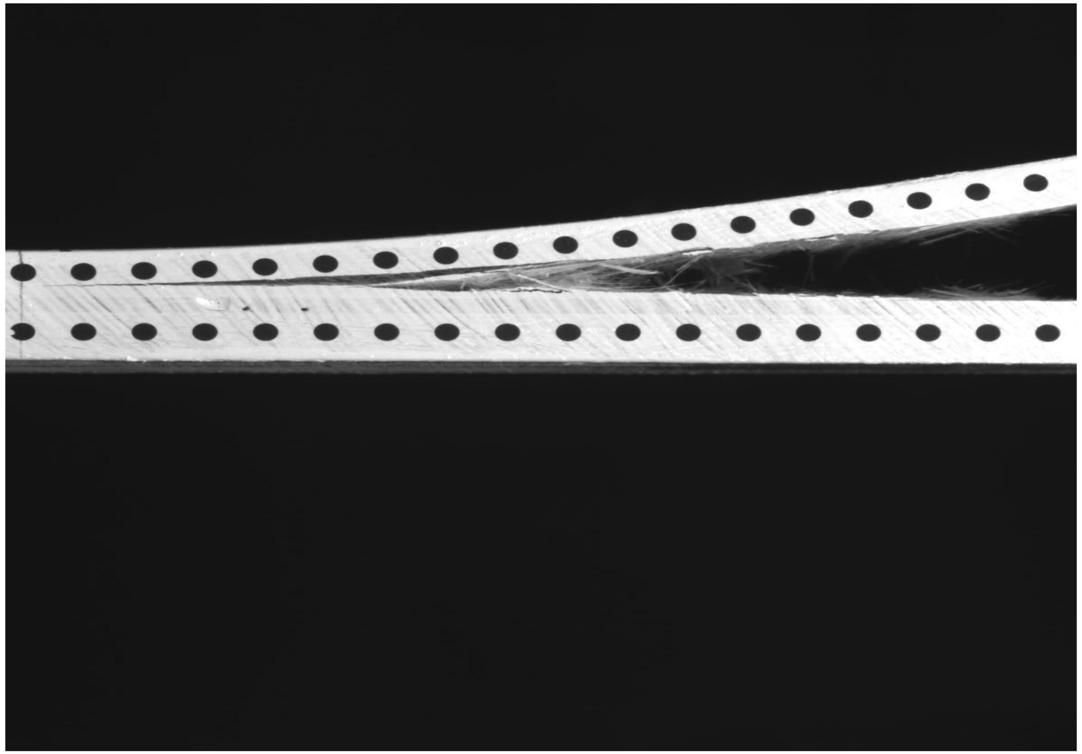


Magnetic tomography

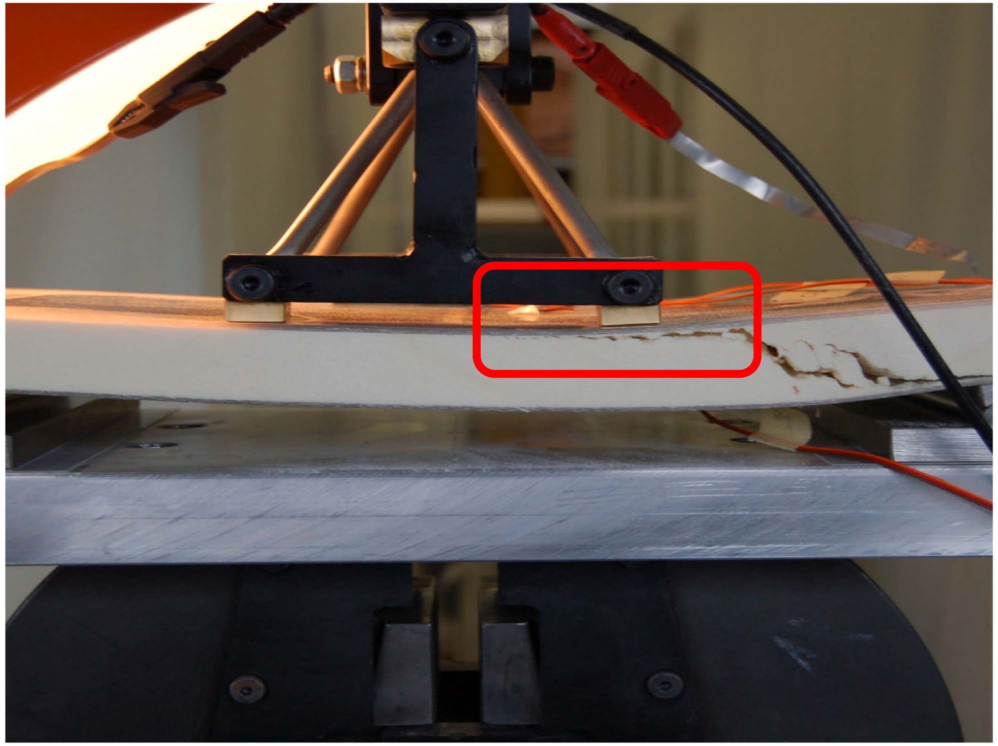


Fracture
(failure as well 😉)

