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Exercise #6: Inelastic SDF systems and their seismic design for ductility

Problem 1

Consider a long reinforced-concrete bridge. The total weight of the superstructure, $2{,}000\ kg/m$, is supported on identical bents $10\ m$ high, uniformly spaced at $40\ m$. Each bent consists of a single circular column $1.5\ m$ in diameter (see Fig. 1.1). The anticipated period of a bridge bent is, $T_n = 1\ s$. Design the longitudinal reinforcement ratio, ρ_t, of the column under the El Centro ground motion (see Fig. 1.2 the response spectra) for 2% damping ratio for the following two cases:

1. to remain elastic; and
2. for an allowable ductility factor of μ =5 (assume, $R_y = \mu = 5$)
3. For the reinforcement ratio, ρ_t you estimated check if the bridge bent will exceed a maximum inelastic displacement of $u_m = 500mm$.

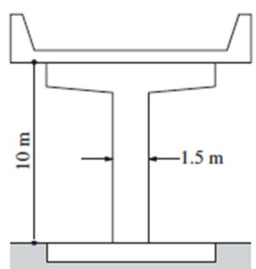

Figure 1.1

Notes:

a) A simplified formula for calculating the flexural resistance of a circular column pier is as follows:

$$M_{Rd} = \frac{2}{3} r^3 \sin^3 \theta\, f'_{cd} + \frac{2}{\pi}(r - c) A_s sin\theta f_{yd}$$

r: radius of column bent cross section.
$f'_{cd} = 30MPa$; $f_{yd} = 420MPa$
c: concrete cover (assume 50mm in this case)
θ: is the angle defining the extension of compression zone (assume $26^o \cong \pi/7$ in this case)
A_s: is the steel reinforcement area.

The longitudinal steel reinforcement ratio can be calculated as follows, $\rho_t = \frac{A_s f_{yd}}{\pi r^2 f'_{cd}}$

b) According to ACI-318-05, the effective stiffness EI for circular columns under lateral load is given by,

$$EI = E_c I_g \left(0.2 + 2\rho_t \gamma^2 \frac{E_s}{E_c}\right)$$

Where:

I_g is the second moment of area of the gross cross section;
E_c and E_s are the elastic moduli of concrete and reinforcing steel, respectively; assume $E_c = 30GPa$ and $E_s = 200GPa$;
ρ_t is the longitudinal reinforcement ratio;
γ is the ratio of the distances from the center of the column to the center of the outermost reinforcing bars and to the column edge; assume $\gamma = 0.9$ for this problem.

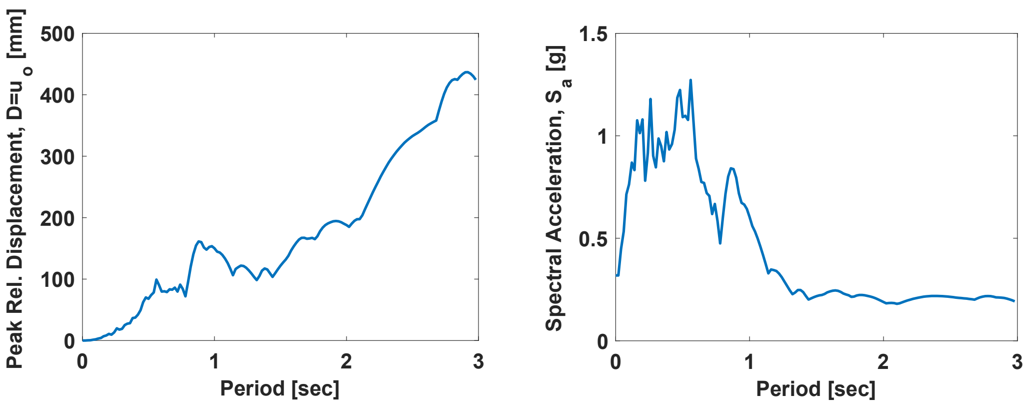

Figure 1.2 response spectra for ElCentro ground motion (2% damping ratio)

Solution

Question 1:

