Exercise 2: Probability of depassivation (carbonation)

Case study of the St. Anthony's Church in Basel

Background and objective

The St. Anthony's Church (Sankt Antonius Kirche) in Basel was built in 1926. Its framework and the frames of the stained-glass windows are made of reinforced concrete. The concrete was placed by tamping or using rods (vibrators did not yet exist at that time), https://www.e-periodica.ch/cntmng?pid=sbz-002:1927:89::378.

According to https://www.antoniuskirche.ch/ueber-uns/kirche :

"It was built in the years 1925-1927 according to the designs of the Swiss architect Karl Moser in formwork-rough exposed concrete. It is thus the first church in Switzerland to be built using this method."

https://commons.wikimedia.org/wiki/Category:Antoniuskirche_%28Basel%29

A measurement campaign to characterize the concrete cover over the reinforcement bars and the depth of carbonation was carried out in 1983. The results of these measurements, their location and the corresponding statistical values are given in Appendix I.

The aim of the exercise is to assess the phenomenon of carbonation and likelihood of depassivation in a reinforced concrete structure and its evolution over the years with a probabilistic approach.

Answer the following questions using the statistical values given in Appendix I (mean and standard deviation of the carbonation depth and the cover thickness for the entire structure, all points combined) and the method described in Appendix II.

Question 1

Determine the characteristic coefficients μ_{10} and σ_{10} of the stochastic process describing the evolution of the carbonation depth as a function of time for the whole structure.

Question 2

Determine the frequency distribution of carbonation depths for the entire structure at 57 years age according to the normal law given in equation (3) Appendix II (calculation with 13 classes: from 0 to 130 mm, bin size = 10 mm, number of measurements = 102). Compare with the histogram of the measurement values shown on figure 2 a), Appendix I.

Question 3

Determine the frequency distribution of carbonation depths for the entire structure at 20- and 57-years age according to the normal law given in equation (3) – Appendix II (calculation with 10 classes, bin size = 10 mm, from 0 to 100 mm at 20 years and 0 to 130 mm at 57 years age). Compare with the concrete cover thickness distribution according to the normal law given in equation (4), Appendix II. What is the likely probability of depassivation at 20 years age?

Question 4

Determine the probability of depassivation for the entire structure after 2, 5, 10, 20, 40 and 57 years of age after equation (7) – Appendix II and Appendix III.

Question 5

Discuss the calculation assumptions and the results obtained. What is the advantage of a probabilistic method compared to a deterministic one for assessing the risk of depassivation?

References

[1] Sentler L. (1984) "Stochastic characterization of carbonation of concrete", in Proceedings 3rd Int. Conf. On The Durability of Building Materials and Components, Technical Research center of Finland, June 1984, pp 569-580.

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Appendix I: St. Anthony's Church in Basel, characterization of the carbonation of the concrete and the thickness of the concrete cover of the reinforcement bars.

Location map of the measurement areas (only outside)