Fracture

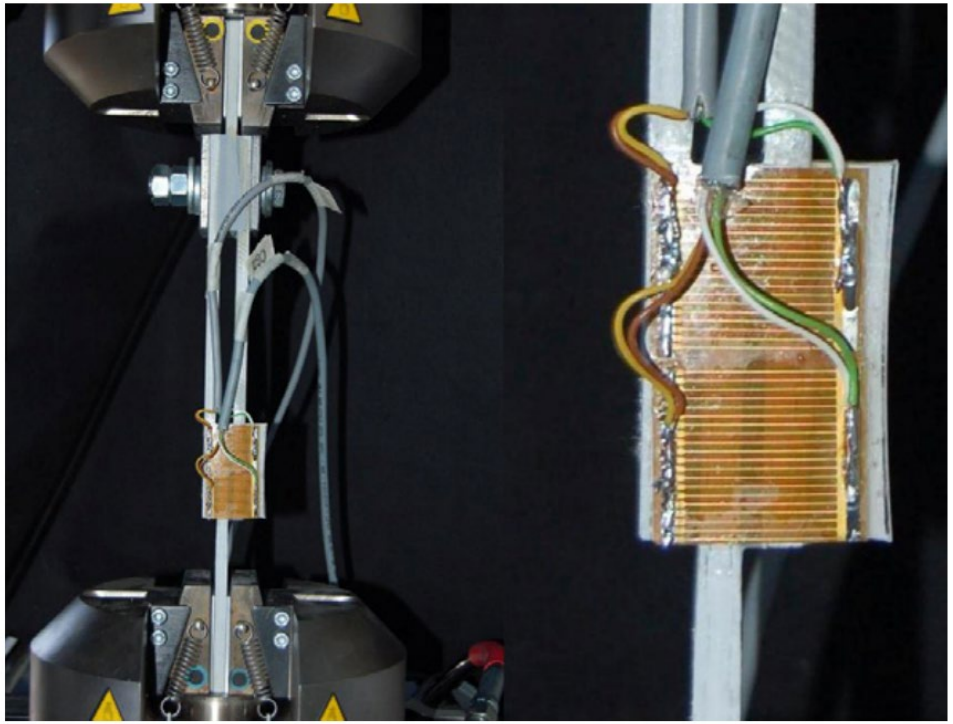


Delamination

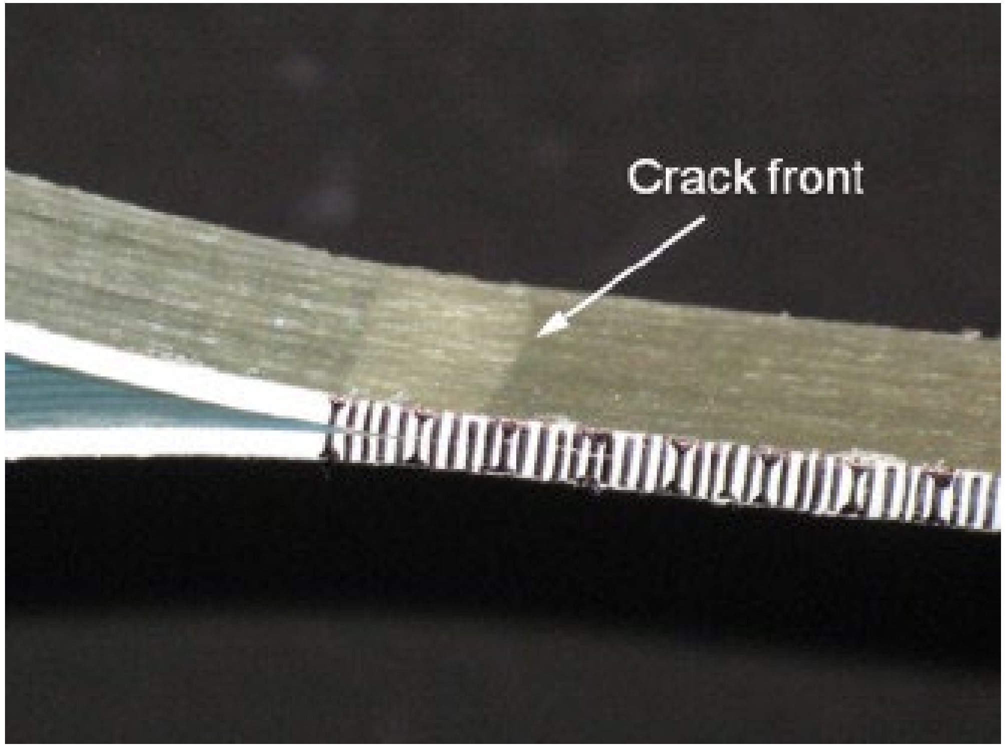


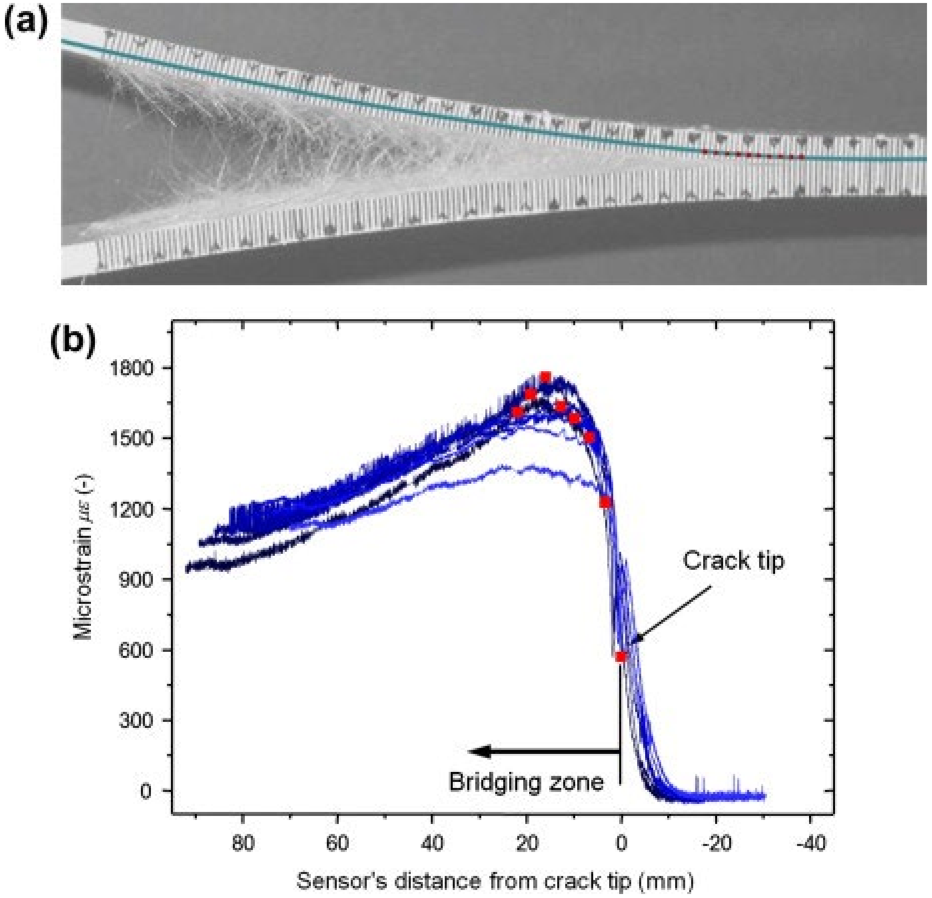
Debonding

Damage monitoring

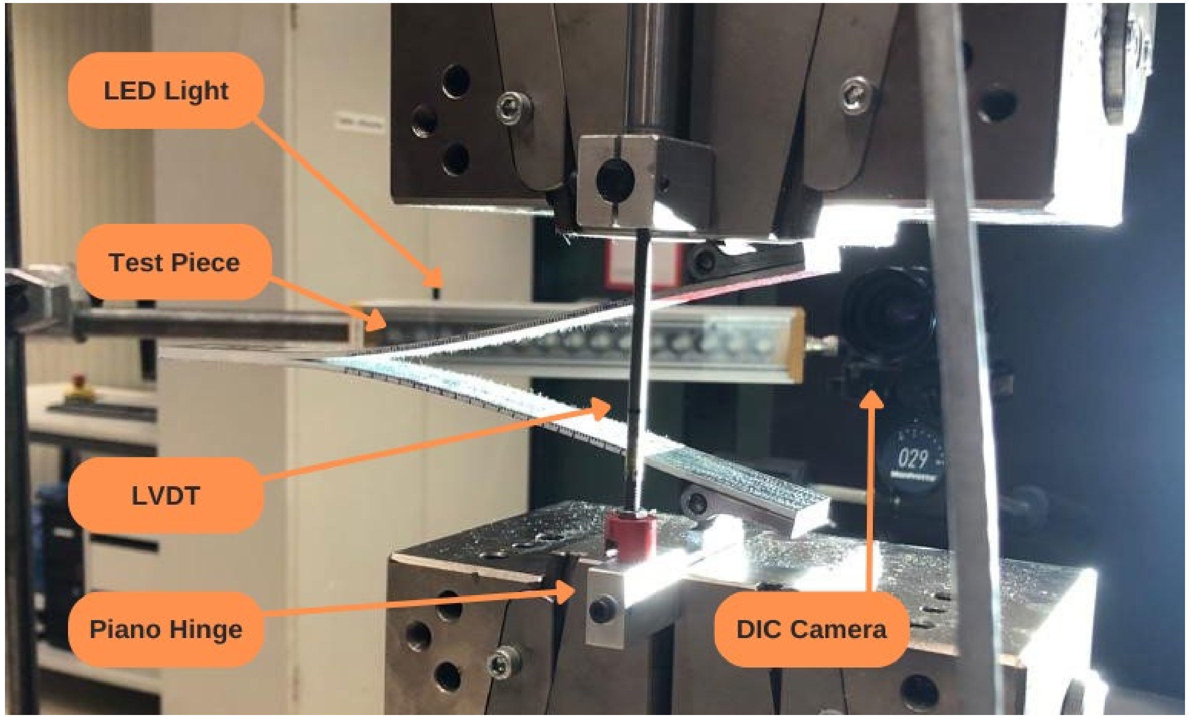


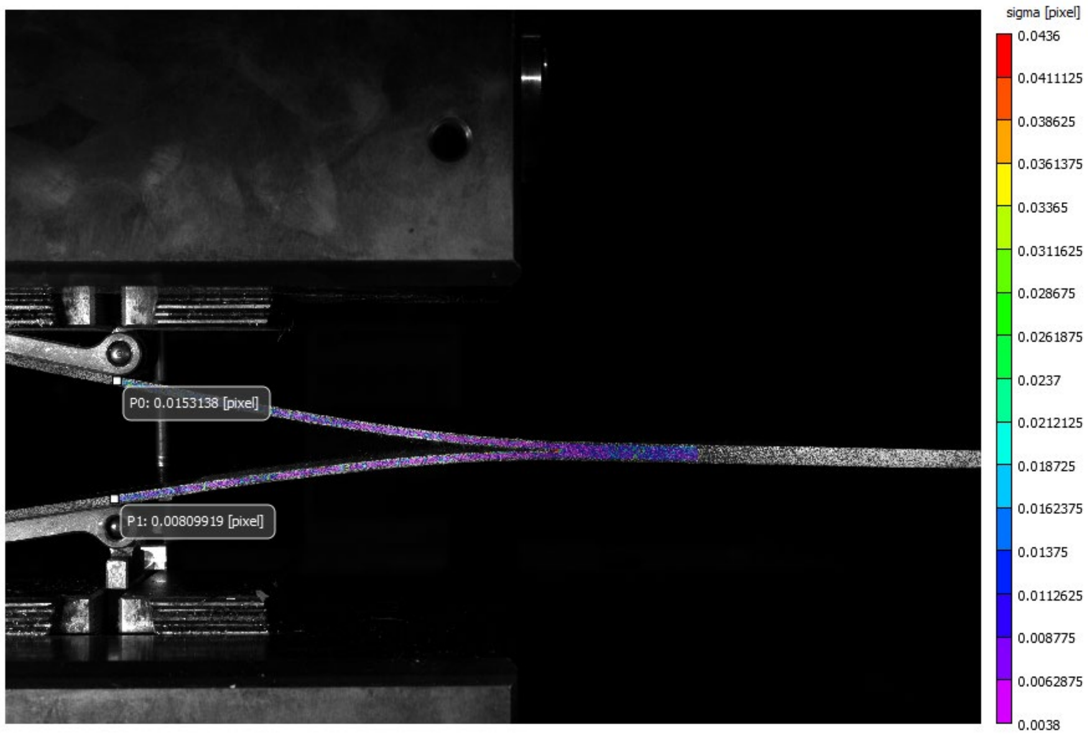
Damage monitoring

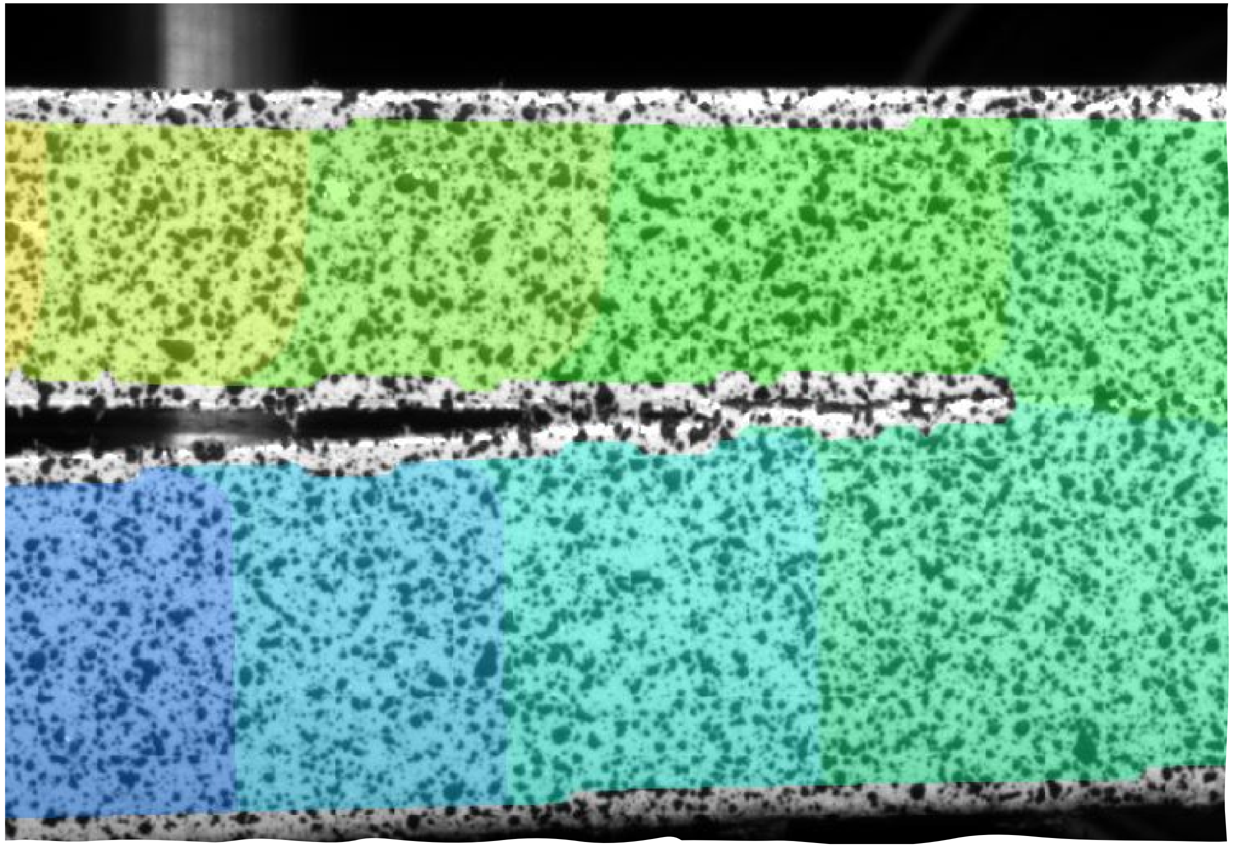


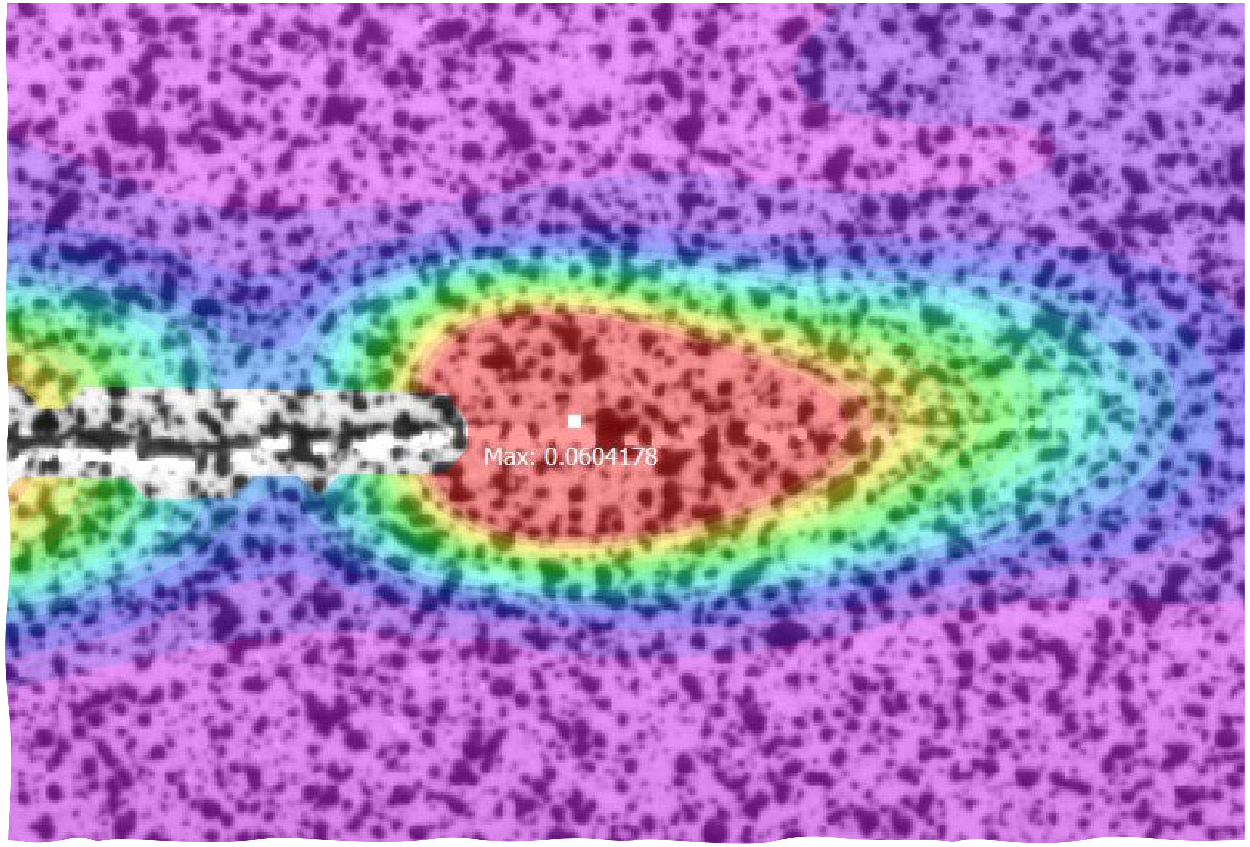


Damage monitoring



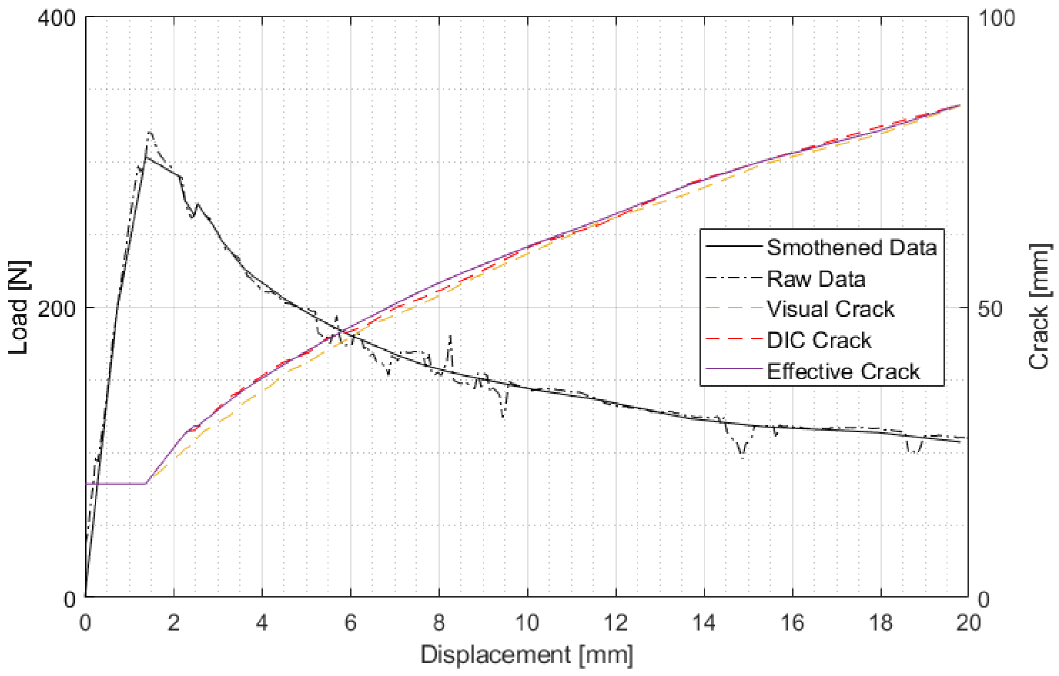


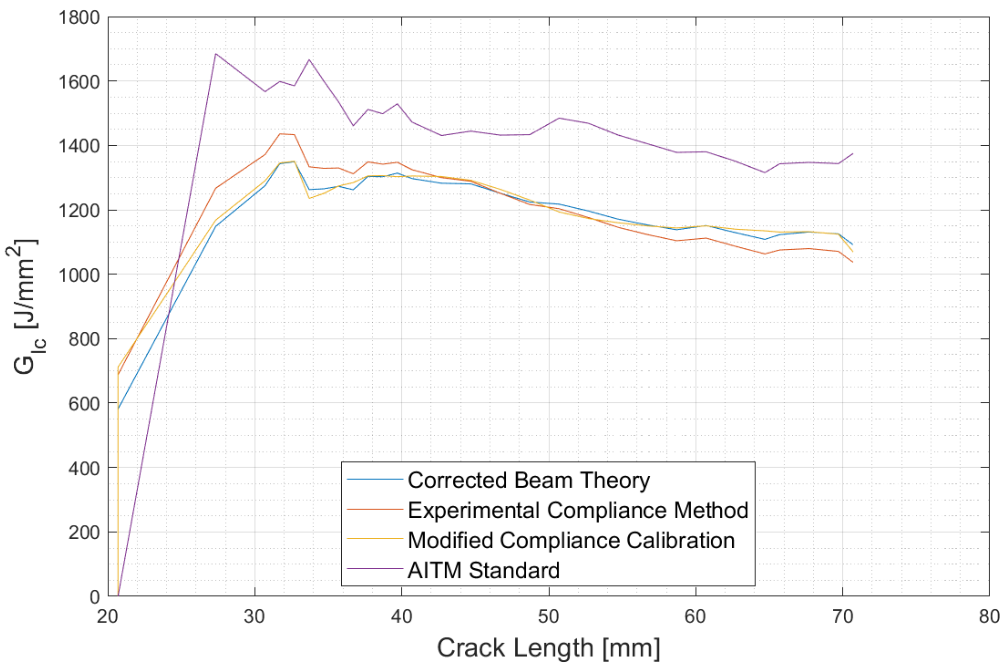




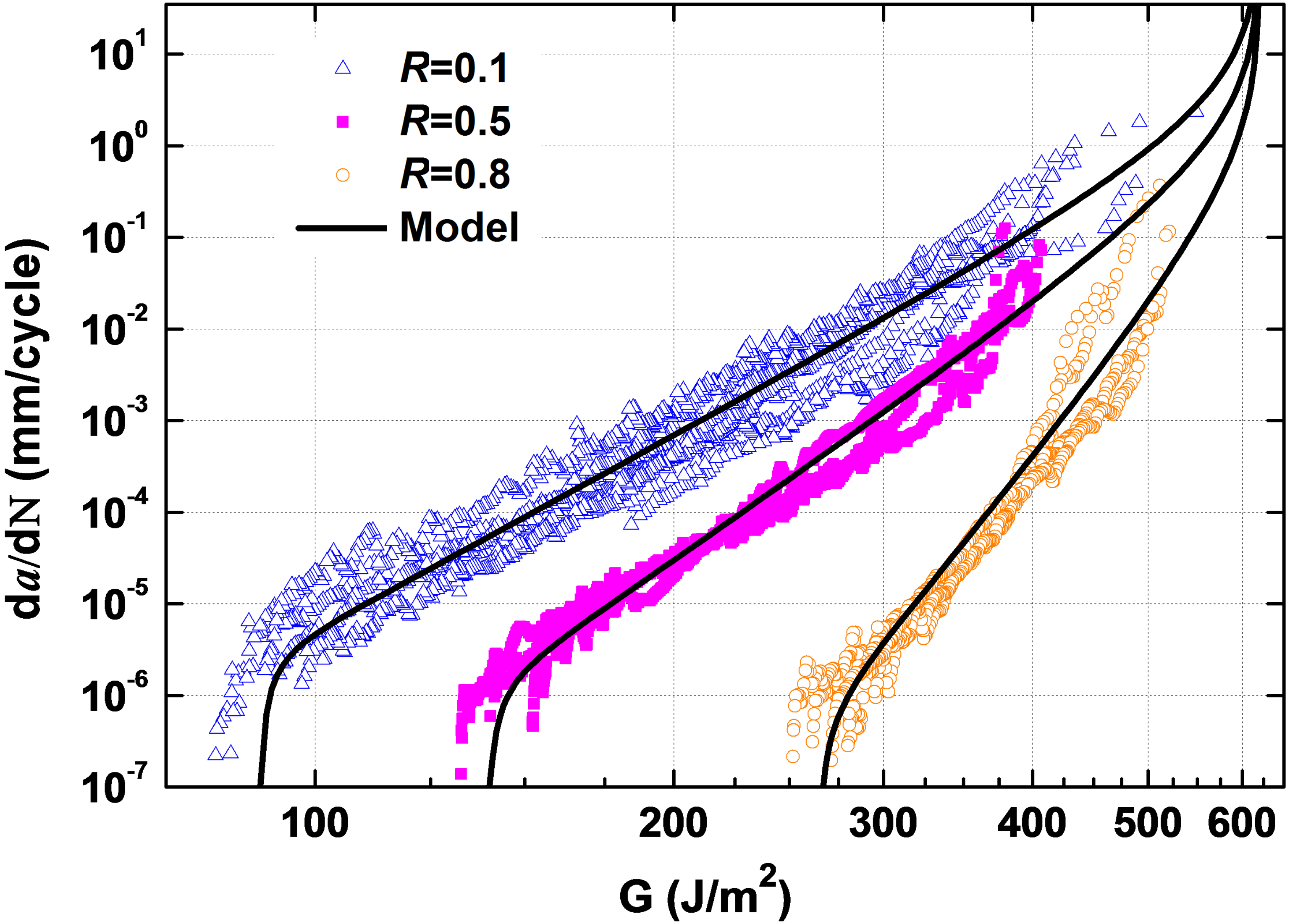
Damage monitoring

Objective (quasi-static)