The SDF system is designed to remain elastic at $T_n = 1\ s$. Therefore, the maximum expected spectral acceleration A for the system to remain elastic can be estimated from the elastic absolute acceleration spectrum of the El Centro record:

$$A = 0.65g$$

The weight of the idealized SDF system is the tributary mass for one bent (i.e., the mass of 40-m length of the superstructure:

$$W = (2'000)(40) = 8x10^4 kg = 784.8kN$$

Therefore, the maximum expected lateral force at a single bent will be:

$$f_o = \frac{W}{g} \cdot A = \frac{784{,}8}{g} \cdot 0.65g = 510.12kN$$

This force is acting laterally to the bent, as such, it causes a moment equal to:

$$M_{Ed} = f_o h = 510.12 \cdot 10m = 5'101.2kNm = 5'101'200kNmm$$

From the notes of the problem,

$$M_{Rd} = \frac{2}{3} r^3 \sin^3\theta\, f'_{cd} + \frac{2}{\pi}(r-c)A_s sin\theta f_{yd} = M_{Ed} \Leftrightarrow$$

$$A_s = \frac{M_{Ed} - \frac{2}{3} r^3 \sin^3\theta\, f'_{cd}}{\frac{2}{\pi}(r-c) sin\theta f_{yd}}$$

$$A_s = \frac{5'101'200kNmm - \frac{2}{3}(750mm)^3 \sin^3\left(\frac{\pi}{7} rad\right) 0.030kN/mm^2}{\frac{2}{\pi}(750mm - 50mm) sin\left(\frac{\pi}{7} rad\right) 0.420kN/mm^2} = 54'329{,}6mm^2$$

Therefore,

$$\rho_t = \frac{A_s f_{yd}}{\pi r^2 f'_{cd}} = \frac{54'329{,}6 \cdot 0.420}{\pi(750^2) \cdot 0.03} = 0.43 = 43\%$$

NOTE: note the extremely large volume of steel reinforcement required to achieve elastic response.

Question 2

We are supposed to design the bridge bents for ductility. Therefore, the anticipated lateral force at yield should be given as follows:

$$R_y = \frac{f_o}{f_y} \Longrightarrow f_y = \frac{f_o}{R_y} = \frac{510.12kN}{5} = 102kN$$

This force is acting laterally to the bent, as such, it causes a moment equal to:

$$M_{Ed} = f_y h = 102kN \cdot 10m = 1'020\ kNm = 1'020'000kNmm$$

From the notes of the problem,

$$M_{Rd} = \frac{2}{3} r^3 \sin^3\theta\, f'_{cd} + \frac{2}{\pi}(r-c) A_s sin\theta f_{yd} = M_{Ed} \Leftrightarrow$$

$$A_s = \frac{M_{Ed} - \frac{2}{3} r^3 \sin^3\theta\, f'_{cd}}{\frac{2}{\pi}(r-c)sin\theta f_{yd}} = \frac{1'020'000kNmm - (\frac{2}{3}750^3\ \sin^3\left(\frac{\pi}{7}\ rad\right)0.03kN/mm^2)}{\frac{2}{\pi}(750-50)sin\left(\frac{\pi}{7}rad\right)0.420kN/mm^2}$$

$$\rightarrow A_s = 4'073.7mm^2$$

Therefore,

$$\rho_t = \frac{A_s f_{yd}}{\pi r^2 f'_{cd}} = \frac{4'073.7 \cdot 0.420}{\pi(750^2)\cdot 0.03} = 0.0323 = 3.23\%$$

Note the significant reduction of the steel reinforcement ratio when the system is designed for a targeted ductility (μ=5 in this case).

Question 3

As the system is designed for ductility, we will have to compute the anticipated yield displacement for the steel reinforcement ratio that we computed in Question 1.

$$u_y = \frac{f_y}{k}$$

From Figure 1-1, the elastic stiffness of the bent is that of a cantilever column,

$$k = \frac{3EI}{h^3} = \frac{3E_c I_g\left(0.2 + 2\rho_t \gamma^2 \frac{E_s}{E_c}\right)}{h^3} = \frac{3\cdot 30\cdot 2.49x10^{11}\left(0.2 + 2\cdot 0.0323\cdot 0.9^2\frac{200}{30}\right)}{10000^3}$$
$$= 12.27kN/mm$$

$$I_g = \frac{\pi r^4}{4} = \frac{\pi 750^4}{4} = 2.49x10^{11}mm^4$$

Therefore,

$$u_y = \frac{f_y}{k} = \frac{102kN}{12.27kN/mm} = 8{,}3mm$$

$$\mu = \frac{u_m}{u_y} \Longrightarrow u_m = 5u_y = 5 \cdot 8.3mm = 41{,}5mm < 500mm$$

Note that in this case an increase in the steel reinforcement is not required to satisfy the deformation limits.