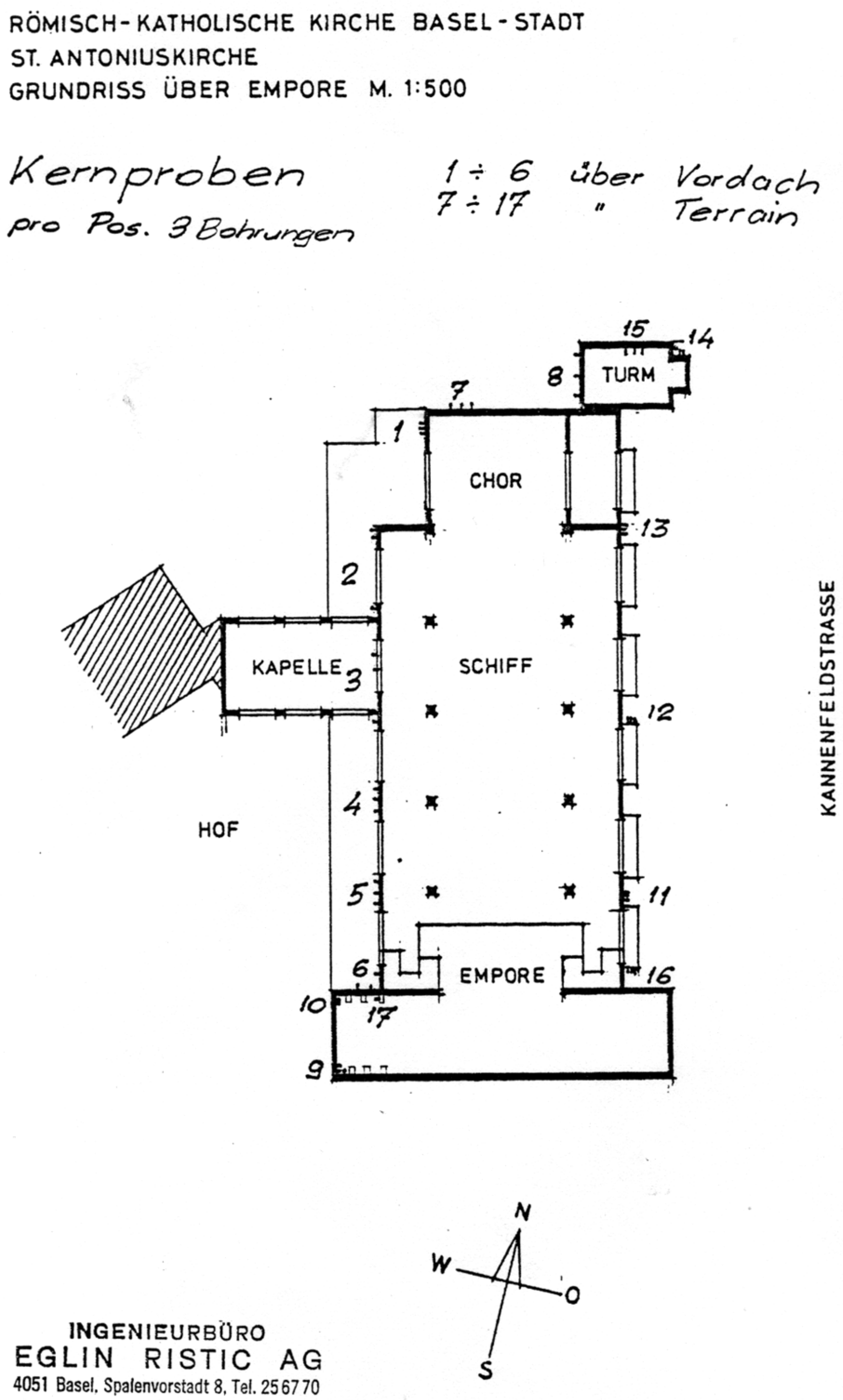
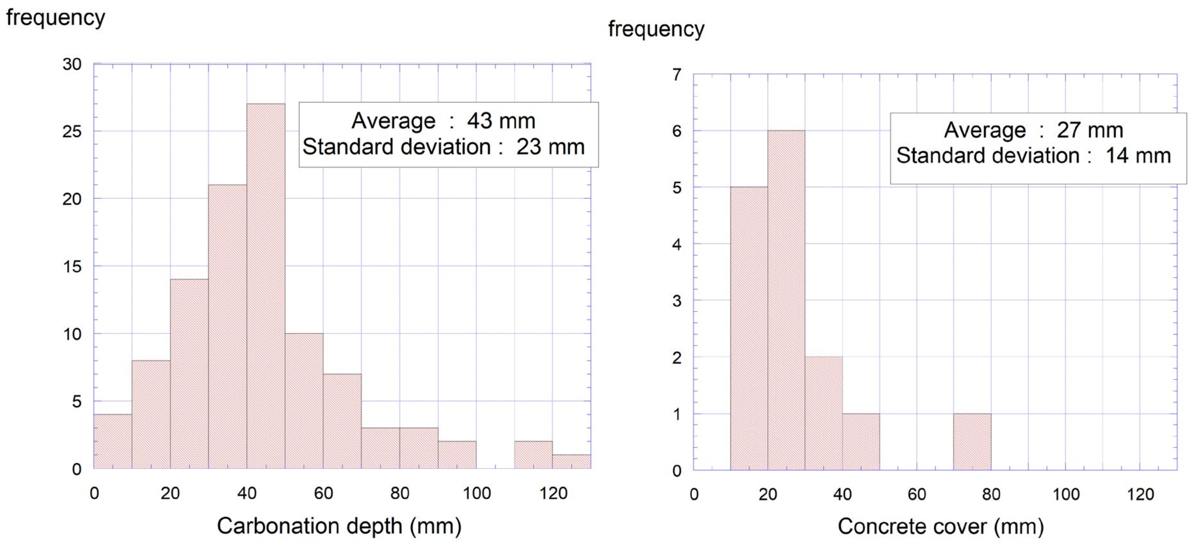


Figure 1: Location map of the measurement areas. Turm = Tower, Chor = Choir, Empore = Mezzanine, Kapelle = Chapel, Schiff = Nave – Aisle.