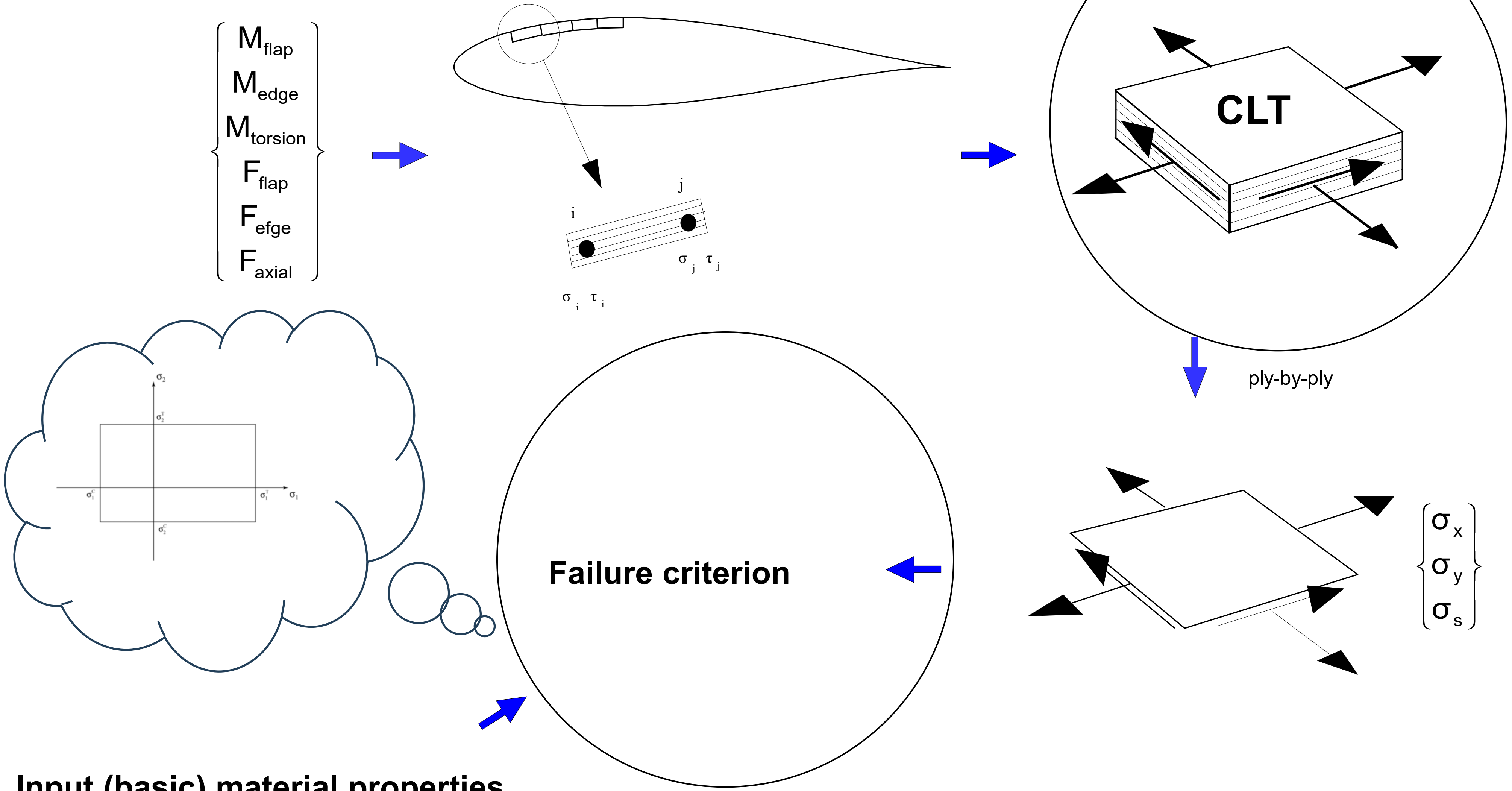


Objective (fatigue)



Failure analysis

Failure analysis (stress/strain)



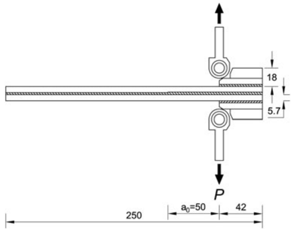
Input (basic) material properties
For each layer or…

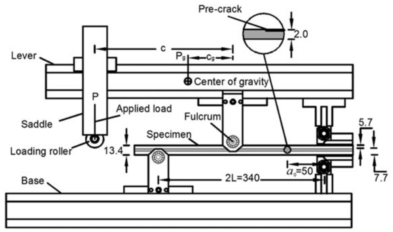
Failure analysis (fracture)

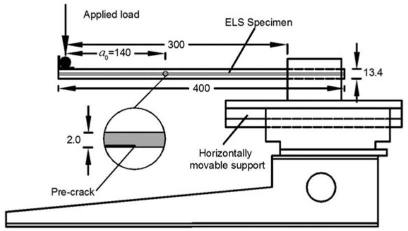
Input:

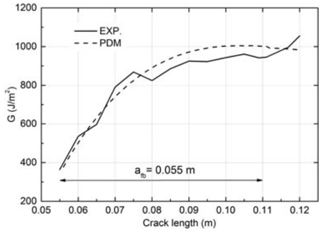
Material

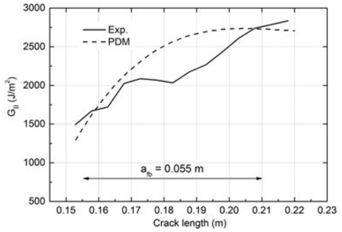
Geometry

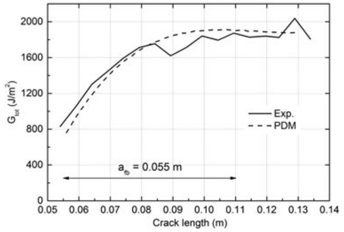




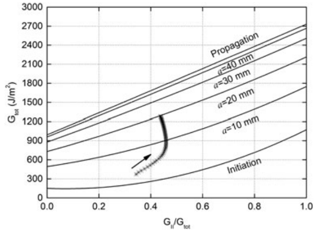




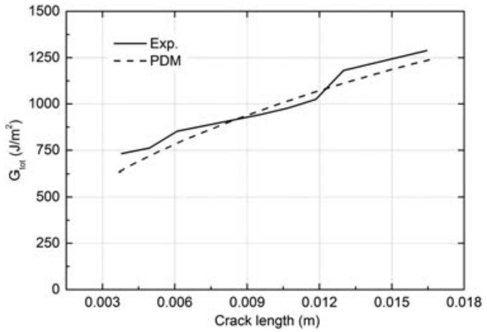


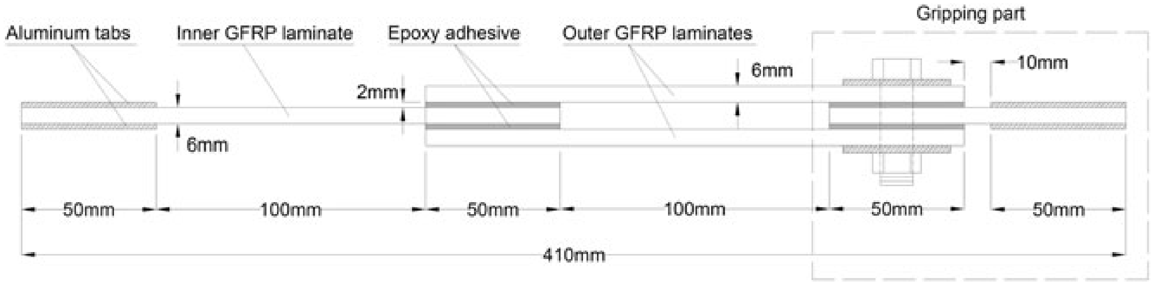


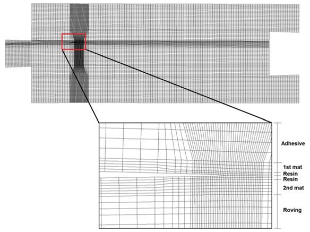
Failure criterion

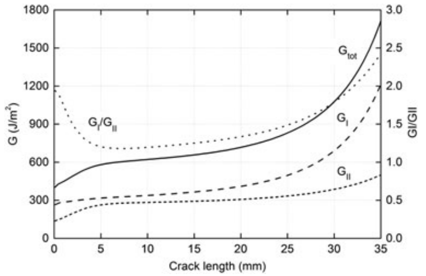


$$G_{tot} = A\left(\frac{G_{II}}{G_{tot}}\right)^2 + B\left(\frac{G_{II}}{G_{tot}}\right) + C$$









Two main approaches for failure analysis (stress/strain)

- a) Maximum stress and strain criteria
  - Not interactive failure criteria associated with failure modes
- b) Quadratic interaction criteria
  - Interactive failure criteria that can be associated or not with failure modes

Strength of (isotropic) materials

Estimate the stress at the time and location when failure occurs

For conventional (isotropic) materials:

- Maximum tensile, compressive or shear stress and observation about the failure is enough
- Usually, one strength parameter is enough…
- Young's modulus is enough since **Poisson's ratio** equals to **0.3** and the uniaxial tensile strength suffice since **shear strength** is taken to be about **50-60%** of the tensile.

Maximum stress/strain

Related to the maximum normal stress theory by Rankine and the maximum shear stress theory by Tresca.

The stresses acting on a point are resolved into the principal normal stresses or the maximum shear stresses. Failure is predicted if any of the stress components is equal to or exceeds the corresponding strength of the material.

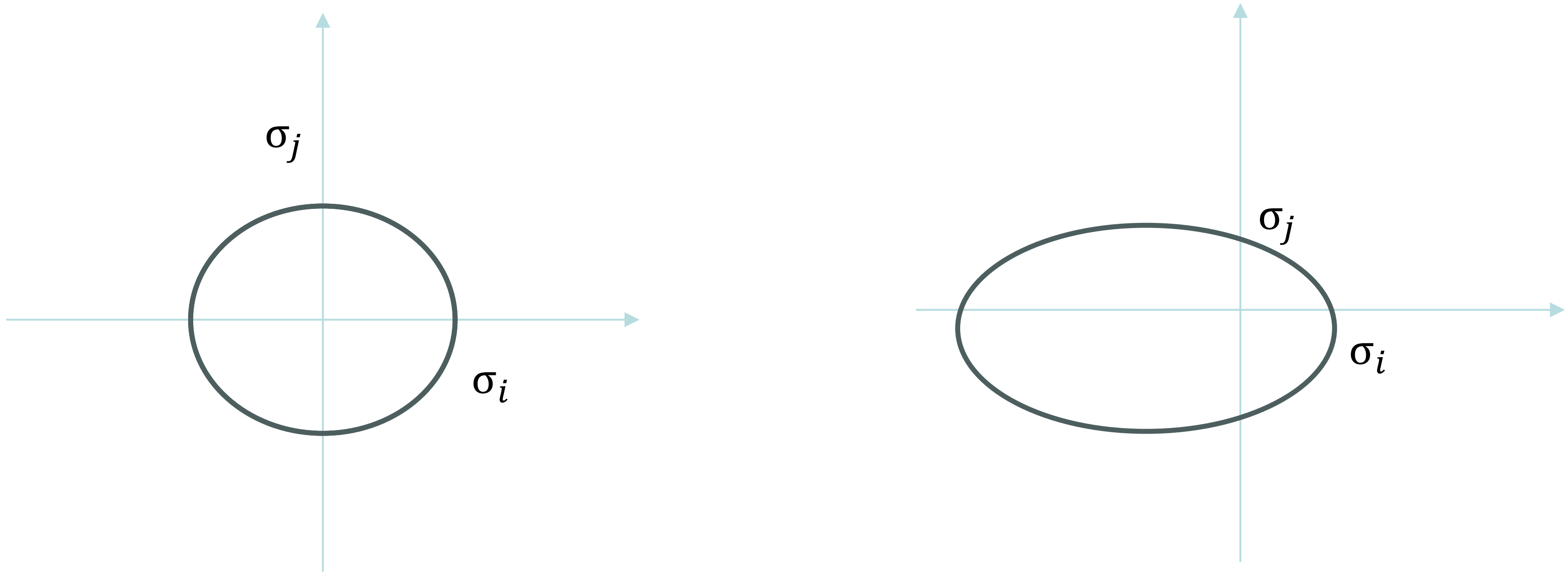
$$-X' < \sigma_1 < X$$
$$-Y' < \sigma_2 < Y$$
$$-S < \sigma_6 < S$$

OR for the maximum strain theory:

$$-\left(\varepsilon_1^c\right)_{ult} < \varepsilon_1 < \varepsilon_{1ult}^t$$
$$-\left(\varepsilon_2^c\right)_{ult} < \varepsilon_2 < \varepsilon_{2ult}^t$$
$$-\left(\varepsilon_6\right)_{ult} < \varepsilon_6 < \varepsilon_{6ult}$$

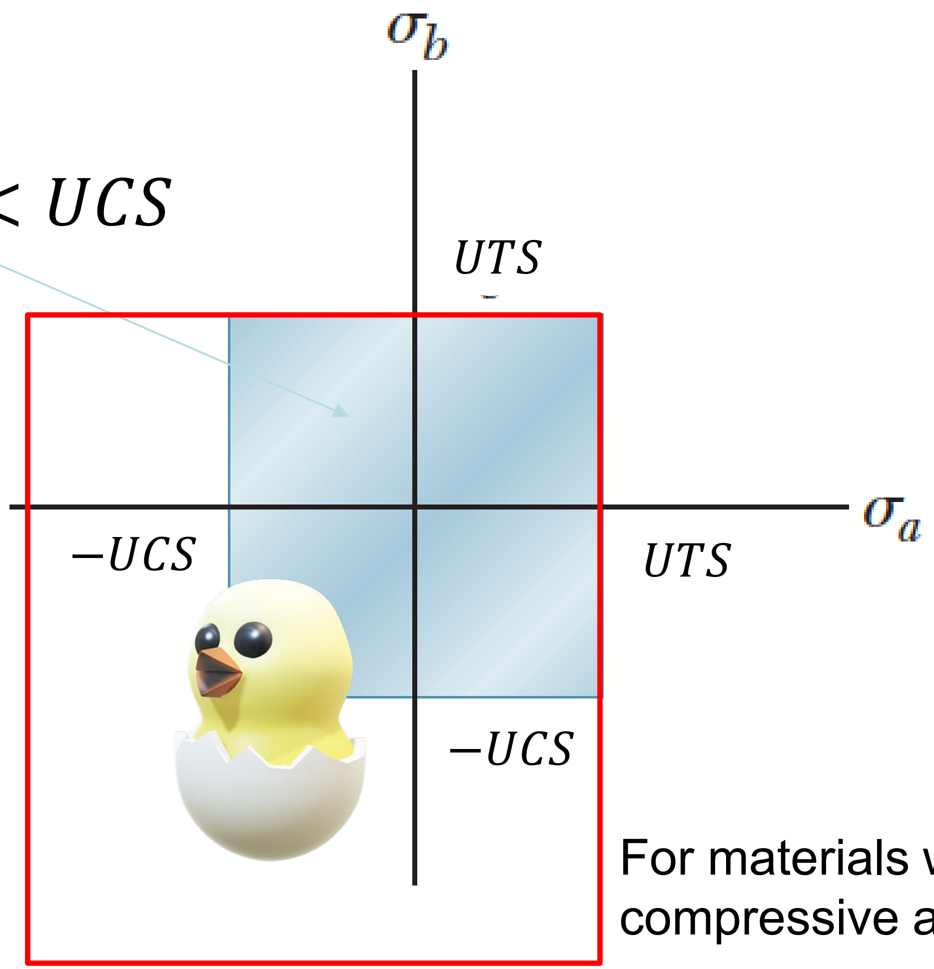
For both cases: Each component of stress (strain) has its own failure condition (criterion) and it is not affected by the other components, i.e., **there is no interaction**

Strength differential



Fracture Criteria for Brittle Materials under Plane Stress —Maximum-Principal stress (Rankine)

$$|\sigma_1| < UTS \ or \ |\sigma_2| < UCS$$



For materials with different compressive and tensile strengths

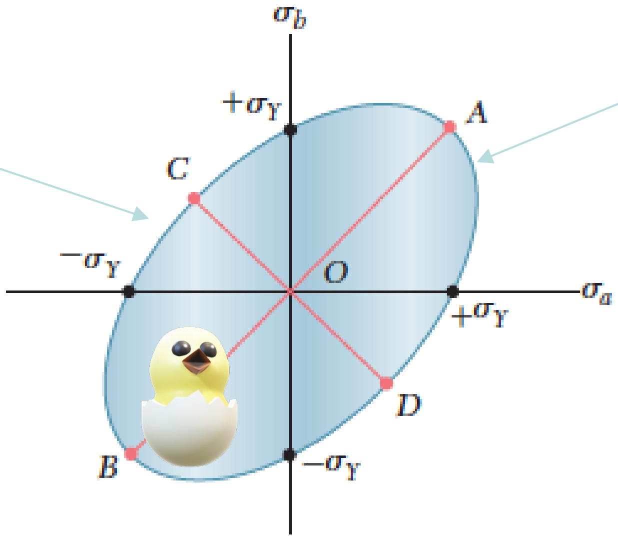
Yield Criteria for Ductile Materials under Plane Stress
Maximum Distortion Energy Criterion (Von Mises)

- For the plane stress condition, the Von Mises criterion is written:

$$\sigma_y{}^2 = (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

Showing the mathematical equation of a rotated ellipse on the $\sigma_1 - \sigma_2$ plane.

$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 < \sigma_Y^2$



$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$

Source: after Austrian Engineer, Richard Von Mises (1883-1953)

Strength of (composite) materials

- **For composite materials:**
  - The one-constant approach for strength or for stiffness is no longer adequate
  - (For the simplest case of the unidirectional laminate) Four elastic constants are needed for the stiffness and six constants for the strength.
  - However, unidirectional composites have highly directionally dependent strengths. Therefore, for any state of applied stress, all stress components must be examined before judgment on the cause of failure can be made.

Failure criteria: Assumptions…

The determination of the "strength" and the behavior of an orthotropic medium is based on the assumptions that the material is:

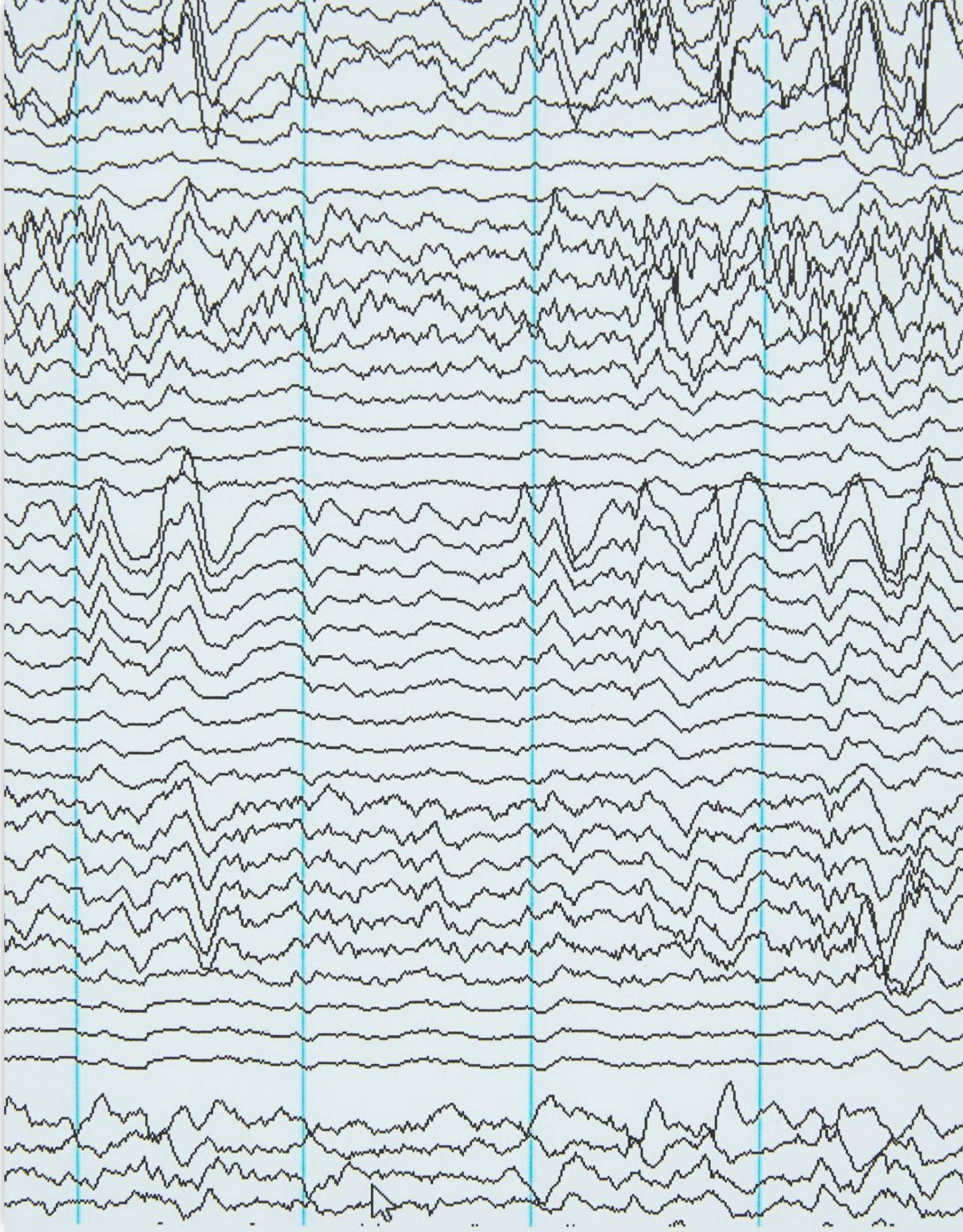
- Homogeneous
- Linear elastic up to failure

For the application of the failure criteria the strength of the material under "uniaxial" loading should be determined experimentally.

Failure criteria

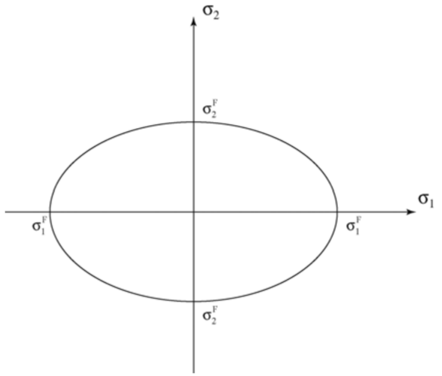
- For the analysis of composite laminates (consisting of a number of unidirectional plies) we need a failure criterion for the unidirectional layer. We would expect successive ply failures as the applied load to a laminate increases.

- The **First Ply Failure (FPF)** is followed by other ply failures until the **Last Ply Failure (LPF)** which would be the ultimate failure of the laminate.



Quadratic interaction criteria

$$F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1$$



And for the strain components:

$$G_{ij}\varepsilon_i\varepsilon_j + G_i\varepsilon_i = 1$$

Where the $F's$ and $G's$ are strength/failure strain parameters analogous to the constants in equations of maximum stress/strain criteria. Failure occurs when equation is met (failure locus).

Use of failure criteria

- Failure criteria constitute the analytical mathematical expressions for the calculation of the failure under complex loading conditions as a function of the “basic” material properties, that can be determined through “simple” tests.

- Failure criteria serve important functions in the design and sizing of composite laminates. **However, the criteria are not intended to explain the mechanics of failure (although…).** The framework should remain the same for different definitions of failure, such as the ultimate strength, the yield point the endurance limit or even a working stress based on reliability considerations.

- **The quadratic criterion will be analyzed here**. It is simple and versatile. It considers interaction between the stress or strain components analogous to the von Mises criterion for isotropic materials.

Tsai-Hill

Criterion that has been proposed by R. Hill* for orthotropic media. It is a generalization of the von-Mises distortional energy yield criterion for isotropic materials.

$$F(\sigma_2-\sigma_3)^2+G(\sigma_3-\sigma_1)^2+H(\sigma_1-\sigma_2)^2+2L\sigma_4^2+2M\sigma_5^2+2N\sigma_6^2=1$$

With:

$$2F=\frac{1}{Y^2}+\frac{1}{Z^2}-\frac{1}{X^2}\ ,\ 2H=\frac{1}{Y^2}+\frac{1}{X^2}-\frac{1}{Z^2}\ ,\ 2G=\frac{1}{X^2}+\frac{1}{Z^2}-\frac{1}{Y^2}$$

$$2L=\frac{1}{R^2}\ ,\quad 2M=\frac{1}{T^2}\ ,\quad 2N=\frac{1}{S^2}$$

X, Y, Z, R, T, S: the material strength under normal and shear loads

Tsai-Hill criterion does not consider for the different strength of the anisotropic material under tensile or compressive loads or positive-negative shear…

*R. Hill, "A Theory of the Yielding and Plastic Flow of Anisotropic Metals" Proc. Royal Soc. A193, pp.281-297 (1948)

Failure tensor polynomial

For orthotropic media, in the local system:

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 +$$
$$2(F_{12}\sigma_1\sigma_2 + F_{13}\sigma_1\sigma_3 + F_{23}\sigma_2\sigma_3) + F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 - 1 = 0$$

There exist 12 independent failure components that should be determined through tests

If the stresses are known for another co-ordination system then:

$$F'_{11}\sigma_1'^2 + F'_{22}\sigma_2'^2 + F'_{33}\sigma_3'^2 + F'_{44}\sigma_4'^2 + F'_{55}\sigma_5'^2 + F'_{66}\sigma_6'^2 +$$
$$2(F'_{12}\sigma'_1\sigma'_2 + F'_{13}\sigma'_1\sigma'_3 + F'_{23}\sigma'_2\sigma'_3) + F'_1\sigma'_1 + F'_2\sigma'_2 + F'_3\sigma'_3 - 1 = 0$$

F'_{ij} unknown, but functions of F_{ij} !

It is always preferable to calculate the stresses for the local system, where the failure components are known.

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix} \begin{Bmatrix}\sigma'_1\\ \sigma'_2\\ \sigma'_6\end{Bmatrix} \longrightarrow$$

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 +$$
$$2(F_{12}\sigma_1\sigma_2 + F_{13}\sigma_1\sigma_3 + F_{23}\sigma_2\sigma_3) + F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 - 1 = 0$$

$m=\cos\theta$, $n=\sin\theta$

Determining the failure components

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 +$$
$$2(F_{12}\sigma_1\sigma_2 + F_{13}\sigma_1\sigma_3 + F_{23}\sigma_2\sigma_3) + F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 - 1 = 0$$

Tensile test in 1-direction

$$\sigma_1 \neq 0,\ \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = \sigma_6 = 0$$

$$F_{11}\sigma_1^2 + F_1\sigma_1 = 1$$ At failure σ_1=X $$F_{11}X^2 + F_1 X = 1$$

Compression in 1-direction

$$F_{11}\sigma_1^2 + F_1\sigma_1 = 1$$ At failure σ_1=-X' $$F_{11}X'^2 - F_1 X' = 1$$

$$F_{11} = \frac{1}{XX'} \quad , \quad F_1 = \frac{1}{X} - \frac{1}{X'}$$

accordingly:

$$F_{22} = \frac{1}{YY'} \quad , \quad F_2 = \frac{1}{Y} - \frac{1}{Y'} \qquad F_{33} = \frac{1}{ZZ'} \quad , \quad F_3 = \frac{1}{Z} - \frac{1}{Z'}$$

Pure shear test on 1-2 plane

For the orthotropic medium the shear strength is not dependent on the sign:

$$R^+ = R^- \quad , \quad T^+ = T^- \quad , \quad S^+ = S^- \longrightarrow F_{44} = \frac{1}{R^2} \quad , \quad F_{55} = \frac{1}{T^2}, \quad F_{66} = \frac{1}{S^2}$$

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 +$$
$$2(F_{12}\sigma_1\sigma_2 + F_{13}\sigma_1\sigma_3 + F_{23}\sigma_2\sigma_3) + F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 - 1 = 0$$

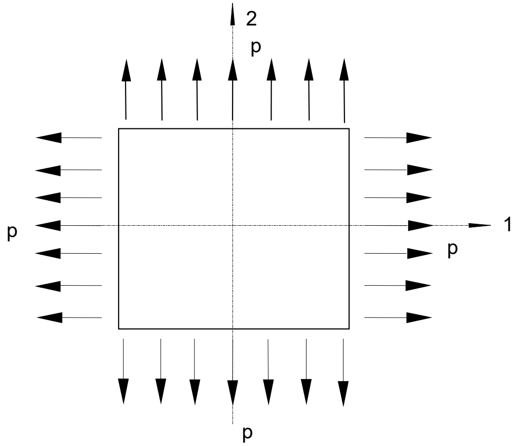
$F_{ij},\ i \neq j$: *stress interaction terms*

With 6 simple tests and 3 “almost” simple the failure components can be measured experimentally.

What can be done with the complex terms, that designate the stress interaction?

Determination of the stress interaction terms leads to different criteria!!!

A. Tsai-Wu[1]



Determine F_{ij} through biaxial tests:

$$(F_{11} + F_{22})p^2 + 2F_{12}p^2 + (F_1 + F_2)p - 1 = 0$$

$$F_{12} = \frac{1}{2p^2}\left\{1 - (F_{11} + F_{22})p^2 - (F_1 + F_2)p\right\}$$

Apparently, same F_{12} value should be measured for different loading, e.g., σ_1=p and σ_2=3p

1. S. W. Tsai and E. M. Wu: A General Theory of Strength for Anisotropic Materials, AFML-TR-71-12 (1971)

B. Tsai-Hahn[1] Theoretical determination of F_{ij} terms through the equations:

$$F_{ij} = -\frac{1}{2}\sqrt{F_{ii}F_{jj}} \qquad i,j = 1,\ldots,3 \qquad i \neq j \qquad e.g.\ F_{12} = -\frac{1}{2}\sqrt{F_{11}F_{22}}$$

C. Narayanaswami-Adelman[2]

$$F_{ij} = 0 \qquad i,j = 1,\ldots,3 \qquad i \neq j$$

F_{ij} should be neglected because of their low values. Good for in-plane off-axis uniaxial laoding. Problems occur when complex plane stress field exists or for 3D stress fields.

D. Wu-Stachurski[1]

$$F_{ij} = \frac{-F_{ii}F_{jj}}{F_{ii} + F_{jj}} \qquad i,j = 1,\ldots,3 \qquad i \neq j$$

Empirical, based on experimental data on specific material.

E. Hoffman[3]-EPFS[4]

$$F_{ij} = \frac{1}{2}\left(F_{kk} - F_{ii} - F_{jj}\right) \qquad i,j,k = 1,\ldots,3 \qquad i \neq j \neq k$$

---

1. R. Y. Wu and Z. Stachurski, J COMPOS MATER 18, p.456 (1984)

Other failure criteria

- Independent conditions and interactive criteria can be combined to allow different formulations for different failure modes. These types of criteria are sometimes referred to as partly interactive criteria.