Carbonated depth

Table 1: Mean and standard deviation of the carbonated depth in the different zones (in 1983, at 57 years age of the building).

Zone	Mean [mm]	Standard deviation [mm]
West façade	48	24
East façade	39	26
North façade	34	9
West façade - Portal	36	15
Bell tower – West side	31	15
Bell tower – East side	40	12
Bell tower – North side	67	29
Whole structure	43	23



a) b)

Figure 2: Histogram of the statistical distribution of: a) carbonation depth in 1983 after 57 years and b) concrete cover, for the whole structure, with 102 measurements for the depth of carbonation and 15 measurements for the concrete cover.

Appendix II: Probabilistic modeling of the evolution of the carbonation depth and the probability of depassivation as a function of time.

The classical deterministic modeling of the evolution of the carbonation depth as a function of time is of the type (where K is a coefficient depending on the quality of the cover concrete and the exposure, and d is the average carbonation depth, at time t):

$$d = K\sqrt{t} \tag{1}$$

According to Sentler (1984), the advancement of the carbonation front can be modeled probabilistically, using a normal law with characteristic parameters (mean μ_1 and standard deviation σ_1) depending on time t as:

$$\mu_1(t) = \mu_{10} \cdot \sqrt{t} \quad \text{and} \quad \sigma_1(t) = \sigma_{10} \cdot \sqrt[4]{t} \tag{2}$$

In this case, the shape and position of the statistical distribution of carbonation depth values is time dependent. Accordingly, this statistical distribution shifts and deforms over time. For a time t, the frequency f_1 of the carbonated depth d is:

$$f_1(d,t) = \frac{1}{\sigma_1\sqrt{2\pi}}\exp\left(-\frac{(d-\mu_1)^2}{2\sigma_1^{\,2}}\right) = \frac{1}{\sigma_{10}\sqrt{2\pi\sqrt{t}}}\exp\left(-\frac{(d-\mu_{10}\sqrt{t})^2}{2\sigma_{10}^{\,2}\sqrt{t}}\right) \tag{3}$$

One recognizes the characteristic form of the distribution function of the normal law, with the difference that the mean and the standard deviation are functions of time.

The statistical distribution of the reinforcement cover thickness c remains constant over time, with μ_2 its mean and σ_2 its standard deviation. This statistical distribution is assumed to follow a normal law of frequency $f_2(c, t)$:

$$f_2(c,t) = \frac{1}{\sigma_2\sqrt{2\pi}}\exp\left(-\frac{(c-\mu_2)^2}{2\sigma_2^{\,2}}\right) \tag{4}$$

As a deterministic criterion for depassivation, it can be assumed that "the carbonated depth is greater than or equal to the concrete cover of the reinforcement bars", i.e.:

$$d \geq c \tag{5}$$

In probabilistic terms, the same criterion can be used. It then applies to random variables:

$$p(d \geq c) \tag{6}$$

Finally, from equations (3), (4) and (6), it can be shown that the probability of depassivation as a function of time is:

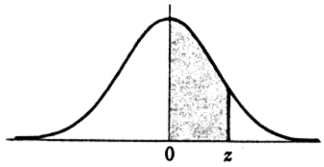
$$p(d \geq c) = p(d - c \geq 0) = 1 - \left(p(z) \leq \beta\right) \quad \text{with } \beta = \frac{\mu_2 - \mu_{10}\sqrt{t}}{\sqrt{\sigma_{10}^{\,2}\sqrt{t} + \sigma_2^{\,2}}} \tag{7}$$

where z(1,0) is the normal reduced variable.

The calculation of the probability for a reduced normal variable can be made using the numerical values given in Appendix III.

Appendix III: Areas under the reduced normal curve between 0 and z.