  – Simple Puck criterion
  – Modified Puck criterion
  – Hashin criterion
  – …

Simplified Puck criterion

- Failure due to longitudinal loads and the matrix failure mode due to transverse and shear loads are considered separately.
  - The criticality of the stress state in the fiber direction is predicted with the maximum stress condition:
  - Matrix failure is predicted using the expression:
  - Tensile or compressive failure stresses are used depending on the stress state:
  - Failure is predicted to occur when either f_f or f_m reaches the value one. Hence the criterion function is defined as:

$$f_f = \left| \frac{\sigma_1}{X} \right|$$

$$f_m = \left(\frac{\sigma_2}{Y} \right)^2 + \left(\frac{\tau_{12}}{S} \right)^2$$

$$\sigma_1 \geq 0 \Rightarrow X = X_t; \sigma_1 < 0 \Rightarrow X = X_c$$

$$\sigma_2 \geq 0 \Rightarrow Y = Y_t; \sigma_2 < 0 \Rightarrow Y = Y_c$$

$$f = \max\left(f_f, f_m\right)$$

Modified Puck criterion

- Identical to the simplified Puck's criterion except for the condition for matrix failure:

$$f_m = \frac{\sigma_2^2}{Y_t Y_c} + \frac{\tau_{12}^2}{S^2} + \left(\frac{1}{Y_t} - \frac{1}{Y_c} \right) \sigma_2$$

Hashin criterion

- Fiber failure:

$$f_f = \left(\frac{\sigma_1}{X_\tau}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2, \quad \sigma_1 \geq 0 \qquad f_f = -\frac{\sigma_1}{X_c}, \quad \sigma_1 < 0$$

- In the case of tensile transverse stress the expression for predicting matrix failure is:

$$f_m = \left(\frac{\sigma_2}{Y_t}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2, \quad \sigma_2 \geq 0$$

- A more complicated expression is used when the transverse stress is compressive:

$$f_m = \left(\frac{\sigma_2}{2S}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2 + \left[\left(\frac{Y_c}{2S}\right)^2 - 1\right]\frac{\sigma_2}{Y_c}, \quad \sigma_2 < 0$$

- The more critical component is used to describe the failure mode:

$$f = \max\left(f_f, f_m\right)$$

Which one is the best?

Experimental evidence:

In certain cases, FTP provides more accurate results compared to other failure criteria

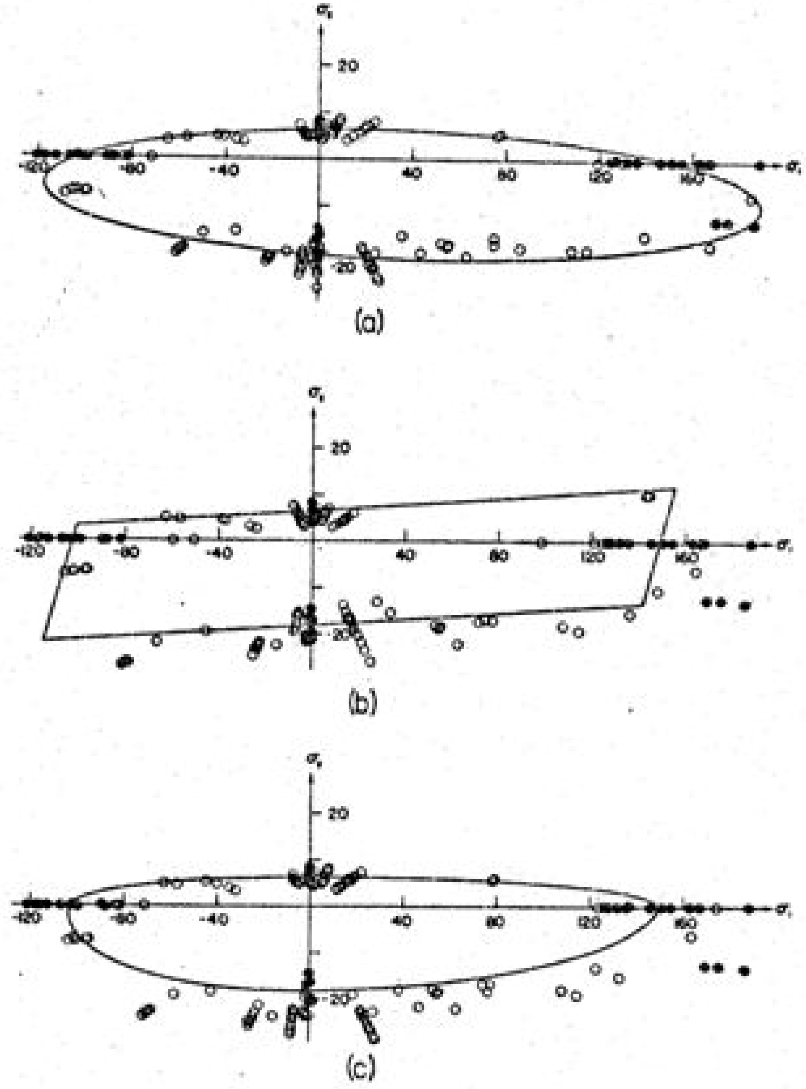


FIG. 18. Failure data convoluted onto $\sigma_1\sigma_2$ plane; stresses in {ksi}: (a) by tensor polynomial failure criterion; (b) by maximum strain failure criterion; (c) by modified von Mises–Hill failure criterion.

E. M. Wu: Phenomenological Anisotropic Failure Criterion, in COMPOSITE MATERIALS **2**, AP, Broutman, Krock Eds., 1974

Use of failure criteria

- Failure prediction
- Off-axis strength prediction
- …

Substituting, Hill's criterion can be written as: (on the local system):

$$\frac{\sigma_1^2}{X^2}+\frac{\sigma_2^2}{Y^2}+\frac{\sigma_3^2}{Z^2}-\left(\frac{1}{Y^2}+\frac{1}{Z^2}-\frac{1}{X^2}\right)\sigma_2\sigma_3-\left(\frac{1}{X^2}+\frac{1}{Z^2}-\frac{1}{Y^2}\right)\sigma_1\sigma_3-\left(\frac{1}{Y^2}+\frac{1}{X^2}-\frac{1}{Z^2}\right)\sigma_1\sigma_2+\frac{\sigma_4^2}{R^2}+\frac{\sigma_5^2}{T^2}+\frac{\sigma_6^2}{S^2}=1$$

0 0 0 0 0

And for plane stress (plane 1-2):

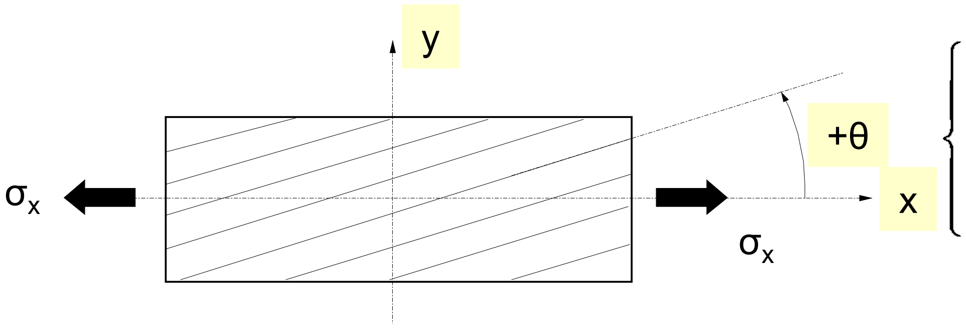
$$\frac{\sigma_1^2}{X^2}+\frac{\sigma_2^2}{Y^2}-\left(\frac{1}{Y^2}+\frac{1}{X^2}-\frac{1}{Z^2}\right)\sigma_1\sigma_2+\frac{\sigma_6^2}{S^2}=1$$

And for a transversely isotropic medium (UD):

Failure prediction

$$\frac{\sigma_1^2}{X^2}+\frac{\sigma_2^2}{Y^2}-\frac{\sigma_1\sigma_2}{X^2}+\frac{\sigma_6^2}{S^2}=1$$

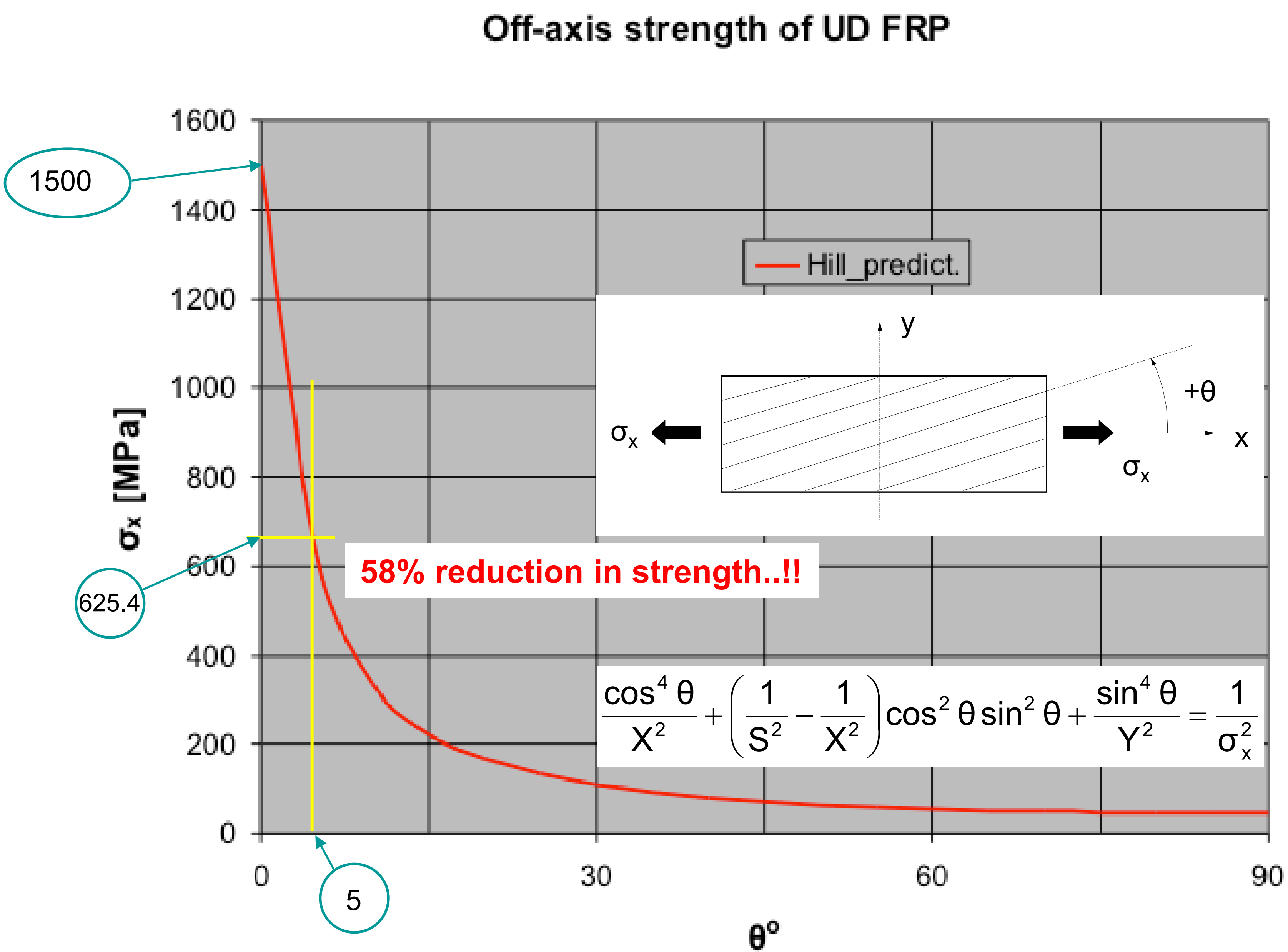
Example: Prediction of failure of an off-axis lamina under uniaxial tensile load (prediction of the off-axis strength!)



$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\end{Bmatrix}=\begin{bmatrix}m^2 & n^2 & 2mn\\ n^2 & m^2 & -2mn\\ -mn & mn & m^2-n^2\end{bmatrix}\begin{Bmatrix}\sigma_x\\ 0\\ 0\end{Bmatrix}$$

m=cosθ, n=sinθ

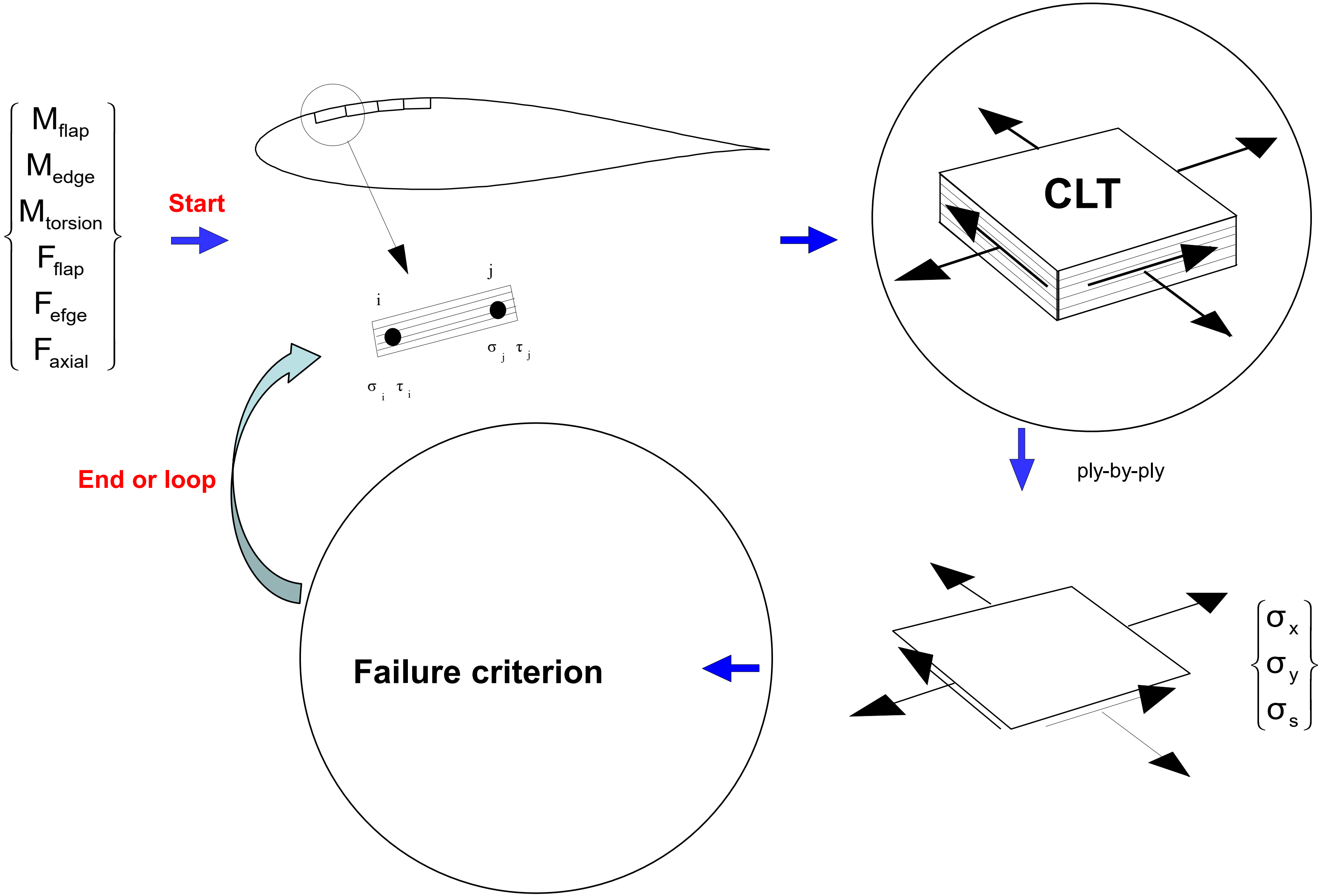
$$\frac{\cos^4\theta}{X^2}+\left(\frac{1}{S^2}-\frac{1}{X^2}\right)\cos^2\theta\sin^2\theta+\frac{\sin^4\theta}{Y^2}=\frac{1}{\sigma_x^2}$$



$$\frac{\cos^4\theta}{X^2} + \left(\frac{1}{S^2} - \frac{1}{X^2}\right)\cos^2\theta\sin^2\theta + \frac{\sin^4\theta}{Y^2} = \frac{1}{\sigma_x^2}$$

Implementation of failure criteria in a design cycle

Or - How and where do we use a failure criterion

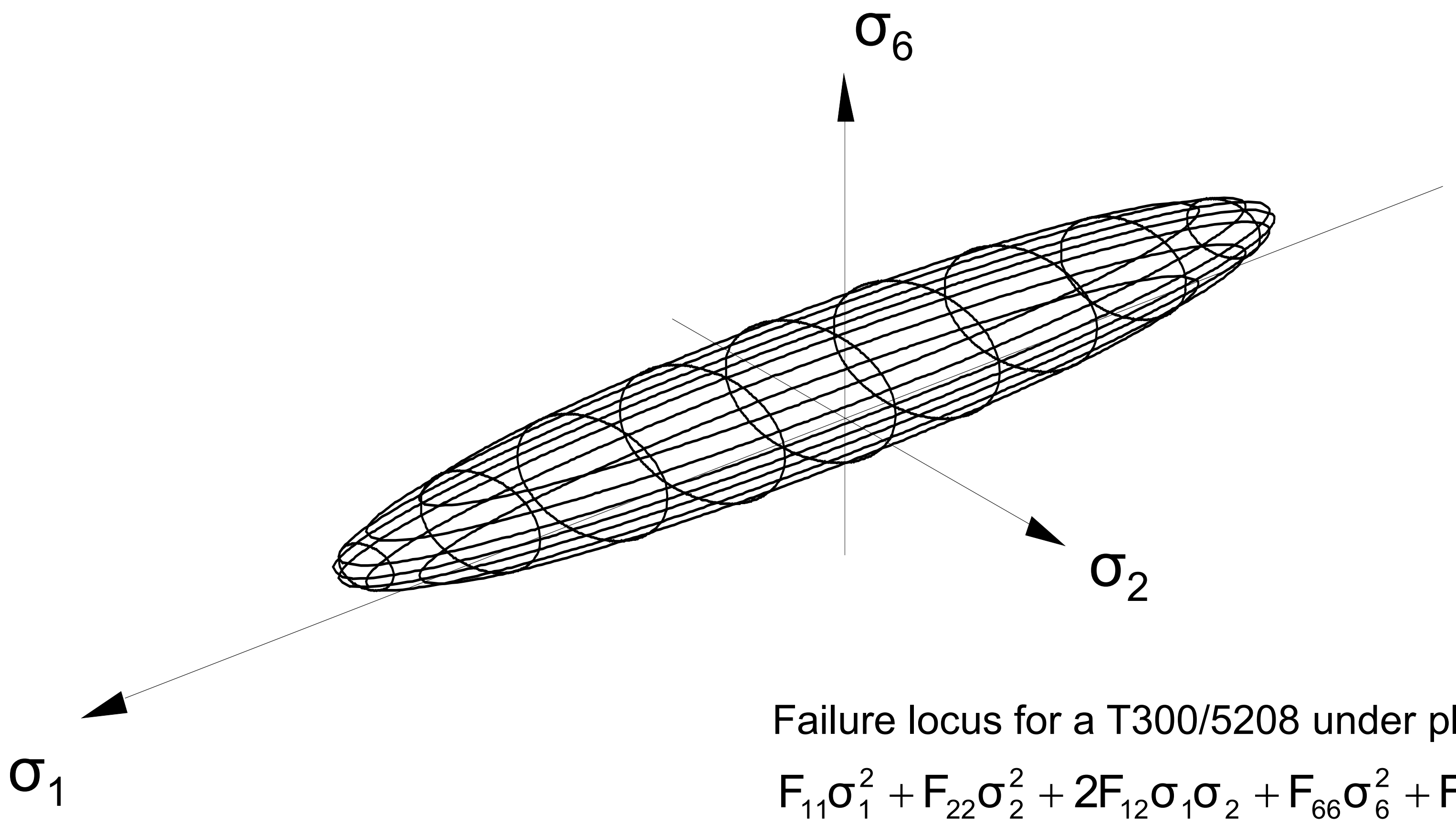


Failure Loci

Failure criteria can assist material selection procedures

A failure locus offers a tool for the evaluation of failure criteria by comparing with available experimental data.

A failure locus, especially for anisotropic media, helps the designer to understand the material capacity for transferring loads – it helps in material selection at preliminary design stages



Failure locus for a T300/5208 under plane stress (1-2)

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 2F_{12}\sigma_1\sigma_2 + F_{66}\sigma_6^2 + F_1\sigma_1 + F_2\sigma_2 - 1 = 0$$

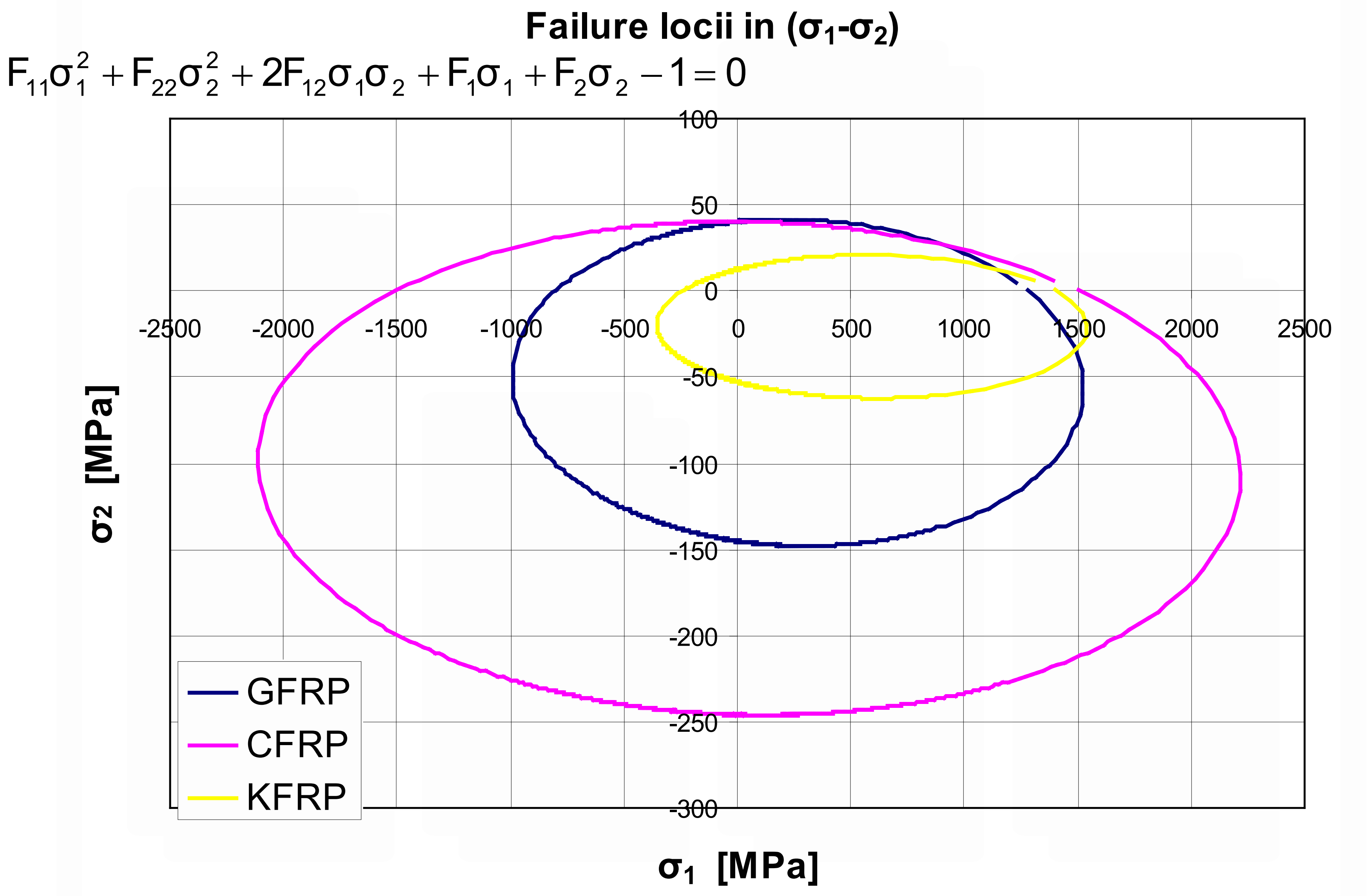
For the transversely isotropic medium (UD) only 5 failure stresses should be determined: X, X´, Y, Y´, S. (the same for the orthotropic medium)

Failure stresses, in MPa, of (UD) composite materials

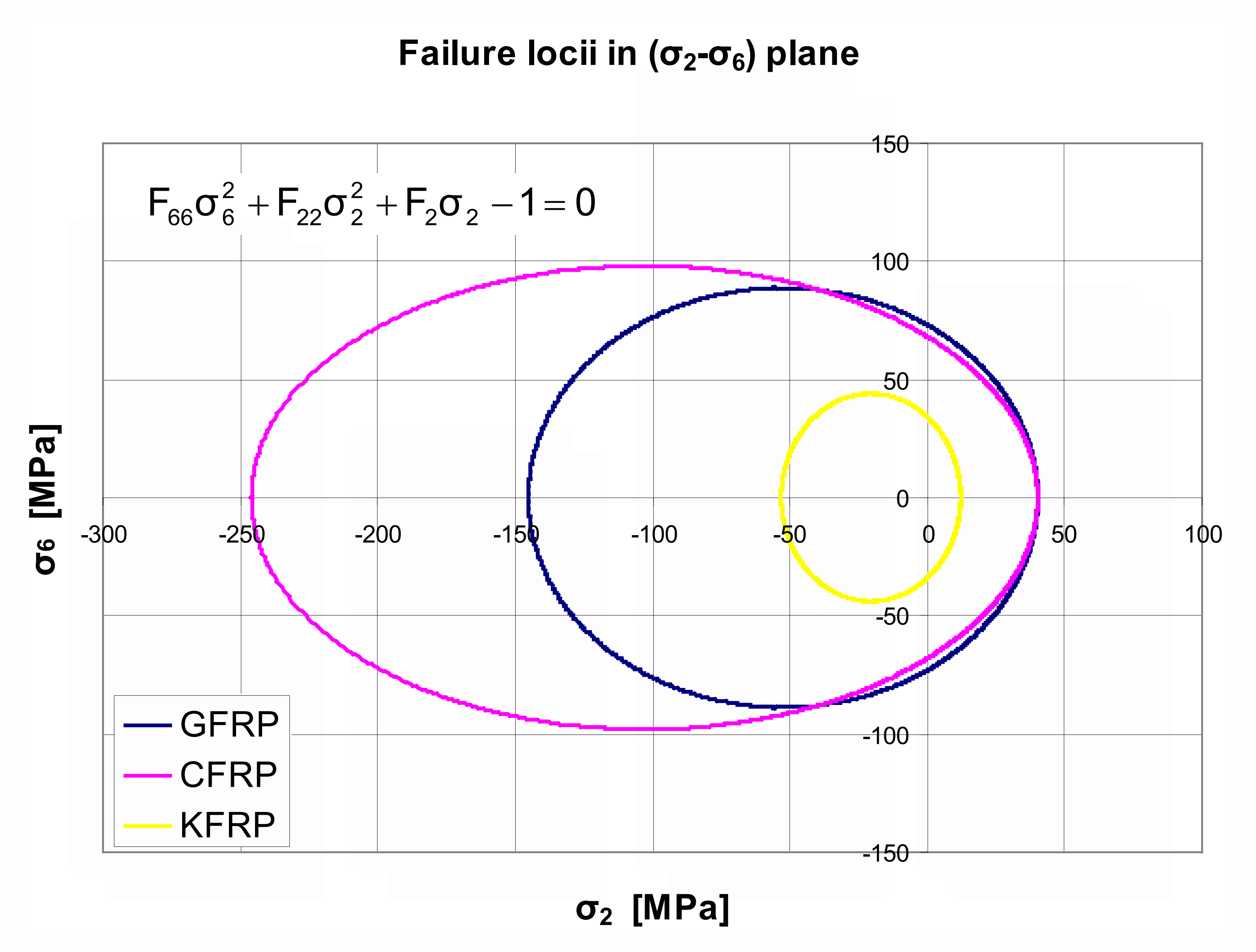
Commercial name	Material	X	X´	Y	Y´	S
T300/5208	Gr/Ep	1500	1500	40	246	68
B(4)/5505	B/Ep	1260	2500	61	202	67
AS/3501	Gr/Ep	1447	1447	52	206	93
Scotchply 1002 (3M)	Gl/Ep	1062	610	31	118	72
Kevlar 49	Ar/Ep	1400	235	12	53	34

Material selection

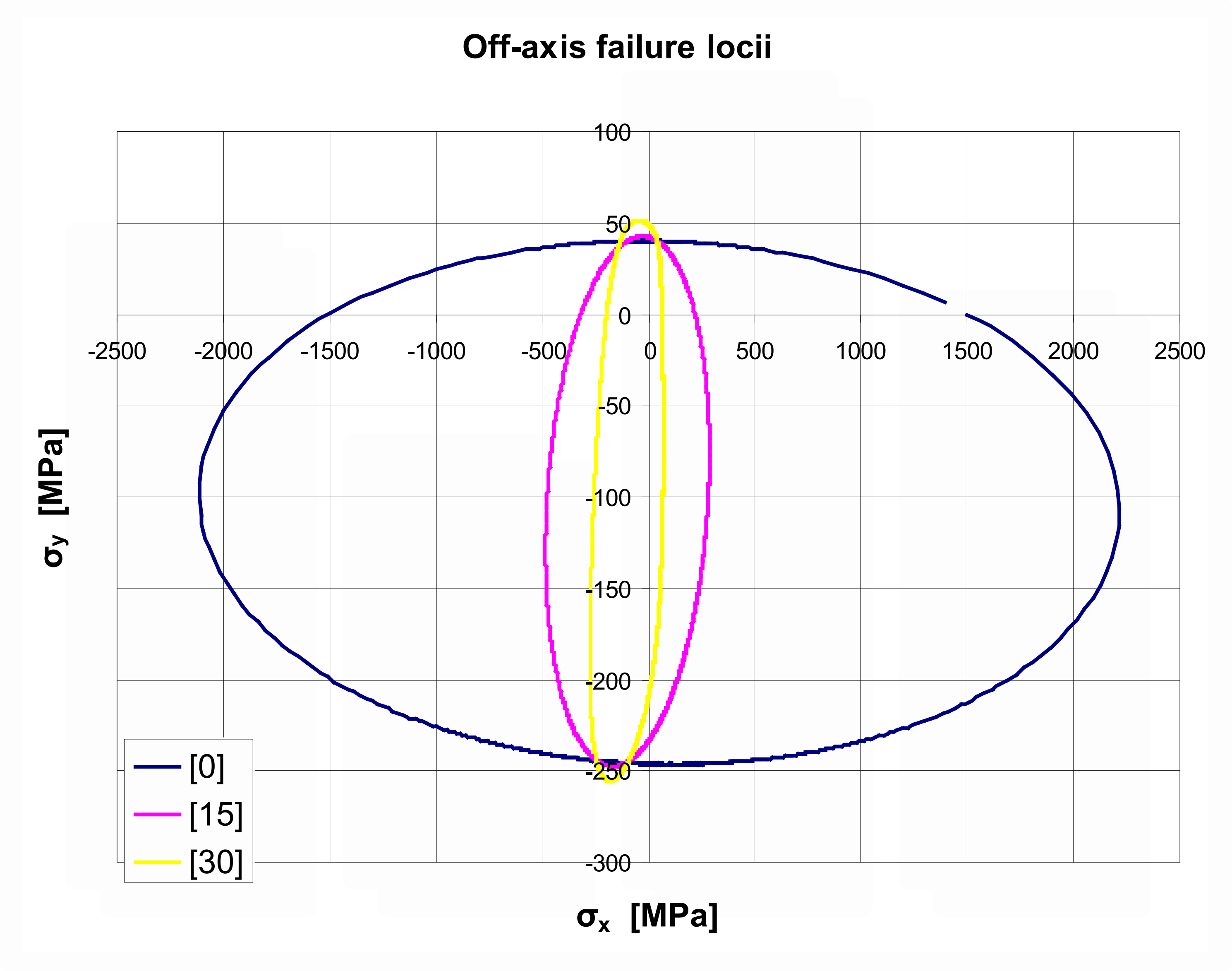
Representative failure locii for different composite materials ($F_{12}=-F_{11}/2$).