Aires sous la courbe normale réduite comprises entre 0 et *z*



z	0	1	2	3	4	5	6	7	8	9
0,0	,0000	,0040	,0080	,0120	,0160	,0199	,0239	,0279	,0319	,0359
0,1	,0398	,0438	,0478	,0517	,0557	,0596	,0636	,0675	,0714	,0754
0,2	,0793	,0832	,0871	,0910	,0948	,0987	,1026	,1064	,1103	,1141
0,3	,1179	,1217	,1255	,1293	,1331	,1368	,1406	,1443	,1480	,1517
0,4	,1554	,1591	,1628	,1664	,1700	,1736	,1772	,1808	,1844	,1879
0,5	,1915	,1950	,1985	,2019	,2054	,2088	,2123	,2157	,2190	,2224
0,6	,2258	,2291	,2324	,2357	,2389	,2422	,2454	,2486	,2518	,2549
0,7	,2580	,2612	,2642	,2673	,2704	,2734	,2764	,2794	,2823	,2852
0,8	,2881	,2910	,2939	,2967	,2996	,3023	,3051	,3078	,3106	,3133
0,9	,3159	,3186	,3212	,3238	,3264	,3289	,3315	,3340	,3365	,3389
1,0	,3413	,3438	,3461	,3485	,3508	,3531	,3554	,3577	,3599	,3621
1,1	,3643	,3665	,3686	,3708	,3729	,3749	,3770	,3790	,3810	,3830
1,2	,3849	,3869	,3888	,3907	,3925	,3944	,3962	,3980	,3997	,4015
1,3	,4032	,4049	,4066	,4082	,4099	,4115	,4131	,4147	,4162	,4177
1,4	,4192	,4207	,4222	,4236	,4251	,4265	,4279	,4292	,4306	,4319
1,5	,4332	,4345	,4357	,4370	,4382	,4394	,4406	,4418	,4429	,4441
1,6	,4452	,4463	,4474	,4484	,4495	,4505	,4515	,4525	,4535	,4545
1,7	,4554	,4564	,4573	,4582	,4591	,4599	,4608	,4616	,4625	,4633
1,8	,4641	,4649	,4656	,4664	,4671	,4678	,4686	,4693	,4699	,4706
1,9	,4713	,4719	,4726	,4732	,4738	,4744	,4750	,4756	,4761	,4767
2,0	,4772	,4778	,4783	,4788	,4793	,4798	,4803	,4808	,4812	,4817
2,1	,4821	,4826	,4830	,4834	,4838	,4842	,4846	,4850	,4854	,4857
2,2	,4861	,4864	,4868	,4871	,4875	,4878	,4881	,4884	,4887	,4890
2,3	,4893	,4896	,4898	,4901	,4904	,4906	,4909	,4911	,4913	,4916
2,4	,4918	,4920	,4922	,4925	,4927	,4929	,4931	,4932	,4934	,4936
2,5	,4938	,4940	,4941	,4943	,4945	,4946	,4948	,4949	,4951	,4952
2,6	,4953	,4955	,4956	,4957	,4959	,4960	,4961	,4962	,4963	,4964
2,7	,4965	,4966	,4967	,4968	,4969	,4970	,4971	,4972	,4973	,4974
2,8	,4974	,4975	,4976	,4977	,4977	,4978	,4979	,4979	,4980	,4981
2,9	,4981	,4982	,4982	,4983	,4984	,4984	,4985	,4985	,4986	,4986
3,0	,4987	,4987	,4987	,4988	,4988	,4989	,4989	,4989	,4990	,4990
3,1	,4990	,4991	,4991	,4991	,4992	,4992	,4992	,4992	,4993	,4993
3,2	,4993	,4993	,4994	,4994	,4994	,4994	,4994	,4995	,4995	,4995
3,3	,4995	,4995	,4995	,4996	,4996	,4996	,4996	,4996	,4996	,4997
3,4	,4997	,4997	,4997	,4997	,4997	,4997	,4997	,4997	,4997	,4998
3,5	,4998	,4998	,4998	,4998	,4998	,4998	,4998	,4998	,4998	,4998
3,6	,4998	,4998	,4999	,4999	,4999	,4999	,4999	,4999	,4999	,4999
3,7	,4999	,4999	,4999	,4999	,4999	,4999	,4999	,4999	,4999	,4999
3,8	,4999	,4999	,4999	,4999	,4999	,4999	,4999	,4999	,4999	,4999
3,9	,5000	,5000	,5000	,5000	,5000	,5000	,5000	,5000	,5000	,5000