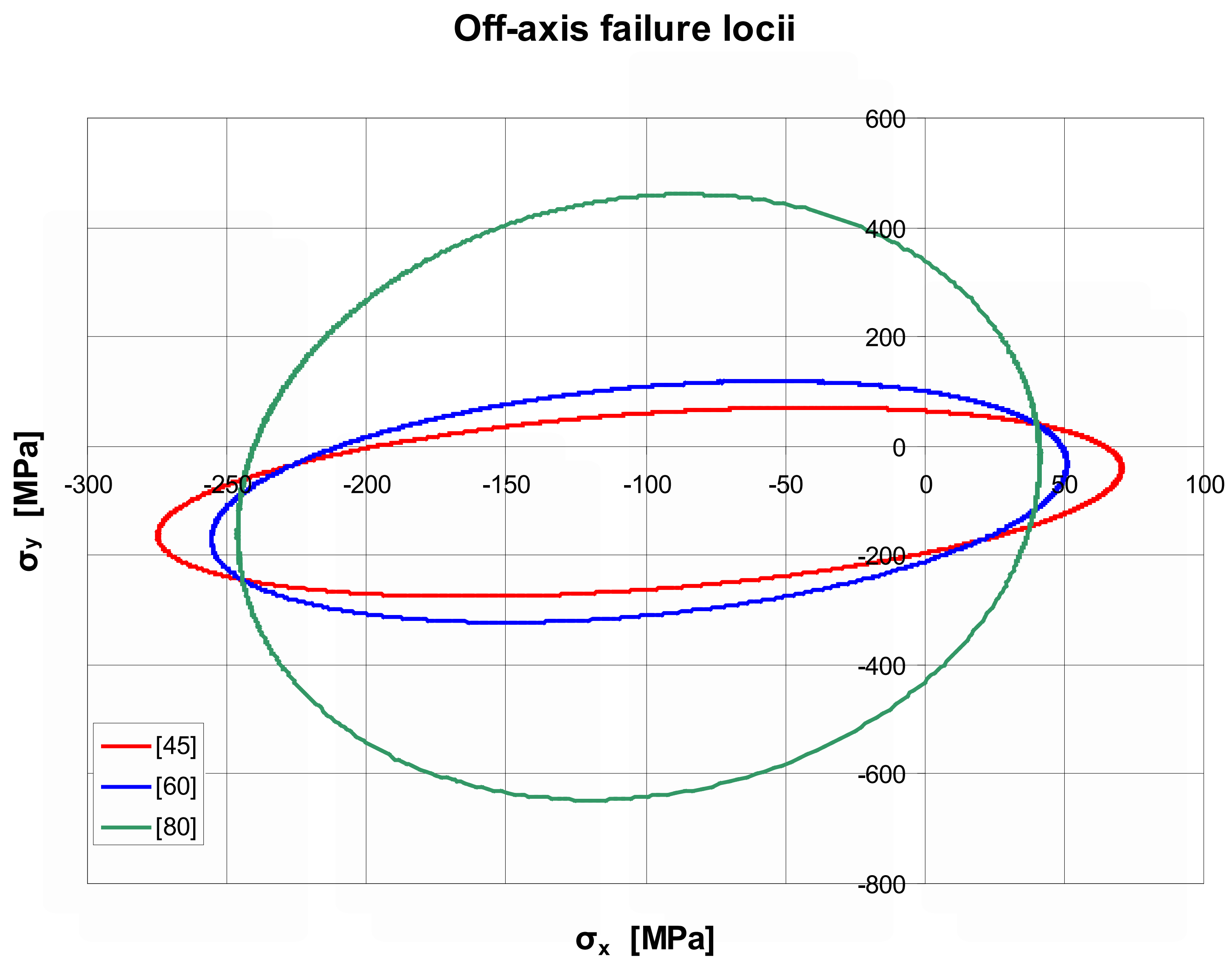
Representative failure locii for different composite materials



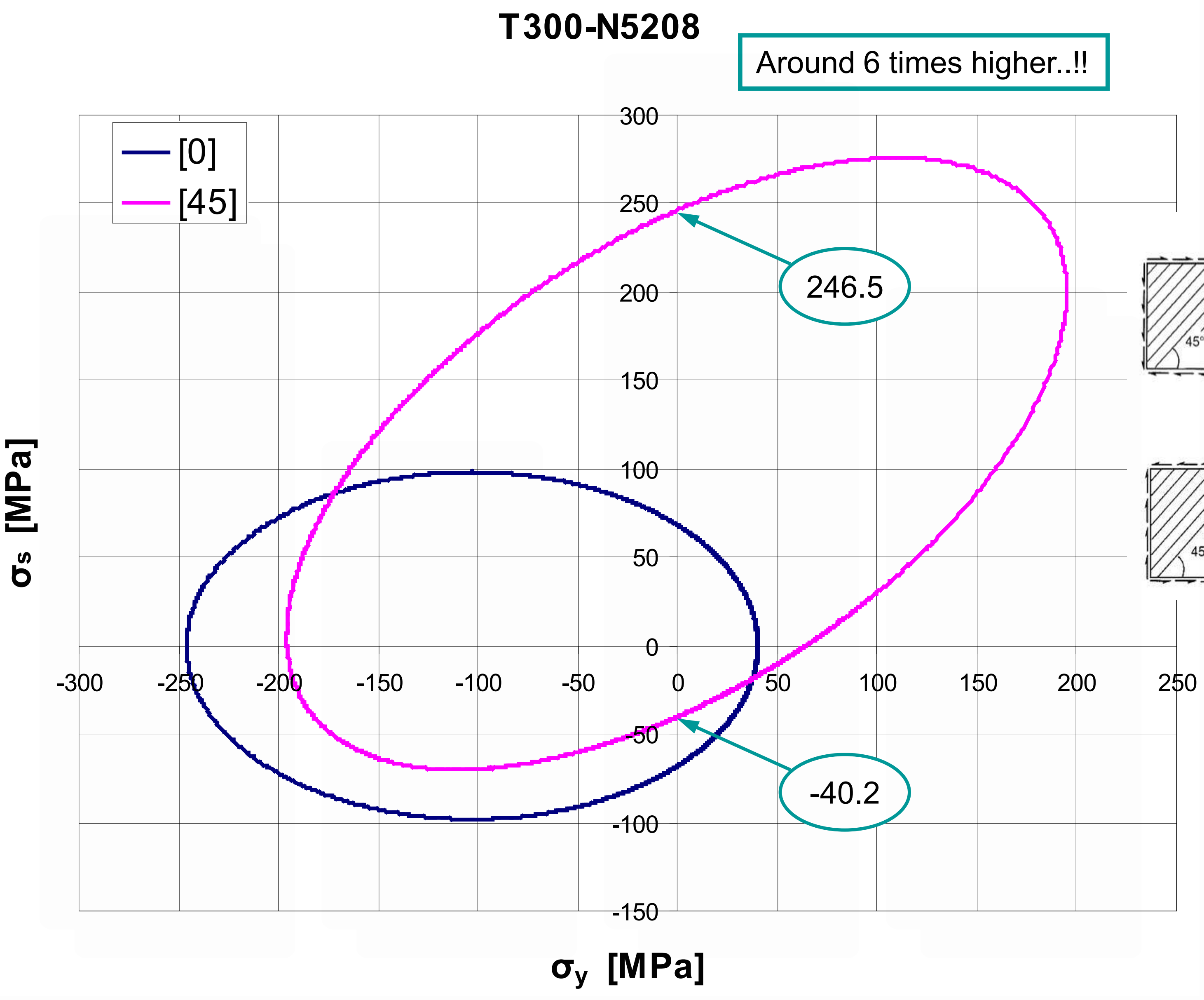
Failure locii for T300/5208 for different off-axis angles (1-2 plane)



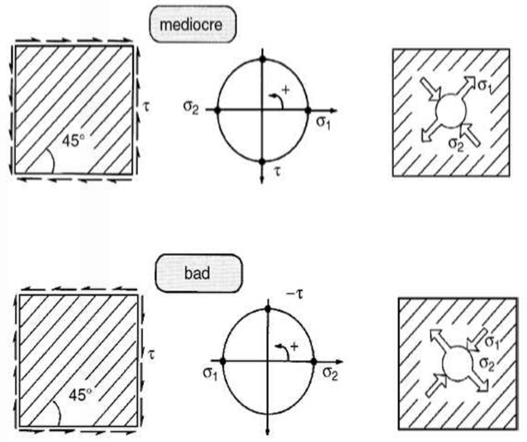
Failure locii for T300/5208 for different off-axis angles (1-2 plane)



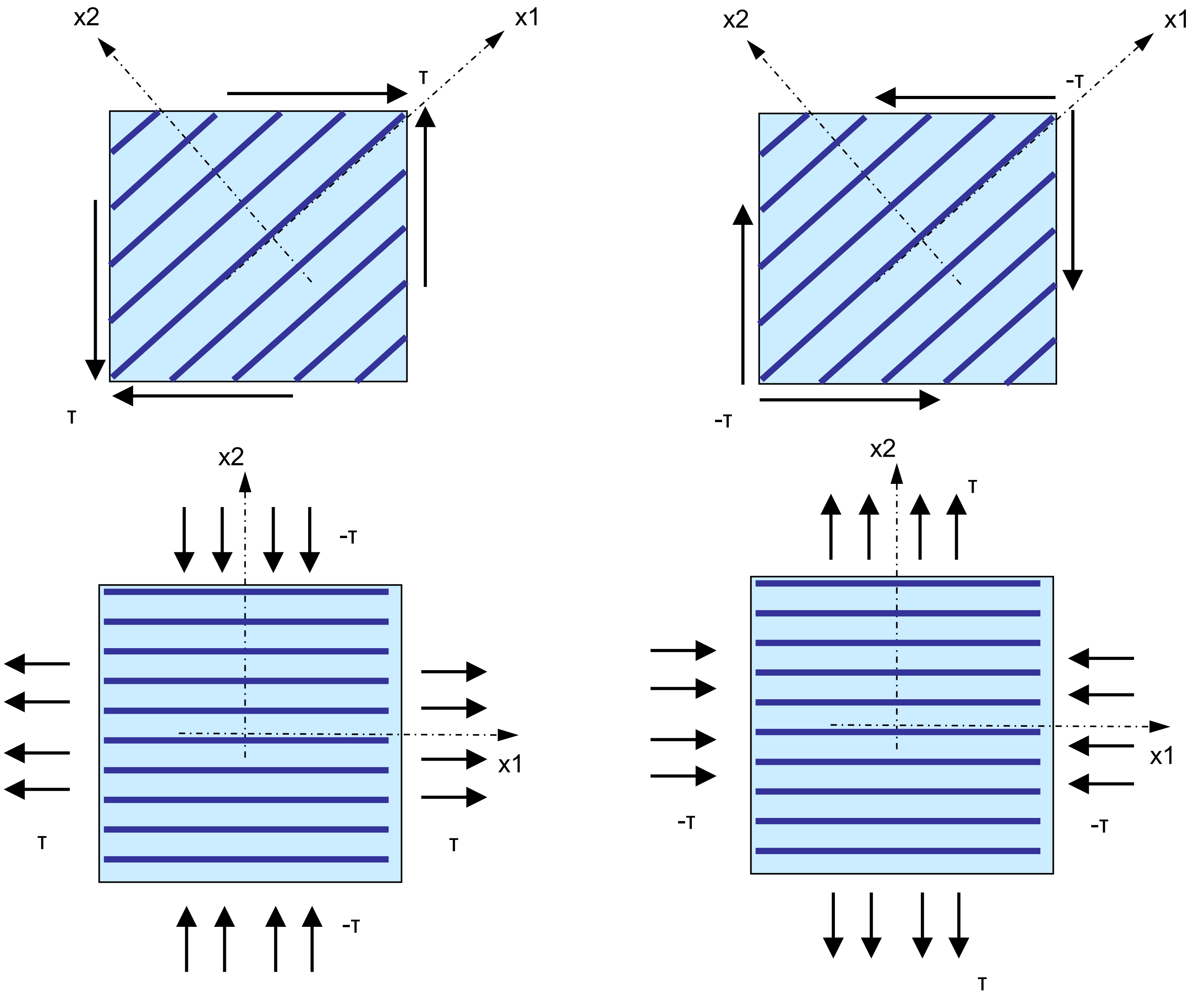
Failure locii for T300/5208 for different off-axis angles (2-6 plane)



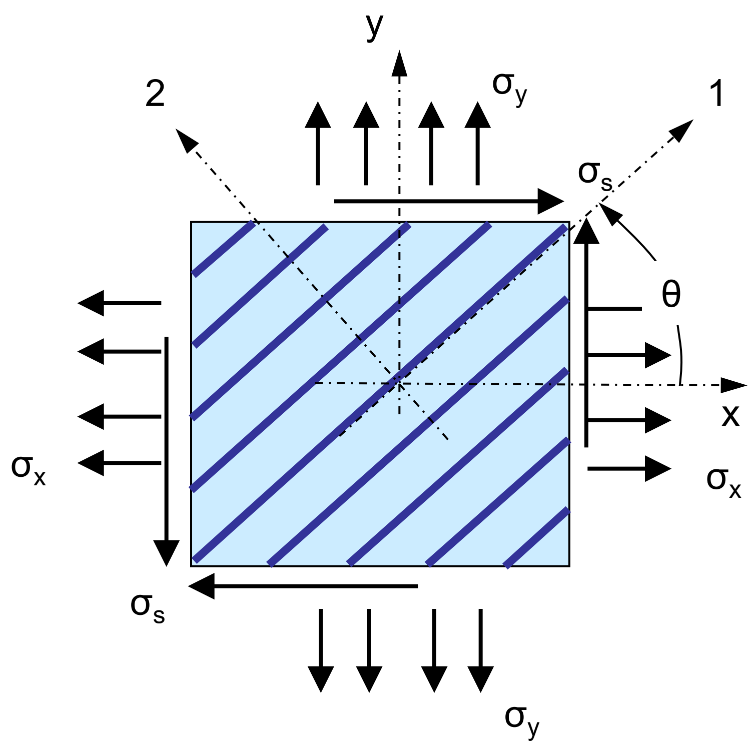
Different “positive” and “negative” shear!



Equivalent stress states for on-axis co-ordinate and 45° off-axis co-ordinate systems



Example I: **Determine the optimum fiber angle for maximum strength under a given plane stress state.**



A. Principal stress orientation and value:

$$\tan 2\theta_\sigma = \frac{2\sigma_s}{\sigma_x - \sigma_y}$$

$$\sigma_{I,II} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \sigma_s^2}$$

B. Minimizing the failure tensor polynomial:

$$f(\sigma_{ij}) = F_{ij}\sigma_i\sigma_j + F_i\sigma_i - 1 \le 0 \quad i,j = x,y,s$$

The result is dependent on the selection of the failure criterion

Consider the next loading case of a laminate made of T300/N5208:

Loading case I: $\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = \begin{Bmatrix} -450 \\ -225 \\ -90 \end{Bmatrix} [MPa]$ → $\begin{Bmatrix} \sigma_I \\ \sigma_{II} \end{Bmatrix} = \begin{Bmatrix} -193.43 \\ -481.57 \end{Bmatrix} [MPa]$ θ_σ=-70.67º

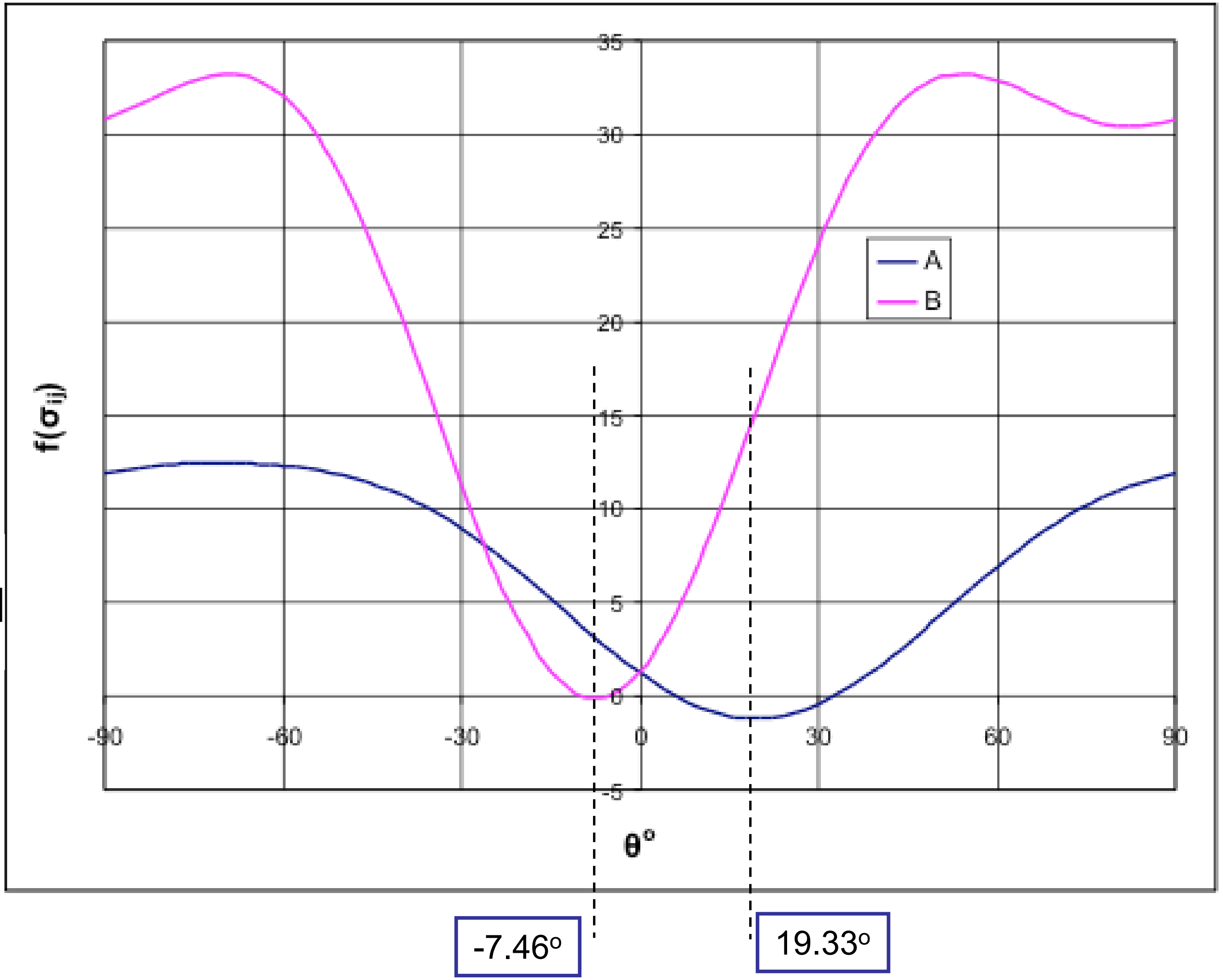
Fibers along an angle of 19.33º

Loading case II: $\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = \begin{Bmatrix} 450 \\ -225 \\ -90 \end{Bmatrix} [MPa]$

↓

$\begin{Bmatrix} \sigma_I \\ \sigma_{II} \end{Bmatrix} = \begin{Bmatrix} 461.79 \\ -236.79 \end{Bmatrix} [MPa]$

θ_σ=-7.46º



Coincidence for both criteria

Discussion

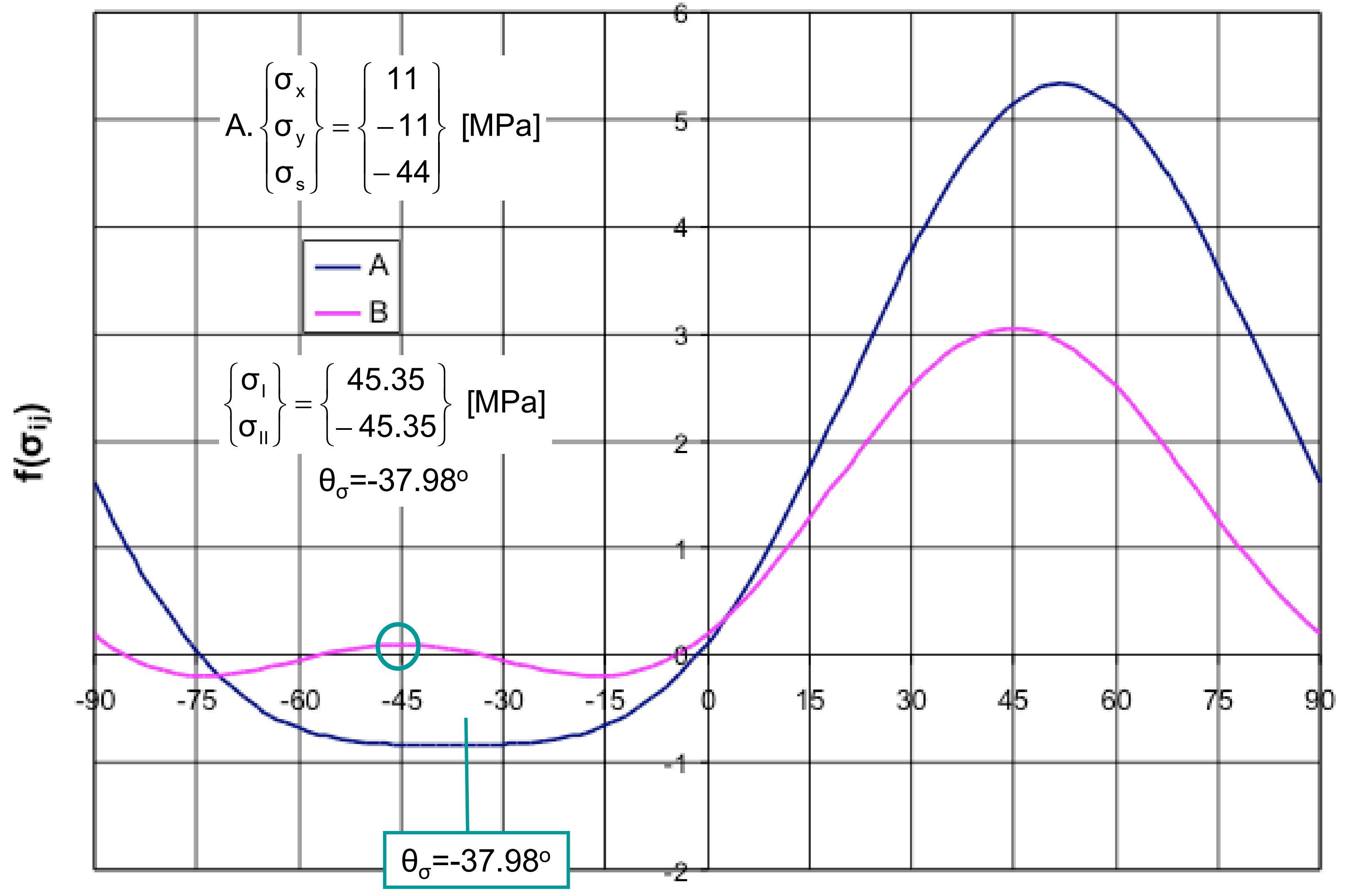
Previous results (for the given material and loading) verify the hypothesis that the fibers should be placed along the direction of the principal normal stresses.

However this is not the case for all materials and any loading case...

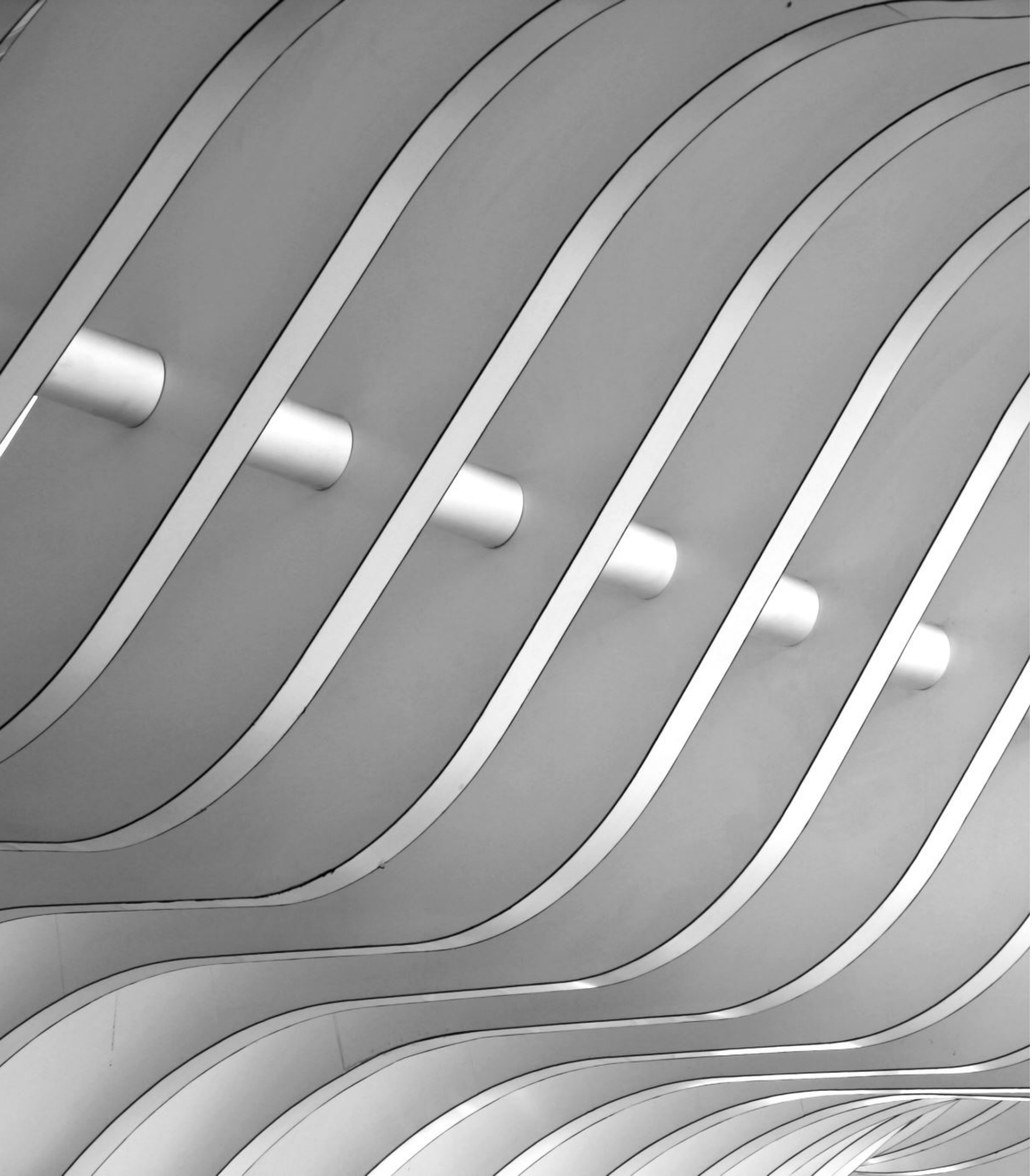
Consider the next loading cases for Kevlar 49:

$$A. \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = \begin{Bmatrix} 11 \\ -11 \\ -44 \end{Bmatrix} [MPa] \longrightarrow \begin{Bmatrix} \sigma_I \\ \sigma_{II} \end{Bmatrix} = \begin{Bmatrix} 45.35 \\ -45.35 \end{Bmatrix} [MPa] \quad \theta_\sigma = -37.98^o$$

$$\text{B.}\ \begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_s\end{Bmatrix} = \begin{Bmatrix}-11\\ -11\\ -44\end{Bmatrix}\ [\text{MPa}] \longrightarrow \begin{Bmatrix}\sigma_I\\ \sigma_{II}\end{Bmatrix} = \begin{Bmatrix}33\\ -55\end{Bmatrix}\ [\text{MPa}] \quad \theta_\sigma=-45^o$$



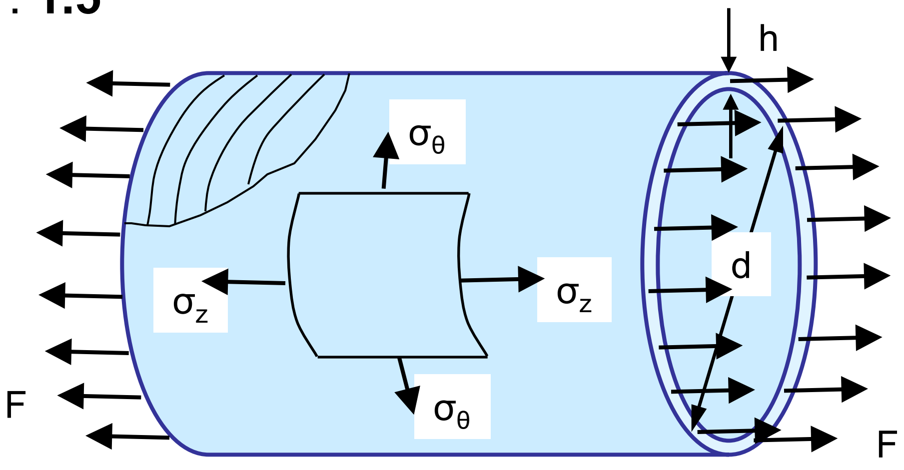
Different estimation from both criteria for a given loading

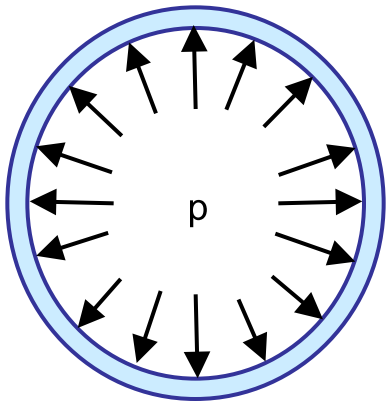


Structural design

***Example II*:** Define the minimum thickness of the walls of a tube with a Diameter *Φ*150 mm that would be fabricated by the filament winding technique.
The direction of the angles (winding angle) is the transverse to the longitudinal axis of the cylinder.

Loads: Internal pressure: **p=2 MPa**, Tensile force **F=20 kN**
Safety factor: **1.5**





$$\sigma_z = \frac{F}{\pi dh}$$

Plane stress

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 2F_{12}\sigma_1\sigma_2 + F_1\sigma_1 + F_2\sigma_2 - 1 = 0$$

$$F_{11}\left(\frac{p'd}{2h}\right)^2 + F_{22}\left(\frac{F'}{\pi dh}\right)^2 + 2F_{12}\left(\frac{p'd}{2h}\right)\left(\frac{F'}{\pi dh}\right) + F_1\left(\frac{p'd}{2h}\right) + F_2\left(\frac{F'}{\pi dh}\right) - 1 \leq 0$$

$$\sigma_\theta = \frac{pd}{2h}$$

$p' = 1.5p$

$F' = 1.5F$

T300/N5208 $h \geq 1.603 \cdot 10^{-3} m$ → 9 layers of 200 μm each

γ_F=1.0 $h \geq 1.069 \cdot 10^{-3} m$ → 6 layers of 200 μm each